# Design of Machine Members - 1
## (DMM-1)



→ Machine consists of machine elements.

→ Each part of the machine has motion with respect to some other part, is called a machine element.

→ The objective of designing a machine element is to ensure that it perserves its operating capacity during the stipulated service life with minimum manu-facturing and operating costs.

→ in order to achieve this objective, the machine element should satisfy the following basic requirements.

→ Strength
→ Rigidity / stiffness
→ Wear Resistance
→ Resilliance
→ Hardness
→ Brittleness
→ Ductility
→ Creep
→ Fatigue
→ toughness
→ Plasticity
→ Elasticity

① Strength :- The Ability of a material that can
② with stand to mechanical load (external).

It may be tensile, compressive in nature.

Eg: Tensile strength, compressive strength.

② Hardness :- Resistance offered by a material to identation/ Penetration.

③ Elasticity :- Ability of a material to regain to its original shape and sizes after deformation, when external load is removed.

④ plasticity :- The property of a material retains the deformation produced under external loads.

⑤ Toughness :- The ability of a material that can absorb energy at the time of failure against fracture is called toughness.

This property is required to with stand Impact loads.

⑥ Creep :- when a part is subjected to loading at High temperatures for a long time, It is called Creep.

## Design procedure

specify functions of element

↓

Determine forces acting on element

↓

select suitable material for element

↓

Determine the failure mode of element

↓

Determine the geometric dimensions of element.

↓

Modify the dimensions for Assembly &
Manufacturing & Design at critical sections

↓

Prepare working drawing of element

## Tolerances:-

Due to inaccuracy of manufacturing methods, It is not possible to machine (Prepare) a Compo-nent to a given dimension. The dimensions are so manufactured so that their dimensions lie between two limits - maximum Limit & minimum limit, Namely

(4) upper limit and lower limit. The difference between two limits is called "Tolerance".

eg: $100^{+0.04}_{+0.00}$ ⇒ upper limit = 100.04

⇒ lower limit = 100.00

→ There are two specifications for tolerances

(a) Unilateral Tolerances:-

- one of the tolerance is zero.

eg: $100^{+0.00}_{-0.04}$ ; $100^{+0.04}_{+0.00}$

(b) Bilateral Tolerances:-

- Variations are given on both tolerances.

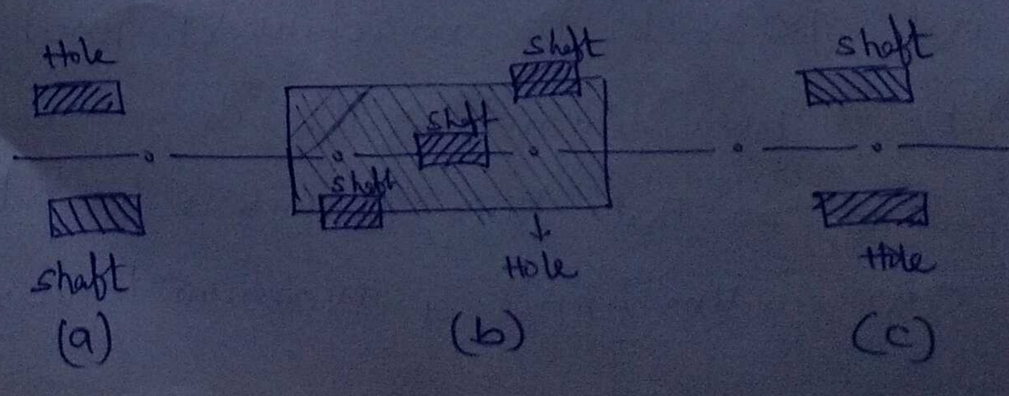
$25^{+0.4}_{-0.4}$ ; $25^{+0.4}_{-0.2}$

Types of fits:-

(a) clearance fit

(b) Transition fit

(c) Interference fit



## BIS system of fits & Tolerances:-

According Bureau of Indian Standards (BIS), tolerances are specified by an Alphabet, (capital/small,) followed by a Number.

Eg: H7, g6.

→ fits is indicated by basic size, followed by symbols for tolerances.

50H8g7 ; 50H8-g7 ; $50\ \frac{H8}{g7}$

Basic size: 50 mm,

Hole: H-type ⇒ Hole Tolerance = IT8

shaft: g-type ⇒ shaft Tolerance = IT7.

## Stress:-

- When a mechanical component is subjected to an external static load, a Resisting force is set up within the component.
- This Internal resisting force per unit area is called "stress". denoted by 'σ'.

$$\sigma = \frac{\text{Resisting force}}{\text{Area}} = \frac{P}{A} ; \left(\frac{N}{m^2}\right)$$

⑥

$$\sigma = \frac{P}{A} \; ; \; \left(N/mm^2\right)$$

→ $1\ MPa = 1 \times 10^6 \frac{N}{m^2} = 1 \times 10^6 \frac{N}{mm^2 \times 10^6}$

→ $1\ MPa = 1 \frac{N}{mm^2}$

**Strain:-** It is the defformation per unit length. (or) change in length to original length.

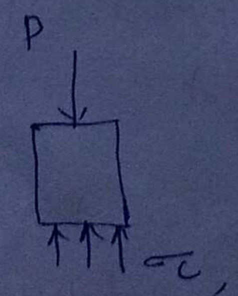
$$\varepsilon = \frac{\delta l}{l} \; ; \quad \delta l = \text{elongation / change in length}, \quad l = \text{original length}.$$

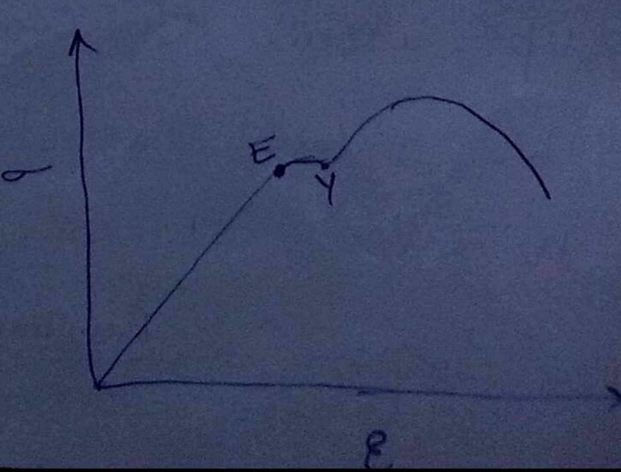
$\varepsilon \propto \sigma$ (Hooks law)

$\varepsilon = E\sigma$

→ Relation b/w strain & strain $\boxed{\varepsilon = E\sigma}$

E = Young's modolous. ($N/m^2$, or $N/mm^2$)

## Shear stress & shear strain:-

When the external force acting on a component tends to slide the adjacent planes with respect to each other, the resulting stresses on these planes are called shear stresses. ($\tau$)

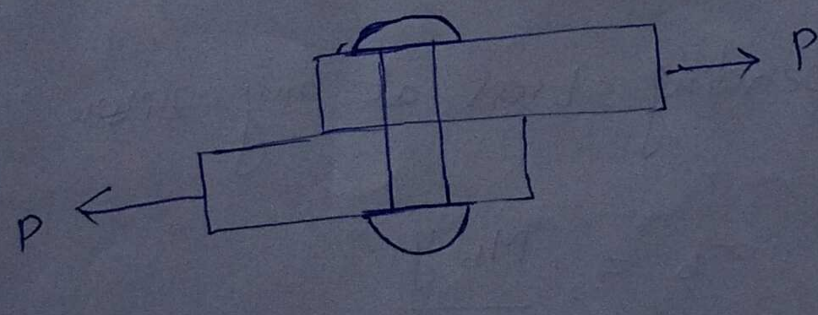
$$\tau = \frac{P}{A} = \frac{\text{Shear load}}{\text{Area}} \left(\frac{N}{m^2}\right)$$

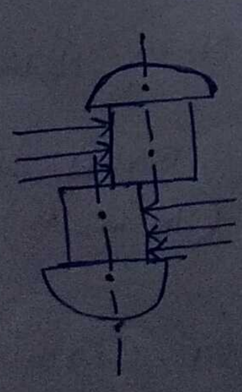
$\rightarrow$ $\tau = G\gamma$

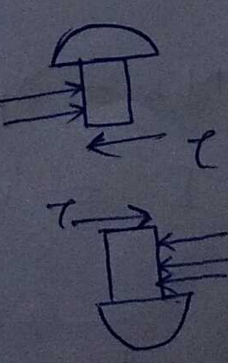
$G$ = Shear modulus / modulus of Rigidity ($N/m^2$)

$\tau$ = shear stress ($N/m^2$, $N/mm^2$)

$\gamma$ = shear strain

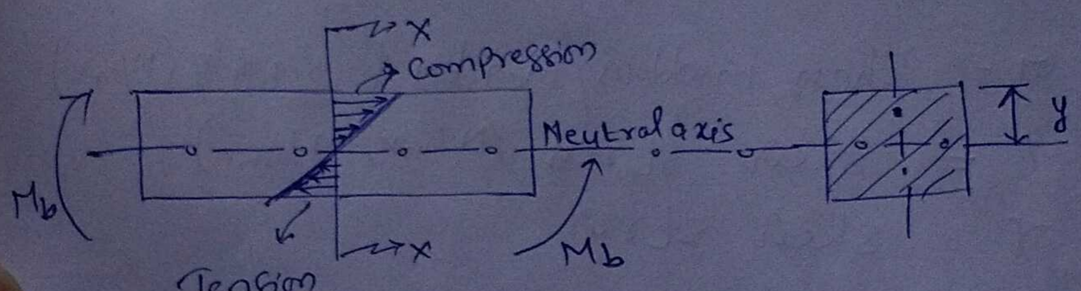




## stresses due to Bending moment:-

A straight beam subjected to bending moment "$M_b$" is shown. The beam is subjected to a combination of tensile stress on one side of neutral axial axis and compressive stress on other side.




∴ The bending stress at any fibre is given by

$$\sigma_b = \frac{M_b\, y}{I}$$

$\sigma_b$ = Bending stress at distance of "y" from N.A

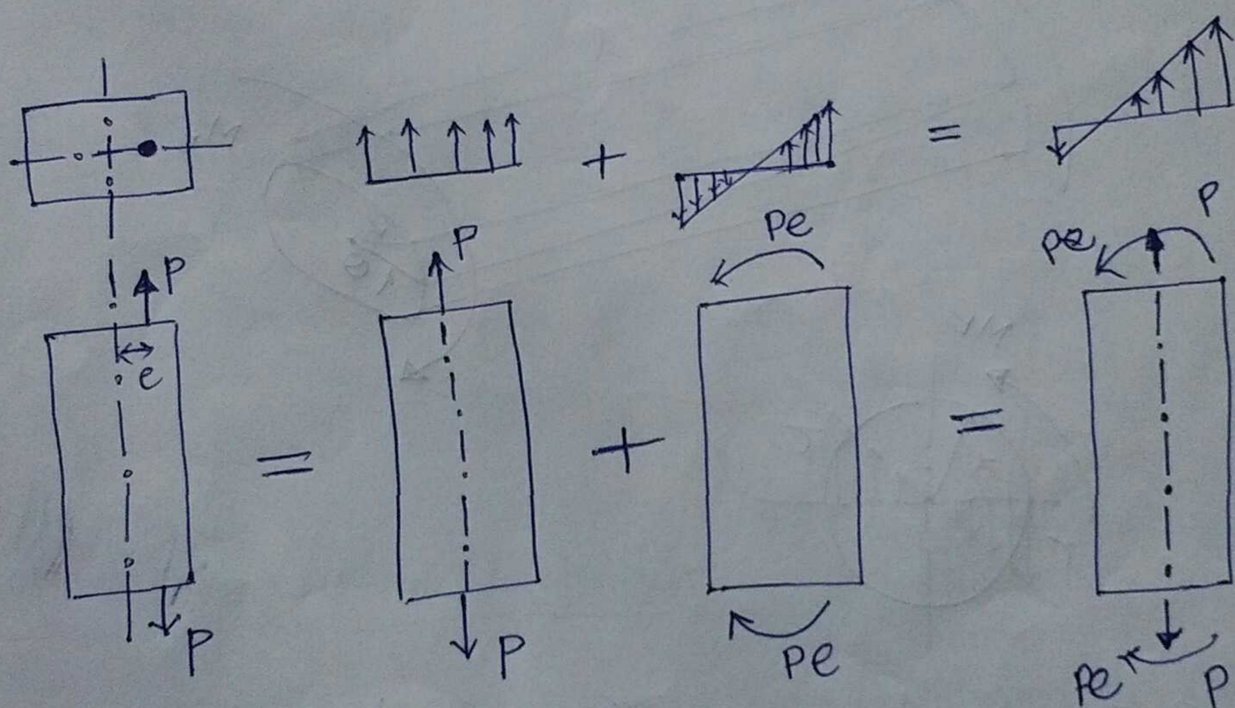
$M_b$ = Applied Bending moment. (N-m / N-mm)

$I$ = moment of Inertia of the cross section about N.A (mm⁴)

$$\rightarrow \quad \boxed{\sigma_b = \frac{32 M_b}{\pi d^3}} \quad \text{for circular c/s.}$$

## Ecentric Axial Loading :-

There are certain mechanical components subjected to an external force (Tensile / compressive), which doesn't pass through the centroid of the cross section.

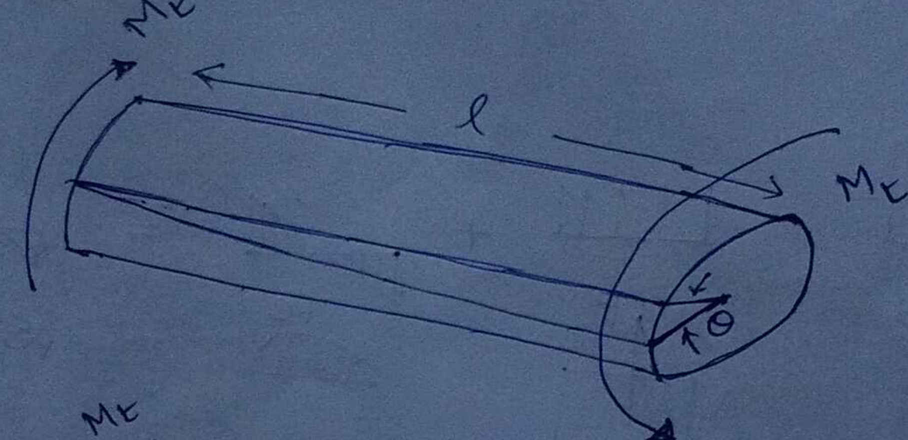


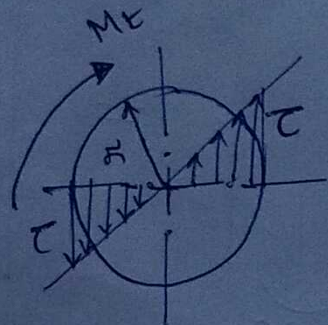

→ ecentric stress $\sigma = \sigma_a + \sigma_b$

* $\boxed{\sigma = \frac{P}{A} + \frac{Pey}{I}}$

## Stresses due to Torsional moment:-

A Transmission shaft, subjected to an external Torque. / Twist moment ($M_t$)




→ Because of twisting moment '$M_t$', Internal stresses are induced to resist the actio of action twist., these Internal are called "torsional shear stresses" ($\tau$).

$$\tau = \frac{M_t r}{J} ;$$

$M_t$ = Twisting moment / Torque applied

$r$ = radial distance from Neutral ax

$J$ = polar moment of Inertia of c/s about axis of rotation.

→ $\boxed{\tau = \frac{16 M_t}{\pi d^3}}$ for circular c/s.

## Factor of safety :-

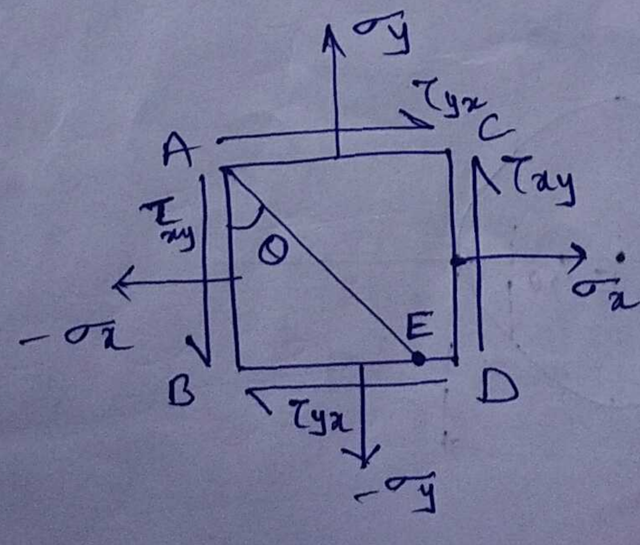
While designing a mechanical member, it is necessary to provide sufficient reserve strength in case of any accidental load. This is achieved by taking a suitable factor of safety.

→ $$\text{Factor of safety} = \frac{\text{failure stress}}{\text{allowable stress}}$$

$$= \frac{\text{failure load}}{\text{working load}}$$

## Principle stresses / Principle planes :-

- They explains about maximum/minimum Normal stresses
- principle plane is a plane carrying zero shear stresses.
- only Normal stresses are acting on principle planes.



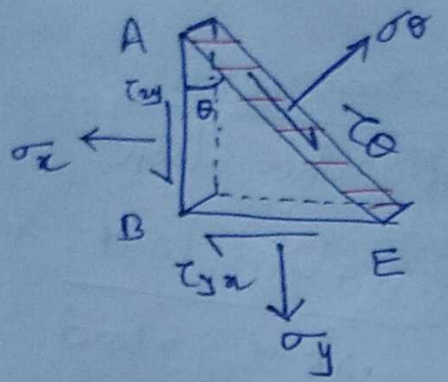
(12)

$$\sigma_\theta = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_\theta = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta - \tau_{xy}\cos 2\theta$$

$\sigma_\theta$ = Resultant of Normal stresses acting on a plane at $\theta$

$\tau_\theta$ = Resultant of Shear stresses acting on a plane at $\theta$

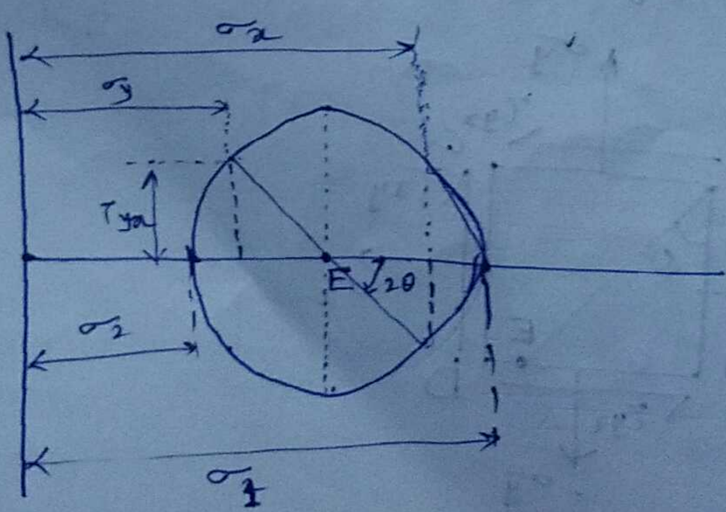



by definition of principle plane; $\tau_\theta = 0$.

$$\therefore \tan 2\theta = \frac{2\tau_{xy}}{\sigma_x-\sigma_y} \Rightarrow \theta = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x-\sigma_y}\right)$$

## Mohr's circle:-

Mohr's circle a most effective method to determi[ne] the principle Normal stresses & principle shear stresses.



$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_1$ = Maximum principle stress

$\sigma_2$ = Minimum Principle stresses.

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\tau_{max}$ = principle shear stresses.

① <u>Maximum principal stress theory</u> (<u>Rankine theory</u>):—

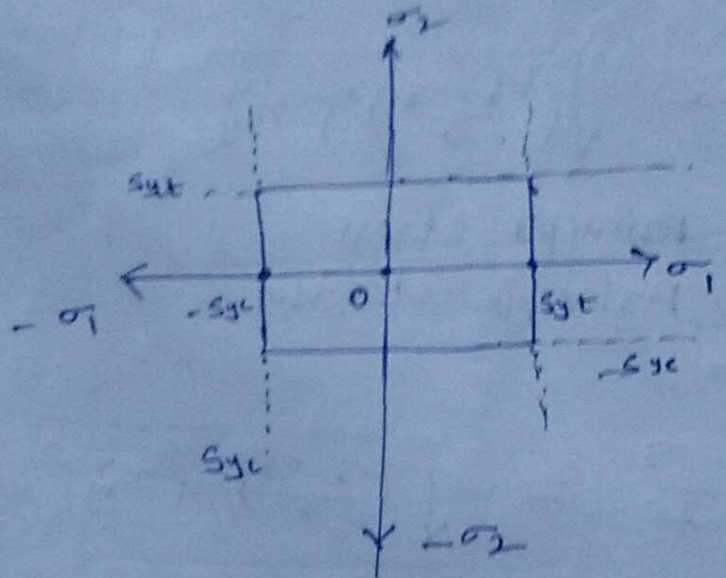
This theory states that failure of Mechanical component subjected to bi-axial or tri-axial stresses occurs when maximum principal stress reaches the yeild or ultimate strength of material.

$\sigma_1 = S_{yt}$ → for Ductile

$\sigma_1 = S_{ut}$ → for Brittle.

i.e $\sigma_1 = \dfrac{S_{yt}}{F.S}$ or $\sigma_1 = \dfrac{S_{ut}}{F.S}$

(1b) Region of safety (for Bi-axial stresses)

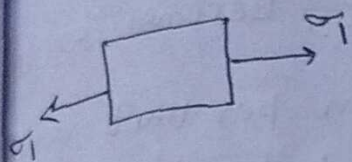


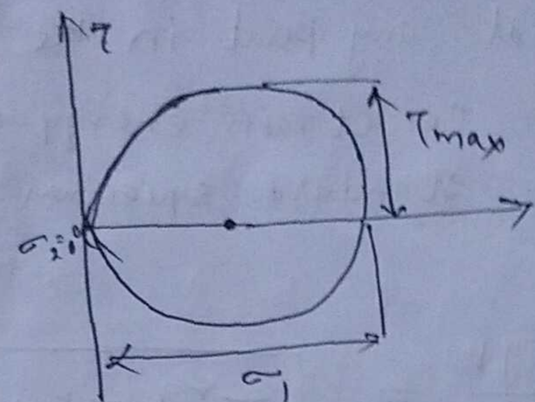

(2) Maximum shear stress theory (Tresca & Guest theory)

"This theory states that failure of mechanical component subjected to bi-axial (or Tri-axial) stresses occurs when the maximum shear stress at any point in the component becomes equal to maximum shear stress in the standard specimen of the tension test, when yeilding starts.

$$\tau_{max} = \frac{\sigma_1}{2(F.S)}$$

$$= \frac{S_{yt}}{2(F.S)}$$



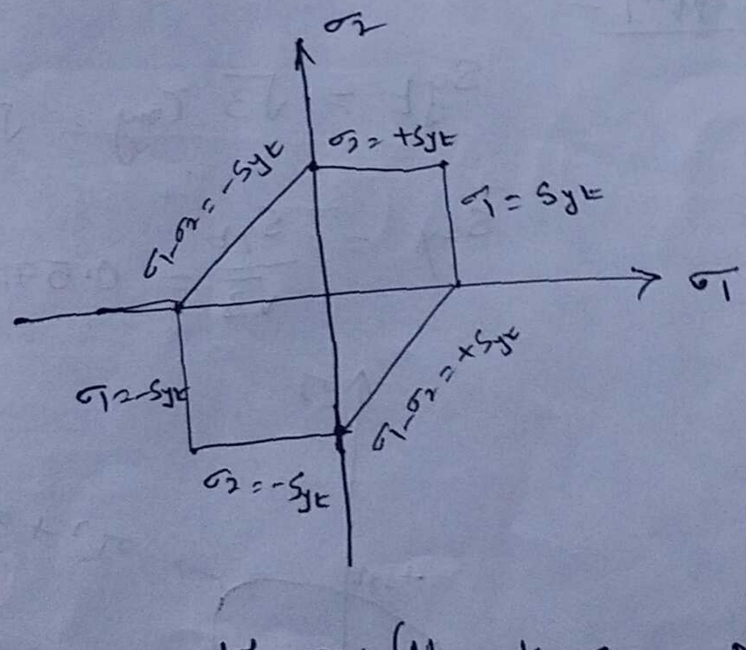


<u>Region of safety:-</u>

$$\sigma_1 - \sigma_2 = \pm S_{yt}$$

$$\sigma_1 = \pm S_{yt}$$

$$\sigma_2 = \pm S_{yt}.$$



<u>Distorsion - Energy theory (Hencky & von Mises):-</u>

"It states that the failure of mechanical member subjected to bi-axial (or Trial-axial) stresses occurs when the strain energy of distorsion per unit volume at any point in the component, becomes equa- to the strain energy of distortion per unit volu in standard spiecemen of tension test, when yeil

$$\frac{S_{yt}}{F.S} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}$$

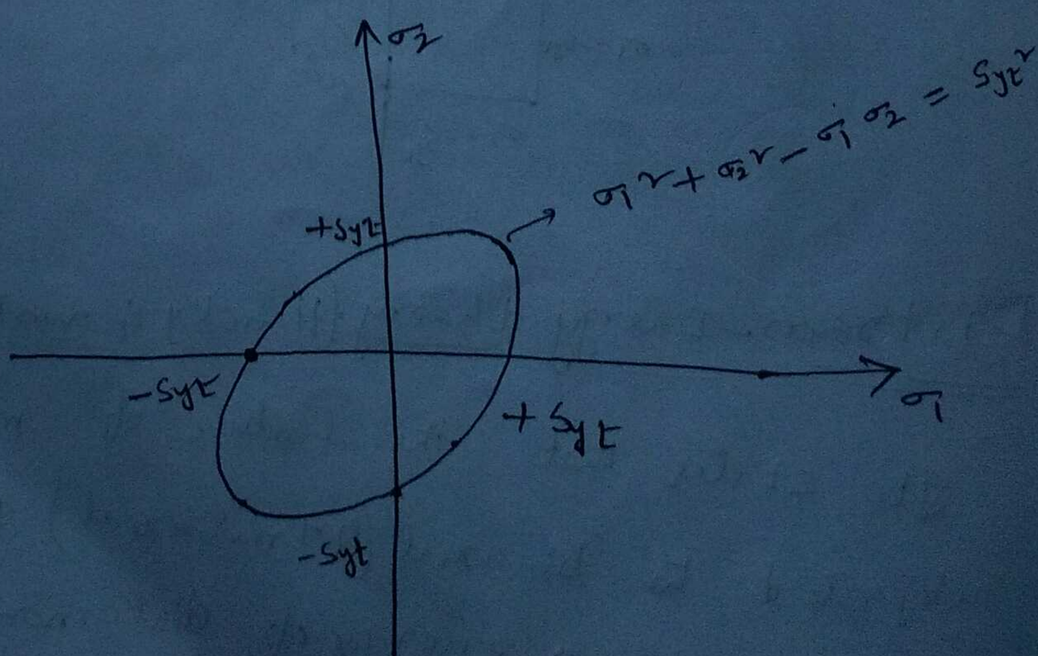
for 3 D,

$$\frac{S_{yt}}{F.S} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2} \quad \text{- for 2 D}$$

Region of safety:-

$$S_{yt} = \sqrt{3}\,\tau_{ny} = \sqrt{3}\,S_{sy}$$

$$S_{sy} = \frac{S_{yt}}{\sqrt{3}} = 0.577\,S_{yt}$$



→ Any moment acting perpendicular to axis of member is called Bending (M)

→ Any moment acting along the axis is called torsion (T),

∴ Equivalent Torque ($T_e$) = $\sqrt{M^2+T^2}$

$$\Rightarrow \tau_{max} = \frac{16\,T_e}{\pi d^3}$$

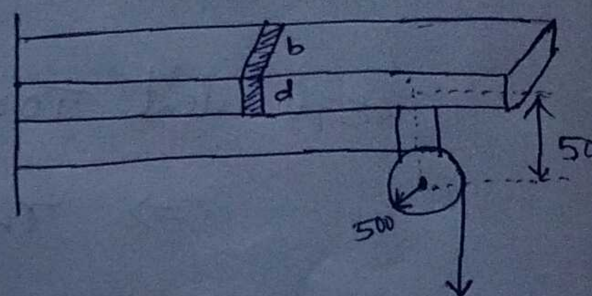
Equivalent Bending moment ($M_e$) = $\frac{1}{2}\left[M+\sqrt{M^2+T^2}\right]$

$$\Rightarrow \sigma_{b\,max} = \frac{32\,M_e}{\pi d^3}.$$

(18)

(1) A Cantilever beam of rectangular section is used to support a pulley as shown. The beam is made of C.I $S_{yt} = 200\ N/mm^2$; F.S = 2.5, d/b = 2. Find the value of "b" According to the maximum Normal stress theory?

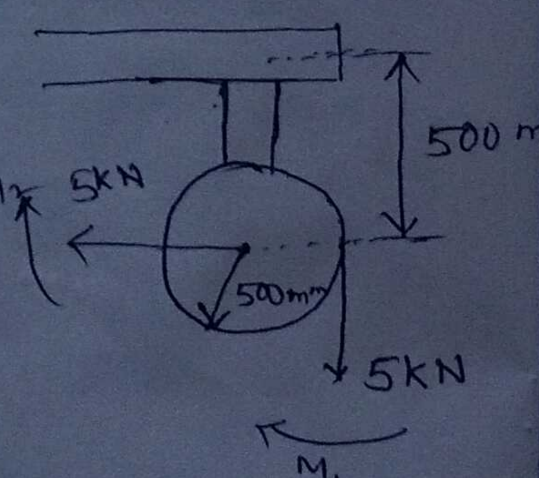
Sol)




According to Maximum Normal stress theory
(or) Rankines theory

$$\sigma_1 = \frac{S_{yt}}{F.S}$$

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$




$M_b = M_1 + M_2$

$= -(5000 \times 500) - (5000 \times 500)$

$M_b = 5 \times 10^6$ N-mm

$$\sigma_b = \frac{M_b\, y}{I} \quad ; \quad I = \frac{d\, b^3}{12} \quad ; \quad y = \frac{b}{2}$$

$$\therefore \sigma_b = \frac{5\times10^6 \times b/2}{\left(\frac{d\,b^3}{12}\right)} = \frac{15\times10^6}{b^3}$$

$$\sigma_1 = \sigma_b$$

According Normal stress theory (Rankines theory)

$$\sigma_1 = \frac{S_{yt}}{F.S} \Rightarrow \frac{15\times10^6}{b^3} = \frac{200}{2.5}$$

$$\Rightarrow b^3 = 187500$$

$$\Rightarrow b = 57.23 \text{ mm.}$$

Note: $\sigma_1 = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

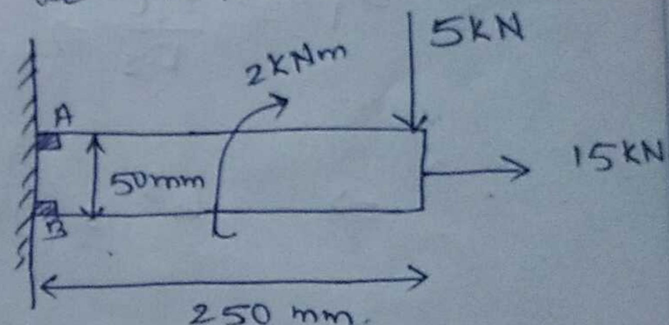
in this Problem $\sigma_y = 0$; '$\sigma_x$' be the Bending stress ($\sigma_b$) There is no shear stress. ($\tau_{xy} = 0$).

$$\therefore \sigma_1 = \sigma_b$$

(20) (2) A cylindrical bar of 50 mm diameter and 250 mm long in fixed at one end, and free end it is loaded as shown, with axial of 15KN, a downward transverse load of 5KN and a twisting moment of 2KN-m. The yeild strength of bar is 343 MPa, then (i) find the operating factor of safety according to maximum shear stress theory.?

(ii) Also find the stresses at A & B?

Sol




Given that (d) = 50mm.

length (l) = 250 mm

Axial load $(P_a)$ = 15 KN = $15\times10^3$ N

Transverse load $(P_t)$ = 5KN = $5\times10^3$ N

Twisting moment (T) = 2 KN-m = $2\times10^6$ N-mm

Yeild strength $(S_{yt})$ = 343 MPa

(i) According to maximum shear stress theory

$$\tau_{max} = \left|\frac{\sigma_1-\sigma_2}{2}\right| = \frac{S_{sy}}{F_s} \quad (or)$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} \quad (\because \text{Mohrs circle})$$

Axial stress / Normal stress $(\sigma_a) = \frac{P_a}{A}$

$$\sigma_a = \frac{4 P_a}{\pi d^2} = \frac{4 \times 15 \times 10^3}{\pi \times (50)^2} = \frac{60,000}{\pi (50)^2}$$

$$\sigma_a = 7.6394 \ N/mm^2$$

Bending stress $(\sigma_b) = \frac{32 M_b}{\pi d^3} =$

$$M_b = P_t \times l = 5000 \times 250 = 125 \times 10^4 \ N\text{-}mm$$

$$\sigma_b = \frac{32 \times 125 \times 10^4}{\pi (50^3)} = 101.859 \ N/mm^2$$

Shear stress $(\tau_{xy}) = \frac{16 T}{\pi d^3} = \frac{16 \times 2 \times 10^6}{\pi \times (50^3)}$

$$\tau_{xy} = 81.4873 \ N/mm^2$$

$$\sigma_x = \sigma_a + \sigma_b \ ; \quad \sigma_y = 0$$
$$= 109.49$$

$$\therefore \tau_{max} = \sqrt{\left(\frac{\sigma_a + \sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{109.49}{2}\right)^2 + (81.48)^2}$$

$$\tau_{max} = 98.17 \ N/mm^2$$

According to Max. S.S $\tau_{max} = \frac{S_{sy}}{F.S.}$

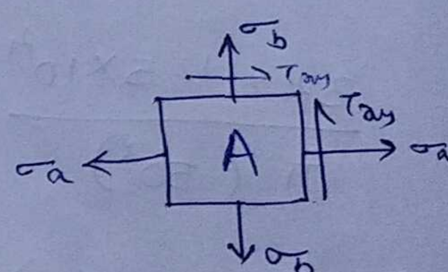
$$\tau_{max} = \frac{S_{yt}}{2 F.S}$$

$$\Rightarrow \quad 98.17 = \frac{343}{2 \times F_s}$$

$$\Rightarrow \quad F.S = 1.7469 \quad \text{(Factor of safety)}$$

(ii)

→ A The element "A" will be in Tension due to bending




$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \left(\frac{\sigma_a + \sigma_b}{2}\right) + \sqrt{\left(\frac{\sigma_a - \sigma_b}{2}\right)^2 + \tau_{xy}^2}$$

$$= \left(\frac{7.63 + 101.85}{2}\right) + \sqrt{(-47.11)^2 + (81.48)^2}$$

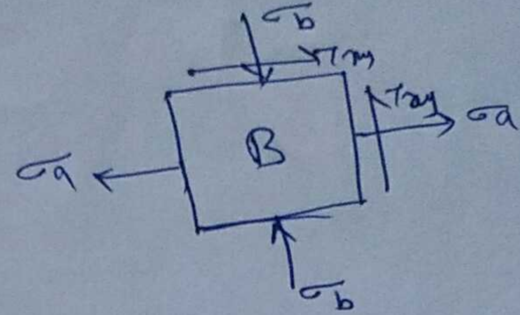
$$\sigma_1 = 54.74 + 94.11 = 148.85 \text{ N/mm}^2$$

$\sigma_1$ = Maximum principle stress on 'A' is $= 148.85 \text{ N/mm}^2$

$$\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 54.74 - 94.11 = -39.37$$

$\sigma_2$ = Minimum principle stress on 'A' is $-39.37 \text{ N/mm}^2$

→ The element 'B' will be ~~Tens~~ compression due to bending




$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \left(\frac{\sigma_a - \sigma_b}{2}\right) + \sqrt{\left(\frac{\sigma_a + \sigma_b}{2}\right)^2 + \tau_{xy}^2}$$

$$= (-47.11) + \sqrt{(54.74)^2 + (81.48)^2}$$

$$\sigma_1 = -47.11 + 98.16 = 51.05 \ N/mm^2$$

$$\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = -47.11 - 98.16 = -145.27 \ N/mm^2$$

———— o ————

# Design against Fluctuating load

Tensile stress $(\sigma_t) = \frac{P}{A}$

Bending Stress $(\sigma_b) = \frac{My}{I}$

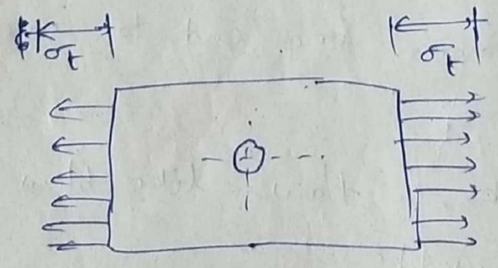
Shear stress $(\tau) = \frac{Tr}{J}$

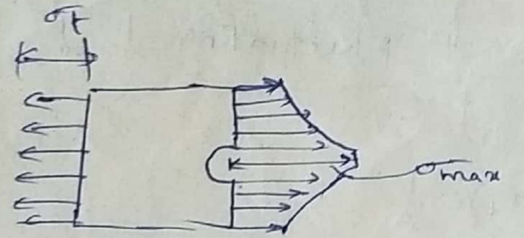
*) These three equations are called elementary equations.
These equations can be used only in the case of, there
are no discontinuities in the cross-section of the
Component.

## Stress Concentration :-

A plate with a small circular hole, subjected
to tensile stress. The distribution of stresses near the
hole can be observed by using Photo-elasticity Technique.
In this method, an identical model of the plate is
made of epoxy resin. The model is placed in a circular
Polariscope and loaded at the edges.

It is observed that there is a sudden rise
in the magnitude of stresses in the vicinity of hole.
The localized stresses in the neighbourhood of hole are
far greater than stresses obtained by elementary
equations.






*) Stress concentration is defined as the localization of high stresses due to the irregularities present in the component and abrupt changes of cross-section.

$$\text{Stress Concentration factor } (K_t) = \frac{(\text{Highest value of actual stresses near discontinuity})}{(\text{Nominal stress obtained by elementary equations for minimum cross-section})}$$

$$K_t = \frac{\sigma_{max}}{\sigma_0} = \frac{\tau_{max}}{\tau_0}$$

where $K_t$ = Theoretical stress concentration factor

$\sigma_{max}$, $\tau_{max}$ are localized stresses at discontinuities

$\sigma_0$, $\tau_0$ are stresses determined by elementary equations.

The Causes of Stress Concentration are as follows:

1) Variation in Properties of Materials:-

In design of machine components, it is assumed that the material is homogeneous throughout the component. In practice, there is variation in material properties from one end to another due to following factors.

a) Internal cracks, & flaws like blow holes.

b) Cavities in welds

c) air holes in steel components

d) Non metallic (or) Foreign inclusions

ii) Load application:-

The loads act either at a Point (or) over a small area on the component. Since the area is small, the Pressure at these Points is excessive. This result in stress concentration.

Eg:- a) contact between the meshing teeth of the driving and driven gear

b) Contact between the cam and follower

c) Contact between the balls and the races of ball bearing

d) Contact between the rail and wheel.

iii) Abrupt changes in section:-

In order to mount gears, sprockets, pulleys & ball bearings on a transmission shaft, steps are cut on the shaft and shoulders are Provided from assembly considerations. These create change of cross-section of the shaft leads to stress concentration.

iv) Discontinuities in the component:-

Certain features of machine components such as oil holes (or) oil grooves, key ways and splines, screw threads result in discontinuities in the cross-sections of component leads to stress concentration.

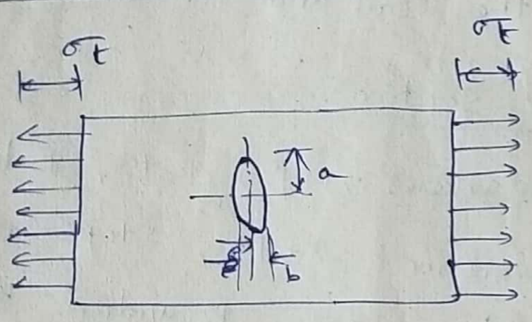
v) Machining Scratches :-

Machining Scratches, Stamp marks, (or) Inspection marks are surface irregularities, which cause stress concentration.

*) It is possible to find out the stress concentration factor for some simple geometric shapes using the Theory of Elasticity.

A flat plate with an elliptical hole and subjected to tensile force, the theoretical stress concentration factor at the edge of hole is given by,

$$\boxed{K_t = 1 + 2\left(\frac{a}{b}\right)}$$




where a = Semi major axis

b = Semi Minor axis

If 'b' approaches zero, the ellipse becomes sharper and sharper. A very sharp crack is indicated and the stress at edge of crack becomes very large.

$\therefore$ as $b \to 0$ ; $K_t = \infty$.

For a circular hole $a = b$

$$K_t = 1 + 2\left(\frac{\cancel{a}}{\cancel{b}}\right)$$

$$= 1 + 2$$

$$K_t = 3.$$

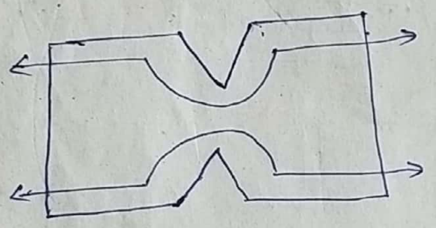
## Reduction of Stress Concentration:-

Reduction of stress concentration is achieved by the following methods.

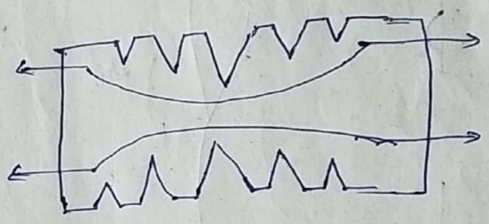
### (i) Additional Notches and Holes in Tension Member:-

A flat plate with a V-notch subjected to tensile force is shown in figure. It is observed that a single notch results in a high degree of stress concentration. The severity of stress concentration is reduced by three methods.

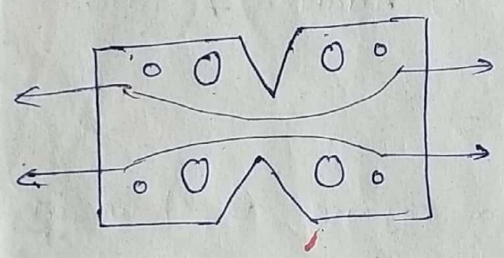
a) Use of multiple notches
b) Drilling additional holes
c) Removal of undesired material



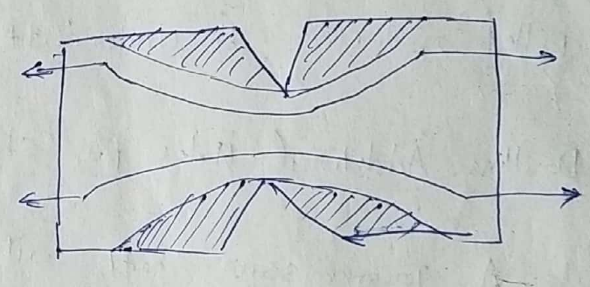
Original Notch.



Multiple Notches



Drilled Holes.

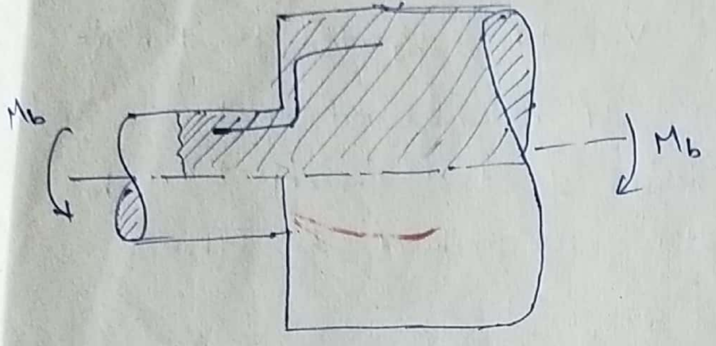


Removal of undesired material

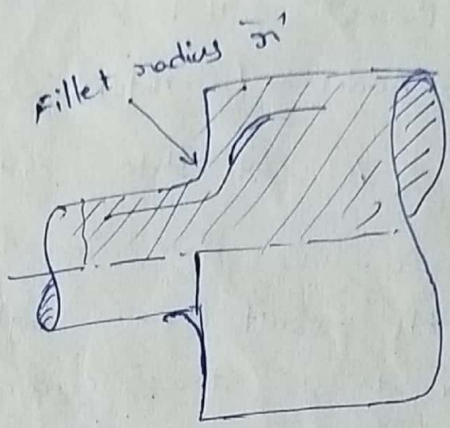
### ii) Fillet Radius, Undercutting & Notch for Member in Bending:-

A bar of circular cross-section with a shoulder and subjected to bending moment as shown in figure.

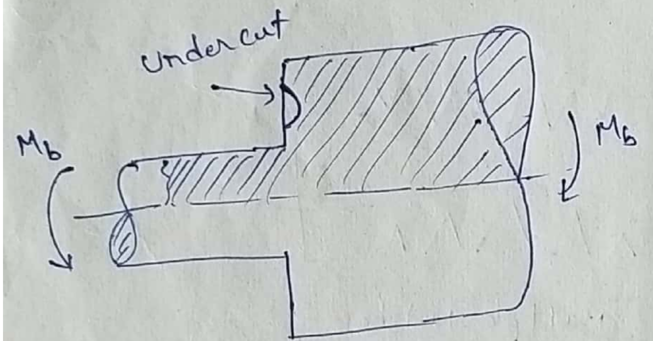
The shoulder creates a change in cross-section of the shaft which results in stress concentration. There are three methods to reduce stress concentration at the base of this shoulder.



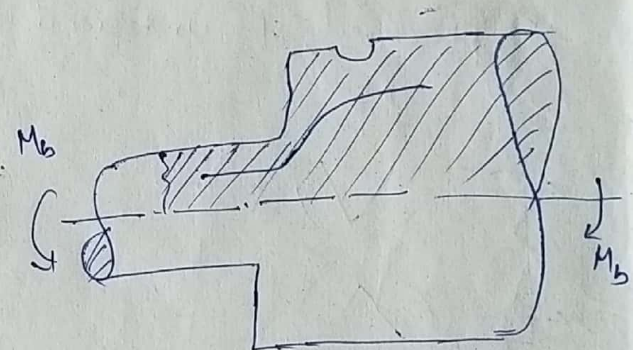

Original component




Fillet radius




Under cutting




Addition of notch

a) Fillet radius  b) Undercutting  c) Addition of notch

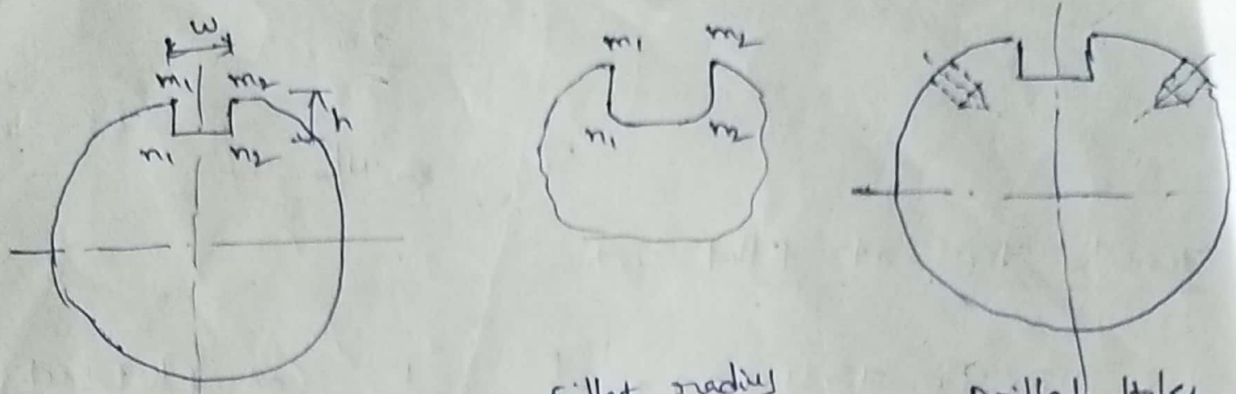
iii) <u>Drilling Additional Holes for shaft :-</u>

A Transmission shaft with a key way as shown in figure. The key way is discontinuity and results in stress concentration at the corners of Keyway and reduces torsional shear strength.

The stress concentration can be reduced by

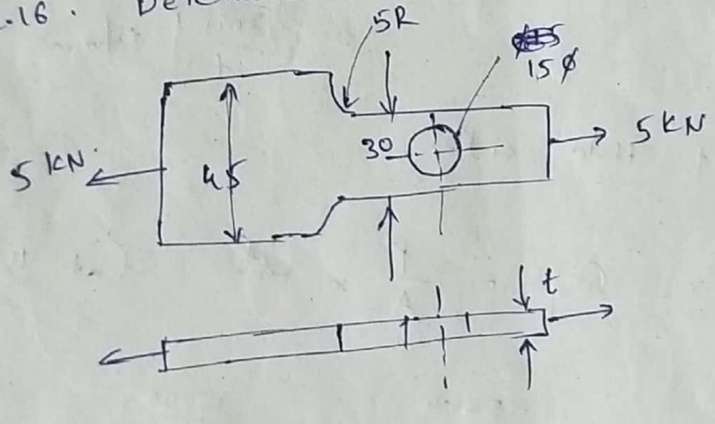
i) Drilling Two symmetrical holes on the sides of Keyway.

ii) Giving fillet radius at the inner corners of the keyway.




Original shaft          Fillet radius          Drilled Holes

Eg:- A flat plate subjected to a Tensile force of 5 kN is shown in figure. The plate material is grey cast Iron FG200 and the factor of safety is 2.5. Take $k_t$ at fillet radius is : 1.8 ; $k_t$ at hole section is : 2.16. Determine the thickness of plate.




Ans:-

Force $P = 5\ kN = 5000\ N$.

FG 200 means Ultimate Tensile strength $S_{ut} = 200\ N/mm^2$

Factor of safety $= 2.5$

At fillet radius $k_t = 1.8$

At Hole section $k_t = 2.16$

Let 't' is thickness of the plate

Permissible Tensile stress $\sigma_{Per} = \frac{S_{ut}}{FOS}$

$$= \frac{200}{2.5}$$

$$\sigma_{Per} = 80\ N/mm^2$$

Tensile stress at fillet section:-

Tensile stress $(\sigma_o) = \frac{P}{A} = \frac{P}{dt}$ (at fillet section)

where $d = 30$

$$\sigma_o = \frac{5000}{30 \times t}$$

But $K_t = \frac{\sigma_{max}}{\sigma_o} \Rightarrow \sigma_{max} = K_t\, \sigma_o.$

Maximum stress at fillet section } $\sigma_{max} = 1.8 \times \frac{5000}{30 \times t}$

$$= \left(\frac{300}{t}\right) N/mm^2 \quad \text{--- (1)}$$

Tensile stress at Hole section:-

Tensile stress $(\sigma_o) = \frac{P}{A} = \frac{P}{\cancel{A}}$ (at Hole section)

$$= \frac{5000}{(30-15)\,t} = \frac{5000}{15t}$$

$$K_t = 2.16$$

Maximum stress at Hole section } $\sigma_{max} = K_t\, \sigma_o$

$$= 2.16 \times \frac{5000}{15t}$$

$$= \left(\frac{720}{t}\right) N/mm^2 \quad \text{--- (2)}$$

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Thickness of plate:-

From equations 1 & 2 the maximum stress is induced at the hole section.

$$\cancel{\frac{720}{t}}$$

Equating maximum stress at hole section to Permissible Tensile stress

$$\frac{720}{t} = 80$$

$$\boxed{t = 9\ mm}$$

## Fatigue stress Concentration Factor :-

When a machine member is subjected to a cyclic (or) Fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor.

$$\left.\begin{array}{l}\text{Fatigue stress}\\ \text{Concentration factor}\end{array}\right\} K_f = \frac{\text{Endurance limit without stress Concentration}}{\text{Endurance limit with stress Concentration}}$$

## Notch Sensitivity :-

It is defined as the susceptibility of a material to succumb to the ~~deg~~ damaging effects of stress raising notches in fatigue loading. The notch sensitivity factor 'q' is defined as

$$q = \frac{\text{Increase of actual stress over nominal stress}}{\text{Increase of theoretical stress over nominal stress}}$$

Let $\sigma_0$ = Nominal stress obtained by elementary equations.

Actual stress $= K_f \sigma_0$

Theoretical stress $= K_t \sigma_0$

durance

Increase of actual stress over nominal stress

$= K_f \sigma_o - \sigma_o$

Increase of theoretical stress over nominal stress

$= K_t \sigma_o - \sigma_o.$

$$\therefore q = \frac{K_f \sigma_o - \sigma_o}{K_t \sigma_o - \sigma_o} = \frac{\cancel{\sigma_o}(K_f - 1)}{\cancel{\sigma_o}(K_t - 1)} = \frac{K_f - 1}{K_t - 1}$$

$$\Rightarrow \quad q(K_t - 1) = (K_f - 1)$$

$$\boxed{K_f = 1 + q(K_t - 1)}$$

i) when the material has no sensitivity to notches,

$q = 0$ ; $\therefore K_f = 1$

ii) when the material ~~has~~ is fully sensitive to notches,

$q = 1$ ; $\therefore K_f = K_t.$

Note:- The magnitude of notch sensitivity factor 'q'

Varies from 0 to 1.

## Endurance limit Approximate Estimation :-

A number of tests are required to prepare one S-N curve and each test takes considerable time. It is, therefore, not possible to get the experimental data of each and every material.

Let $S_e'$ = Endurance limit stress of a rotating beam specimen subjected to reverse bending stress

$S_e$ = Endurance limit stress of a particular mechanical component subjected to reverse bending stress

Relationship b/w Endurance limit & Ultimate tensile strength ($S_{ut}$) is :

For steels, $S_e' = 0.5\,S_{ut}$

For cast Iron & cast steels, $S_e' = 0.4\,S_{ut}$

For Wrought Aluminium alloys, $S_e' = 0.4\,S_{ut}$

For cast aluminium alloys, $S_e' = 0.3\,S_{ut}$

*) The endurance limit of a component is different from the endurance limit of a rotating beam specimen due to number of factors.

i) Surface finish factor ($K_a$):-

The surface of the rotating beam specimen is polished to mirror finish. It is impractical to provide such an expensive surface finish for the actual component.

When the surface finish is poor, the surface scratches serve as stress raisers and result in stress concentration. The endurance limit is reduced due to introduction of stress concentration at these scratches.

ii) Size factor ($K_b$):-

The rotating beam specimen is small with 7.5 mm diameter. The larger the machine part, the greater the probability that a flaw exists somewhere in the component. The chances of fatigue failure originating at any one of these flaws are more. The endurance limit, therefore reduces with increasing the size of the component.

| Diameter (d) | $K_b$ |
|---|---|
| $d \le 7.5$ | 1.00 |
| $7.5 < d \le 50$ | 0.85 |
| $d > 50$ | 0.75 |

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5

## iii) Reliability Factor :- ($K_c$).

The laboratory values of endurance limit are usually mean values. The reliability factor is one for 50% reliability.

| Reliability (%) | $K_c$ |
|---|---|
| 50 | 1.000 |
| 90 | 0.897 |
| 95 | 0.868 |
| 99 | 0.814 |
| 99.9 | 0.753 |
| 99.99 | 0.702 |
| 99.999 | 0.659 |

## iv) Modifying factor to account for stress Concentration:

The endurance limit is reduced due to stress concentration. The modifying factor ($K_d$) to account for ~~s~~ effect of stress concentration is defined as:

$$\boxed{K_d = \frac{1}{K_f}}$$

The above mentioned four factors are used to find out the endurance limit of ~~a~~ actual component.

The relationship between $S_e$ and $S_e'$ is as follows.

$$S_e = K_a K_b K_c K_d S_e' \quad \left(\text{where } K_d = \frac{1}{K_f}\right)$$

where $K_a$ = Surface finish factor

$K_b$ = Size factor

$K_c$ = Reliability factor

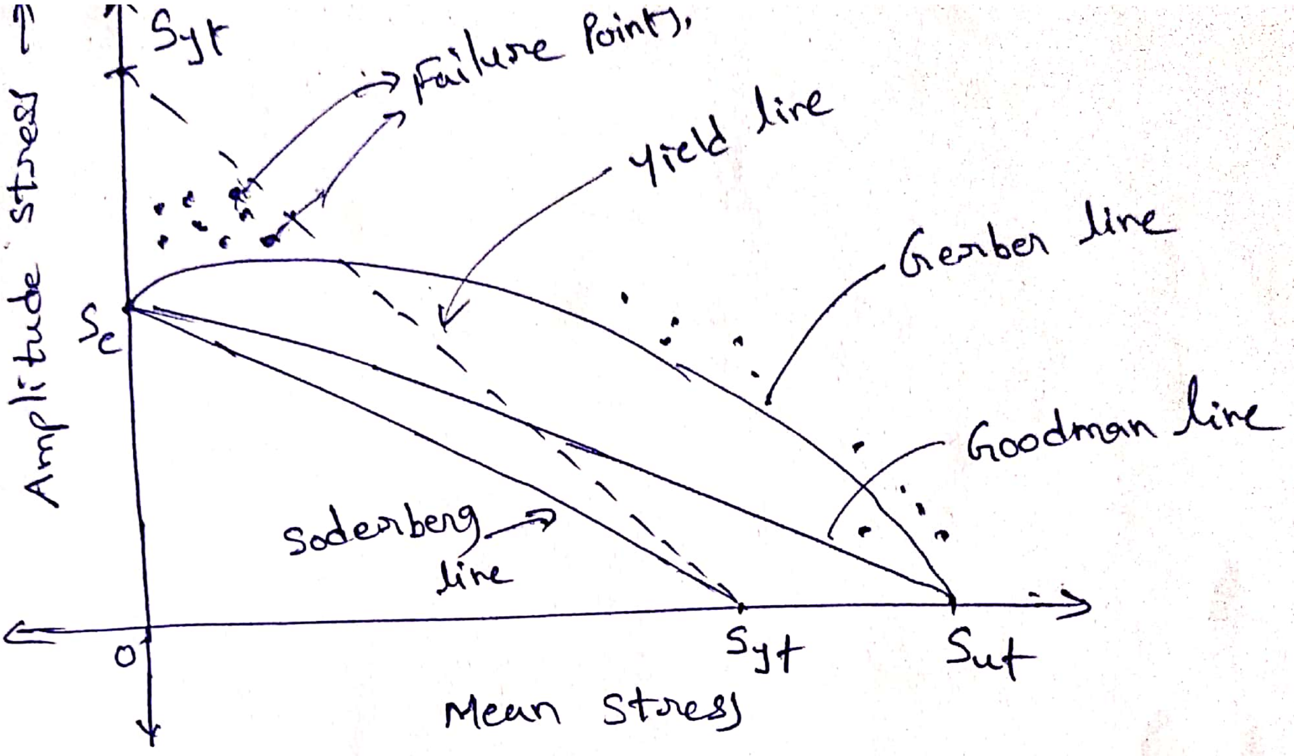
$K_d$ = Modifying factor to account for stress concentration.

## Sodenberg & Goodman Lines:-

When a component subjected to fluctuating (or) Alternating stresses, there is mean stress ($\sigma_m$) as well as amplitude stress ($\sigma_a$).

The mean stress is plotted on the x-axis, the amplitude stress is plotted on the ordinate. The When the amplitude stress ($\sigma_a$) is zero, the load is Purely static and criterion of failure is $S_{ut}$ (or) $S_{yt}$. These limits are plotted on ~~abscissa~~ x-axis.

When the mean stress ($\sigma_m$) is zero, the stress is completely reversing and criterion of failure is endurance limit $S_e'$, it is plotted on Y-axis.




Soderberg line :- A straight line Joining $S_e'$ on the ordinate to $S_{yt}$ on abscissa is called the soderberg line.

$$\boxed{\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_e} = 1}$$

Q, A machine Component is Subjected to flexural stresses which flutuating the between 300 MN/m² and -150 MN/m². Determine the minimum ultimate Strength according to

i, Gerber relation.

ii, Goodman relation.

iii, Soderberg relation.

Take yield Strength is equal to 0.55 ultimate Strength. Endurance Strength is equal to 0.5 ultimate Strength. And 'Fos' is 2.

A, Given that :-

Maximum stress ($\sigma_{max}$) = 300 MN/m²

Minimum stress ($\sigma_{min}$) = -150 MN/m²

$S_{yt} = 0.55\, S_{ut}$

$S_e = 0.5\, S_{ut}$

FOS = 2

$S_{ut} = ?$

i, Gerber Relation:-

$$\boxed{\frac{\sigma_a}{S_e} + FOS\left(\frac{\sigma_m}{S_{ut}}\right)^2 = \frac{1}{FOS}} \rightarrow (1)$$

Now,

$\sigma_a$ = Amplitude Stress = $\dfrac{\sigma_{max} - \sigma_{min}}{2}$

$$\sigma_a = \frac{300 - (-150)}{2} = 225 \text{ MN/m}^2$$

$\sigma_m$ = Mean stress = $\dfrac{\sigma_{max} + \sigma_{min}}{2}$

$= \dfrac{300 - 150}{2} = 75 \, MN/m^2$

Now, from eq (1),

$$\Rightarrow \frac{225}{0.5 S_{ut}} + \left[2 \times \left(\frac{75}{S_{ut}}\right)^2\right] = \frac{1}{2}$$

$$\Rightarrow \frac{450}{S_{ut}} + \frac{11250}{(S_{ut})^2} = \frac{1}{2}$$

$$\Rightarrow \frac{450 S_{ut} + 11250}{(S_{ut})^2} = \frac{1}{2}$$

$$\Rightarrow 900 S_{ut} + 22500 = (S_{ut})^2$$

$$\Rightarrow (S_{ut})^2 - 900 S_{ut} - 22500 = 0$$

$a = 1$ ; $b = -900$ ; $C = -22500$

$$\therefore S_{ut} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{900 \pm \sqrt{(900)^2 - [4 \times (-22500) \times 1]}}{2(1)}$$

$$= \frac{900 \pm \sqrt{810000 + 90000}}{2}$$

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$$= \frac{900 \pm 948.88}{2}$$

$$= \frac{900 \pm 948.88}{2} \quad \text{[Take Positive values]}$$

$$\boxed{S_{ut} = 924.34\ MN/m^2}$$

ii, Goodman Relation :-

$$\boxed{\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} = \frac{1}{FOS}} \rightarrow (2)$$

$$\Rightarrow \frac{75}{S_{ut}} + \frac{225}{0.5\ S_{ut}} = \frac{1}{2}$$

$$\Rightarrow \frac{1}{S_{ut}} [75 + 450] = \frac{1}{2}$$

$$\Rightarrow \frac{1}{S_{ut}} [525] = \frac{1}{2}$$

$$\Rightarrow \boxed{S_{ut} = 1050\ MN/m^2}$$

iii, Soderberg relation :-

$$\boxed{\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_e} = \frac{1}{FOS}} \rightarrow (3)$$

$$\Rightarrow \frac{75}{0.55\ S_{ut}} + \frac{\cancel{450}\ 225}{0.5\ S_{ut}} = \frac{1}{2}$$

$$\Rightarrow \frac{136.36}{S_{ut}} + \frac{450}{S_{ut}} = \frac{1}{2}$$

$$\Rightarrow \frac{1}{S_{ut}} [586.36] = \frac{1}{2}$$

$$\Rightarrow \boxed{S_{ut} = 1172\ MN/m^2}$$

//.

Soderberg relations:-

$$\boxed{\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_e} = \frac{1}{FOS}} \rightarrow (1)$$

$\sigma_m$ = Mean stress = $\frac{\text{Mean load}}{\text{cross. section area}}$

Mean load $(W_m) = \frac{W_{max} + W_{min}}{2}$

$= \frac{(250 \times 10^3) + (100 \times 10^3)}{2}$

$= 175 \times 10^3 N$

Area of cross section (A) = bt

$= \boxed{120 \times t \ mm^2}$

$\sigma_a$ = Amplitude stress

$= \frac{\text{Amplitude load}}{\text{cross section area}} = \frac{W_a}{A}$

Amplitude load $= W_a = \frac{W_{max} - W_{min}}{2}$

$= \frac{(250 \times 10^3) - (100 \times 10^3)}{2}$

$= 75 \times 10^3 N$

$$\sigma_m = \frac{W_m}{A} = \frac{175 \times 10^3}{120t}$$

$$\sigma_a = \frac{W_a}{A} = \frac{75 \times 10^3}{120t}$$

now, from eq ①,

$$\Rightarrow \frac{\frac{175\times10^3}{120t}}{300} + \frac{\frac{75\times10^3}{120t}}{225} = \frac{1}{1.5}$$

$$\Rightarrow \frac{\frac{1458}{t}}{300} + \frac{\frac{625}{t}}{225} = \frac{1}{1.5}$$

$$\Rightarrow \frac{4.86}{t} + \frac{2.77}{t} = \frac{1}{1.5}$$

$$\Rightarrow \frac{1}{t}[7.63] = \frac{1}{1.5}$$

$$\Rightarrow \boxed{t = 11.4 \text{ mm}}$$

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3. A 50 mm diameter shaft is made from carbon steel having Ultimate tensile strength is 630 MPa, is subjected to a torque which fluctuates between 2000 NM to −800 NM. Using Soderberg Equation calculate the factor of safety. Assume suitable value for any other data needed.

Sol:- Given data,

diameter of shaft, $d = 50mm$

Ultimate tensile strength, $S_{ut} = 630 \text{ MPa}$

$= 630 \text{ N/mm}^2$.

Maximum Torque. $T_{max} = 2000 \text{ Nm} = 2000 \times 10^3 \text{ N-mm}$

Minimum Torque, $T_{min} = -800 \text{ Nm} = -800 \times 10^3 \text{ Nmm}$

Now,

mean Torque., $T_m = \frac{T_{max} + T_{min}}{2}$

$= \frac{(2000 - 800) \times 10^3}{2} = 600 \times 10^3 \text{ N-mm}$

Amplitude Torque, $T_a = \frac{T_{max} - T_{min}}{2}$

$= \frac{(2000 + 800) \times 10^3}{2} = 1400 \times 10^3 \text{ N-mm}$

Mean shear stress, $(\tau_m) = \frac{16 \times T_m}{\pi d^3}$

$= \frac{16 \times 600 \times 10^3}{\pi (50)^3}$

$= 24.44 \text{ N/mm}^2$

Amplitude shear stress, $(\tau_a) = \frac{16 \times T_a}{\pi d^3}$

$= \frac{16 \times 1400 \times 10^3}{\pi (50)^3} = 57.04 \text{ N/mm}^2$

Assumptions,

$\tau_e' = 0.55\, S_e$

$= 0.55\,(0.5\, S_{ut})$ $\quad [\because S_e = 0.5\, S_{ut}]$

$= 0.55\,(0.5 \times 630)$

$= 173.25\ N/mm^2$

$K_a = 0.87$

$K_b = 0.85$

$K_f = 1$

Assume yield stress of carbon steel, $S_{yt} = 510\ N/mm^2$

$\tau_{yt} = 0.5\, S_{yt}$

$= 0.5 \times 510 = 255\ N/mm^2$

Now,

By Soderberg Equation,

$$\frac{\tau_m}{\tau_{yt}} + \frac{\tau_a}{\tau_e} = \frac{1}{FOS}$$

where $\tau_e = 0.87 \times 0.85 \times 1 \times 173.25$

$= 128.1\ N/mm^2$

then,

$$\frac{24.4}{255} + \frac{57.04}{128.1} = \frac{1}{FOS}$$

$$FOS = \frac{255 \times 128.1}{(24.4 \times 128.1) + (57.04 \times 255)}$$

$$= \frac{32665.5}{17670.8}$$

$$\therefore FOS = \underline{\underline{1.84}}$$

4. A circular bar of 500 mm length is supported freely at its two ends. It is acted upon by a central concentrated cyclic load having a minimum value of 20 kN & maximum load of 50 kN. Determine the diameter of the bar by taking Factor of Safety = 1.5,

Size factor = 0.85 &

surface finish factor = 0.9

The material proportion of the bar are given by Ultimate strength is 650 MPa, Yield strength is 500 MPa & Endurance strength is 350 MPa.

Sol:- Given data.

Let diameter of the bar be "d".

Maximum load, $W_{max} = 50kN = 50 \times 10^3 N$

Minimum load, $W_{min} = 20kN = 20 \times 10^3 N$

$FOS = 1.5$

$K_a = 0.9$

$K_b = 0.85$

$S_{ut} = 650 \; MPa = 650 \; N/mm^2$

$S_{yt} = 500 \; MPa = 500 \; N/mm^2$

$S_e' = 350 \; MPa = 350 \; N/mm^2$

Goodman line :-

$$\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} = \frac{1}{FOS}$$

Soderberg line :-

$$\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_e} = \frac{1}{FOS}$$

where, $S_e = K_a K_b K_c S_e'$

$\therefore S_e = 0.9 \times 0.85 \times 1 \times 350 = 267.75 \; N/mm^2$

Maximum bending moment, $M_{max} = \frac{W_{max} \times l}{4}$

$= \frac{50 \times 10^3 \times 500}{4} = 6.25 \times 10^6$ Nmm

Minimum bending moment, $M_{min} = \frac{W_{min} \times l}{4}$

$= \frac{20 \times 10^3 \times 500}{4} = 2.5 \times 10^6$ N.mm

From bending Equation,

$$\frac{M_{max}}{I} = \frac{\sigma_{max}}{y}$$

$$\sigma_{max} = \frac{M_{max}\, y}{\frac{\pi}{64} d^4}$$

$$\therefore \sigma_{max} = \frac{M_{max} (d/2)}{\frac{\pi}{64} d^{4}}$$

$$= \frac{32 M_{max}}{\pi d^3}$$

$$\therefore \sigma_{max} = \frac{32 \times 6.25 \times 10^6}{\pi d^3} = \frac{63.66 \times 10^6}{d^3} \text{ N/mm}^2$$

similarly,

$$\sigma_{min} = \frac{32 M_{min}}{\pi d^3} = \frac{32 \times 2.5 \times 10^6}{\pi d^3} = \frac{25.46 \times 10^6}{d^3} \text{ N/mm}^2$$

Mean stress, $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$

$$= \frac{\frac{63.66 \times 10^6}{d^3} + \frac{25.46 \times 10^6}{d^3}}{2}$$

$$= \frac{44.56 \times 10^6}{d^3} \text{ N/mm}^2$$

Amplitude stress, $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$

$$\Rightarrow \sigma_a = \frac{19.1 \times 10^6}{d^3} \text{ N/mm}^2$$

i) Goodman line :-

$$\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} = \frac{1}{FOS}$$

$$\frac{\left(\frac{44.56 \times 10^6}{d^3}\right)}{650} + \frac{\left(\frac{19.1 \times 10^6}{d^3}\right)}{267.75} = \frac{1}{1.5}$$

$$\frac{68.55 \times 10^3}{d^3} + \frac{71.33 \times 10^3}{d^3} = \frac{1}{1.5}$$

$$d^3 = 209.82 \times 10^3$$

$$d = 59.42 \text{ mm}$$

ii) Soderberg line :-

$$\frac{\left(\frac{44.56 \times 10^6}{d^3}\right)}{500} + \frac{\left(\frac{19.1 \times 10^6}{d^3}\right)}{267.75} = \frac{1}{1.5}$$

$$\frac{89.12 \times 10^3}{d^3} + \frac{71.33 \times 10^3}{d^3} = \frac{1}{1.5}$$

$$d^3 = 24067 \times 10^3$$

$$d = 62.2 \text{ mm}$$

Taking larger value of diameter

∴ Diameter of bar "d" = 62.2 mm.

Q.5. A STEEL ROD is subjected to a Reversal axial load of 180KN. Find the diameter of Rod. The material ultimate Tensile strength 1070 mpa and yield strength of 910mpa. The Endurance limit Se in the Reverse bending may be assumed to be half of the ultimate Tensile Strength other correction factor may be taken as follows

for axial load = 0.7

For surface = 0.8

For size = 0.85

For stress concentration = 1

Factor of Safety = 2

ultimate tensile strength (sut) 1070 Mpa

yield Strength ($S_{yt}$) = 910 MPa

$$S_e' = \frac{1}{2} S_{ut}$$

$$K_a = 0.8$$

$$K_b = 0.85$$

$$F.O.S = 2$$

$$K_e' = 0.7$$

$$K_f = 1$$

$$K_d = 1$$

$$S_e = K_a \cdot K_b \cdot K_c \cdot K_d \cdot K_e \cdot S_e'$$

$$S_e = 254.66 \text{ N/mm}^2$$

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maximum bending moment

$$\sigma_{max} = \frac{W_{max}}{\frac{\pi}{4}d^2}$$

$$= \frac{180}{0.785 \times d^2} = \frac{229.18 \times 10^3}{d^2}$$

$$\sigma_{min} = \frac{W_{min}}{\pi/4\, d^2}$$

$$= \frac{-229.18 \times 10^3}{d^2}$$

mean stress ($\sigma_m$) $= \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{229.18 \times 10^3 + (-229.18 \times 10^3)}{2}$

$$= 0$$

Amplitude stress ($\sigma_a$) $= \frac{\sigma_{max} + \sigma_{min}}{2}$

$$= \frac{229.18 \times 10^3 - (-229.18 \times 10^3)}{2}$$

$$= \frac{229.18 \times 10^3}{2}$$

soderberg Equation

$$\frac{\sigma_m}{Syt} + \frac{\sigma_a}{Se} = \frac{1}{F.O.S}$$

$$\frac{0}{910} + \frac{229.18 \times 10^3}{d^2 (254.66)} = \frac{1}{2}$$

$$d^2 = 1800$$

$$d = 42.4\ mm$$

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6 Q) A Cantilever beam made of Calbon Steel of circular cross-section is subjected to a load which varies from $-F$ to $3F$. Determine the maximum load that this member can withstand for an indefinite life using a factor of safety 2. The theortical stress concentration factor is 1.42 and notch sensitivity 0.9. Assume the following values

Ultimate stress = 550 Mpa

yield stress = 470 mpa

endurance limit = 275 Mpa

size factor = 0.85

surface finish factor = 0.89

minimum load ($W_{min}$) = $-F$

maximum load ($W_{max}$) = $3F$

F.O.S = 2

Stress Concⁿ $K_t$ = 1.42

notch sensitivity $q$ = 0.9

ultimate stress ($S_{ut}$) = 550 N/mm²

yield stress $S_{yt}$ = 470 N/mm²

Surface finish $K_a$ = 0.89

size. $K_b$ = 0.85

$$K_f = 1+q(K_t-1)$$

$$= 1+0.9(1.42-1)$$

$$= 1.378$$

$$K_d = \frac{1}{K_f} = \frac{1}{1.378} = 0.725$$

$$S_e = K_a \cdot K_b \cdot K_c \cdot K_d \, S_e'$$

$$= 0.89 \times 0.85 \times 1 \times 0.725 \times 275$$

$$= 150.82 \ N/mm^2$$

mean load $W_m = \frac{3F - F}{2}$

$$= F$$

Amplitode load $W_a = \frac{3F + F}{2}$

$$= 2F$$

$L = 125mm$

mean bending load $(M_m) = W_m \times l$

$$= F \times 125$$

$$= 125F$$

Amplitode bending load $(M_a) = W_a \times l$

$$= 250F$$

mean bending stress $(\sigma_m) = \frac{M_m}{\frac{\pi}{32} d^3}$

$$= \frac{32 \times 125F}{\pi (d^3)} = 0.57F \ N/mm^2$$

Amplitude bending stress $(\sigma_a) = \frac{M_a}{\frac{\pi}{32} \times d^3}$

$$= \frac{32 \times 250F}{\pi \times d^3} = 1.15F \ N/mm^2$$

i) Good Man line

$$\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} = \frac{1}{F.O.S}$$

$$\frac{0.57F}{550} + \frac{1.15F}{150.82} = \frac{1}{2}$$

$$1.03 \times 10^{-3} F + 4.62 \times 10^{-3} F = 1/2$$

$$F = 57.3 \text{ N}$$

iii) Soderberg line

$$\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_e} = \frac{1}{F.O.S}$$

$$\frac{0.57F}{470} + \frac{1.15F}{150.82} = \frac{1}{2}$$

$$8.83 \times 10^{-3} F = \frac{1}{2}$$

$$F = 56.08 \text{ N}$$

Taking larger of two values, we have $F = 57.3 \text{ N}$

A Simply Supported beam has a concentrated load at the centre which fluctuates from a value of P to 4P. The span of a beam is 500mm and its crosssectional is circular with a diameter of 60mm. Taking for the beam material an ultimate stress of 700MPa, a yield stress of 500MPa, endurance limit of 330MPa for reversed bending and factor of safety is 1.3. Calculate the maximum value of P. Take Size factor of 0.85 and surface finish factor of 0.9.

Sol Given data :-

Maximum load ($W_{max}$) = 4P

minimum load ($W_{min}$) = P

Span of the beam ($l$) = 500mm

Diameter of the beam ($d$) = 60mm

ultimate stress ($S_{ut}$) = 700MPa = 700 $N/mm^2$

yield stress ($S_{yt}$) = 500MPa = 500 $N/mm^2$

endurance limit ($S_e'$) = 330MPa = 330 $N/mm^2$

factor of safety (FOS) = 1.3

Surface finish factor ($k_a$) = 0.9

Size factor ($k_b$) = 0.85

* Maximum bending moment $(M_{max}) = \frac{W_{max} \cdot l}{4}$

$$= \frac{4P(500)}{4}$$

$$= 500P$$

Minimum bending moment $(M_{min}) = \frac{W_{min} \cdot l}{4}$

$$= \frac{P(500)}{4}$$

$$= 125P$$

* $\frac{M_{max}}{I} = \frac{\sigma_{max}}{y}$

$$\frac{M_{max}}{\frac{\pi}{64} d^4} = \frac{\sigma_{max}}{\frac{d}{2}}$$

$$\frac{500P}{\frac{\pi}{64}(60)^4} = \frac{\sigma_{max}}{\frac{60}{2}} \Rightarrow \frac{500P}{636172.5} = \frac{\sigma_{max}}{30}$$

$$\sigma_{max} = 0.0235P \; N/mm^2$$

$$\frac{M_{min}}{I} = \frac{\sigma_{min}}{y}$$

$$\frac{125P}{\frac{\pi}{4}(60)^4} = \frac{\sigma_{min}}{\frac{60}{2}}$$

$$\frac{125P}{636172.5} = \frac{\sigma_{min}}{30}$$

$\sigma_{min} = 0.00589P \ N/mm^2$

Now find mean stress and amplitude stress

Mean stress $(\sigma_m) = \dfrac{\sigma_{max} + \sigma_{min}}{2}$

$= \dfrac{0.0235P + 0.00589P}{2}$

$\sigma_m = 0.0147P \ N/mm^2$

Amplitude stress $(\sigma_a) = \dfrac{\sigma_{max} - \sigma_{min}}{2}$

$= \dfrac{0.0235P - 0.00589P}{2}$

$\sigma_a = 0.0088P \ N/mm^2$

Goodman line

$$\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} = \frac{1}{FOS}$$

$$\frac{0.0147P}{700} + \frac{0.0088P}{S_e} = \frac{1}{1.3}$$

$S_e = K_a \cdot K_b \cdot K_c \cdot K_d \cdot S_e'$

$S_e = 0.9 \times 0.85 \times 1 \times 1 \times 330$

$S_e = 252.45 \ N/mm^2$

# UNIT-3

## COTTER & KNUCKLE JOINTS

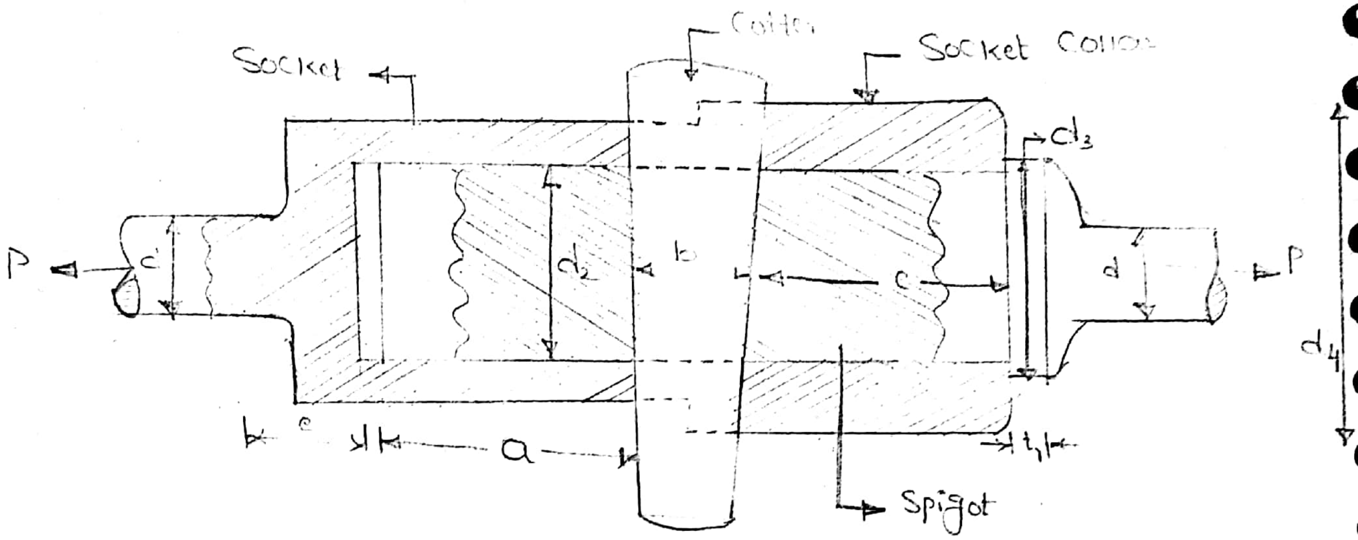
A Cotter is a flat wedgeshaped piece of rectangular c/s and it width is tapered from one end to another for an easy adjustment. The cotter is usually made of mild steel (or) wrought iron. A Cotter joint is temporary fastening and is used to connect rigidly two co-axial rods or bars which are subjected to axial tensile or compressive forces. It is usually used in connecting a piston rod to the cross-head of a reciprocating steam engine.

Types of Cotter joints :-

1. Socket and Spigot Cotter joint 2. Sleeve and Cotter joint
3. Gib and Cotter joint.

Socket and Spigot Cotter joint :-

A Socket and Spigot Cotter joint, one end of the rod (say A) is provided with a socket type of end and the other end of the other rod (say B) is inserted into a socket. The end of the rod which goes into a socket is also called spigot. A rectangular hole is made in the socket and spigot. A cotter is then driven tightly through h in order to make the temporary connection b/w the two rods. The load is usually acting axially, but it changes its direction and hence the cotter joint must be designed to carry both tensile and compressive loads. The compressive load is taken up by collar on the spigot.




$P$ = load carried by the rods

$D$ = Diameter of the rods

$d_1$ = outside diameter of socket

$d_2$ = Diameter of spigot (or) inside diameter of socket

$d_3$ = outside diameter of spigot collar

$d_4$ = Diameter of socket collar

$t_1$ - Thickness of ~~spigot~~ Spigot collar

$c$ = Thickness of socket collar

$b$ = Mean width of cotter

$t$ = Thickness of cotter $= \frac{d_2}{4}$

$l$ = length of cotter

$a$ = Distance from end of the slot to the end of rod.

$\sigma_t$ = Permissible tensile stress for the rod material

$\tau$ = Permissible shear stress for the cotter material

$\sigma_c$ = Permissible crushing stress for the cotter material

1. failure of the rods in tension :-

The rods may fail in tension due to the tenile load P.

Area resisting tearing $P_s = \frac{\pi}{4} d^2$

Tearing strength of the rods $= \frac{\pi}{4} \times d^2 \times \sigma_t$

$$P = \frac{\pi}{4} d^2 \sigma_t$$

from this equation, diameter of the rods (d) may be determined.

2. failure of Spigot in tension across the weakest section :-

Area resisting tearing of the Spigot across the Slot

$$= \frac{\pi}{4} (d_2)^2 - d_2 \times t$$

Tearing strength of the Spigot across the Slot

$$= \left[\frac{\pi}{4} (d_2)^2 - d_2 \times t\right] \times \sigma_t$$

$$P = \left[\frac{\pi}{4} (d_2)^2 - d_2 \times t\right] \sigma_t$$

3. failure of the Socket in tension across the Slot :-

Resisting Area of Socket across the Slot

$$= \frac{\pi}{4} \left[d_1^2 - d_2^2\right] - (d_1 - d_2) t$$

Tearing strength of the Socket across the Slot

$$= \left[\frac{\pi}{4} (d_1^2 - d_2^2) - (d_1 - d_2) t\right] \sigma_t$$

$$P = \left[\frac{\pi}{4} (d_1)^2 - (d_2)^2 - (d_1 - d_2) t\right] \sigma_t$$

4. Failure of the rod (or) cutter in crushing :-

Area that resists crushing of a rod or cutter $= d_2 \times t$

Crushing strength $= d_2 \times t \times \sigma_c$

$P = d_2 \times t \times \sigma_c$

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5. failure of the Socket Collar in Crushing :-

Area that resists Crushing of Socket Collar $= (d_4 - d_2) t$

Crushing strength $= (d_4 - d_2) t \times \sigma_c$

$$P = (d_4 - d_2) t \, \sigma_c$$

6. failure of Spigot Collar in Crushing :-

Area that resists Crushing of the Collar

$$= \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right]$$

Crushing strength of the Collar

$$= \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right] \sigma_c$$

$$P = \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right] \sigma_c$$

7. failure of Cotter in Shear :-

Since the Cotter is in double shear, therefore shearing area of the Cotter $= 2bt$

Shearing strength of the Cotter $= 2bt\tau$

$$P = 2bt\tau$$

8. failure of socket end in Shearing :-

resists shearing of socket Collar $= 2(d_4 - d_2)c$

Shearing strength of socket Collar $= 2(d_4 - d_2)c \times \tau$

$$P = 2(d_4 - d_2) c \tau$$

9. failure of rod end in Shear :-

rod end is in double shear, therefore the area resisting shear of the rod end $= 2ad_2$

Shear strength of the rod end $= 2ad_2 \times \tau$

$$P = 2ad_2\tau$$

10. Failure of the Spigot Collar in shearing :-

Area that resist shearing of the Collar $= \pi d_2 t_1$

Shearing strength of the Collar $= \pi d_2 t_1 \tau$

$$P = \pi d_2 t \tau$$

11. Failure of Cotter in bending

$$\sigma_b = \frac{P(d_4 + 0.5 d_2)}{2tb^2}$$

① Design and draw a Cotter joint to support a load vary from 30 kN in Compression to 30 kN in tension. The material used is Carbon Steel for which the following allowable stresses may be used. The load is applied statically. Tensile stress = Compressive stress = 50 MPa, Shear stress = 35 MPa, Crushing stress = 90 MPa.

$P = 30\,kN = 30 \times 10^3\,N$, $\sigma_t = 50\,MPa = 50\,N/mm^2$, $\tau = 35\,MPa = 35\,N/mm^2$

$\sigma_c = 90\,MPa = 90\,N/mm^2$

Diameter of the rods :-

$d$ = Diameter of the rod

$$P = \frac{\pi}{4} d^2 \times \sigma_t$$

$$30 \times 10^3 = \frac{\pi}{4} d^2 \times 50$$

$$d = 27.6 \text{ say } 28\,mm$$

Diameter of Spigot and thickness of Cotter :-

$d_2$ - Diameter of Spigot

$t$ = Thickness of Cotter $\frac{d_2}{4}$

$$P = \left[\frac{\pi}{4} d_2^2 - d_2 \times t\right] \sigma_t$$

$$30 \times 10^3 = \left[\frac{\pi}{4} d_2^2 - d_2 \cdot \frac{d_2}{4}\right] \sigma_t$$

$$= \left[\frac{\pi}{4} d_2^2 - \frac{d_2^2}{4}\right] \sigma_t$$

$$30\times10^3 = \left[\frac{\pi}{4} d_2^2 - \frac{1}{4} d_2^2\right]\times 50$$

$d_2 = 33.4$ say $34\,mm$

Thickness of Cotter $t = \frac{d_2}{4} = \frac{34}{4} = 8.5\,mm$

Let us now Check the induced Crushing stress, load (P)

$$P = d_2 \times t \times \sigma_c$$

$$30\times10^3 = 34\times 8.5 \times \sigma_c$$

$$\sigma_c = 103.8\ N/mm^2$$

Since this value of $\sigma_c$ is more than the given value $\sigma_c = 90\,N/mm^2$ therefore the dimension $d_2 = 34\,mm$ $t = 8.5\,mm$ are not safe. Now let us find the values of $d_2$ and $t_2$ by substituting the value of $\sigma_c = 90\ N/mm^2$ in the above expression

$$P = d_2 \times t \times \sigma_c$$

$$30\times10^3 = d_2 \times \frac{d_2}{4} \times \sigma_c$$

$$d_2^2 = 1333$$

$d_2 = 36.5$ say $40\,mm$

$$t = \frac{d_2}{4} = \frac{40}{4} = 10\,mm$$

3. Outside diameter of Socket

$$P = \left[\frac{\pi}{4}\left\{(d_1)^2 - (d_2)^2\right\} - (d_1 - d_2)t\right]\sigma_t$$

$$= \left[\frac{\pi}{4}\left\{d_1^2 - (40)^2\right\} - (d_1 - 40)\,10\right]50$$

$$d_1^2 - 12.7 d_1 - 1854.6 = 0$$

$$d_1 = \frac{12.7 \pm \sqrt{(12.7)^2 + 4\times 1854.6}}{2}$$

$= 49.9$ Say

Width of Cotter :-

b = width of Cotter

$$P = 2btT$$

$$30\times10^3 = 2\times b\times 10\times 35$$

$$b = 43 \text{ mm}$$

Diameter of Socket Collar :-

$$P = 2(d_4 - d_2)t\times T$$

$$30\times10^3 = 2(d_4 - 40)\,10\times 35$$

$$d_4 = 73.3 \text{ Say } 75 \text{ mm}$$

Thickness of Socket Collar :-

$$P = 2(d_4 - d_2)c\times T$$

$$30\times10^3 = 2(75 - 40)\,c\times 35$$

$$C = 12 \text{ mm}$$

Distance from end of Slot to end of the rod :-

$$P = 2ad_2T$$

$$30\times10^3 = 2a(40)\times 35$$

$$a = 10.7 \text{ Say } 11 \text{ mm}$$

Diameter of Spigot Collar :-

$$P = \frac{\pi}{4}\left[d_3^2 - d_2^2\right]\times\sigma_c$$

$$30\times10^3 = \frac{\pi}{4}\left[d_3^2 - (40)^2\right]\times 90$$

$$d_3 = 45 \text{ mm}$$

Thickness of Spigot Collar :-

$t_1$ = Thickness of Spigot Collar

$$P = \pi d_2 \times t_1$$

$$30 \times 10^3 = \pi \times 40 \times t_1$$

$$t_1 = 6.8 \text{ mm} \quad \text{Say } 8 \text{ mm}$$

length of Cotter

$$l = 4d$$

$$= 4 \times 28 = 112 \text{ mm}$$

The dimension e is taken as

$$e = 1.2 d$$

$$e = 1.2 \times 28 = 33.6 \text{ mm} \quad \text{Say } 34 \text{ mm}$$

## Sleeve and Cotter joint :-

Sleeve and Cotter joints are used to Connect two round rods or bars. In this type of joint, a Sleeve or muff is used over the two rods and then two Cotters are inserted in the holes Provided for them in the Sleeve and rods. The taper of Cotter is usually 1 in 24. It may be noted that the taper Sides of the two Cotter should face each other. The clearance is so adjusted that when the Cotters are driven in, the two rods Come Closer to each Other thus making the joint tight.

out. dia of Sleeve $d_1 = 2.5 d$

ins. dia of Sleeve $d_2 = 1.25 d$

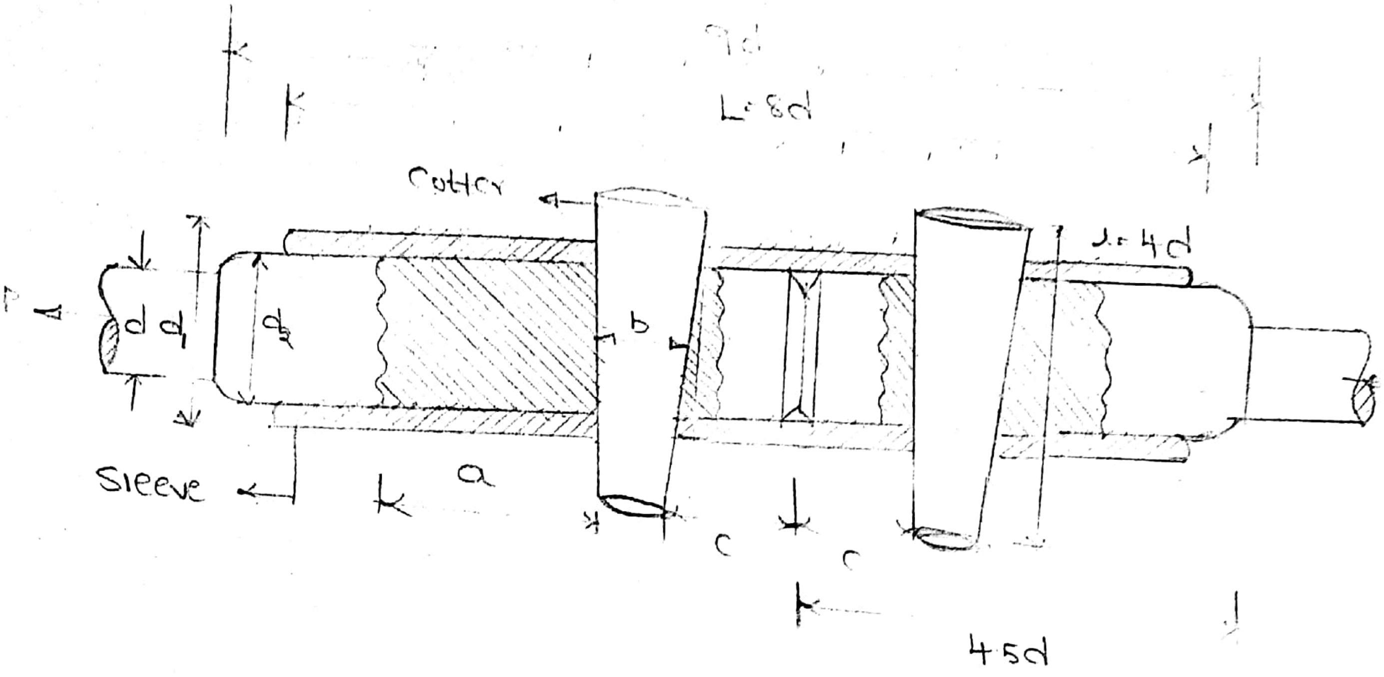
length of Sleeve $L = 8d$

Thickness of Cotter $t = \frac{d_2}{4}$

width of Cotter $b = 1.25 d$

length of Cotter $l = 4d$

Distance of the rod end (a) from the begining to the Cotter hole




P = load carried by the rods

d = Diameter of the rods

$d_1$ = outside diameter of sleeve

$d_2$ = Inside diameter of sleeve

t = Thickness of cotter

$l$ = length of cotter

b = width of cotter

a = Distance of the rod end from the begining to the cotter hole.

c = Distance of the rod end from its end to cotter hole

$\sigma_t$ = permissible tensile stress

$\sigma_c$ = permissible crushing stress

$\tau$ = permissible shear stress

1. Failure of the rods in tension :-

Area resisting tearing $= \frac{\pi}{4} d^2$

tearing strength of the rods $= \frac{\pi}{4} d^2 \times \sigma_t$

$$P = \frac{\pi}{4} d^2 \times \sigma_t$$

2. Failure of the rod in tension across the weakest section :-

Area resisting tearing of the rod across the slot $= \frac{\pi}{4} d_2^2 - d_2 \times t$

tearing strength of the rod across the slot $= \left[\frac{\pi}{4} d_2^2 - d_2 \times t\right] \sigma_t$

$$P = \left[\frac{\pi}{4} d_2^2 - d_2 \times t\right] \sigma_t$$

3. Failure of the sleeve in tension across the slot :-

Resisting area of sleeve across the slot $= \frac{\pi}{4}\left[(d_1)^2 - (d_2)^2\right] - (d_1 - d_2)t$

Tearing strength of sleeve across the slot

$$= \left[\frac{\pi}{4}\left[d_1^2 - d_2^2\right] - (d_1 - d_2)t\right]\sigma_t$$

4. Failure of the rod or cotter in crushing :-

Crushing resistance area $= d_2 \times t$

Crushing strength $= d_2 \times t \times \sigma_c$

$$P = d_2 \times t \times \sigma_c$$

5. Failure of cotter in shear :-

Since the cotter is in double shear, therefore shearing area of the cotter

$= 2bt$

Shear strength of the cotter $= 2bt \times \tau$

$$P = 2bt\tau$$

6. Failure of rod end in shear :-

Since the rod end is in double shear, therefore area resisting shear of the rod end $= 2ad_2$

Shear strength of the rod $= 2ad_2 \times \tau$

$$P = 2ad_2\tau$$

7. Failure of sleeve end in shear :-

Resisting area of sleeve $= 2(d_1 - d_2)c$

Shear strength of the sleeve $= 2(d_1 - d_2)c \times \tau$

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$$P = 2(d_1 - d_2)\,\tau \times c$$

① Design a sleeve and cotter joint to resist a tensile load of 60KN all the parts of the joint are made of the same material with the following allowable stresses are, $\sigma_t = 60\,mpa$, $\tau = 70\,mpa$, $\sigma_c = 125\,Mpa$

Given data

$$P = 60 \times 10^3\,N$$

$$\sigma_t = 60\,mpa = 60\,N/mm^2$$

$$\sigma_c = 125\,mpa = 125\,N/mm^2$$

$$\tau = 70\,mpa = 70\,N/mm^2$$

diameter of rods :-

$$P = \frac{\pi}{4} d^2\,\sigma_t$$

$$60 \times 10^3 = \frac{\pi}{4} d^2 \times 60$$

$$d = 35.7 \text{ say } 36\,mm$$

Inside diameter of sleeve & thickness of cotter :-

$$P = \left[\frac{\pi}{4} d_2^2 - d_2 \times t\right] \sigma_t$$

$$60 \times 10^3 = \left[\frac{\pi}{4}(d_2^2) - \frac{d_2^2}{4}\right] \sigma_t$$

$$d_2 = 43.2 \text{ say } 44\,mm$$

thickness of cotter $t = \frac{d_2}{4} = \frac{44}{4} = 11\,mm$

Let us now check the induced crushing stress in the cotter

$$P = \sigma_c \times d_2 \times t$$

$$60 \times 10^3 = \sigma_c \times 44 \times 11$$

$$\sigma_c = 124\,N/mm^2$$

Since the induced crushing stress is less than the given value of $125\,N/mm^2$ therefore the dimensions $d_2$ and $t$ are within safe limits.

outside diameter of Sleeve :-

$d_1$ = outside dia. of Sleeve

$$P = \left[\frac{\pi}{4}[(d_1)^2 - (d_2)^2] - (d_1 - d_2)t\right]\sigma_t$$

$$60\times10^3 = \left[\frac{\pi}{4}[d_1^2 - (44)^2] - (d_1 - 44)11\right]60$$

$$d_1^2 - 14d_1 - 2593 = 0$$

$$d_1 = 58.4\text{ mm say } 60\text{ mm}$$

width of Cotter :-

b = width of Cotter

$$P = 2bt\tau$$

$$60\times10^3 = 2\times b\times11\times70$$

$$b = 38.96 \text{ say } 40\text{ mm}$$

Distance of the rod from the beginning to the Cotter hole :-

a = Required distance

$$P = 2ad_2\tau$$

$$60\times10^3 = 2\times a\times44\times70$$

$$a = 9.74 \text{ say } 10\text{ mm}$$

Distance of the rod end from its end to the Cotter hole :-

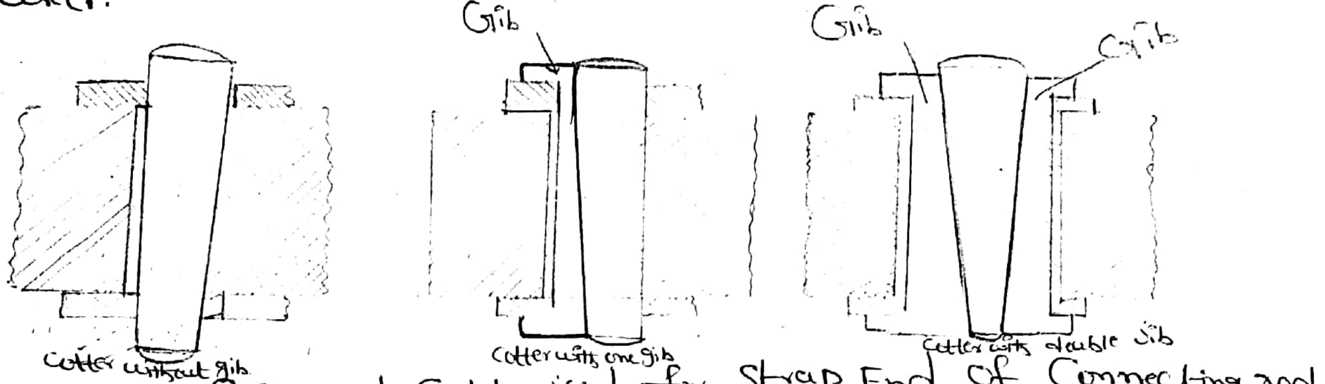
$$P = 2(d_1 - d_2)c\tau$$

$$60\times10^3 = 2(60-44)c\times70$$

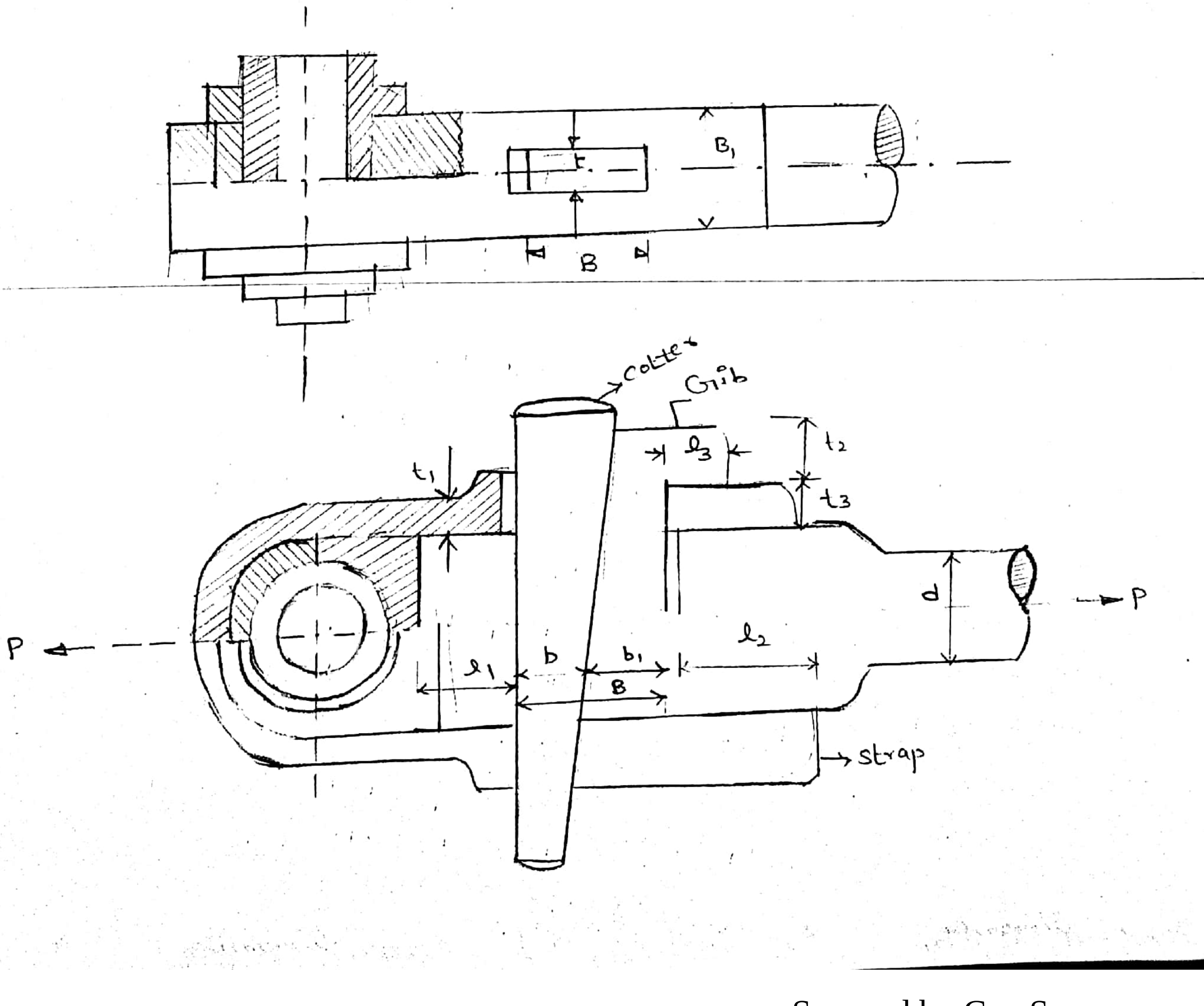
$$c = 26.78 \text{ say } 28\text{ mm}$$

## Gib and Cotter joint :-

A Gib Cotter joint is usually used in strap end (or big end) of a Connecting rod. when the Cotter alone (ie without gib) is driven the friction b/w its ends and the inside of the Slots in the strap tends to Cause the Sides of the strap to Spring open outwards as shown in fig ① (dotted line). In order to prevent this gibs as shown in fig②, fig③, are used which hold together the ends of the strap. Moreover Gibs provide a large bearing surface for the cotter to slide on, due to the increased holding power. Thus the tendency of cotter to slacken back owing to friction is considerably decrease. The jib also enables parallel holes to be used.




## Design of a Gib and Cotter joint for strap End of Connecting rod



Consider a gib and Cotter joint for strap end (or big end) of a Connecting rod as shown in fig. The Connecting rod is subjected to tensile and Compressive loads.

Let

$P$ = max. thrust (or) pull in the Connecting rod

$d$ = Dia. of the adjacent end of the round part of the rod

$B_1$ = width of the strap.

$B$ = Total width of gib and Cotter.

$t$ = Thickness of Cotter

$t_1$ = Thickness of the strap at the thinnest part

$\sigma_t$ = permissible tensile stress for the material of strap

$\tau$ = Permissible shear stress for the material of the Cotter & gib

The width of strap ($B_1$) is generally taken equal to the diameter of the adjacent end of the round part of the rod ($d$).

Thickness of Cotter $\quad t = \dfrac{\text{width of strap}}{4} = \dfrac{B_1}{4}$

Thickness of gib = Thickness of Cotter ($t$)

Height ($t_2$) and length of gib head ($l_3$) = Thickness of Cotter ($t$)

Failure of the strap in tension :-

Assuming that no hole is provided for lubrication, that area resist the failure of the strap due to tearing $= 2B_1t_1$

Tearing strength of the strap $= 2B_1t_1\sigma_t$

$$P = 2B_1t_1\sigma_t$$

from this equation, the thickness of the strap at the thinnest part ($t_1$) may be obtained. when an Oil hole is provide in the strap, then its weakening effect should be Considered.

The thickness of the strap at the Cotter ($t_3$) is increased such that area of c/s of the strap at the Cotter hole is not less than the area of the strap at the thinnest part. In other words

$$2t_3(B_1-t) = 2B_1t_1$$

2. Failure of the gib and Cotter in Shearing :-

Since the gib and Cotter are in double shear, therefore area resisting failure

$$= 2Bt$$

resisting strength $= 2Bt\tau$

Equating this to the load (P) we get,

$$P = 2Bt\tau$$

width $b_1 = 0.55B$ $b = 0.45B$

$t_4 = 1.15t_1$ to $1.5t_1$

$l_1 = 2t_1$ and $l_2 = 2.5t_1$

① The big end of a connecting rod, is subjected to a max. load of 50 kN the diameter of the circular part of the rod adjacent to the strap end is 75 mm. Design the joint, assuming permissible tensile stress for the material of ~~the strap as 25 mpa~~ and permissible shear stress for the material of cotter and gib as 20 mpa.

$P = 50kN = 50\times10^3 N$, $d = 75$ mm, $\sigma_t = 25$ mpa, $\tau = 20$ mpa

width of the strap :-

$B_1$ = width of the strap

width of the strap is equal to the diameter of end of rod

$$B_1 = d = 75 \text{ mm}$$

Thickness of the cutter $t = \frac{B_1}{4} = \frac{75}{4} = 18.5$ say 20 mm

Thickness of gib = Thickness of Cutter = 20 mm

Height ($t_2$) and length of gib head ($l_3$) = thickness of cotter = 2.

2. Thickness of the strap at the thinnest part :-

$t_1$ = thickness of the strap at the thinnest part

$$P = 2B_1 t_1 \sigma_t$$

$$50 \times 10^3 = 2 \times 75 \times t_1 \times 25$$

$$t_1 = 13.3 \text{ say } 15 \text{ mm}$$

3. Thickness of the strap at the Cotter :-

$$2t_3(B_1 - t) = 2t_1 B_1$$

$$2t_3(75-20) = 2 \times 15 \times 75$$

$$t_3 = 20.45 \text{ say } 21 \text{ mm}$$

4. Total width of gib and Cotter :-

B = Total width of gib and Cotter

$$P = 2B \times t \times \tau$$

$$50 \times 10^3 = 2B \times 20 \times 20$$

$$B = 62.5 \text{ say } 65 \text{ mm}$$

Since one gib is used therefore width of gib

$$b_1 = 0.55B = 0.55 \times 65 = 35.75 \text{ say } 36 \text{ mm}$$

$$b = 0.45B = 0.45 \times 65 = 29.25 \text{ say } 30 \text{ mm}$$

the others dimensions are fixed as follows

$$t_4 = 1.25 t_1 = 18.75 \text{ say } 20 \text{ mm}$$

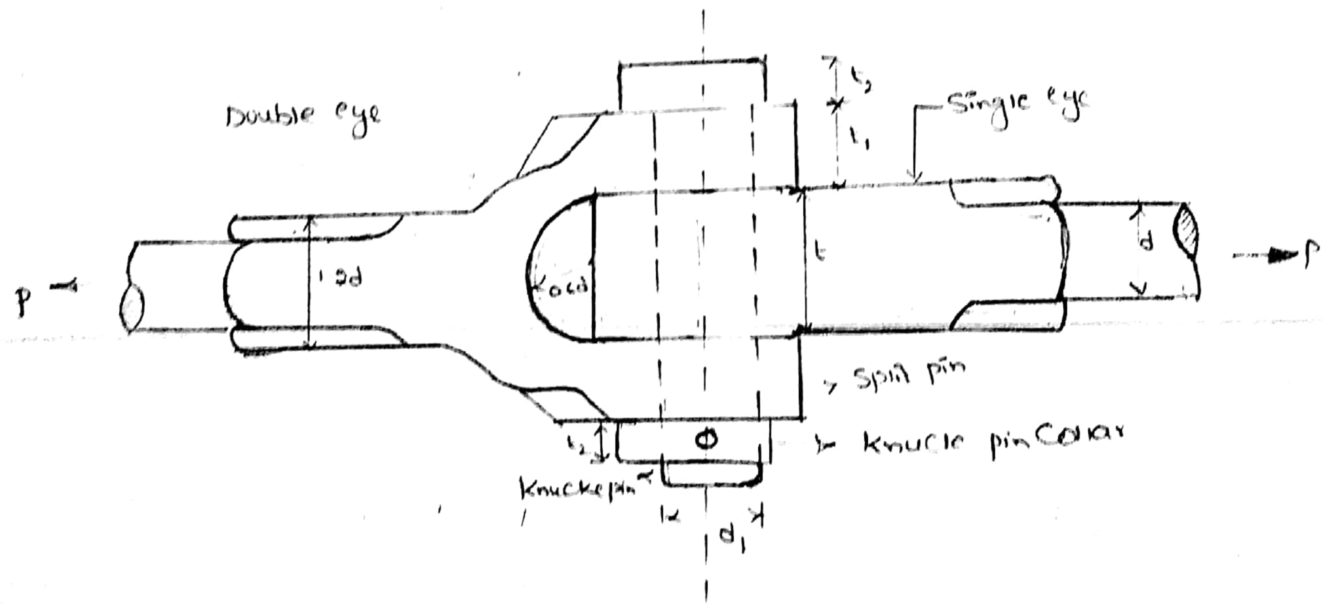
$$l_1 = 2t_1 = 30 \text{ mm}$$

$$l_2 = 2.5 t_1 = 37.5 \text{ say } 40 \text{ mm}$$

Knuckle joint :-

A knuckle joint is used to connect two rods which are under the action of tensile loads. However, if the joint is guided, the rods may support a compressive load. A knuckle joint may be readily disconnected for adjustments or repairs. It's use may be found in the link of cycle chain.

In knuckle joint, one end of one of the rods is made into an eye and the end of the other rod is formed into a fork with an eye in each of the fork leg. The knuckle pin passes through both the eye hole and the fork holes. and may be secured by means of a collar and taper pin or split pin. The knuckle pin may be prevented from rotating in the fork by means of a small stop, pin, peg or snug. In order to get a better quality of joint, the sides of the fork and eye are machined, the hole is accurately drilled and pin turned. The material used for the joint may be steel or wrought iron.




If $d$ is the diameter of rod, then diameter of pin

$$d_1 = d$$

outer diameter of eye

$$d_2 = 2d$$

Diameter of knuckle pin head and collar

$$d_3 = 1.5d$$

Thickness of single eye or rod end,

$$t = 1.25d$$

Thickness of fork $t_1 = 0.75d$

Thickness of pin head $t_2 = 0.5d$

Methods of failure of knuckle joint :-

$P$ = Tensile load acting on the rod

$d$ = Diameter of the rod

$d_1$ = Diameter of the pin

$d_2$ = Outer diameter of eye

$t$ = Thickness of single eye

$t_1$ = Thickness of fork

$\sigma_t, \tau \& \sigma_c$ = permissible stresses for the joint material in tension, shear & crushing

Failure of the solid rod in tension :-

tensile strength of the rod $= \frac{\pi}{4} d^2 \sigma_t$

$$P = \frac{\pi}{4} d^2 \sigma_t$$

Failure of the knuckle pin in shear :-

Since the pin is in double shear, therefore c/s area of the pin under shearing

$$= 2 \times \frac{\pi}{4}(d_1)^2$$

Shear strength of the pin

$$= 2 \times \frac{\pi}{4}(d_1)^2 \tau$$

$$P = 2 \times \frac{\pi}{4}(d_1)^2 \tau$$

3. Failure of the single eye (or) rod end in tension :-

area resisting tearing $= (d_2 - d_1)t$

Tearing strength of single eye or rod end

$$= (d_2 - d_1)t\,\sigma_t$$

$$P = (d_2 - d_1)t\,\sigma_t$$

4. Failure of the single eye or rod end in shearing :-

area resisting shearing $= (d_2 - d_1)t$

Shearing strength of single eye or rod end

$$= (d_2 - d_1)t\,\tau$$

$P = (d_2 - d_1) t \tau$

5. Failure of the Single eye or rod end in Crushing :-

Area resisting Crushing $= d_1 \times t$

Crushing strength of single eye or rod end

$= d_1 \times t \times \sigma_c$

$P = d_1 t \sigma_c$

6. Failure of the forked end in tension :-

Area resisting tearing $= (d_2 - d_1) 2t_1$

Tearing strength of forked end $= (d_2 - d_1) 2t_1 \times \sigma_t$

$P = (d_2 - d_1) \times 2t_1 \times \sigma_t$

7. Failure of the forked end in shear :-

Area resisting shearing $= (d_2 - d_1) 2t_1$

Shearing strength of the forked end $= (d_2 - d_1) 2t_1 \tau$

$P = (d_2 - d_1) 2t_1 \tau$

8. Failure of the forked end in Crushing :-

Area resisting Crushing $= d_1 \times 2t_1$

Crushing strength of the forked end $= d_1 \times 2t_1 \times \sigma_c$

$P = d_1 \times 2t_1 \times \sigma_c$

---

(1) Design a knuckle joint to transmit 150 KN. The design stresses may be taken as 75 MPa in tension, 60 MPa, in shear and 150 MPa in Compr

$P = 150\,KN, = 150 \times 10^3\,N, \sigma_t = 75\,MPa = 75\,N/mm^2, \tau = 60\,MPa = 60\,N/$

$\sigma_c = 150\,MPa = 150\,N/mm^2$

1. Failure of the solid rod in tension

$d =$ Diameter of the rod

$P = \frac{\pi}{4} d^2 \sigma_t$

$150 \times 10^3 = \frac{\pi}{4} \times d^2 \times 75$

$d = 50.4$ say $52\,mm$

Diameter of knuckle pin

$d_1 = d = 52\,mm$

outer diameter of eye $d_2 = 2d = 2 \times 52 = 104$ mm

Diameter of knucklepin head and Collar

$$d_3 = 1.5d = 1.5 \times 52 = 78 \text{ mm}$$

Thickness of single eye or rod end

$$t = 1.25d = 1.25 \times 52 = 65 \text{ mm}$$

$$t_1 = 0.75d = 0.75 \times 52 = 39 \text{ say } 40 \text{ mm}$$

$$t_2 = 0.5d = 0.5 \times 52 = 26 \text{ mm}$$

2. Failure of the knuckle pin in shear :-

$$P = 2 \times \frac{\pi}{4} (d_1)^2 \tau$$

$$150 \times 10^3 = 2 \times \frac{\pi}{4} (52)^2 \tau$$

$$\tau = 35.3 \text{ Mpa}$$

3. Failure of the single eye or rod end in tension :-

$$P = (d_2 - d_1) t \sigma_t$$

$$150 \times 10^3 = (104 - 52) 65 \sigma_t$$

$$\sigma_t = 44.4 \text{ Mpa}$$

4. Failure of the single eye or rod end in shearing :-

$$P = (d_2 - d_1) t \tau$$

$$150 \times 10^3 = (104 - 52) 65 \times \tau$$

$$\tau = 44.4 \text{ Mpa}$$

5. Failure of single eye or rod end in crushing :-

$$P = d_1 t \sigma_c$$

$$150 \times 10^3 = 52 \times 65 \times \sigma_c$$

$$\sigma_c = 44.4 \text{ Mpa}$$

6. Failure of the forked end in tension :-

$$P = (d_2 - d_1) 2 t_1 \sigma_t$$

$$150 \times 10^3 = (104 - 52) 2 \times 40 \times \sigma_t$$

$$\sigma_t = 36 \text{ Mpa}$$

7. Failure of the forked end in shear :-

$$150 \times 10^3 = (d_2 - d_1) 2 t_1 \tau$$

$$= (104 - 52) 2 \times 40 \times \tau$$

$$\tau = 36 \text{ mpa}$$

8. Failure of the forked end in Crushing :-

$$150 \times 10^3 = d_1 \times 2t_1 \times \sigma_c$$

$$= 52 \times 2 \times 40 \times \sigma_c$$

$$\sigma_c = 36 \text{ Mpa}$$

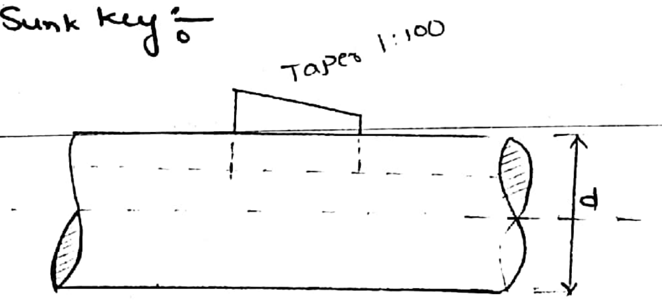
From above, we see that the induced stresses are less than the given design stresses, therefore the joint is safe

Key :- A key is a piece of mildsteel inserted between the shaft and hub to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft keys are used as temporary fastening and are subjected to considerable crushing & shearing stresses. A keyway is a slot or recess in a shaft and hub of pulley to accommodate a key.

Types of keys :-

1. Sunk key :- The sunk key are provided half in the keyway of the shaft and half in the keyway of the hub of the pulley. The sunk keys are of the following types

a. Rectangular Sunk key :-




width of the key $w = \frac{d}{4}$ ; thickness of key $t = \frac{d}{6}$

$d$ = dia. of shaft (or) dia. of hole in the hub

b. Square Sunk key :- The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal

$$w = t = \frac{d}{4}$$

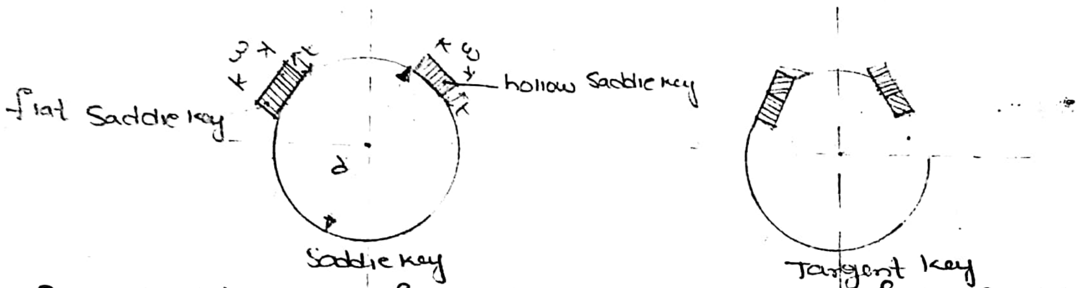
c. Parallel sunk key :- The parallel sunk key may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

d. Gib-head key :- It is a rectangular sunk key with a head at one end known as gib head. It is usually provided to facilitate the removal of key.

2. Saddle keys :-

The saddle key are of the following two types

1. flat saddle key  2. Hollow saddle key




A flat saddle key is a taper key which fits in a keyway in the hub and is flat on the shaft as shown in fig. It is likely to slip round the shaft under load. Therefore it is used for comparatively light loads.

A hallow saddle key is taper key which fits in a keyway in the hub and bottom of the key is shaped to fit the curved surface of the shaft. Since hallow saddle keys hold on by friction, therefore these are suitable for light loads. It is usually used as a temporary ~~fastening~~ in fixing and setting eccentrics, cams etc.

3. Tangent keys :- The tangent keys are fitted in pair at right angles. each key is to withstand torsion in one direction only. These are used in large heavy duty shafts

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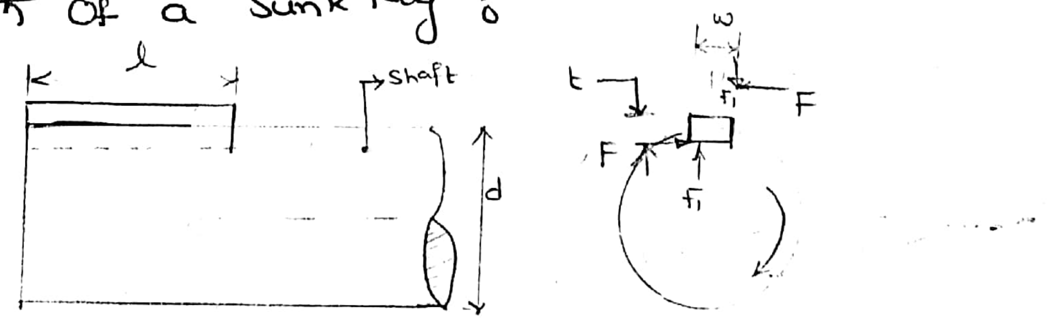
4. Round Keys :- round keys are circular in c/s in section a fit into holes drilled partly in the shaft and partly in the hub. It have the advantage that their keyway may be drilled and reame after the mating parts have been assembled. Round keys are usual considered to be most appropriate for low power drives.

5. Splines :- Sometimes, key are made integral with the shaft which fit in the keyway broached in the hub. Such shafts are know as splind shafts. These shafts usually have four, six, ten (or) sixtee splines. The splined shafts are relatively stronger than shafts having a single keyway.

Strength of a Sunk key :-




forces ($F_1$) due to fit of the key in its keyway

Forces (F) due to the torque transmitted by the shaft.

T = Torque transmitted by the shaft

F = Tangential force acting at the circumference of the sh

d = diameter of shaft

l = length of key

w = width of key

t = Thickness of key

$\tau$ & $\sigma_c$ = Shear and crushing stresses for the material of ke

A little consideration will show that due to the power transmitte by the shaft, the key may fail due to shearing (or) crushing

Consider shearing of the key, the tangential shearing force acting at the circumference of the shaft.

$$F = \text{Area resisting shearing} \times \text{Shear stress} = l \times w \times \tau$$

Torque transmitted by the shaft

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2}$$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft

$$F = \text{Area resisting crushing} \times \text{crushing stress} = l \times \frac{t}{2} \times \sigma_c$$

Torque transmitted by the shaft

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

The key is equally strong in shearing and crushing

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

$$\frac{w}{t} = \frac{\sigma_c}{2\tau}$$

The permissible crushing stress for the usual key material is atleast twice the permissible shear stress. Therefore from equation, we have $w = t$. In other words, a square key is equally strong in shearing and crushing

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft

Shearing strength of key

$$T = l \times w \times \tau \times \frac{d}{2} \quad \text{— ①}$$

Torsional shear strength of the shaft

$$T = \frac{\pi}{16} \tau_1 d^3 \quad \text{— ②}$$

from equations ① & ②  $\left[w = \frac{d}{4}\right]$

$$l w \tau \frac{d}{2} = \frac{\pi}{16} \tau_1 d^3$$

$$l = 1.571 d \times \frac{\tau_1}{\tau}$$

when the key material is same as that of the shaft $\tau_1 =$

$$l = 1.571 d.$$

① Design the rectangular key for a shaft of 50mm diameter. The shearing and crushing stresses for the key material are 42Mpa & 70Mpa

Given $d = 50mm$, $\tau = 42 Mpa = 42 N/mm^2$, $\sigma_c = 70Mpa = 70 N/mm^2$

from the table we find that for a shaft of 50mm diameter

width of key $w = 16$ mm

thickness of key $t = 10$ mm

length of key obtained by considering key in shearing & crushing

$$T = l w \tau \frac{d}{2} \quad \text{[torque transmitted]}$$

$$= l \times 16 \times 42 \times 25 = 16800 l \text{ N-mm} \quad -①$$

$$T = \frac{\pi}{16} \tau d^3 \quad \text{[Torsional shear strength of key]}$$

$$T = 1.03 \times 10^6 \text{ N-mm} \quad -②$$

from the equations ① & ②

$$l = \frac{1.03 \times 10^6}{16800}$$

$$l = 61.31 \text{ mm}$$

Consider crushing of the key

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

$$= l \times \frac{10}{2} \times 70 \times \frac{50}{2}$$

$$= 8750 l \text{ N-mm} \quad -③$$

from the equations ② & ③

$$l = \frac{1.03 \times 10^6}{8750} = 117.7 \text{ mm}$$

Taking larger of two values

$l = 117.7$ say $120$ mm

# Keys

→ A key is a piece of mild steel Inserted between the shaft and hub or boss of the pulley to Connect these together in Order to Prevent relative motion between them.

→ A keyway is a slot or recess in a shaft and hub off the Pulley to accommodate a key.

→ keys are Used as temporary fastenings and are Subjected to Considerable Crushing and shearing stresses.

## Types of keys:

1. Sunk key:

The Sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley.

The Sunk keys are of following types.

* a. Rectangular Sunk key:

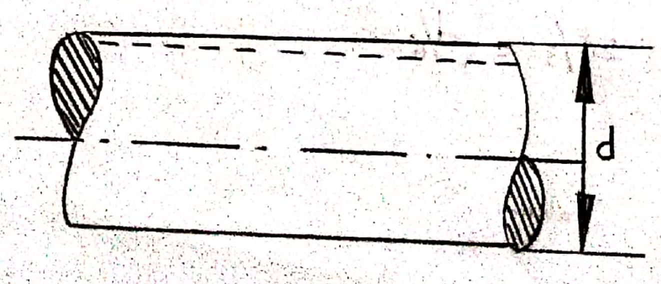
The ~~diameter~~ dimensions of Rectangular keys are

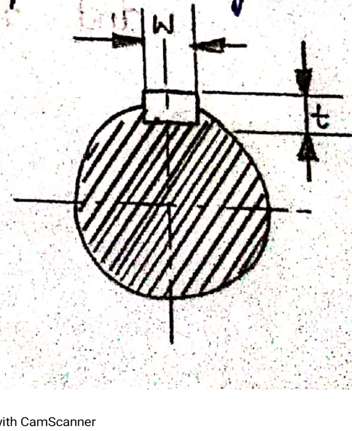
Width of the key $w = d/4$;

Thickness of the key $t = 2w/3 = d/6$

$d$ = diameter of the shaft or diameter of the hole in the hub.

The key has taper 1 in 100 on the top Side only.



2. Square Sunk key:

The only difference between a rectangular Sunk key and Square Sunk is that its width and thickness are Equal

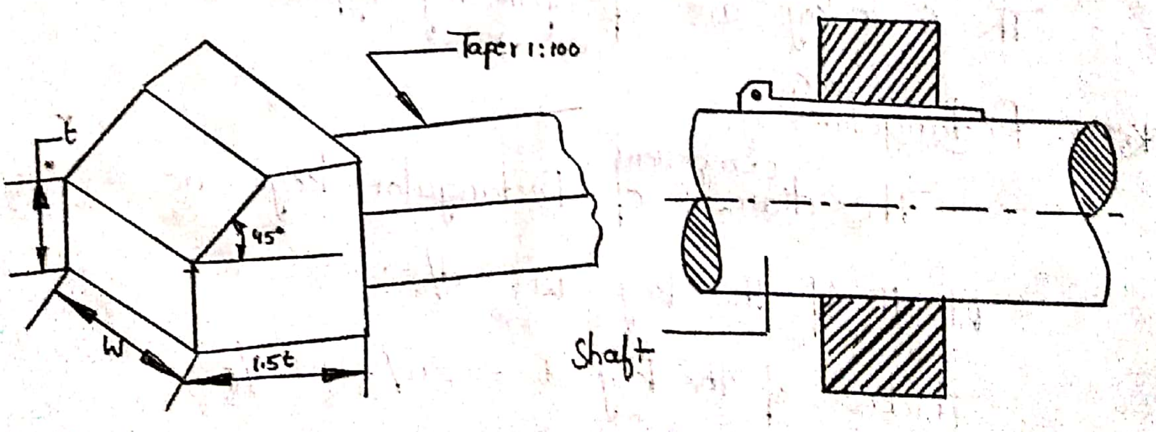
i.e $w = t = d/4$

3. Parallel Sunk key:

The Parallel Sunk keys may be of rectangular or Square Section Uniform in width and thickness thoughout. it it may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating Piece is required to Slide along the shaft.

4. Gib-Head key:

It is a rectangular Sunk key with a head at one end known as a gib head. It is Usually provided to facilitate the removal of key.




The Usual Proportions of the gib Head key are

Width, $w = d/4$

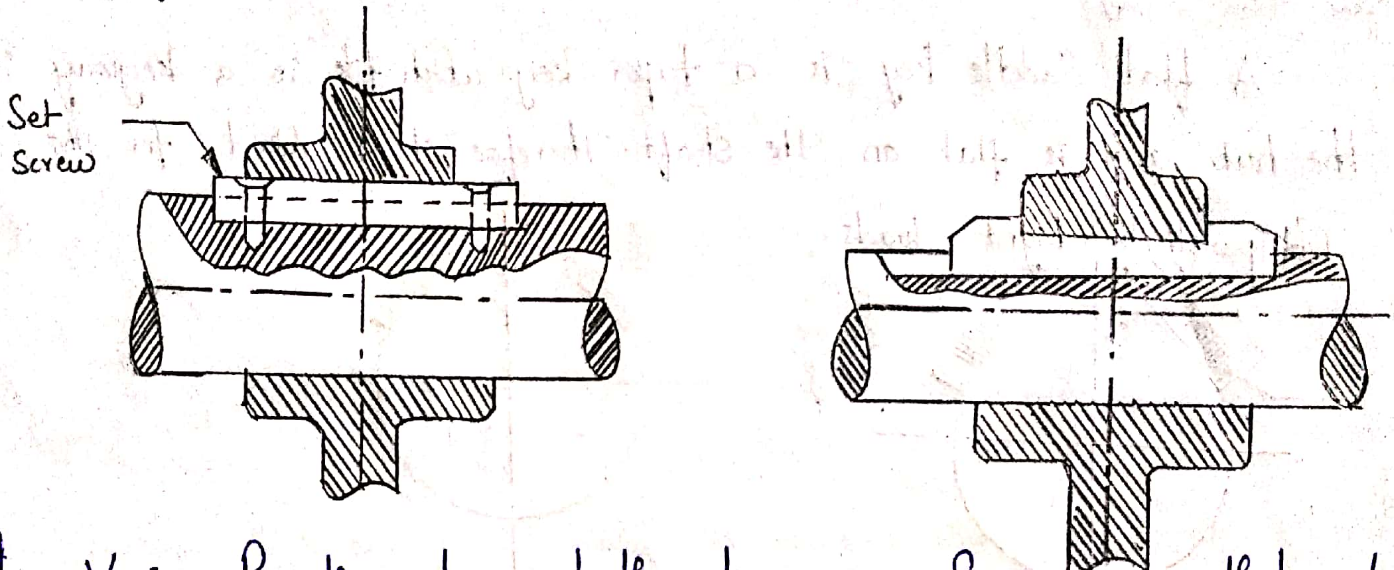
and thickness at large end

$t = 2w/3 = d/6$

## ther key:

A key attached to One member of a pair and which permits relative and axial movement is known as feather key.

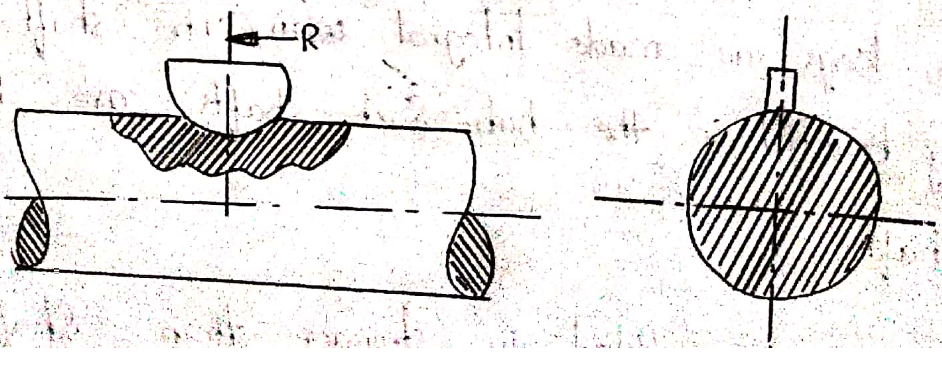
It is a Special type of parallel Sunk key which transmits a turning moment and also permits axial movement.




The Various Proportions of a feather key are Same as that of Rectangular Sunk key and gib head key.

## 6. Woodruff-key:

The Woodruff key is an Easily adjustable key. It is a piece from a Cylindrical disc having Segmental Cross-Section in front View. This key is largely used in machine tool and the automobile Construction.



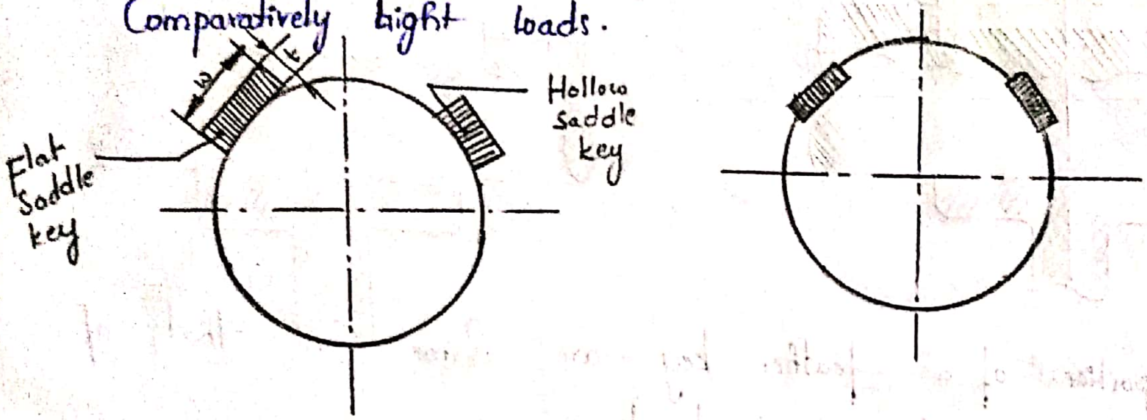
## Saddle keys:

The Saddle keys are of the following two types.

1. Flat Saddle key.
2. Hollow Saddle key.

### * Flat Saddle key:

A flat Saddle key is a taper key which fits in a keyway in the hub and is flat on the shaft. Therefore it is Used for the Comparatively light loads.




### * Hollow Saddle key:

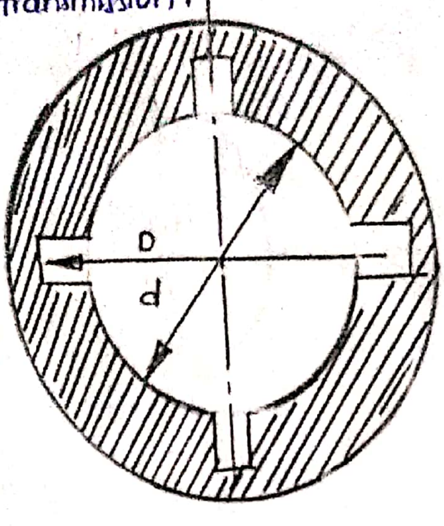
A Hollow Saddle key is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the Curved surface of the Shaft.

## Splines:

→ Sometimes, keys are made Integral with the shaft which fits in the keyway broached in the hub. Such shafts are known as Splined shaft.

→ Splined shafts are Relatively stronger than Shafts having a Single keyway.

The Splined shafts are Used when the force to transmitted is large in Proportion to the Size of the shaft as in the automobile transmission.



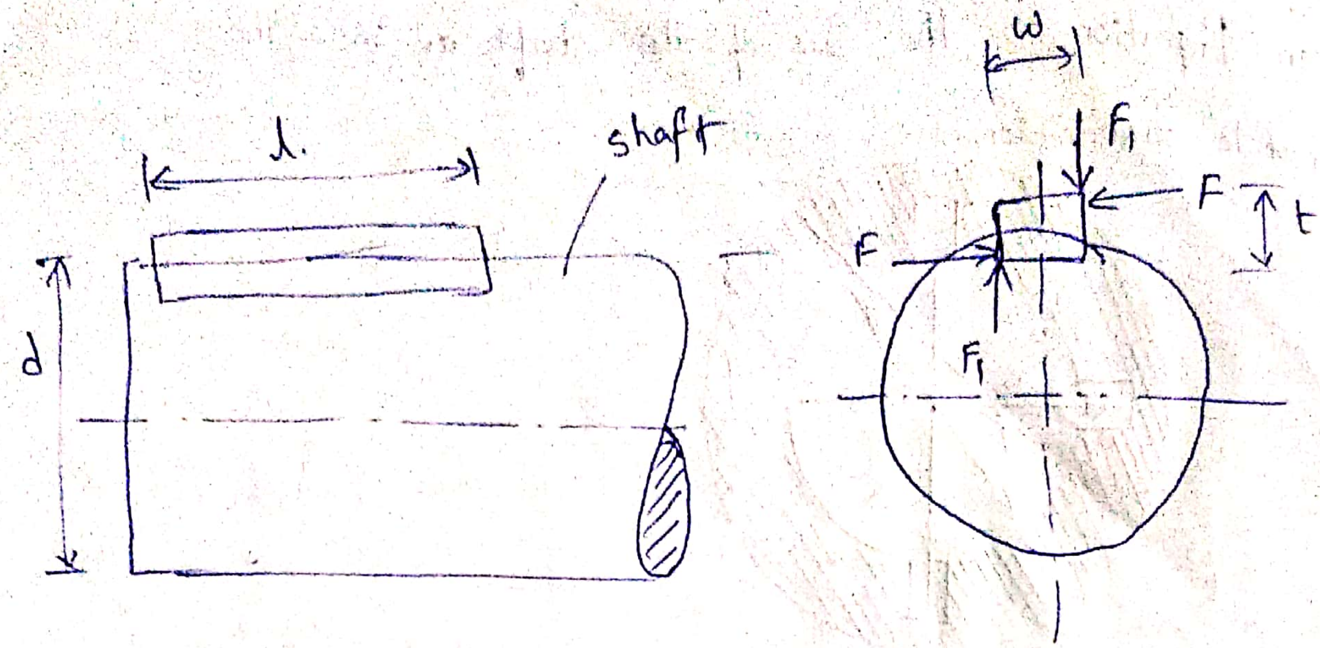

## Forces acting on Sunk key :-

When a key is used in transmitting torque from a shaft to rotor (or) hub, the following two types of forces act on the key.

1) Forces ($F_1$) due to fit of the key in its keyway. These forces Produce compressive stresses in the key which are difficult to determine in magnitude.

2) Forces (F) due to the torque transmitted by the shaft. These forces Produce shearing and compressive stresses in the key.

In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.




## Strength of a Sunk key:-

Let $T$ = Torque transmitted by shaft

$F$ = Tangential force acting at the circumference of the shaft

$d$ = diameter of shaft

$l$ = length of key

$w$ = width of key

$t$ = Thickness of key

$\tau$ & $\sigma_c$ = shear and crushing stresses for the material of key.

Due to Power transmitted by the shaft, the key may fail due to shearing (or) crushing.

n order to find the length of key to transmit full Power of shaft, the shear Strength of the key is equal to torsional shear strength of shaft

shear strength of key, $T = l \times w \times \tau \times \frac{d}{2}$ — (3)

Torsional shear strength of shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3 \quad \text{— (4)}$$

($\tau_1$ = shear stress of shaft material)

Equating (3) & (4)

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3.$$

$$l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau}$$

Take $w = \frac{d}{4}$

$$l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{\left(\frac{d}{4}\right)\tau} = 1.571\, d \times \frac{\tau_1}{\tau}$$

$$l = 1.571\, d \times \frac{\tau_1}{\tau}$$

When the key material is same as that of shaft, then $\tau = \tau_1$.

$$\boxed{l = 1.571 d}$$

Considering shearing of the key, Tangential shearing force acting at the circumference of sha[ft]

F = Area resisting shearing × shear stress

$= l \times w \times \tau$

∴ Torque transmitted by shaft

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2} \quad \text{—(1)}$$

Considering crushing of the key, the tangential crushing force acting at the circumference of shaft

F = Area resisting ~~shearing~~ crushing × crushing stress

$= l \times \frac{t}{2} \times \sigma_c$

∴ Torque transmitted by shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \text{—(2)}$$

if the key is equally strong in shearing & crushing

$$\cancel{l} \times w \times \tau \times \frac{d}{\cancel{2}} = \cancel{l} \times \frac{t}{2} \times \sigma_c \times \frac{d}{\cancel{2}}$$

$$\boxed{\frac{w}{t} = \frac{\sigma_c}{2\tau}}$$

For square key, w = t ∴ The permissible crushing stress for key material is atleast twice the permissible shear stress.

45 M
yield

# Shafts



*) A shaft is rotating machine element which is used to transmit Power from one place to another.

*) The Power is delivered to the shaft by some tangential force and the resultant torque set up within the shaft Permits the Power to be transferred to the another shaft.

*) Various members such as Pulleys, gears etc are mounted on shafts. These members along with the forces exerted upon them causes the shaft to bending.

## Standard sizes of shafts:-

25 mm to 60 mm with 5 mm steps ;

60 mm to 110 mm with 10 mm steps ;

110 mm to 140 mm with 15 mm steps ;

140 mm to 500 mm with 20 mm steps.

The standard length of the shafts are 5m, 6m, 7m.

## Stresses in shafts:-

1) Shear stresses due to transmission of torque

2) Bending Stresses due to forces acting upon machine elements like gears, Pulleys etc, as well as due to weight of shaft itself.

3) Stresses due to combined torsional & Bending loads.

## Design of shafts:-

The shafts may be designed on the basis of

1) Strength  2) Rigidity & stiffness

on the basis of strength, the following cases may be considered.

a) shafts subjected to twisting moment (or) Torque only.

b) shafts subjected to bending moment only

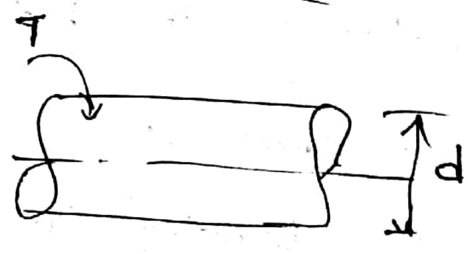
c) shafts subjected to combined twisting & bending moments.

d) shafts subjected to axial loads in addition to combined torsional and bending loads.

### Shafts subjected to Twisting Moment only:-

When the shaft is subjected to twisting moment, then the diameter of shaft may be obtained by Torsion equation.

$$\frac{T}{J} = \frac{\tau}{r}$$




T = Twisting moment (or) Torque

J = Polar moment of Inertia

$\cancel{J = \frac{\pi}{32} d^4}$

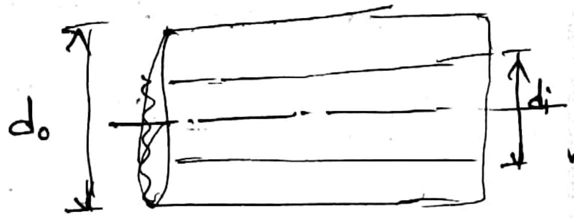
$\tau$ = Torsional shear stress

$r$ = Distance from neutral axis to outer fibre $= d/2$

For Solid shaft ; $J = \frac{\pi}{32} d^4$

$$\frac{T}{\frac{\pi}{32} d^4} = \frac{\tau}{d/2} \Rightarrow \boxed{T = \frac{\pi}{16} \times \tau \times d^3}$$

For Hollow shaft ;




$d_o$ → outside diameter

$d_i$ → Inside diameter

Polar moment of Inertia ; $J = \frac{\pi}{32} (d_o^4 - d_i^4)$

$$r = \frac{d_o}{2}$$

$$\frac{T}{\frac{\pi}{32} (d_o^4 - d_i^4)} = \frac{\tau}{\left(\frac{d_o}{2}\right)}$$

$$T = \frac{\pi}{16} \times \tau \times \frac{d_o^4 - d_i^4}{d_o}$$

$$\Rightarrow T = \frac{\pi}{16} \times \tau \times \frac{d_o^4}{d_o} \left(1 - \frac{d_i^4}{d_o^4}\right)$$

$$\Rightarrow \boxed{T = \frac{\pi}{16} \times \tau \times (d_o)^3 (1 - K^4)}$$

where $K = \frac{d_i}{d_o}$

*) **Note:-** 1) Torque 'T' can be obtained as follows:

$$P = \frac{2\pi NT}{60} \Rightarrow T = \frac{60 \times P}{2\pi N}$$

2) In case of belt drives, Twisting moment (or) Torque (T) is given by

$$T = (T_1 - T_2)R$$

$$\cancel{\frac{T_1}{T_2} = 2.3\log}\ 2.3\log\left(\frac{T_1}{T_2}\right) = \mu\theta$$

where $T_1$, $T_2$ are Tensions in tight side & slack side of belt respectively.

$R \rightarrow$ Radius of Pulley.

'$\mu$' $\rightarrow$ coefficient of friction b/w Pulley & belt

'$\theta$' $\rightarrow$ angle of lap in radians.

**Eg:-** Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and factor of safety as 8. If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameter is 0.5

**Egs**

$P = 20\ kW = 20 \times 1000\ W$ ; $N = 200\ rpm$

Ultimate shear stress $(\tau_u) = 360\ MPa = 360\ N/mm^2$

$F.O.S = 8$ ; $K = \frac{d_i}{d_o} = 0.5$

Allowable shear stress $(\tau) = \frac{\tau_u}{F.O.S} = \frac{360}{8} = 45\ N/mm^2$

Scanned by CamScanner

Diameter of the solid shaft :

Let d = dia. of solid shaft

Torque Transmitted by shaft,

$$T = \frac{60 \times P}{2\pi N} = \frac{60 \times 20 \times 1000}{2\pi \times 200} = 955 \text{ N-m}$$

$$T = 955 \times 10^3 \text{ N-mm}$$

Torque Transmitted by solid shaft

$$T = \frac{\pi}{16} \times \tau \times d^3$$

$$\Rightarrow 955 \times 10^3 = \frac{\pi}{16} \times 45 \times d^3$$

$$d = 47.6 \text{ mm (or) } 50 \text{ mm}$$

Diameter of hollow shaft:-

Let $d_i$ = Inside diameter

$d_o$ = outside diameter

Torque Transmitted by hollow shaft

$$T = \frac{\pi}{16} \times \tau \times \left(\frac{d_o^4 - d_i^4}{d_o}\right)$$

$$\Rightarrow T = \frac{\pi}{16} \times \tau \times d_o^3 (1 - k^4)$$ where $k = \frac{d_i}{d_o}$

$$\Rightarrow 955 \times 10^3 = \frac{\pi}{16} \times 45 \times d_o^3 (1 - (0.5)^4)$$

$$d_o = 48.6 \text{ mm (or) } 50 \text{ mm}$$

$$d_i = 0.5 d_o = 0.5 \times 50 = 25 \text{ mm}$$

## Shafts subjected to Bending moment only:-

When the shaft is subjected to a bending moment only, then the maximum stress is given by the bending equation.

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

where $M$ = Bending moment

$I$ = Moment of Inertia of cross-sectional area of the shaft about the axis of rotation.

$\sigma_b$ = Bending stress

$y$ = Distance from neutral axis to the outer-most fibre

For round solid shaft,

$$I = \frac{\pi}{64} d^4 \quad ; \quad y = \frac{d}{2}$$

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\Rightarrow \frac{M}{\frac{\pi}{64} d^4} = \frac{\sigma_b}{(d/2)}$$

$$\Rightarrow \cancel{\sigma_b = \frac{32 M}{\pi d^3}} \qquad \boxed{M = \frac{\pi}{32} \times \sigma_b \times d^3}$$

For Hollow shaft,

$$I = \frac{\pi}{64} (d_o^4 - d_i^4) = \frac{\pi}{64} d_o^4 (1 - K^4)$$

where $K = \frac{d_i}{d_o}$

$$y = \frac{d_o}{2}$$

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\Rightarrow \frac{M}{\frac{\pi}{64} d_o^4 (1-k^4)} = \frac{\sigma_b}{\left(\frac{d_o}{2}\right)}$$

$$\boxed{M = \frac{\pi}{32} \times \sigma_b \times d_o^3 (1-k^4)}$$

## Shafts subjected to combined Twisting moment & Bending Moment :-

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of two moments simultaneously. The following two theories are widely used when shaft subjected to various types of combined stresses.

1) Maximum shear stress theory (or) Guest's Theory. It is used for ductile materials such as mild steel.

2) Maximum normal stress theory (or) Rankine's Theory. It is used for brittle materials such as cast Iron.

Let $\tau$ = shear stress induced due to twisting moment

$\sigma_b$ = Bending stress induced due to bending moment

(i) According to Maximum shear stress Theory,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$\Rightarrow \tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + \left(\frac{32T}{\pi d^3}\right)^2}$$

$$= \left(\frac{32}{\pi d^3} \times \frac{1}{2}\right)\sqrt{M^2 + T^2}$$

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{M^2+T^2}$$

$$\Rightarrow \boxed{\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2+T^2}} = T_e$$

$T_e = \sqrt{M^2+T^2}$ is known as equivalent twisting moment $(T_e)$.

ii) According to maximum normal stress Theory,

$$(\sigma_b)_{max} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$= \frac{1}{2}\left(\frac{32M}{\pi d^3}\right) + \frac{1}{2}\sqrt{\frac{32M}{\pi d^3} + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{32M}{\pi d^3}\left(\frac{1}{2}\left(M + \sqrt{M^2+T^2}\right)\right)$$

$$\boxed{\frac{\pi}{32} \times (\sigma_b)_{max} \times d^3 = \frac{1}{2}\left(M + \sqrt{M^2+T^2}\right) = M_e}$$

$M_e = \frac{1}{2}\left(M+\sqrt{M^2+T^2}\right)$ is known as equivalent bending moment.

For hollow shaft ;

$$T_e = \sqrt{M^2+T^2} = \frac{\pi}{16} \times \tau (d_o)^3 (1-k^4)$$

$$M_e = \frac{1}{2}\left(M+\sqrt{M^2+T^2}\right) = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1-k^4)$$

Note:- It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of two values is adopted.

Shafts subjected to Fluctuating Loads:-

In actual practice, the shafts are subjected to fluctuating torque and bending moments. In order to design such shafts ~~like~~ ~~to~~ the combined shock and fatigue factors must be taken into account for the computed twisting moment (T) and bending moment.

A shaft ~~is~~ subjected to combined bending & Torsion, the equivalent twisting moment,

$$\boxed{T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}}$$

Equivalent bending moment,

$$\boxed{M_e = \frac{1}{2}\left[(K_m \times M) + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right]}$$

where $K_m$ = combined shock & fatigue factor for bending,

$K_t$ = Combined Shock & fatigue factor for Torsion.

## Shafts subjected to Axial load in addition to Combined and Bending loads:-

When the shaft is subjected to an axial load (F) in addition to torsion and bending loads as in Propeller shafts of ships and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress ($\sigma_b$).

For solid shaft, $\frac{M}{I} = \frac{\sigma_b}{y} \Rightarrow \sigma_b = \frac{M \cdot y}{I}$

$$\Rightarrow \sigma_b = \frac{M \times \frac{d}{2}}{\frac{\pi}{64} d^3} = \frac{32 M}{\pi d^3}$$

$$\text{Stress due to axial load} = \frac{F}{\frac{\pi}{4} \times d^2} = \frac{4F}{\pi d^2}$$

$\therefore$ Resultant stress for solid shaft,

$$\sigma_1 = \frac{32 M}{\pi d^3} + \frac{4F}{\pi d^2} = \frac{32}{\pi d^3}\left(M + \frac{F \times d}{8}\right)$$

$$\sigma_1 = \frac{32 M_1}{\pi d^3} \quad \left(\text{where } M_1 = M + \frac{Fd}{8}\right)$$

For hollow shaft,

$$\sigma_b = \frac{32 M}{\pi d_o^3 (1 - k^4)}$$

$$\text{Stress due to axial load} = \frac{F}{\frac{\pi}{4} \times d_o^2} = \frac{4F}{\pi d_o^2}$$

The resultant stress,

$$\sigma_1 = \frac{32M}{\pi d_o^3(1-k^4)} + \frac{4F}{\pi d_o^2(1-k^2)}$$

$$= \frac{32M}{\pi d_o^3(1-k^4)} + \frac{4F \times 8d_o(1+k^2)}{\pi d_o^3 \times 8(1-k^2)(1+k^2)}$$

$$= \frac{32M}{\pi d_o^3(1-k^4)} + \frac{32Fd_o(1+k^2)}{8\pi d_o^3(1-k^4)}$$

$$= \frac{32}{\pi d_o^3(1-k^4)}\left[M + \frac{Fd_o(1+k^2)}{8}\right]$$

$$= \frac{32M_1}{\pi d_o^3(1-k^4)} \quad \left(\text{where } M_1 = M + \frac{Fd_o(1+k^2)}{8}\right)$$

*) In case of long shafts (slender shafts) subjected to compressive loads, a factor known as column factor ($\alpha$) must be introduced to take column effect into account.

stress due to compressive load,

$$\boxed{\sigma_c = \frac{\alpha \times 4F}{\pi d^2}} \rightarrow \text{For Round shaft}$$

$$\boxed{\sigma_c = \frac{\alpha \times 4F}{\pi d_o^2(1-k^2)}} \rightarrow \text{For hollow shaft}$$

when slenderness ratio $\left(\frac{L}{k}\right)$ is less than 115. Then

column factor, $\alpha = \dfrac{1}{1-0.0044\left(\frac{L}{k}\right)}$.

when the slenderness ratio $\left(\frac{L}{k}\right)$ is more than 115.

Column factor, $\boxed{\alpha = \frac{\sigma_y \left(\frac{L}{k}\right)^2}{C \pi^2 E}}$

where $L$ = Length of shaft b/w bearings

$k$ = Least radius of gyration

$\sigma_y$ = Compressive yield point stress of shaft material

$C$ = coefficient in Euler's formula depending upon the end conditions

$C$ = 1, for hinged ends

= 2.25, for fixed ends

= 1.6, for ends that are partly restrained as in bearings.

*) For hollow shaft subjected to fluctuating torsional, bending load, along with an axial load, the equations for equivalent twisting moment ($T_e$) & Equivalent bending moment ($M_e$) may be written as:

$$T_e = \sqrt{\left[(k_m \times M) + \frac{\alpha F d_o (1+k^2)}{8}\right]^2 + (k_t \times T)^2}$$

$$= \frac{\pi}{16} \times \tau (d_o^3)(1-k^4) \quad \&$$

$$M_e = \frac{1}{2}\left[(k_m \times M) + \frac{\alpha F d_o (1+k^2)}{8} + \sqrt{\left[(k_m \times M) + \frac{\alpha F d_o (1+k^2)}{8}\right]^2 + (k_t T)^2}\right]$$

$$= \frac{\pi}{32} \times \sigma_b (d_o)^3 (1-k^4)$$

Note:- For solid shaft, $K=0$, $d_o = d$.

When the shaft carries no axial load, then $F=0$ &

When the shaft carries axial tensile load, then $\alpha = 1$

## Design of shafts on the basis of Rigidity:-

1) Torsional rigidity:- Torsional rigidity is important in the case of camshaft of an IC engine where the timing of the valves would be effected.

$$\frac{T}{J} = \frac{G\theta}{l} \Rightarrow \theta = \frac{TL}{GJ}$$

where $\theta$ = Torsional deflection (or) angle of twist in radians

$T$ = Twisting moment

$J$ = Polar moment of Inertia $= \frac{\pi}{32} d^4$

$J = \frac{\pi}{32}(d_o^4 - d_i^4)$

$G$ = Modulus of rigidity for the shaft material

$L$ = Length of the shaft

2) Lateral rigidity:-

It is important in case of transmission of shafts running at high speed, where small lateral deflection would cause huge out of balance forces. The lateral rigidity is also important for maintaining proper bearing clearances & for correct gear teeth alignment. When the shaft is of variable cross section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam; i.e.,

$$\boxed{\frac{d^2y}{dx^2} = \frac{M}{EI}}$$

# Couplings

shaft couplings are used in machinery for following Purpose.

i) To Provide for the connection of shafts of units
ii) To reduce the transmissin of shock loads from one shaft to another
iii) To Provide for misalignment of the shafts (or) to introduce mechanical flexibility.
iv) To introduce Protection against overloads.

## Requirements of a good shaft coupling:-

i) It should be easy to connect (or) Disconnect
ii) It should transmit the full Power from one shaft to the other shaft without losses.
iii) It should hold the shafts in Perfect alignment.
iv) It should reduce the transmission of shock loads from one shaft to another shaft.
v) It should have no Projecting Parts.

## Types of shaft couplings:-

shaft couplings are divided into two main groups.

1) **Rigid coupling:-** It is used to ~~two~~ connect two shafts which are perfectly aligned.

Eg:-
a) Sleeve (or) muff coupling
b) clamp (or) slip muff (or) compression coupling
c) Flange coupling

2

2) Flexible coupling:- It is used to connect two shafts having both lateral & angular misalignment.

Ex:- a) Bushed pin type coupling
b) Universal coupling
c) oldham coupling

Sleeve (or) Muff coupling:-

It is the simplest type of rigid coupling, made of cast Iron. It consists of hollow cylinder whose inner diameter is the same as that of the shaft. It is fitted over the ~~keys~~ ends of two shafts by means of a gib head key The power is transmitted from one shaft to the other shaft by means of key & sleeve.

usual proportions of cast Iron sleeve couplings are as follows.

outer diameter of the sleeve, $D = 2d + 13$ mm

Length of the sleeve, $L = 3.5\,d$

where 'd' is the diameter of shaft.

i) Design of sleeve:-

the sleeve is designed by considering it as hollow shaft.

$T$ = Torque transmitted by the coupling

$\tau_c$ = Permissible shear stress for the material of the sleeve

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \times D^3 (1 - k^4)$$

From this expression, the induced shear stress in the sleeve $\left(\text{where } k = \frac{d}{D}\right)$ may be checked.

2) Design for key:-

The length of key is atleast equal to the length of the sleeve.

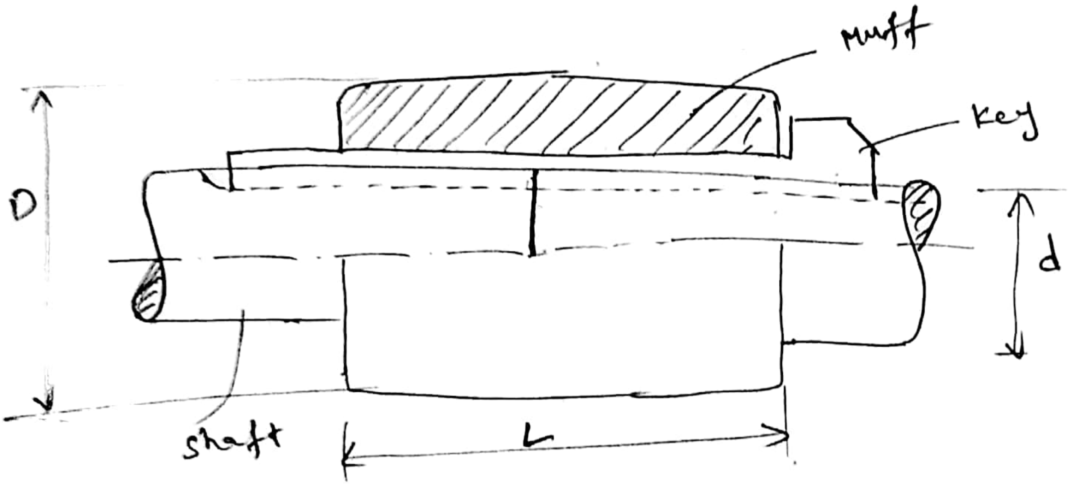
$$l = \frac{L}{2} = \frac{3.5d}{2}$$

After fixing the length of key in each shaft, the induced shearing & crushing stresses may be checked.

We know that Torque transmitted,

$$T = l \times w \times \tau \times \frac{d}{2} \quad \text{(Considering shearing of key)}$$

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \text{(Considering crushing of key)}$$




Sleeve (or) Muff coupling

Eg:- Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 rpm. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa & 80 MPa resply. The material for the muff is Cast Iron for which the allowable shear stress may be assumed as 15 MPa.

Ans:- $P = 40\,kW = 40 \times 10^3\,W$ ; $N = 350$ rpm.

Allowable shear stress $(\tau_s) = 40$ MPa $= 40\,N/mm^2$

Allowable crushing stress $(\sigma_{cs}) = 80$ MPa $= 80\,N/mm^2$

Muff allowable shear stress $(\tau_c) = 15$ MPa $= 15\,N/mm^2$

1) Design of shaft:-

Let $d$ = Diameter of shaft

Torque transmitted by shaft, key & Muff.

$$T = \frac{P \times 60}{2\pi N} = \frac{40 \times 10^3 \times 60}{2\pi \times 350} = 1100\ N\text{-}m$$

$$T = 1100 \times 10^3\ N\text{-}mm$$

$$\tau_s = \frac{16T}{\pi d^3}$$

$$\Rightarrow 40 = \frac{16 \times 1100 \times 10^3}{\pi d^3}$$

$$\Rightarrow d^3 = \frac{16 \times 1100 \times 10^3}{40 \times \pi}$$

$$d = 52\ mm \text{ (or) } 55\ mm$$

2) Design for sleeve :-

outer diameter of the muff

$$D = 2d + 13 = (2\times55) + 13 = 123 \text{ (say) } 125 \text{ mm}$$

Length of the muff

$$L = 3.5d = 3.5 \times 55 = 192.5 \text{ mm (or) } 195 \text{ mm}$$

Let us check induced shear stress in the muff.

Let '$\tau_c$' is induced shear stress.

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right)$$

$$\Rightarrow 1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125}\right]$$

$$\tau_c = 2.97 \text{ N/mm}^2$$

Induced shear stress in the muff is less than the permissible shear stress of 15 N/mm². ∴ Design is safe.

3) Design for key :-

width of key $w = \frac{d}{4}$

$$= \frac{55}{4} = 13.75 \cong 14 \text{ mm}$$

Crushing stress for the key material is twice the shearing stress. Therefore, square key may be used.

$$t = w = 14 \text{ mm}$$

Length of key in each shaft ; $l = \frac{L}{2} = \frac{195}{2} = 97.5 \text{ mm}$

Let us check induced shear & crushing stresses ⑨

$$T = l \times w \times \tau_s \times \frac{d}{2}$$

$$\Rightarrow 1100 \times 10^3 = 97.5 \times 14 \times \tau_s \times \frac{55}{2}$$

$$\tau_s = 29.304 \ N/mm^2$$

$$T = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2}$$

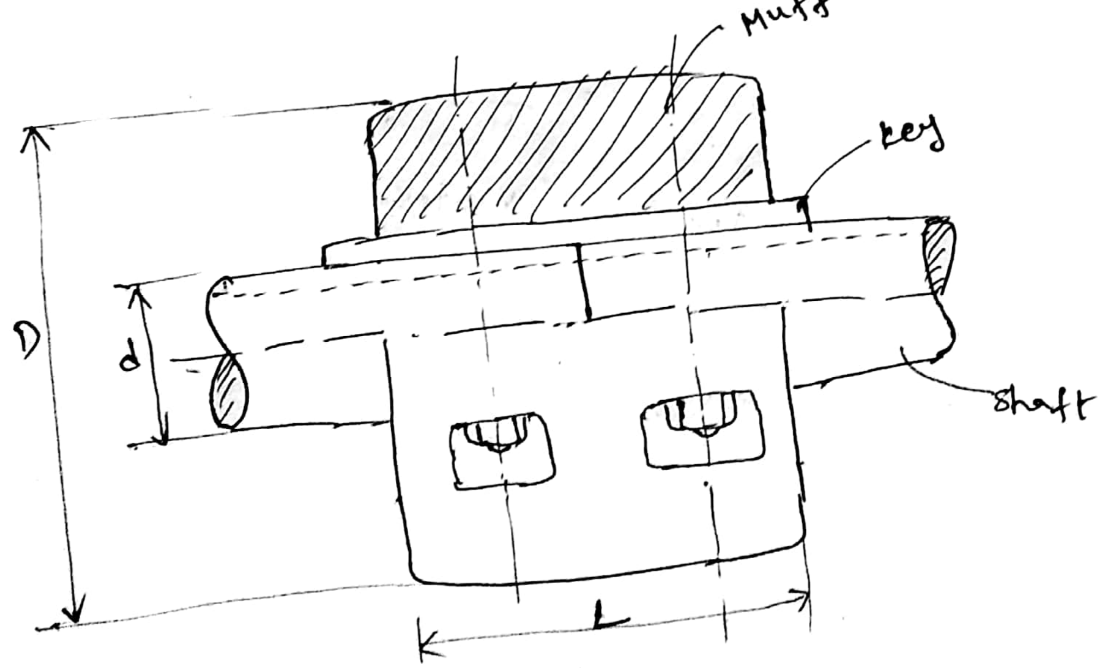
$$\Rightarrow 1100 \times 10^3 = 97.5 \times \frac{14}{2} \times \sigma_{cs} \times \frac{55}{2}$$

$$\sigma_{cs} = 58.60 \ N/mm^2$$

| | Permissible |
|---|---|
| $\tau_s = 29.304 \ N/mm^2$ | $\tau_s = 40 \ N/mm^2$ |
| $\sigma_{cs} = 58.6 \ N/mm^2$ | $\sigma_{cs} = 80 \ N/mm^2$ |

Induced shear stresses are less than Permissible shear stresses. ∴ Design is safe.

Clamp (or) Compression (or) split muff coupling:-



It is also known as split muff coupling. In this case the muff (or) sleeve is made into two halves and are bolted together. The shaft ends are made to a butt each other and single key is fitted directly into the keyways of both the shafts.

Let $d$ = diameter of shaft

Diameter of muff (or) sleeve $D = 2d + 13$ mm

Length of the muff (or) sleeve $= L = 3.5\,d$

1) Design of muff & key:-

The muff & key design is similar to muff coupling.

2) Design of clamping bolt:-

Let $T$ = Torque transmitted by the shaft

$d$ = Diameter of shaft

$d_b$ = Root (or) Effective diameter of bolt

$n$ = Number of bolts.

$\sigma_t$ = Permissible tensile stress for bolt material

$\mu$ = Coefficient of friction b/w muff & shaft

$L$ = Length of muff

Force exerted by each bolt $= \frac{\pi}{4}(d_b)^2\,\sigma_t$

Force exerted by the bolts on each side of shaft $= \frac{\pi}{4}(d_b)^2\,\sigma_t \times \frac{n}{2}$

Let 'p' be the pressure on the shaft & muff surface due to the force, then uniform pressure distribution over the surface,

$$P = \frac{\text{Force}}{\text{Projected Area}} = \frac{\frac{\pi}{4}(d_b^2)\,\sigma_t\left(\frac{n}{2}\right)}{\frac{1}{2}(L\times d)}$$

∴ Frictional force between each shaft & muff,

$$F = \mu \times \text{Pressure} \times \text{Area}$$

$$= \mu \times \frac{\frac{\pi}{4}(d_b)^2\,\sigma_t\left(\frac{n}{2}\right)}{\frac{1}{2}(L\times d)} \times \frac{1}{2}\pi d \times L$$

$$= \mu \times \frac{\pi^2}{8}(d_b)^2\,\sigma_t \times n$$

∴ Torque transmitted by the coupling

$$T = F \times \frac{d}{2} = \mu \times \frac{\pi^2}{8}(d_b)^2\sigma_t \times n \times \frac{d}{2}$$

$$\boxed{T = \frac{\pi^2}{16} \times \mu (d_b)^2 \sigma_t \times n \times d}$$

Eg:- Design a clamp coupling to transmit 30 kW at 100 rpm. The allowable shear stress for the shaft and key is 40 MPa and the number of bolts connecting the two halves are six. The permissible tensile stress for the bolts is 70 MPa. The coefficient of friction is 0.3.

A) $P = 30\,kW = 30\times10^3\,W$ ; $N = 100$ rpm ; $n = 6$

Allowable shear stress for shaft & key ; $\tau = 40\,MPa = 40\,N/mm^2$

Permissible Tensile stress for bolts ; $\sigma_t = 70\,N/mm^2$

$\mu = 0.3$

1) Design of shaft

$$T = \frac{60 \times P}{2\pi N} = \frac{\cancel{\phantom{x}} 60 \times 10^3 \times 60}{2\pi \times 100} = 2865 \text{ N-m}$$

$$T = 2865 \times 10^3 \text{ N-mm}$$

From Torsion equation; $T = \frac{\pi}{16} \times \tau \times d^3$

$$\Rightarrow 2865 \times 10^3 = \frac{\pi}{16} \times 40 \times d^3$$

$$d = 71.4 \text{ (or) } 75 \text{ mm}$$

2) Design of muff

$$D = 2d + 13 = (2 \times 75) + 13 = 163 \text{ (or) } 165 \text{ mm}$$

$$L = 3.5d = 3.5 \times 75 = 262.5 \text{ mm}$$

3) Design of key

$$W = \frac{d}{4} = \frac{75}{4} \approx 18.75 \text{ (say) } 19 \text{ mm}$$

$$t = \frac{d}{6} = \frac{75}{6} \approx 12.5 \text{ (say) } 13 \text{ mm}$$

4) Design for bolts

$$T = \frac{\pi^2}{16} \mu (d_b)^2 \sigma_t (n \times d)$$

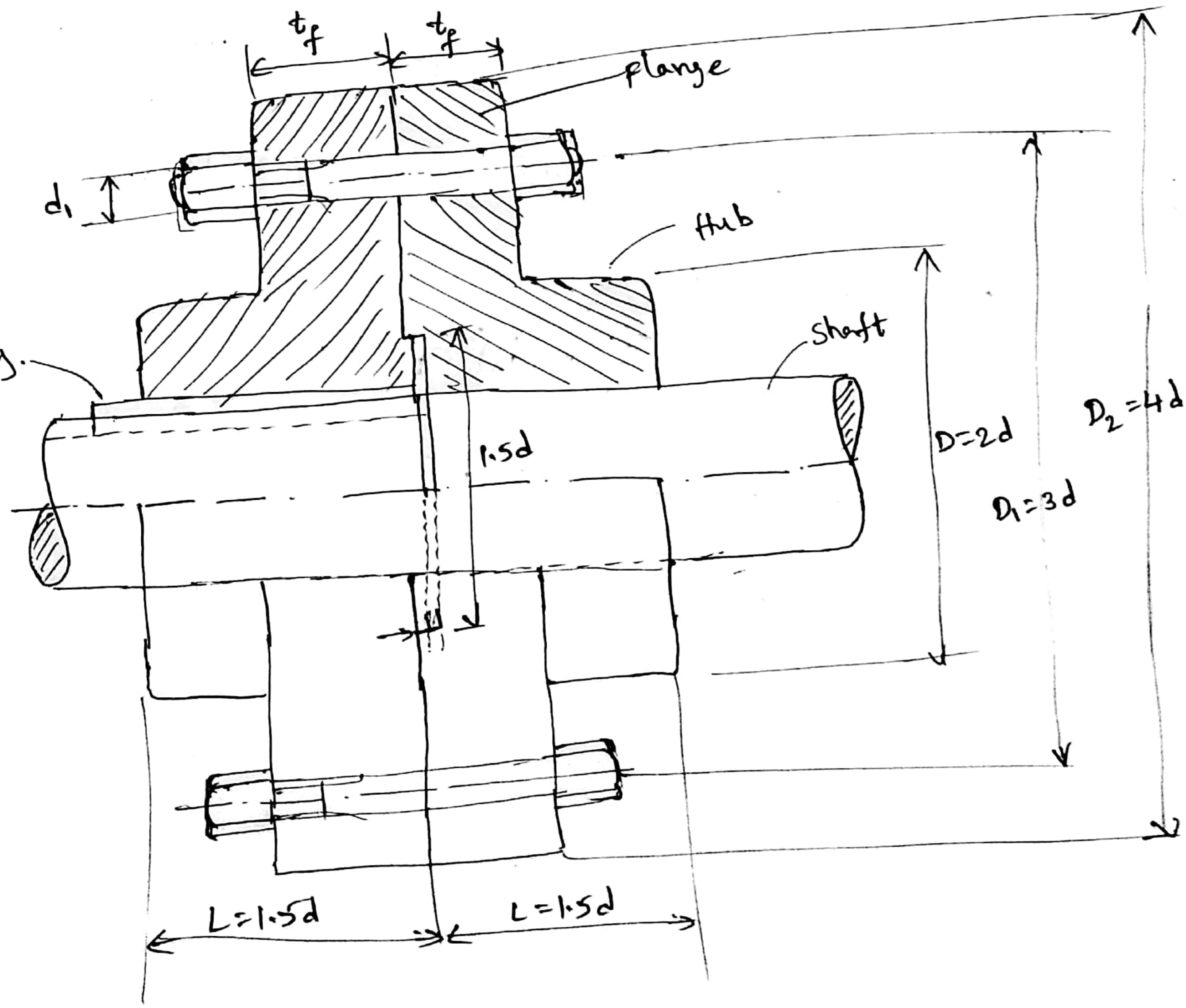
$$2865 \times 10^3 = \frac{\pi^2}{16} \times \mu \times (d_b)^2 \times 70 \times 6 \times 75$$

$$d_b = 22.2 \text{ mm} \cong 23 \text{ mm}$$

## Flange coupling :-

A flange coupling usually applies to a coupling having two seperate cast Iron flanges. Each flange is mounted on the shaft end and keyed to it. The faces are turned up at right angles to the axis of the shaft. one of the flange has Projected Portion and the other flange has a corresponding recess. The two flanges are coupled by means of bolts & ~~and~~ nuts.

i) Unprotected type flange coupling :-



In unprotected type flange coupling, each shaft is keyed to the boss of a flange with a counter sunk key & the flanges are coupled together by means of bolts. Generally, three, four (or) six bolts are used.

If 'd' is the diameter of the shaft (or) Inner dia. of hub, then outside diameter of Hub, $D = 2d$

Length of Hub, $L = 1.5d$

Pitch circle diameter of flange, $D_1 = 3d$

outside diameter of flange,

$D_2 = 4d$

Thickness of flange, $t_f = 0.5d$

Number of bolts $= 3$, for d up to 40 mm

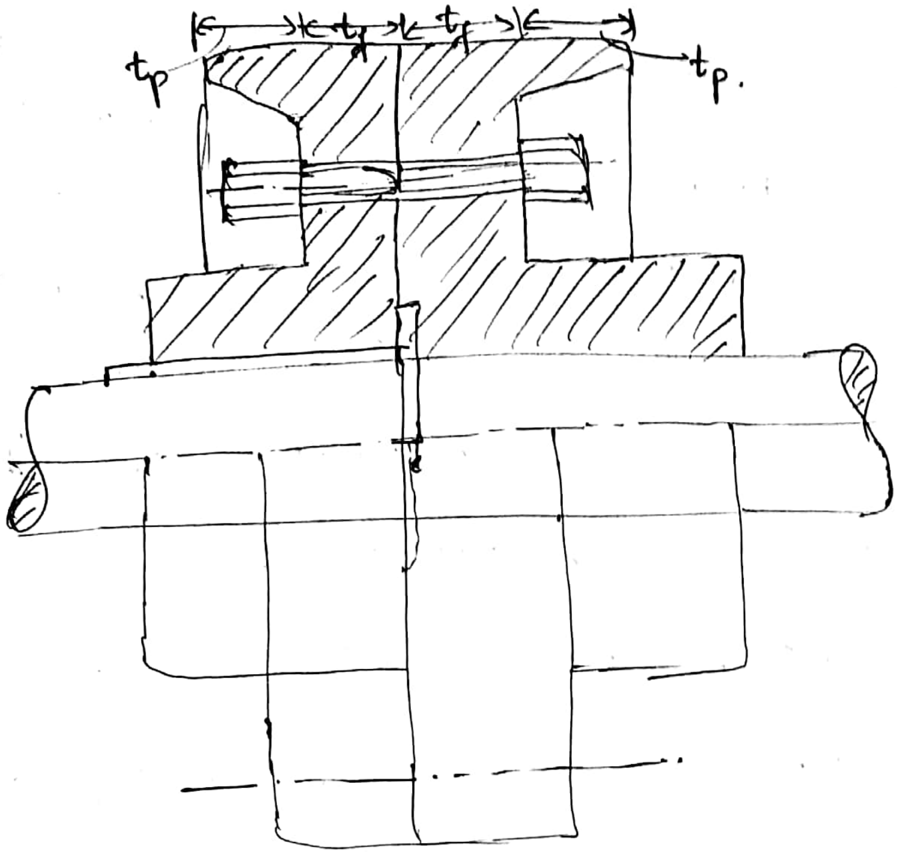
$= 4$, for 'd' up to 100 mm

$= 6$, for 'd' up to 180 mm

2) Protected type flange coupling:-

In this, the protruding bolts and nuts are protected by flanges on the two halves of the coupling, in order to avoid danger to the workman.

The thickness of the protective circumferential flange ($t_p$) is taken as $0.25d$. Remaining proportions are same as unprotected type flange coupling.




Protective type flange coupling

3) Marine type flange coupling:-

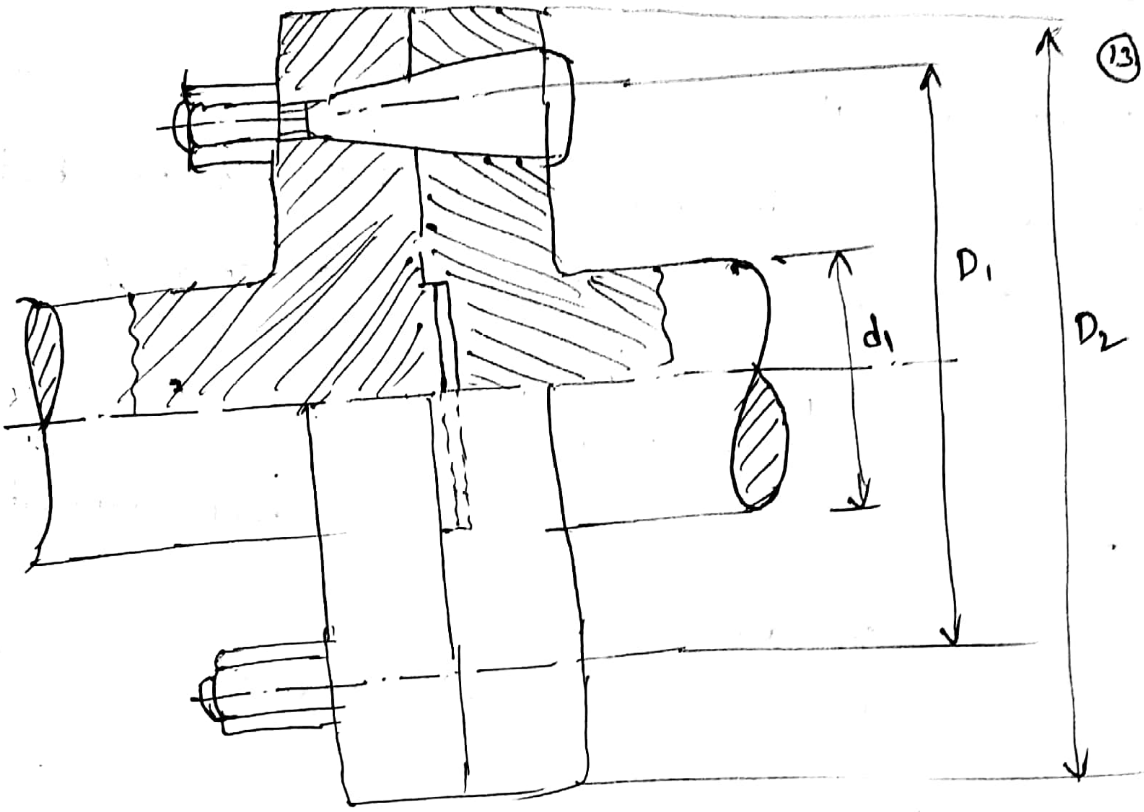
In this coupling, the flanges are forged integral with the shafts. The flanges are held together by means of tapered headless bolts, numbering from four to twelve depending ~~of~~ upon the diameter of shaft.

Thickness of flange $= d/3$

Taper of bolt $=$ 1 in 20 to 1 in 40

Pitch circle diameter of bolts $= D_1 = 1.6\,d$

outside diameter of flange $= D_2 = 2.2\,d$




Marine type flange coupling

## Design of flange coupling :-

Let $d$ = Diameter of shaft (or) Inner dia. of hub.

$D$ = Outer diameter of Hub

$d_1$ = Nominal (or) outside dia. of bolt

$D_1$ = Diameter of bolt circle

$n$ = no. of bolts

$t_f$ = Thickness of flange

$\tau_s$, $\tau_b$ and $\tau_k$ = Allowable shear stress for shaft, bolt and key material resply.

$\tau_c$ = Allowable shear stress for the flange material.

$\sigma_{cb}$ & $\sigma_{ck}$ = Allowable crushing stress for bolt & key material respectively.

1) **Design for Hub:-**

The hub is designed by considering it as hollow shaft, transmitting the same torque (T) as that of solid shaft

$$\therefore T = \frac{\pi}{16} \times \tau_c \left( \frac{D^4 - d^4}{D} \right)$$

By using above equation we can check induced shear stress in the hub.

Length of the hub (L) = 1.5 d

2) **Design for key:-**

The material of key is usually the same as that of shaft. The length of key is taken equal to the length of the Hub.

3) **Design for flange:-**

The flange at the junction of the hub is under shear while transmitting the torque. Torque transmitted =

T = shear force × Radius of Hub

= shear stress of flanges × circumference of Hub × Thickness of Hub × Radius of Hub

$$= \tau_c \times \pi D \times t_f \times \frac{D}{2}$$

$$\boxed{T = \frac{\pi D^2}{2} \times \tau_c \times t_f}$$

usually $t_f = \frac{d}{2}$

$\therefore$ By using above relation, we can check induced shearing stress in the flange.

## 4) Design for bolts:-

The bolts are subjected to shear stress due to the torque transmitted. The no. of bolts (n) depends upon the diameter of shaft & pitch circle diameter of bolts ($D_1$).

Load on each bolt $= \tau_b \times \frac{\pi}{4} d_1^2$

$\therefore$ Total load on all bolts $= \frac{\pi}{4} d_1^2 \times \tau_b \times n$

Torque Transmitted, $T = \frac{\pi}{4}(d_1^2) \times \tau_b \times n \times \left(\frac{D_1}{2}\right)$.

$\therefore$ Diameter of bolt ($d_1$) may be obtained from above equation.

Now diameter of bolt may be checked in crushing.

Area resisting crushing of all bolts $= n \times d_1 \times t_f$

Crushing strength of all the bolts $= (n \times d_1 \times t_f)\, \sigma_{cb}$

$\therefore$ Torque $(T) = (n \times d_1 \times t_f \times \sigma_{cb}) \frac{D_1}{2}$

From above equation, the induced crushing stress in the bolts may be checked.

Eg:- Design a Cast Iron Protective type flange Coupling to transmit 15 KW at 900 rpm. from an electric motor to a compressor. The service factor may be assumed as 1.35. The following permissible stresses may be used.

shear stress for shaft, bolt & key material = 40 MPa

Crushing stress for bolt & key = 80 MPa

shear stress for cast Iron = 8 MPa

Draw a neat sketch of the coupling. (Consider number of bolts = 3)

Ans:- $P = 15\,KW = 15\times10^3\,W$ ; $N = 900$ rpm

service factor = 1.35 ;

$\tau_s = \tau_b = \tau_k = 40\,MPa = 40\,N/mm^2$

$\sigma_{cb} = \sigma_{ck} = 80\,MPa = 80\,N/mm^2$

$\tau_c = 8\,MPa = 8\,N/mm^2$ ; $n = 3$.

i) Design of hub:

Torque transmitted by shaft $(T) = \frac{P\times60}{2\pi N} = \frac{15\times10^3\times60}{2\pi\times900}$

$T = 159.13$ N-m.

Since service factor is : 1.35 ;

∴ The maximum torque transmitted by the shaft

$T_{max} = 1.35\times159.13 = 215\,N\text{-}m = 215\times10^3\,N\text{-}mm$

Torque transmitted by the shaft $(T) = \frac{\pi}{16}\times\tau_s\times d^3$

$\Rightarrow 215\times10^3 = \frac{\pi}{16}\times40\times d^3$

$d = 30.1$ (Say) 35 mm

outer diameter of the hub ; $D = 2d = 2\times35$

$D = 70$ mm

Length of hub $(L) = 1.5d = 1.5 \times 35 = 52.5$ mm (17)

Let us check induced shear stress for hub material.

Maximum Torque Transmitted by ~~key~~ hub $(T)_{max}$

$$T_{max} = \frac{\pi}{16} \times \tau_c \left( \frac{D^4 - d^4}{D} \right)$$

$$\Rightarrow 215 \times 10^3 = \frac{\pi}{16} \times \tau_c \left( \frac{(70)^4 - (35)^4}{70} \right)$$

$$\tau_c = 3.4 \text{ N/mm}^2 = 3.4 \text{ MPa}$$

$\therefore \tau_c <$ Permissible shear stress $= 8$ MPa.

$\therefore$ Design of hub is safe.

2) <u>Design for key:-</u>

Since the crushing stress for the key material is twice its shear stress (i.e., $\sigma_{ck} = 2\tau_k$). $\therefore$ Square key is used.

$$\text{Width of key } (w) = \frac{d}{4} = \frac{35}{4} = 8.75 \text{ mm} \cong 9 \text{ mm}$$

For square key; $w = t = 9$ mm

Length of key = Length of hub ~~to~~

$$l = L = 52.5 \text{ mm}$$

Now check the induced stresses in the key considering it in shearing & crushing.

Key under shearing; Maximum Torque Transmitted

$$T_{max} = l \times w \times \tau_k \times \frac{d}{2}$$

$$215 \times 10^3 = 52.5 \times 9 \times \tau_k \times \frac{35}{2}$$

$$\tau_k = 26 \text{ N/mm}^2 < 40 \text{ N/mm}^2$$

Key is in crushing, Maximum Torque transmitted ($T_{max}$)

$$T_{max} = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2}$$

$$\Rightarrow 215 \times 1000 = 52.5 \times \frac{9}{2} \times \sigma_{ck} \times \frac{35}{2}$$

$$\sigma_{ck} = 52 \; N/mm^2 < 80 \; N/mm^2$$

$\therefore$ Design is safe.

3) Design for flange:-

Thickness of flange ($t_f$) = $0.5d = 0.5 \times 35 = 17.5$ mm

Let us check the induced shear stress in the flange.

$$\text{Maximum Torque } (T_{max}) = \frac{\pi D^2}{2} \times \tau_c \times t_f$$

$$\Rightarrow 215 \times 10^3 = \frac{\pi \times (70)^2}{2} \times \tau_c \times 17.5$$

$$\tau_c = 1.6 \; N/mm^2 = 1.6 \; MPa < 8 \; MPa.$$

Design of flange is safe.

4) Design for bolts:-

Let $d_1$ = Nominal dia. of bolts

$n = 3$.

Pitch circle diameter of bolts $D_1 = 3d = 3 \times 35 = 105$ mm

Bolts are subjected to shear stress due to torque transmitted.

$$T_{max} = \frac{\pi}{4} (d_1)^2 \times \tau_b \times n \times \frac{D_1}{2}$$

$$\Rightarrow 215 \times 10^3 = \frac{\pi}{4} (d_1)^2 \times 40 \times 3 \times \frac{105}{2}$$

~~$d_1 = 43.43$ (or)~~ $d_1 = 6.6$ mm

Outer diameter of the flange, $D_2 = 4d = 140$ mm

Thickness of Protective Circumference flange $t_p = 0.25d$

$t_p = 8.75$ (or) 10 mm

## Flexible coupling:-

Flexible coupling is used to join the butting ends of shafts when they are not in exact alignment.

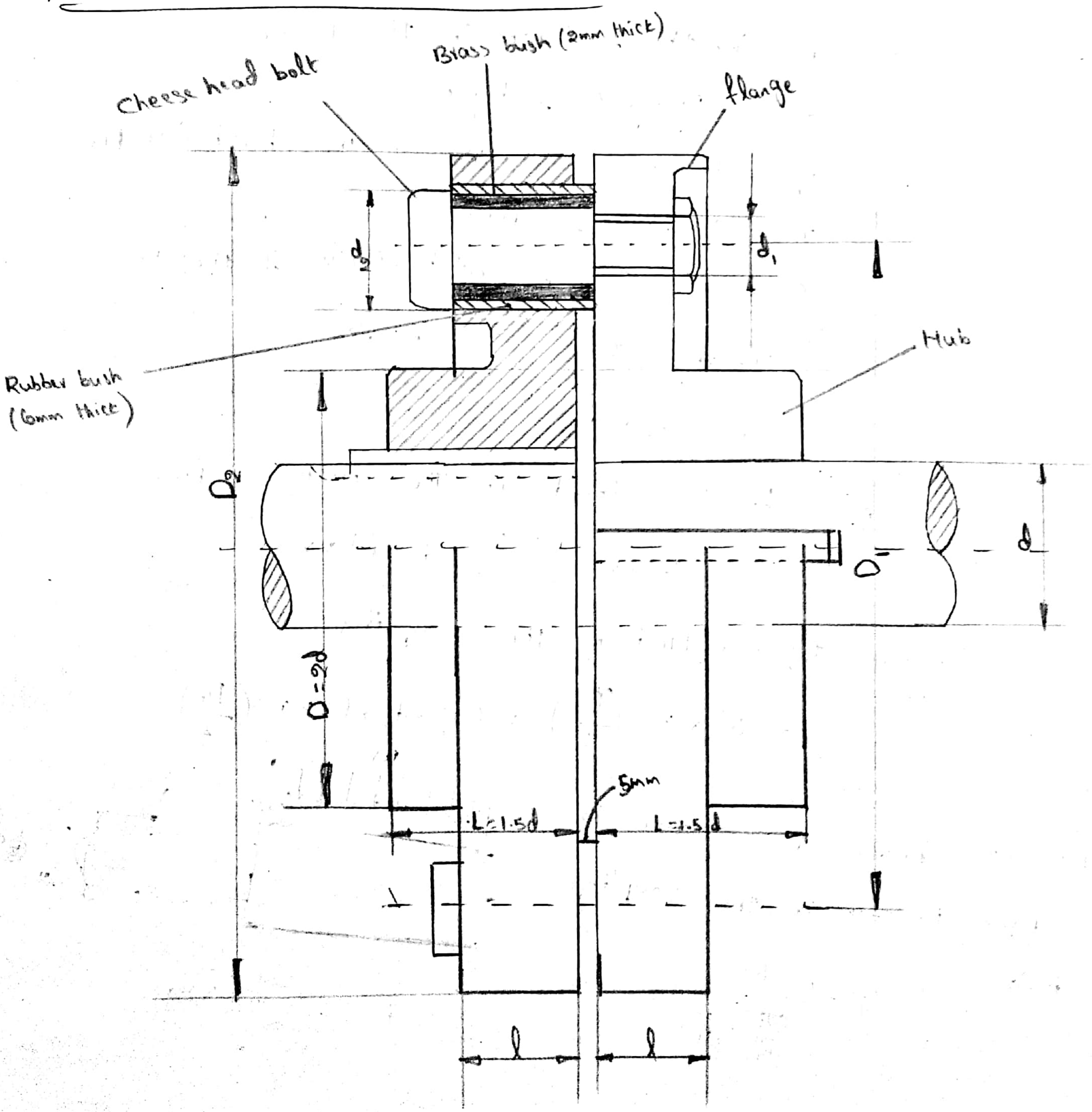
Different types of flexible coupling are:

(i) Bushed pin flexible coupling

(ii) Oldham's coupling

iii) Universal coupling

## Bushed pin flexible coupling:-



A bushed Pin flexible Coupling is a modification of the rigid type of flange Coupling. The coupling bolts are known as pins. The rubber (or) Leather bushes are used over the pins. The two halves of couplings are dissimilar in construction. A clearance of 5 mm is left between the face of the two halves of Coupling. There is no rigid connection b/w them and drive takes place through medium of the Compressible rubber (or) Leather bush.

Let $l$ = Length of bush in the flange

$d_2$ = Diameter of bush

$P_b$ = Bearing pressure on the bush (or) Pin

$n$ = Number of Pins

$D_1$ = Diameter of Pitch circle of the Pins

Bearing load acting on each pin,

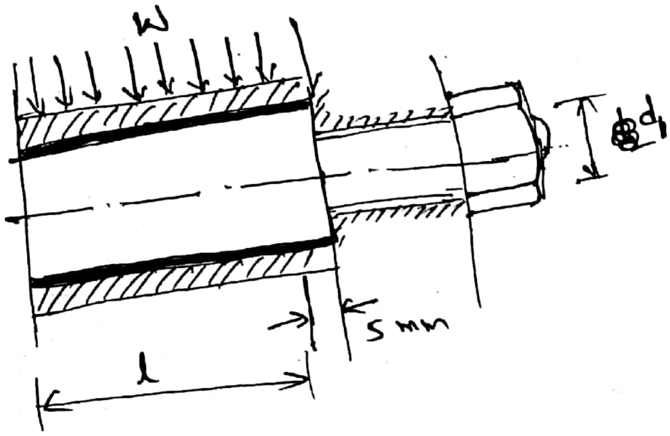
$$W = P_b \times d_2 \times l$$

Total bearing load on the bush (or) Pins

$$= W \times n = (P_b \times d_2 \times l) \times n$$

Torque transmitted by the coupling

$$T = W \times n \times \left(\frac{D_1}{2}\right) = (P_b \times d_2 \times l) \times n \times \left(\frac{D_1}{2}\right)$$




Direct shear stress due to Pure torsion in the coupling halves,

$$\tau = \frac{W}{\frac{\pi}{4}(d_1^2)}$$

Since the pin and the rubber (or) Leather bush is not rigidly held in the left hand flange, therefore the tangential load (W) at the enlarged portion will exert a bending action on the pin. The bush portion of the pin acts as a cantilever beam of length $l$. Assuming a uniform distribution of the load W along the bush, the maximum bending moment on the pin,

$$M = W\left(\frac{l}{2} + 5\right)$$

$$\text{Bending stress, } \sigma = \frac{M}{Z} = \frac{W\left(\frac{l}{2}+5\right)}{\frac{\pi}{32}(d_1^3)}$$

Since the pin is subjected to bending & shear stress, therefore design must be checked either of maximum principal stress (or) Maximum shear stress.

$$\text{Maximum principal stress} = \frac{1}{2}\left(\sigma + \sqrt{\sigma^2 + 4\tau^2}\right)$$

$$\text{Maximum shear stress on pin} = \frac{1}{2}\left(\sqrt{\sigma^2 + 4\tau^2}\right)$$

The maximum principal stress varies from 28 to 42 MPa.

**Eg:-** Design a bushed-pin type of flexible coupling to connect a pump shaft to a motor shaft transmitting 32 kW at 960 rpm. The overall torque is 20 percent more than mean torque. The material properties are as follows:

1) The allowable shear & crushing stress for shaft & key material is 40 MPa and 80 MPa resply.

b) Allowable shear stress for cast Iron is: 15 MPa. (2)

c) The allowable bearing pressure for rubber bush is: 0.8 N/mm²

d) The material of the pin is same as that of shaft and key.

Draw neat sketch of coupling.

Ans:- $P = 32\,kW = 32 \times 1000\,W$ ; $N = 960$ rpm

$T_{max} = 1.2\,T_{mean}$

$\tau_s = \tau_k = 40\,MPa = 40\,N/mm^2$

$\sigma_{cs} = \sigma_{ck} = 80\,MPa = 80\,N/mm^2$

$\tau_c = 15\,MPa = 15\,N/mm^2$ ; $P_b = 0.8\,N/mm^2$

1) Design for Pins and rubber bush :-

First find out the diameter of shaft 'd'.

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{32 \times 10^3 \times 60}{2\pi \times 960} = 318.3\,N\text{-}m$$

$$T_{max} = 1.2\,T_{mean} = 1.2 \times 318.3 = 382\,N\text{-}m$$

$$T_{max} = 382 \times 10^3\,N\text{-}mm$$

Maximum Torque transmitted by the shaft ($T_{max}$)

$$T_{max} = \frac{\pi}{16} \times \tau_s \times d^3$$

$$\Rightarrow 382 \times 10^3 = \frac{\pi}{16} \times 40 \times d^3$$

$d = 36.5$ mm (or) 40 mm

In rigid type flange coupling No. of bolts used for 40 mm dia shaft are '3'.

In the flexible coupling, we shall use the no. of pins as $n = 6$

Diameter of Pins, $\boxed{d_1 = \frac{0.5d}{\sqrt{n}}}$

$$d_1 = \frac{0.5 \times 40}{\sqrt{6}} = 8.2 \text{ mm} \cong 9 \text{ mm}$$

In order to allow for the bending stress induced in the pin, the diameter of Pin ($d_1$) may be taken as

$\boxed{d_1 = 18 \text{ mm}}$

$\therefore$ overall diameter of rubber bush, $\boxed{d_2 = d_1 + 16}$

$$\Rightarrow d_2 = 18 + 16 = 34 \text{ mm}$$

Diameter of Pitch circle of Pins,

$$\boxed{D_1 = 2d + d_2 + 12}$$

$$\Rightarrow D_1 = 2(40) + 34 + 12 = 126 \text{ mm}$$

Let $l$ = length of bush in the flange,

Bearing load acting on each Pin

$$\boxed{W = P_b \times l \times d_2}$$

$$\Rightarrow W = 0.8 \times l \times 34$$

$$\Rightarrow W = 27.2 l$$

Maximum Torque transmitted by coupling ($T_{max}$),

$$\boxed{T_{max} = W \times n \times \frac{D_1}{2}}$$

$$\Rightarrow 382 \times 10^3 = 27.2 l \times 6 \times \frac{126}{2}$$

$$l = 37.15 \text{ mm} \cong 38 \text{ mm}$$

$W = 8 \times 27.2\, l$

$\Rightarrow W = 27.2 \times 38 = 1033.6$ N

$\therefore W = 1034$ N

$\therefore$ Direct shear stress due to Pure Torsion,

$$\tau = \frac{W}{\frac{\pi}{4} d_1^2} = \frac{1034}{\frac{\pi}{4} \times (18)^2}$$

$$\tau = 4.06 \text{ N/mm}^2$$

The maximum bending moment on the pin,

$$M = W\left(\frac{l}{2} + 5\right) = 1034\left(\frac{38}{2} + 5\right)$$

$$M = 24816 \text{ N-mm}$$

Bending stress $(\sigma_b) = \frac{32M}{\pi d_1^3} = \frac{32 \times 24816}{\pi \times (18)^3}$

$$\Rightarrow \sigma_b = \cancel{\text{1404.2.9 N/mm}^2}\ 43.34 \text{ N/mm}^2$$

Maximum Principal stress

$$= \frac{1}{2}\left(\sigma_b + \sqrt{\sigma_b^2 + 4\tau^2}\right)$$

$$= \frac{1}{2}\left(43.34 + \sqrt{(43.34)^2 + 4(4.06)^2}\right)$$

$$= \cancel{\text{22.2688 N/mm}^2}\ 43 \text{ N/mm}^2$$

Maximum shear stress

$$= \frac{1}{2}\left(\sqrt{\sigma_b^2 + 4\tau^2}\right) = \frac{1}{2}\sqrt{(43.34)^2 + 4(4.06)^2}$$

$$= 5.226 \text{ N/mm}^2$$

$\therefore$ Maximum Principal stress & maximum shear stress are within limits. $\therefore$ Design is safe.

2) Design for Hub:-

Outer diameter of Hub ; $\boxed{D = 2d}$

$\Rightarrow D = 2(40) = 80$ mm

Length of the Hub ; $L = 1.5d = 1.5 \times 40$

$\Rightarrow L = 60$ mm

Now, checking for Induced shear ~~& crush~~ stresses for Hub.

$$T_{max} = \frac{\pi}{16} \times \tau_i \left( \frac{D^4 - d^4}{D} \right)$$

$$\Rightarrow 382 \times 10^3 = \frac{\pi}{16} \times \tau_i \left( \frac{(80)^4 - (40)^4}{80} \right)$$

$$\Rightarrow \tau_i = 4.05 \ N/mm^2 < 15 \ N/mm^2$$

$\tau_i < \tau_{permissible}$

∴ Design of Hub is safe.

3) Design for key:-

For key $\boxed{\sigma_{ck} = 2\tau_k}$

square key can be used.

$$w = t = \frac{d}{4} = \frac{40}{4} = 10 \text{mm}$$

∴ width of key (w) = 10 mm

Thickness of key (t) = 10 mm

Length of the key (L) = 1.5d = 1.5 × 40 = 60 mm

Let us check induced shear stresses in key.

Consider key is in shear,

$$T_{max} = l \times w \times \tau_{ki} \times \frac{d}{2}$$

$$\Rightarrow 382 \times 10^3 = 60 \times 10 \times \tau_{ki} \times \frac{40}{2}$$

$$\Rightarrow \tau_{ki} = 31.83 < 40 \ N/mm^2$$

$$\tau_{ki} < \tau_{permissible}$$

Consider key is in crushing,

$$T_{max} = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2}$$

$$\Rightarrow 382 \times 10^3 = 60 \times \frac{10}{2} \times \sigma_{ck} \times \left(\frac{40}{2}\right)$$

$$\Rightarrow \sigma_{ck} = 63.66 \ N/mm^2 < 80 \ N/mm^2$$

Design of key is safe.

4) <u>Design for flange:-</u>

The thickness of flange; $\boxed{t_f = 0.5d}$

$$\Rightarrow t_f = 0.5 \times 40 = 20mm$$

Let us check induced shear stresses in flange.

$$T_{max} = \frac{\pi D^2}{2} \times \tau_c \times t_f$$

$$\Rightarrow 382 \times 10^3 = \frac{\pi \times (80)^2}{2} \times \tau_c \times 20$$

$$\tau_c = 1.9 \ N/mm^2 < 15 \ N/mm^2$$

∴ Design for flange is safe.

# Springs

* A spring is defined as an elastic machine element, which deflects under the action of the load and return to its original shape when the load is removed.

Applications & functions of springs are:

(i) Springs are used to absorb shocks and vibrations.
Eg:- Vehicle suspension springs, Railway buffer springs etc.

(ii) Springs are used to store energy.
Eg:- Springs used in clocks, toys, circuit breakers etc.

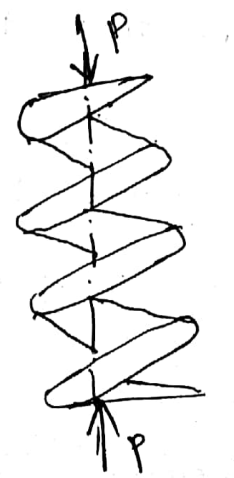
(iii) Spring are used to measure force
Eg:- Springs used in weighing balances & scales.

(iv) Springs are used to apply force and control motion.
Eg:- Cam & follower, Engine valve mechanism, clutches etc.

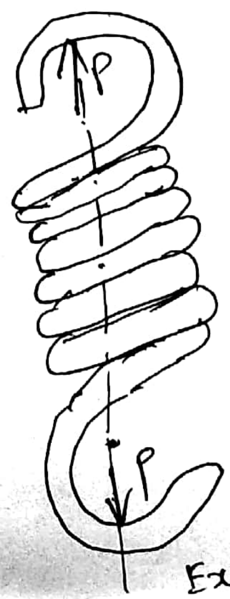
## Types of Springs:-

### Helical springs:-

Helical spring is made from a wire, usually of circular cross-section which is bent in the form of a helix.




Compression spring




Extension spring

*) Helical compression spring, the external force tends to shorten the spring.

*) Helical Extension spring, the external force tends to lengthen the spring.

In both the cases, the external force acts along the axis of the spring and induces shear stresses in the spring wire.

Helical springs are also classified as closely-coiled helical spring and open coiled helical spring.

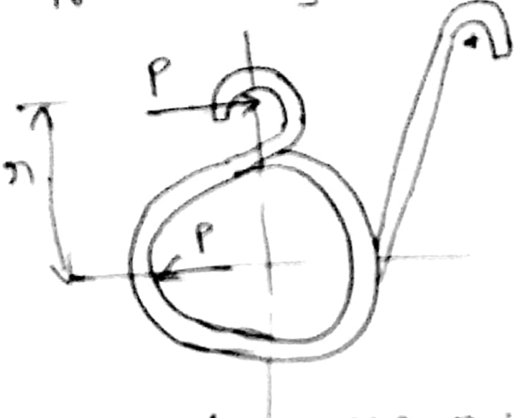
(i) A helical spring is said to be closely coiled spring, when the spring wire is coiled so close that the plane containing each coil is almost right angles to the axis of the helix. The helix is very small, it is less than 10°.

(ii) A helical spring is said to be open-coiled spring, when the spring wire is coiled in such a way, that there is large gap b/w adjacent coils. The helix angle is large, it is usually more than 10°.

<u>Helical Torsion Spring:-</u>

The construction of this spring is similar to that of compression (or) Extension spring, except that the ends are formed in such away that the spring is loaded by a torque about the axis of the coils. It is used to transmit torque to a particular component in the machine.

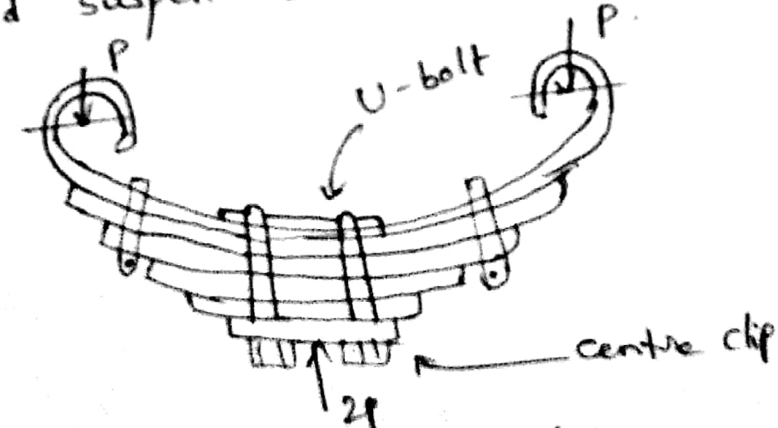
Helical torsion spring is used in door-hinges, brush holders, door locks etc., The bending moment induces bending stresses in the wire. Helical Torsion spring is not subjected to torsional shear stresses, it is subjected to bending stresses.




Helical Torsion spring.

Multi leaf springs:-

A multi leaf (or laminated spring consists of a Series of flat plates, usually of semi-elliptical shape. The flat plates called leaves have varying lengths. The leaves are held together by means of U-bolts & centre clip. The longest leaf is called master leaf, is bent at the two ends to form spring eyes. Multi leaf springs are widely used in automobile & rail road suspensions.




Semi-elliptic Leaf spring

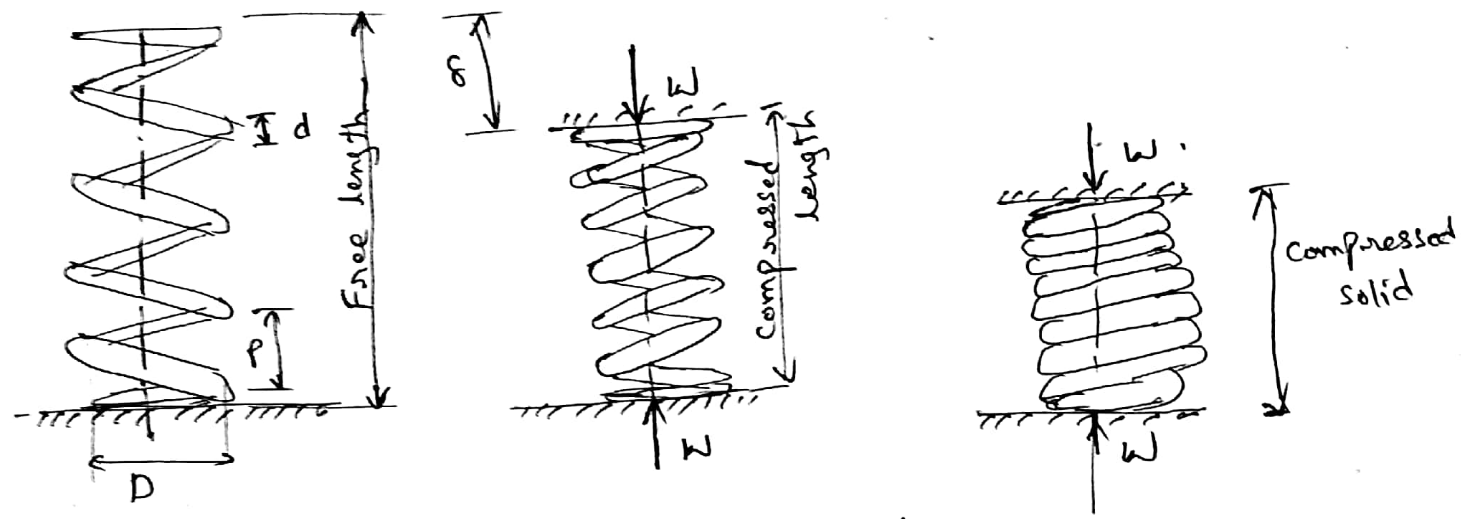
Terminology used in compression springs:-

Let $d$ = wire diameter of spring

$D_i$ = Inside diameter of spring coil

$D_o$ = outside diameter of spring coil

$D$ = mean coil diameter $= \frac{D_i + D_o}{2}$




compression spring nomenclature

1) Solid length:- when the compression spring is compressed until the coil come in contact with each other, the spring length is called solid length.

Let $n'$ = Total no. of coils.

Solid length of spring, $L_s = n'.d$

2) Free length:- It is the length of the spring in the free (or) unloaded condition.

Free length of the spring,

$L_f$ = Solid length + Maximum Compression + clearance between adjacent coils.

$= n'd + \delta_{max} + 0.15\,\delta_{max}$

The following relation may also be used to find the free length of the spring.

$$L_F = n'd + \delta_{max} + (n'-1) \times 1 \text{ mm.}$$

where '1 mm' is the clearence b/w two adjacent coils.

3) Spring Index :- It is the ratio of mean diameter of the coil to diameter of the wire.

$$C = \frac{D}{d}.$$

spring Index indicates the relative sharpness of the curvature of the Coil. It is usualy varies from 4 to 12 and for close tolerance springs, cyclic loading it is varies from 6 to 9.

4) Spring rate :- It is defined as the load required per unit deflection of the spring.

$$K = \frac{W}{\delta}$$ where $k$ = spring rate
$W$ = load
$\delta$ = Deflection of spring.

5) Pitch :- The Pitch of the Coil is defined as the axial distance b/w adjacent Coils in uncompressed state. It is represented by 'p'.

$$'p' = \frac{\text{Free length}}{n'-1}$$

Pitch of the coil may also be obtained by

$$p = \frac{L_F - L_S}{n'} + d.$$

~~Act~~

## 6) Active and Inactive coils:-

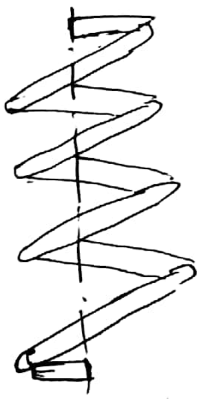
*) Active coils are the coils in the spring which contribute to spring action, support the external force and deflects under the action of force.

*) A portion of the end coils, which is in contact with the seat, does not contribute to spring action and are called inactive coils.

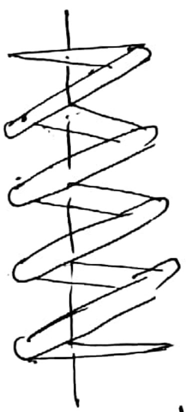
The number of inactive coils = Total coils − no. of active coils

$$= n' - n$$

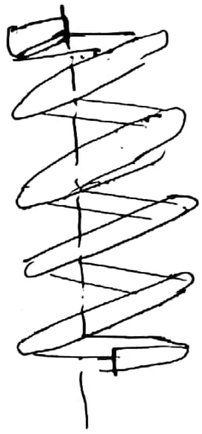
## Styles of End:-



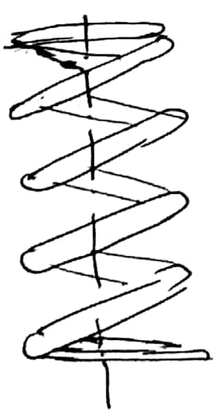
a) Plain ends



b) plain and ground ends



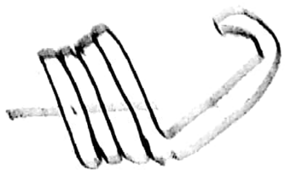
c) Square ends.



d) square and ground ends

| Type of Ends | no. of active turns |
|---|---|
| plain Ends | $n'$ |
| plain Ends (ground) | $(n' - \frac{1}{2})$ |
| square Ends | $(n' - 2)$ |
| square ends (ground) | $(n' - 2)$ |

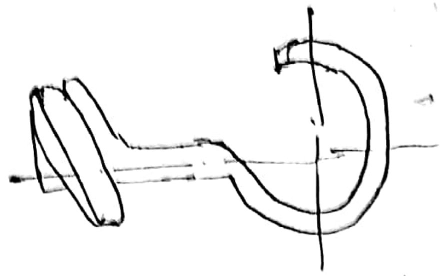
End styles of Helical compression spring



a) V- Hook

b) Rectangular hook

c) Full hook



d) Extended hook

End styles of Helical Extension springs.

*) ~~The~~ For helical extension ends, all coil are active coils.

Stresses in Helical springs of circular wire:-

A helical spring made from a circular cross-section, 'D and 'd' are mean coil diameter and wire diameter. The number of active coils in the spring is: "n".

G = Modulus of rigidity of spring material

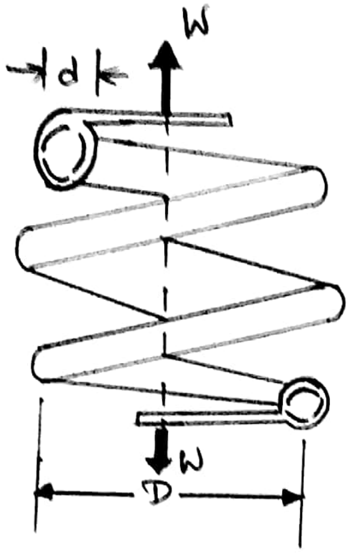
W = axial load on the spring

$C$ = spring Index = $\frac{D}{d}$

P = Pitch of the coils

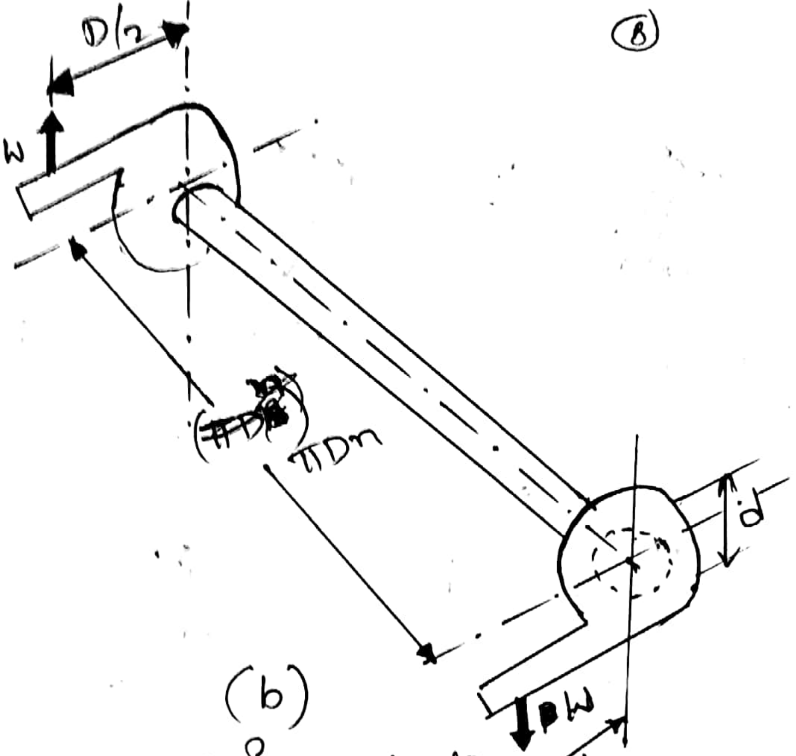
$\tau$ = Maximum shear stress induced in the wire

$\delta$ = Deflection of spring, as a result of load W.




(a)

Helical Spring




(b)

Helical Spring-unbent

When the wire of the helical spring is uncoiled and straightened it takes the shape of the bar as shown in figure.

The dimensions of the equivalent ~~the~~ bar as follows.

(i) The diameter of the bar is equal to wire diameter of the spring (d).

ii) The bar is fitted with a bracket of length $\frac{D}{2}$.

iii) Length of the bar = $\pi D n$

The force 'W' acting at the end of the bracket induces torsional shear stress in the bar.

$\therefore$ Twisting moment (T) = $W \times \frac{D}{2}$

Torsional shear stresses in the bar

$$\tau_1 = \frac{16\,T}{\pi d^3} = \frac{16 \times W \times \frac{D}{2}}{\pi d^3} = \frac{8WD}{\pi d^3}$$

When the equivalent bar is bent in the form of helical coil, there are additional stresses due to following factors.

(i) There is direct (or) Transverse shear stress in spring wire due to the load W.

ii) Stress due to curvature of wire.

Direct shear stress due to load 'W'.

$$\tau_2 = \frac{\text{Load}}{\text{Cross sectional area of wire}} = \frac{W}{\left(\frac{\pi}{4} \times d^2\right)}$$

$$\tau_2 = \frac{4W}{\pi d^2} = \frac{4W(2Dd)}{\pi d^2 (2Dd)} = \frac{8WD}{\pi d^3}\left(\frac{0.5d}{D}\right)$$

Resultant shear stress induced in the wire

$$\tau = \tau_1 \pm \tau_2 = \frac{8WD}{\pi d^3} \pm \cancel{\frac{4W}{\pi d^2}}\ \frac{8WD}{\pi d^3}\left(\frac{0.5d}{D}\right)$$

$$\cancel{\tau = \frac{8WD}{\pi d^3} +}$$

The positive sign is used for inner edge of wire and negative sign is used for the outer edge of the wire. Since the stress is maximum at the inner edge of the wire.

∴ Maximum shear stress = Torsional shear stress + Direct shear stress

$$= \frac{8WD}{\pi d^3} + \frac{8WD}{\pi d^3}\left(\frac{0.5d}{D}\right)$$

$$= \frac{8WD}{\pi d^3}\left(1 + \frac{0.5d}{D}\right)$$

$$= \frac{8WD}{\pi d^3} \times K_s$$

where $K_s$ = shear stress factor $= 1 + \frac{0.5d}{D}$

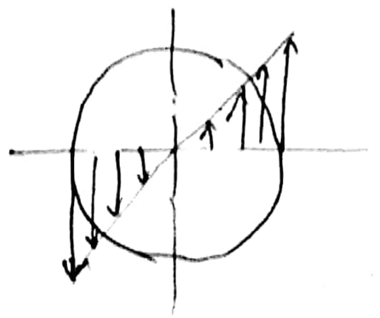
$$\boxed{K_s = 1 + \frac{1}{2C}}$$

+) A Wahl derived the equation for resultant stress which includes torsional shear stress, direct shear stress and stress concentration due to curvature.

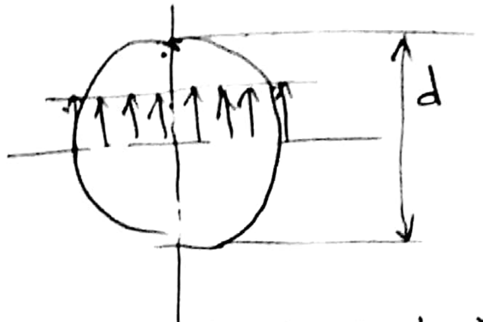
$$\tau = K\left(\frac{8WD}{\pi d^3}\right)$$

where 'K' is called Wahl factor.

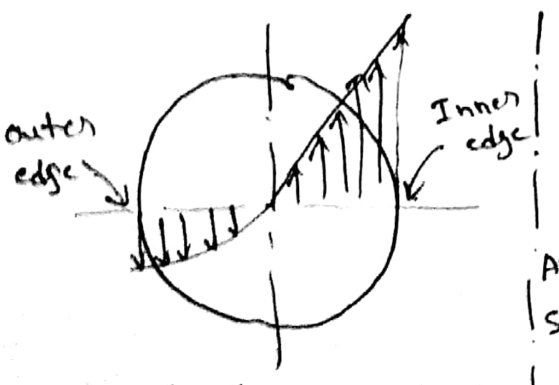
$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$



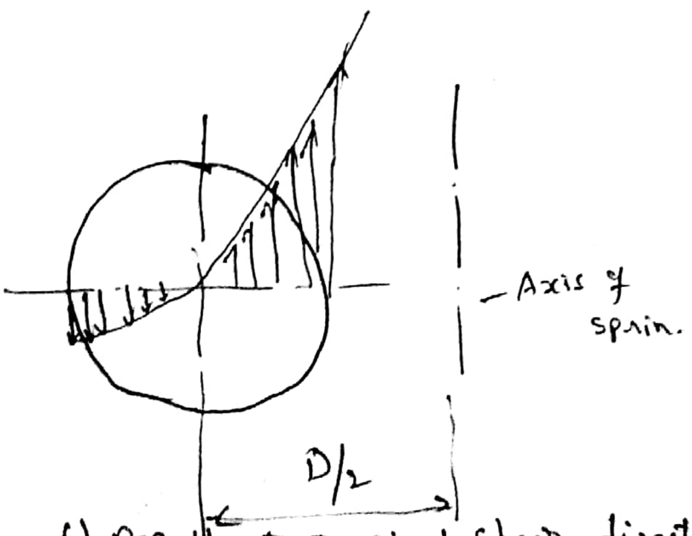
a) Torsional shear stress diagram




b) Direct shear stress diagram

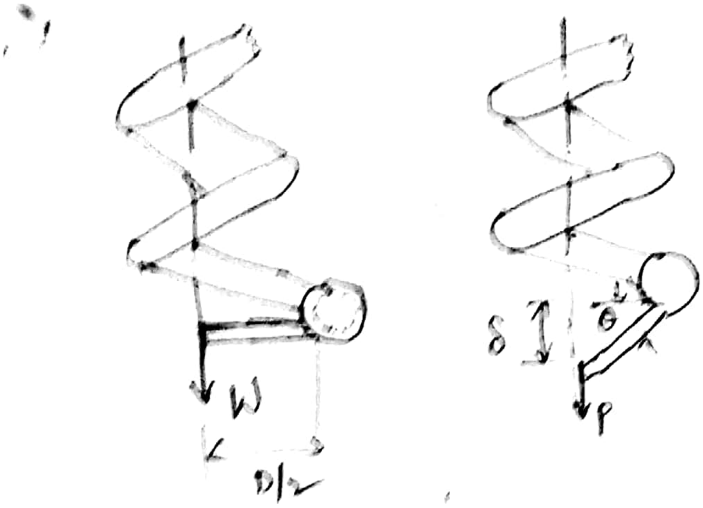



c) Resultant Torsional shear & Direct shear stress diagram




d) Resultant Torsional shear, direct shear and curvature shear stress diagram

Deflection of Helical spring of circular wire :-




length of wire = $\pi D n$

Deflection of spring

We know that $\dfrac{T}{J} = \dfrac{G\theta}{l}$

$$\Rightarrow \theta = \frac{Tl}{GJ} = \frac{\left(W \times \frac{D}{2}\right)\pi D n}{G \times \frac{\pi}{32} d^4}$$

$$\theta = \frac{16 W D^2 n}{G d^4}$$

Axial deflection of spring ; $\delta = \theta \times \dfrac{D}{2}$

$$\delta = \left(\frac{16 W D^2 n}{G d^4}\right) \times \frac{D}{2} = \frac{8 W D^3 n}{G d^4}$$

$$\delta = \frac{8 W C^3 n}{G d}$$

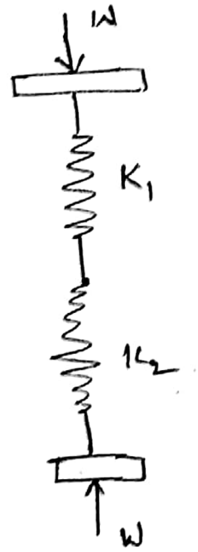
Spring rate , $K = \dfrac{W}{\delta} = \dfrac{G d}{8 C^3 n}$

Energy stored in spring $= \frac{1}{2} W \delta$

**Note:-** When a helical spring is cut into two parts, the parameter G, d, D remains same & 'n' becomes $\frac{n}{2}$. Therefore the stiffness (k) will be double when 'n' becomes $\frac{n}{2}$.

## Springs in series and Parallel:-

Springs in Series:




$$\delta = \delta_1 + \delta_2$$

$$\delta = \frac{W}{K}$$

$$\delta_1 = \frac{W}{K_1} \; ; \; \delta_2 = \frac{W}{K_2}$$

$$\frac{W}{K} = \frac{W}{K_1} + \frac{W}{K_2}$$

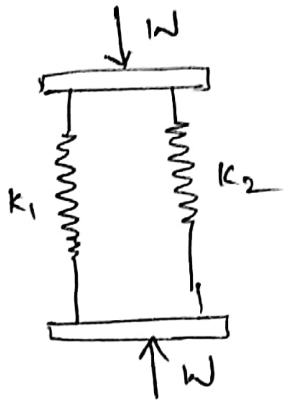
$$\Rightarrow \boxed{\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}}$$

where K → combined stiffness of all spring

$K_1$ → stiffness of spring 1

$K_2$ → stiffness of spring 2

Springs in Parallel




$$W = K\delta$$

$$W_1 = K_1\delta \; ; \; W_2 = K_2\delta$$

$$W = W_1 + W_2$$

$$\Rightarrow K\delta = K_1\delta + K_2\delta$$

$$\Rightarrow \boxed{K = K_1 + K_2}$$

Note:- In springs are in series, the ~~E~~ deflection of each spring is different & same load acting on all springs, For springs are in parallel, the acting load is distributed among all springs & deflection in each spring is same.

Design against Fluctuating load:-

Let us consider a spring subjected to an external fluctuating force, which changes its magnitude from $W_{max}$ to $W_{min}$.

The mean force $W_m = \dfrac{W_{max} + W_{min}}{2}$

Amplitude force (or) load $= \dfrac{W_{max} - W_{min}}{2}$

The mean shear stress $(\tau_m) = k_s\left(\dfrac{8 W_m D}{\pi d^3}\right)$

where $k_s$ = shear stress factor $= 1 + \dfrac{1}{2C}$

Amplitude shear stress $(\tau_a) = K\left(\dfrac{8 W_a D}{\pi d^3}\right)$

where $K$ = wahl factor $= \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$

In general, the spring wires are subjected to pulsating shear stresses which vary from zero to $S'_{se}$.

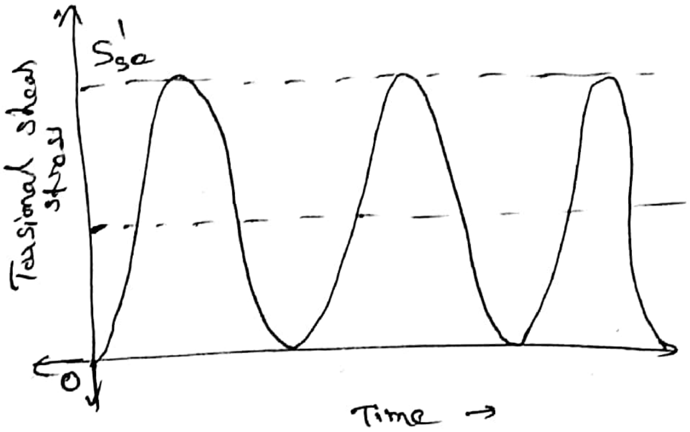
'$S'_{se}$' is endurance limit in shear.

HJ Elmendorf suggested following relations.

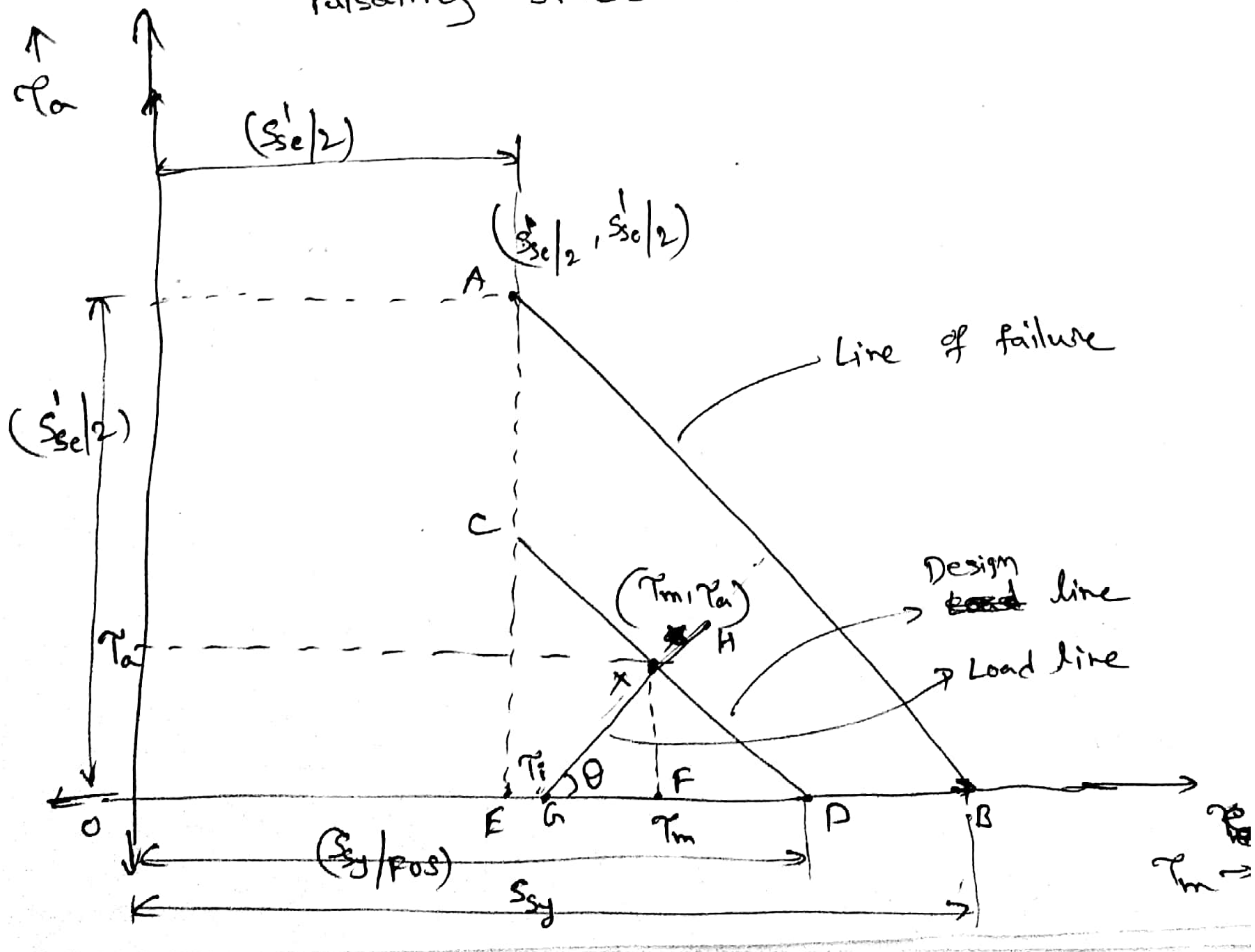
$$S'_{se} = 0.21\ Sut \quad S_{sy} = 0.42\ Sut$$ } For Patented & cold-drawn steel wire

$$S'_{se} = 0.22\ Sut \quad S_{sy} = 0.45\ Sut$$ } For oil-hardened and tempered steel wire.

where Sut = Ultimate strength value




Pulsating stress cycle



The fatigue diagram for the spring is shown in fig. The mean stress ($\tau_m$) on x-axis & amplitude shear stress ($\tau_a$) on y-axis. ~~A~~ Point 'A' with coordinates $\left(\frac{S'_{se}}{2}, \frac{S'_{se}}{2}\right)$ indicates the failure point of the spring wire in fatigue test with pulsating stress cycle. Point 'B' indicates the failure under static condition, when mean stress reaches the ~~yield~~ torsional yield strength ($S_{sy}$).

Line $\overline{AB}$ is called line of failure.

To consider the effect of factor of safety, a line $\overline{DC}$ which is parallel to line $\overline{AB}$ & point 'D' on y-axis such away that

$$\overline{OD} = \frac{S_{sy}}{F.O.S}$$

~~AEF~~

The line $\overline{GH}$ is called load line. It is drawn from a point 'G' on x-axis at a distance '$\tau_i$' from origin. The line $\overline{GH}$ is constructed in such a way that its slope '$\theta$' given by

$$\tan\theta = \frac{\tau_a}{\tau_m}.$$

The point of intersection b/w design line $\overline{DC}$ & load line $\overline{GH}$ is X. The coordinates of 'X' are: ($\tau_m$, $\tau_a$)

$\Delta XFD$, ~~AE~~ $\Delta AEB$ are similar

$$\frac{\overline{XF}}{\overline{FD}} = \frac{\overline{AE}}{\overline{EB}}$$

$$\frac{\overline{XF}}{\overline{OD} - \overline{OF}} = \frac{\overline{AE}}{\overline{OB} - \overline{OE}}$$

$$\frac{\tau_a}{\left(\frac{S_{sy}}{F.o.s}\right) - \tau_m} = \frac{\left(S'_{se}/2\right)}{S_{sy} - \left(\frac{S'_{se}}{2}\right)}$$

## Energy stored in Helical spring:-

Let $W$ = Load applied on the spring

$\delta$ = Deflection

Energy stored in spring $(U) = \frac{1}{2} \times W \times \delta$ —(1)

Maximum shear stress $\Big\} = \tau = K\left(\frac{8WD}{\pi d^3}\right) \Rightarrow W = \frac{\pi d^3 \tau}{8K \cdot D}$

$$\delta = \frac{8WD^3 n}{Gd^4} = \frac{8D^3 n}{Gd^4} \times \left(\frac{\pi d^3 \tau}{8KD}\right)$$

$$\delta = \frac{\pi \tau D^2 n}{KdG}$$

$\therefore$ From equation (1); $U = \frac{1}{2} \times W \times \delta$

$$\Rightarrow U = \frac{1}{2} \times \left(\frac{\pi d^{\cancel{3}} \tau}{8KD}\right) \times \left(\frac{\pi \tau D^2 n}{K\cancel{d}G}\right)$$

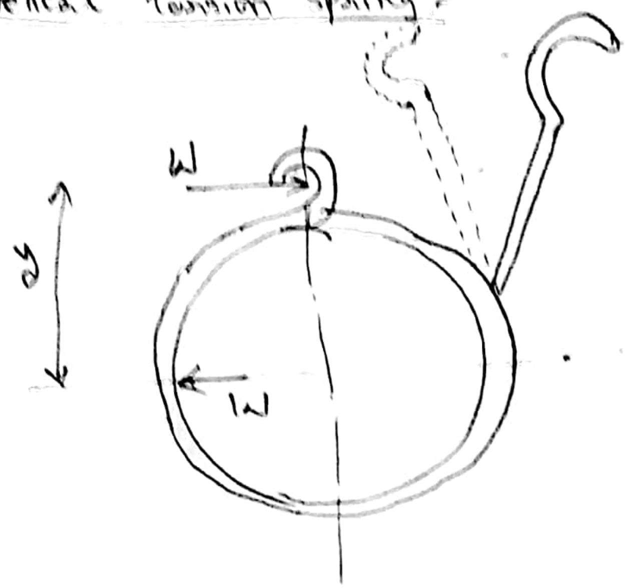
$$U = \frac{\tau^2}{4K^2 G} (\pi D n)\left(\frac{\pi}{4} d^2\right) = \frac{\tau^2}{4K^2 G} \times V$$

where $V$ = volume of spring

= Length of spring × c/s Area of spring wire

$$V = (\pi D n) \times \left(\frac{\pi}{4} d^2\right)$$

Helical Torsion Spring:




The helical Torsion springs may be ~~from~~ made from round, rectangular (or) square wire. These are used for transmitting torque. The Primary stress in helical torsion spring is bending stresses. The helical springs are used for transmitting small torque as in door hinges, brush holders, electric motors etc,

the wire is under Pure bending according to A.M.Wahl, the bending stress in helical torsion spring made of round wire is:

$$\sigma_b = K \times \frac{32M}{\pi d^3} = K \times \frac{32 W y}{\pi d^3}$$

where $K$ = Wahl's stress factor = $\frac{4C^2 - C - 1}{4C^2 - 4C}$

$C$ = spring Index

$M$ = Bending moment = $W \times y$

$W$ = Load acting on the spring

$y$ = Distance of load from the spring axis

$d$ = Diameter of spring wire

Total angle of twist (or) angular deflection

$$\theta = \frac{Ml}{EI} = \frac{M \times \pi D n}{E \times (\pi d^4/64)} = \frac{64\, M.D.n}{E d^4}$$

where $l$ = Length of the wire = $\pi D n$

$E$ = Young's modulus.

$I$ = moment of Inertia = $\frac{\pi}{64} d^4$

$D$ = Diameter of spring

$n$ = no. of turns

$M$ = Bending moment = $W \times y$

Deflection, $\delta = \theta \times y = \frac{64\, M.D.n}{E d^4} \times y$

When the spring is made of rectangular wire having width 'b' and thickness 't', then

$$\sigma_b = K \times \frac{6M}{t.b^2} = K \times \frac{6W \times y}{t b^2}$$

where $K = \frac{3C^2 - C - 0.8}{3C^2 - 3C}$

Angular deflection, $\theta = \frac{12\pi . M . D n}{E . t . b^3}$

$$\delta = \theta . y = \frac{12\pi . M . D . n}{E t b^3} \times y.$$

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## Concentric (or) composite springs:-

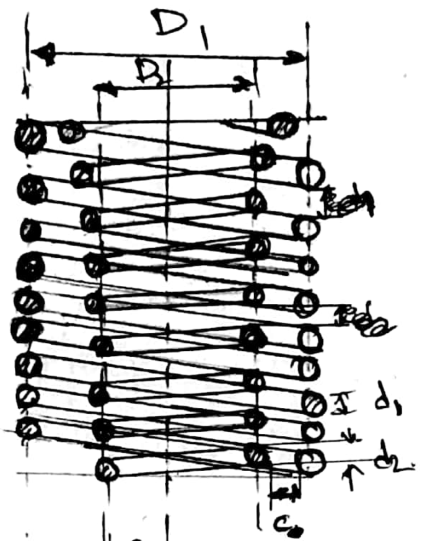


Fig (a)

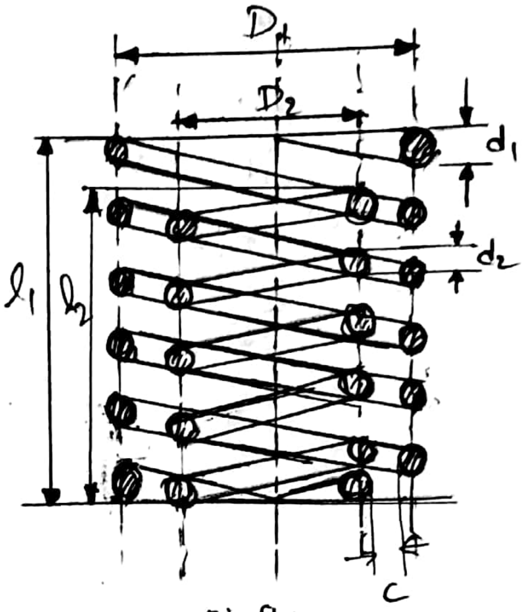


Fig (b)

These springs are used for

(i) To obtain greater spring force within a given space.

(ii) To ensure the operation of a mechanism in the event of failure of one of the springs.

*) Fig (a) represents ~~spri~~ concentric springs of equal length & compressed equally. Such springs are used in automobile clutches, valve springs in aircrafts, etc.,

*) Fig (b) represents a concentric spring of different lengths in which the shorter spring begins to act only after the longer spring is compressed to a certain amount. These springs are used in governors of variable speed engines to take care of the variable centrifugal force.

Let $W$ = axial load

$W_1$ = load shared by outer spring

$W_2$ = load shared by inner spring

$d_1$ = Diameter of spring wire of outer spring

$d_2$ = Diameter of spring wire of Inner spring

$D_1$ = Mean diameter of outer spring

$D_2$ = Mean Diameter of Inner spring

$\delta_1$ = Deflection of outer spring

$\delta_2$ = Deflection of Inner spring

$n_1$ = Number of active turns of outer spring

$n_2$ = Number of active turns of Inner spring.

Assume both the springs are made of same material, then maximum shear stress induced in both the springs are same. i.e.

$$\tau_1 = \tau_2$$

$$\frac{8 W_1 D_1 K_1}{\pi d_1^3} = \frac{8 W_2 D_2 K_2}{\pi d_2^3}$$

when stress factor $K_1 = K_2$ then

$$\frac{W_1 D_1}{d_1^3} = \frac{W_2 D_2}{d_2^3} \quad \text{—} (1)$$

If both the springs are effective throughout their working range. Therefore deflections are equal.

$$\delta_1 = \delta_2$$

$$\frac{8W_1(D_1^3)n_1}{d_1^4 G} = \frac{8W_2 D_2^3 n_2}{d_2^4 G}$$

$$\frac{W_1 D_1^3 n_1}{d_1^4} = \frac{W_2 D_2^3 n_2}{d_2^4} \quad \text{--- (2)}$$

When both the springs are compressed until the adjacent coils meet, then the solid length of both the springs is equal. i.e,

$$n_1 d_1 = n_2 d_2$$

~~Te~~ The equation (2) can be written as

$$\frac{W_1 D_1^3 n_1}{d_1^4} = \frac{W_2 D_2^3 n_2}{d_2^4}$$

$$\frac{W_1 D_1^3 (n_1 d_1)}{d_1^4 \cdot d_1} = \frac{W_2 D_2^3 (n_2 d_2)}{d_2^4 \cdot d_2}$$

$$\frac{W_1 D_1^3}{d_1^5} = \frac{W_2 D_2^3}{d_2^5} \quad \text{--- (3)}$$

$$\frac{(3)}{(1)} \Rightarrow \frac{W_1 D_1^3}{d_1^5} \times \frac{d_1^3}{W_1 D_1} = \frac{W_2 D_2^3}{d_2^5} \times \frac{d_2^3}{W_2 D_2}$$

$$\Rightarrow \frac{D_1^2}{d_1^2} = \frac{D_2^2}{d_2^2}$$

$$\Rightarrow \frac{D_1}{d_1} = \frac{D_2}{d_2} = C = \text{spring Index}$$

i.e, the springs should be designed in such a way that the spring Index for both the springs is same.

(97)

From equation (1); $\frac{W_1 D_1}{d_1^4} = \frac{W_2 D_2}{d_2^4}$ $\left(\because \frac{D_1}{d_1} = \frac{D_2}{d_2} = C\right)$

$\Rightarrow \frac{W_1}{d_1^3} = \frac{W_2}{d_2^3}$

$\Rightarrow \boxed{\frac{W_1}{W_2} = \left(\frac{d_1}{d_2}\right)^3}$ —— (4)

From fig(a); Radial clearance b/w two springs

$$c = \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right)$$

Usually radial clearance b/w two springs is taken as

$$= \frac{d_1 - d_2}{2}$$

$\therefore c = \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right) = \frac{d_1 - d_2}{2}$

$\Rightarrow \frac{D_1}{2} - \frac{D_2}{2} - \frac{d_1}{2} - \cancel{\frac{d_2}{2}} = \frac{d_1}{2} - \cancel{\frac{d_2}{2}}$

$\Rightarrow \frac{D_1 - D_2}{2} = \frac{d_1}{2} + \frac{d_1}{2}$

$\Rightarrow \boxed{\frac{D_1 - D_2}{2} = d_1}$ —— (5)

But $D_1 = Cd_1$ & $D_2 = C \cdot d_2$ where 'C' spring Index.

From equation (5); $\frac{Cd_1 - Cd_2}{2} = d_1$

$\Rightarrow Cd_1 - Cd_2 = 2d_1$

$\Rightarrow Cd_1 - 2d_1 = Cd_2$

$\Rightarrow d_1(C-2) = Cd_2$

$\Rightarrow \boxed{\frac{d_1}{d_2} = \frac{C}{C-2}}$ —— (6)

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springs are used as flexible joint in b/w two parts (or) bodies.

*) To cushion, absorb (or) control Energy due to shock

Eg: car springs (or) Railway buffers

springs supports and vibration dampers.

*) To control motion

→ Maintaining contact b/w two elements (cam & follower)

→ creation of the necessary pressure in a friction device (a brake (or) clutch)

→ Restoration of machine part to its normal position when the applied force is withdrawn (a governor (or) valve)

*) To measure forces.

→ spring balances, gauges

*) To store Energy

→ In clocks (or) starters

Commonly used spring materials:-

Hard drawn wire: This is cold drawn, cheapest spring steel. Normally used for low stress and static load. The material is not suitable at sub zero temperatures (or) at temperatures above 120°c.

Oil - Temperature wire:- It is a cold drawn, quenched, tempered and general purpose spring steel. Not suitable for fatigue (or) sudden loads, at sub zero temperatures and at temperatures above 180°c.

__Chrome vanadium:-__ This alloy spring steel is used for high stress conditions and at high temperature up to 220°c. It is good for fatigue resistance long endurance and for shock & Impact loads.

__Chrome silicon:-__ This material can be used for highly stressed springs. It offers excellent service for long life, shock loading and for temperature upto 250°c.

__Music wire:-__ This spring material is most widely used for small springs. It is the toughest and has highest tensile strength and can ~~to~~ withstand repeated loading at high stresses. Cannot be used at sub zero temperatures (or) at temperatures above 120°c.

__Phosphor Bronze/spring Brass:-__ It has good Corrosion resistance and electrical conductivity. Commonly used for contacts in electrical switches. spring brass can be used at subzero temperatures.