Fluid Statics : UNIT-1 PART-A

→ Fluid mechanics is that branch of science which deals with the behaviour of the fluids (liquids (or) gases) at rest as well as in motion.

→ Fluid statics – Study of fluids at rest.

→ Fluid Kinematics – Study of fluids in motion, where Pressure forces not considered.

→ Fluid Dynamics – Study of fluids in motion, where pressure forces are also considered.

Properties of fluids :-

Density (or) Mass density :- It is defined as the ratio of the mass of a fluid to its volume. The density of liquids may be considered as constant while that of gases changes with the variation of pressures and temperatures.

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}$$

→ $\rho_{water} = 1000\ kg/m^3$

→ $\rho_{mercury} = 13600\ kg/m^3$.

$$\boxed{\therefore \rho = \frac{m}{V}}$$

Specific weight (or) weight density :- It is the ratio between the weight of a fluid to its volume.

$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{m \times g}{V} = \rho \times g. \text{ in } N/m^3.$$

$$\boxed{\therefore w = \rho \times g.}$$

Specific Volume :- It is defined as the volume of a fluid occupied by a unit mass (or) volume of fluid is called specific volume.

$$v = \frac{\text{Volume of fluid}}{\text{mass of fluid}} = \frac{1}{\rho}. \text{ in } m^3/kg.$$

$$\boxed{v = \frac{1}{\rho}}$$

Specific gravity :- It is the ratio of the weight density (or density) of fluid to weight density (or density) of a Standard fluid.

→ For liquids, the standard fluid is taken water.

→ For gases, the standard fluid is taken air.

→ Specific gravity also called relative density.

$$\boxed{S \text{ for liquids} = \frac{\rho_{liquid}}{\rho_{water}} \qquad S \text{ for gases} = \frac{\rho_{gas}}{\rho_{air}}}$$

Example:

$S_{mercury} = 13.6$, $\rho_{mercury} = 13.6 \times \rho_w = 13.6 \times 1000 = 13600 \; kg/m^3$.

① One lit of Crude oil weighs 9.6 N, Calculate its specific weight, density, specific gravity.

Sol:-

Volume of crude oil $= 1$ lit

$= 1 \times 10^{-3} \; m^3$.

weight of crude oil $(m \times g) = 9.6 \, N$.

(i) Sp. weight $(\omega) = \frac{m \times g}{V} = \frac{9.6}{1 \times 10^{-3}} = 9600 \; N/m^3$

(ii) density $(\rho) = \frac{\omega}{g} = \frac{9600}{9.81} = 978.6 \; kg/m^3$

(iii) Sp. gravity $(S) = \frac{\rho_{crude\ oil}}{\rho_{water}}$

$= \frac{978.6}{1000}$

$= 0.978.$

② A fan delivers $4\,m^3$ of air per sec. at 20°C and 1.25 bar. Assuming mol. wt. of air is 28.97. Calculate

(i) m_{air} (ii) ρ_{air} (iii) v_{air} (iv) ω_{air}

Sol: Volume flow rate = $4\,m^3/s$.

Temperature $(T) = 20°C + 273 = 293\,K$

pressure $(P) = 1.25\,bar = 1.25\times10^5\,Pa$ (or) N/m^2.

Molecular weight $M_{air} = 28.97$.

From ideal gas relation,

$$PV = RT$$

$R = \frac{R_u}{M} = \frac{8314}{28.97} = 287.02\ J/kg\,K.$

$R_u \rightarrow$ Universal gas Const

$R \rightarrow$ characteristic gas constant.

(iii) $$v_{air} = \frac{R\cdot T}{P}$$

$$= \frac{287.02\times293}{1.25\times10^5}$$

$$= 0.6727\ m^3/kg.$$

(ii) $$\rho_{air} = \frac{1}{v_{air}} = \frac{1}{0.6727} = 1.486\ kg/m^3.$$

(i) $$m_{air} = \text{Volume flow rate}\times\rho_{air}$$

$$= 4\times1.486$$

$$= 5.944\ kg/s.$$

(iv) $$\omega_{air} = \rho_{air}\times g$$

$$= 1.486\times9.81$$

$$= 14.57\ N/m^3.$$

Viscosity :-

→ It is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

→ When two layers of a fluid, a distance 'dy' apart, move one layer over another at different velocities say "u" and "u+du" as shown in Fig. The viscosity together with relative velocity causes a shear stress acting between the fluid layers.

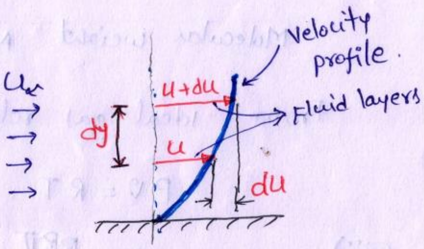


Fig:- Velocity variation near solid boundary.

→ The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.

→ This shear stress (τ) is proportional to the rate of change of velocity with respect to 'y'.

$$\tau \propto \frac{du}{dy} \quad ; \quad \boxed{\tau = \mu \frac{du}{dy}}$$

Where,

μ → Constant of proportionality is known as co-efficient of viscosity (or) dynamic viscosity (or) Absolute viscosity (or) simply viscosity.

$\frac{du}{dy}$ → The rate of shear strain (or) rate of shear deformation (or) velocity gradient.

$$\boxed{\mu = \frac{\tau}{\frac{du}{dy}}} \quad \text{in } N\text{-}s/m^2.$$

→ The viscosity is also defined as the shear stress required to produce unit rate of shear strain.

→ M.K.S Unit of $\mu = \frac{Kgf \cdot s}{m^2}$

→ CGS unit of $\mu = \frac{dyne \cdot s}{cm^2}$

→ SI unit of $\mu = \frac{N \cdot s}{m^2}$

1 poise = 1 $\frac{dyne \cdot s}{cm^2}$
1 poise = $\frac{1}{10}$ $N s/m^2$
1 centipoise (cP) = $\frac{1}{100}$ poise.

Variation of viscosity (μ) with Temperature :-

→ Temperature affects the viscosity.

→ The viscosity of liquids decreases with increase of temperature.

→ The viscosity of gases increases with the increase of temperature.

* Viscous forces in a fluid are due to cohesive forces and molecular momentum transfer.

In liquids :- The cohesive forces predominates the molecular momentum transfer, due to closely packed molecules and with the increase in temperature, the cohesive forces decreases with the result of decreasing viscosity.

$$\boxed{\mu = \mu_0 \frac{1}{1+\alpha t+\beta t^2}}$$

where,

$\mu \rightarrow$ viscosity of liquid at "$t°C$" in poise

$\mu_0 \rightarrow$ viscosity of liquid at "$0°C$" in poise

$\alpha, \beta \rightarrow$ constant values.

$t \rightarrow$ Temperature

For water, $\mu_0 = 1.79 \times 10^{-3}$ poise

$\alpha = 0.03368$

$\beta = 0.000221$.

In gases :- In gases, the molecular momentum transfer increases with the temperature which results decreases the viscosity.

$$\boxed{\mu = \mu_0 + \alpha t - \beta t^2}$$

where,

$\mu \rightarrow$ viscosity of gas at "$t°C$" in poise

$\mu_0 \rightarrow$ viscosity of gas at "$0°C$" in poise

$\alpha, \beta \rightarrow$ constants

$t \rightarrow$ temperature

For air,

$\mu_0 = 0.000017$

$\alpha = 0.000000056$

$\beta = 0.1189 \times 10^{-7}$.

Kinematic Viscosity :-

It is defined as the ratio of dynamic viscosity to density of the fluid. It is denoted by ν (nu).

$$\boxed{\nu = \frac{\mu}{\rho}}$$

→ M.K.S unit of $\nu \rightarrow m^2/s$

→ C.G.S unit of $\nu \rightarrow cm^2/s$ (or) stokes

→ S.I unit of $\nu \rightarrow m^2/s$.

1 stoke = 1 cm^2/s

1 stoke = 10^{-4} m^2/s

1 Centistoke (CS) = $\frac{1}{100}$ stokes

Newton's law of viscosity :-

It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain.

$$\tau \propto \frac{du}{dy} \Rightarrow \tau = \mu \frac{du}{dy}$$

Types of fluid :-

1. Ideal fluid
2. Real fluid
3. Newtonian fluid
4. Non-Newtonian fluid
5. plastic fluid
6. Thixotropic fluid.

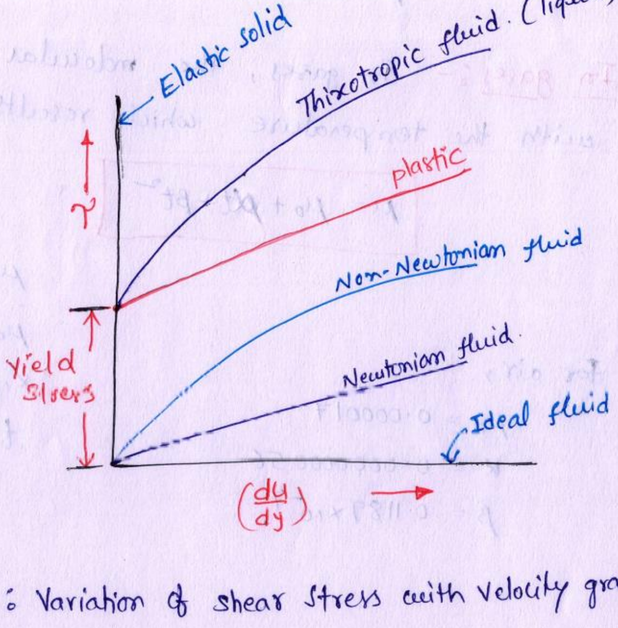


Fig : Variation of shear stress with velocity gradient.

Ideal fluid ;- A fluid, which is compressible and having no viscosity is known as ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids which exist have some viscosity.

Real fluid ;- A fluid, which possesses viscosity, is known as Real fluid. All the fluids in actual practice are real fluids.

Newtonian fluid ;- It is a real fluid, in which the shear stress is directly proportional to the the rate of deformation (shear strain).

Ex:- water, air, glycerine, kerosene, alcohol, Benzene, Hexane etc.

Non-Newtonian fluid ;- It is also a real fluid, in which the shear stress is not proportional to the rate of deformation. This fluid does not obey the Newton's law of viscosity.

Ex:- slurries, suspensions, gels, collids, etc.

Ideal plastic ;- It is a fluid, and has definite yield stress. In this fluid a constant linear relation between shear stress and the rate of deformation.

Thixotropic fluid ;- It is a non-newtonian fluid, has a non-linear relationship between the shear stress and the rate of deformation, beyond an initial yield stress.

Ex:- printer's ink.

NOTE: The fluids with which engineers most often have to deal are Newtonian, that is, their viscosity is not dependent on the rate of angular deformation (shear strain), and the term "fluid-Mechanics" generally refers only to Newtonian fluids.

Problems on Viscosity

① The velocity distribution for a flow over a flat plate is given as $u = \frac{3}{2}y - y^{3/2}$, where u is the point velocity in m/s. at a distance "y" meter above the plate. Determine the shear stress at $y = 9$ cm. Assume, dynamic viscosity as 8 poise.

Sol:-
$\mu = 8$ poise $= \frac{8}{10}$ N·s/m² $= 0.8$ Ns/m²

$y = 9$ cm $= 0.09$ m

$u = \frac{3}{2}y - y^{3/2}$

$\tau_{at\, y=0.09} = ?$

$\frac{du}{dy} = \frac{3}{2} - \frac{3}{2}(y)$.

at $y = 0.09$m,

$\frac{du}{dy} = \frac{3}{2} - \frac{3}{2}(0.09)$

$= 1.05$

$\therefore \tau = \mu \frac{du}{dy}$

$= 0.8 \times 1.05$

$= 0.84$ N/m²

② In a stream of glycerin in motion, at a certain point the velocity gradient is 0.25 m/s. The mass density of fluid is 1268.4 Kg per cubic metre and "ν" is 6.30×10^{-4}. square metre per sec. Calculate shear stress at that point.

Sol:-
$\rho = 1268.4$ kg/m³

$\nu = 6.30 \times 10^{-4}$ m²/s.

$\mu = \nu \times \rho = 1268.4 \times 6.30 \times 10^{-4} = 0.799$ N·s/m²

$\frac{du}{dy} = 0.25$ m/s.

$\tau = \mu \frac{du}{dy} = 0.25 \times 0.799 = 0.199$ N/m².

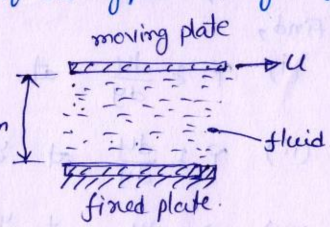
③ A plate 0.025 mm distance from a fixed plate, moves at 50 cm/s and require a force of 1.471 N/m² to maintain this speed. Determine the fluid viscosity between plates in poise. Also find power required to maintain this speed if moving plate having surface area 1.5 m².

Sol:-

$dy = 0.025\,mm = 0.025\times10^{-3}\,m$.

$u = 50\,cm/s = 0.5\,m/s$.

$\tau = 1.471\,N/m^2$



$du = u_{up} - u_{below} = 50 - 0 = 0.5 - 0 = 0.5\,m/s$.

(1) $\tau = \mu \dfrac{du}{dy} \Rightarrow \mu = \dfrac{\tau}{\left(\frac{du}{dy}\right)} = \dfrac{1.471}{\left(\frac{0.5}{0.025\times10^{-3}}\right)} = 7.355\times10^{-5}\,Ns/m^2$

$\mu = 7.355\times10^{-5}\,Ns/m^2$

$= 7.355\times10^{-5}\times10$ poise

$\left\{\begin{array}{l} 1\text{ poise} = \frac{1}{10}\,Ns/m^2 \\ 1\,Ns/m^2 = 10\text{ poise}. \end{array}\right.$

$\mu = 7.355\times10^{-4}$ poise.

(2) power = Force × velocity

= (Shear Stress × Area) × Velocity of the plate (upper plate).

$= \tau \times A \times u$

$= 1.471\times1.5\times0.5$

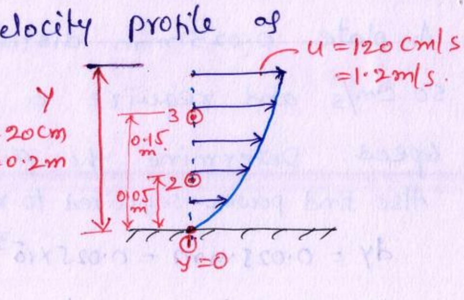
$= 1.10325\,W$.

④ The velocity profile at a viscous fluid over a plate is parabolic with vertex 20 cm from the plate, where the velocity is 120 cm/s. Calculate the velocity gradient and shear stress at distance of 0, 5 and 15 cm from the plate. Take the viscosity of the fluid is 6 poise.

Sol:- For parabolic equation, the velocity profile of

$u = ay^2 + by + c$ —— (I)

$\mu = 6$ poise $= 0.6\ Ns/m^2$



Find,

(i) τ & $\frac{du}{dy}$ at $y = 0$ cm $= 0$ m

(ii) τ & $\frac{du}{dy}$ at $y = 5$cm $= 0.05$m

(iii) τ & $\frac{du}{dy}$ at $y = 15$cm $= 0.15$m.

Apply the following Boundary Conditions, in eqn(I).

a) at $y = 0$, $u = 0$

b) at $y = 0.2$m $u = 1.2$ m/s.

c) at $y = 0.2$m $\frac{du}{dy} = 0$.

→ Now, apply B.C (a) in eqn (I), then we have $\boxed{c = 0}$.

$\therefore u = ay^2 + by$ —— (II) ; $\frac{du}{dy} = 2ay + b$ —— (III).

→ apply B.C (b) in eqn(II), then we have $1.2 = 0.04a + 0.2b$ —— (III).

→ apply B.C (c) in eqn(III), then we have $0 = 0.4a + b$

$\therefore b = -0.4a$ —— (V)

Now, substitute eqn(V) in eqn (IV),

$1.2 = 0.04a + 0.2(-0.4a)$

$1.2 = 0.04a - 0.08a$

$\boxed{a = -30}$

Now, substitute "a = -30" in eqn (V),

$b = -0.4(-30)$

$\boxed{b = 12}$ Now substitute "a, b and c" values in eqn (I),

$$\boxed{u = -30y^2 + 12y}$$

$$u = -30y^2 + 12y$$

$$\frac{du}{dy} = -60y + 12$$

(i) at $y = 0\,m$, $\frac{du}{dy} = \underline{12}$ per sec.

$$\tau = \mu \frac{du}{dy} = 0.6 \times 12 = \underline{\underline{7.2}}\ Ns/m^2$$

(ii) at $y = 0.2\,m$, $\frac{du}{dy} = -60(0.2) + 12 = 0$.

$$\tau = \mu \frac{du}{dy} = \underline{0}.$$

(iii) at $y = 1.5\,m$, $\frac{du}{dy} = -60(1.5) + 12 = \underline{\underline{102}}$ per sec.

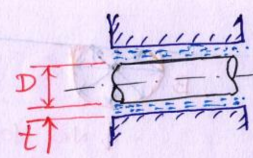
$$\tau = \mu \frac{du}{dy} = 0.6 \times 102 = \underline{\underline{61.2}}\ Ns/m^2.$$

(5) An oil of viscosity 5 poise is used for lubrication between a shaft and sleeve. The diameter of the shaft is 0.5m and it rotates at 200 r.p.m. Calculate the power lost (absorbed by oil) in oil for a sleeve length of 100mm. The thickness of oil film is 1.0mm.

Sol:- $\mu = 5$ poise $= 0.5\ Ns/m^2$

Speed $N = 200$ rpm.

dia. $D = 0.5\,m$.



→ Tangential velocity of shaft, $u = \frac{\pi D N}{60} = 5.235\,m/s$.

change of velocity $= u - 0$

$du = 5.235\,m/s$

→ $\tau = \mu \frac{du}{dy} = 0.5 \times \frac{5.235}{1 \times 10^{-3}} = 2617.5\ N/m^2$

change of distance $= t$

$dy = t = 1\,mm$

→ shear force on shaft $= \tau \times A$

$= 2617.5 \times 0.157$

$= 410.95\,N$

Area $= \pi \cdot D \cdot L$

$= \pi \times 0.5 \times 0.1$

$= 0.157\ m^2$

→ Torque on shaft $= \text{Force} \times \frac{D}{2} = 410.95 \times \frac{0.5}{2} = 102.74\ N\cdot m$.

Power lost $=$ Torque $\times$ angular velocity

$= \text{Torque} \times \frac{2\pi N}{60}$

$= \frac{2\pi N T}{60} = \frac{2 \times \pi \times 200 \times 102.74}{60} = \underline{\underline{2150\ W}}.$

Surface Tension :-

→ It is defined as the tensile force acting on the surface of a liquid in contact with a gas (or) on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.

MKS system → Kgf/m.

→ It is denoted by "σ".

SI system → N/m.

⇒ Let consider three molecules in a fluid as shown in Figure (1).

A – The resultant force acting on molecule A is Zero.

B – Acted upon by upward and downward forces which are Unbalanced. Thus the net resultant force on molecule "B" is acting in the downward direction.

C – The resultant force on molecule 'C' is acting in downward direction due to unbalance.

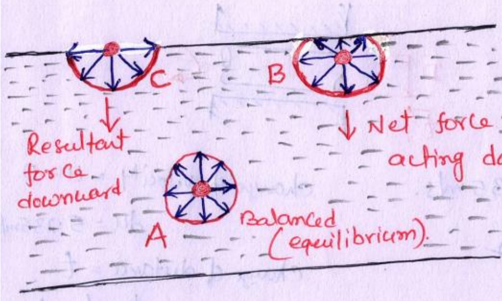


Fig: (1).

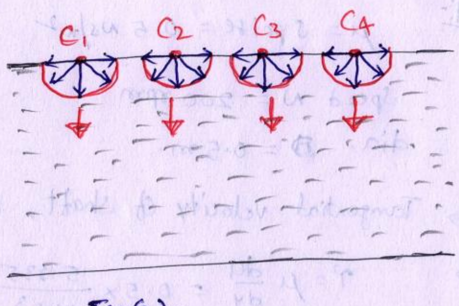


Fig (2).

⇒ All the molecules on the free surface experience a downward force. Thus the free surface of the liquid acts like a very thin film under tension of the surface of the liquid. [as shown in Fig (2)].

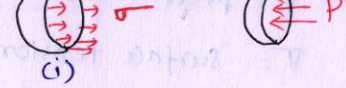
(a) Surface tension on liquid droplet.

Consider a small spherical droplet of liquid of radius "r".

Let, σ = surface tension of the liquid droplet

P = pressure intensity inside the droplet

d = diameter of droplet.



Let, the droplet cut into two halves. The forces acting on one half will be tensile force due to surface tension and pressure force.

→ Tensile force acting around around the circumference of the cut portion as shown in Figure (i) $= \sigma \times \pi d$.

→ pressure force on the area as in Fig (ii) is, $= P \times \frac{\pi}{4} d^2$.

These forces will be equal and opposite under equilibrium condition,

$$P \times \frac{\pi}{4} d^2 = \sigma \pi d.$$

$$\boxed{P = \frac{4\sigma}{d}.}$$

Equation shows that, the decrease of diameter of the droplet, pressure intensity inside the droplet increases.

(b) Surface tension on hollow bubble.

A hollow bubble like soap bubble in air has two surfaces in contact with air, one side and other side. These two surfaces are subjected to surface tension. In such case we have,

$$P \times \frac{\pi}{4} d^2 = 2 (\sigma \times \pi d)$$

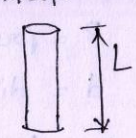
$$\boxed{P = \frac{8\sigma}{d}.}$$

(c) Surface tension on liquid jet.

Consider a liquid jet of diameter "d" and length "L" as shown in fig.

Let P = pressure intensity inside the liquid jet

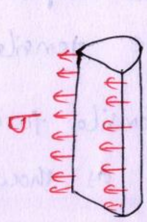
σ = surface tension of the liquid.



Consider the equilibrium of the semijet,

→ Force due to pressure = $P \times$ area of semijet

$= P \times L \times d.$



→ Force due to surface tension $= \sigma \times 2L$

Under equilibrium condition,

$$P \times L \times d = \sigma \times 2L$$

$$\boxed{P = \frac{2\sigma}{d}}$$

Problems on Surface tension.

① The surface tension of water in contact with air is given as 0.0725 N/m. The pressure outside the droplet of water of diameter 0.02 mm is atmospheric (10.32 N/cm²). Calculate the pressure within the droplet of water.

Sol:- diameter of droplet $(d) = 0.04 \times 10^{-3}$ m.

Pressure outside the droplet $= 10.32$ N/cm² $= 10.32 \times 10^{4}$ N/m²

surface tension $(\sigma) = 0.0725$ N/m.

→ The pressure inside the droplet, $P = \frac{4\sigma}{d} = \frac{4 \times 0.0725}{0.04 \times 10^{-3}} = 7250$ N/m²

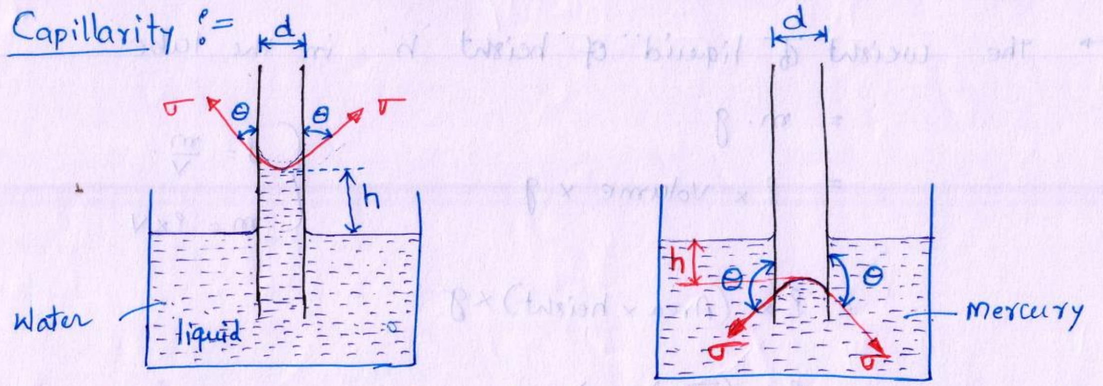
→ Total pressure inside the droplet, = P + Pressure outside the droplet.

$= 7250 + (10.32 \times 10^{4})$

$= 110450$ N/m²

$P_{total} = 11.045 \times 10^{4}$ N/m²

Capillarity :-



(Adhesion > cohesion)

(a) Capillary Rise → $\theta < 90^\circ$

(Cohesion force > Adhesion force)

(b) Capillary fall → $\theta > 90^\circ$

→ Capillarity :- It is defined as a phenomenon of rise (or) fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid.

→ Rise of liquid surface – Capillary rise

Fall of liquid surface – Capillary fall

→ If Adhesion force > Cohesion force

The liquid will wet the glass surface and the liquid level will rise. ($\theta < 90^\circ$).

→ If Cohesion force > Adhesion force

The liquid does not wet the glass surface and the liquid level will fall. ($\theta > 90^\circ$).

→ Let

h = The capillary Rise (or) fall

σ = Surface tension

θ = Angle between surface tension the vertical

→ The weight of liquid of height 'h' in the tube

$= m \cdot g$

$= \rho \times \text{Volume} \times g$

$\left\{ \begin{array}{l} \rho = \frac{m}{V} \\ m = \rho \times V \end{array} \right.$

$= \rho \times (\text{Area} \times \text{height}) \times g$

$= \rho \times \left(\frac{\pi}{4} d^2 \times h\right) \times g$

$= \frac{\pi}{4} d^2 \cdot h \cdot \rho \cdot g$.

→ The vertical component of the surface tension force

$= (\sigma \times \cos\theta) \cdot \pi d$.

$\left\{ \begin{array}{l} \sigma = \frac{F}{L} \\ F = \sigma \times L \end{array} \right.$

→ Under equillibrium, the weight of liquid column will be balanced by surface tension force "σ".

Weight of liquid = surface tension force.

$\frac{\pi}{4} d^2 h \rho g = \sigma \cos\theta \cdot \pi d$.

$$h = \frac{\sigma \times \cos\theta \times \pi d}{\frac{\pi}{4} d^2 \times \rho \times g}$$

$$\therefore \boxed{h = \frac{4 \sigma \cos\theta}{\rho g d}}$$

The contact angle (θ) for water and smooth glass will be zero. Thus the value of $\cos\theta = 1$, then

$$\boxed{h = \frac{4\sigma}{\rho g d}}$$

Applications of Capillarity:

1) Drawing of ink to the nib from the bottom in a fountain pen.
2). Shipping liquid from the container through straw
3) Sponges ~~are~~ used to absorb water to clean.

Q). Why does the water rise and the mercury fall in a capillary tube?

A) → Fluid molecules have two type of forces. ① Cohesion ② Adhesion.

1) Cohesion → Intermolecular forces b/w same molecules

2) Adhesion → intermolecular force b/w different molecules

Capillary rise in water,

- put the capillary tube inside the water.
- Water molecules have higher adhesive force compared its cohesive force.
- So, they will attract towards the surface of capillary and thus gradually increase the level of water in capillary compared to general level of water.
- This is the reason, shape (Meniscus) in this case is concave. (Rise).

Capillary fall in Mercury,

- Now, put the capillary tube in mercury.
- Mercury have higher cohesive force compared to adhesive forces.
- So the molecules of mercury are attracted towards its own molecules and will not stick with capillary wall.
- level of mercury will fall because of higher attractive forces of surface molecules with inside molecules of mercury.
- This is the reason, shape in this case is convex. (fall). (Meniscus).

Problems on Capillarity

① Calculate the capillarity in a glass tube of 3.0 mm diameter when immersed vertically in (a) water and (b) mercury. Take surface tension for mercury and water 0.0725 N/m and 0.52 N/m respectively in contact with air.

Sol: Capillarity rise or fall $(h) = \dfrac{4\sigma \cos\theta}{\rho g d}$ $\left\{ \cos\theta = 1 \right\}$ for water $(\theta = 0^\circ)$

for mercury $(\theta = 130^\circ)$ $\cos\theta = -0.642$

Diameter of glass tube $(d) = 3\,mm = 0.3\,cm = 0.03\,m$.

$\sigma_{water} = 0.52\,N/m$

$\sigma_{mercury} = 0.0725\,N/m$.

$\therefore \rho_{water} = 1000\,kg/m^3$

$\rho_{mercury} = \rho_w \times Sp.\,gr.\,mercury$

$= 1000 \times 13.6$

$= 13600\,kg/m^3$.

(i) water (Capillary rise),

$$h = \frac{4 \times 0.52 \times 1}{1000 \times 9.81 \times 0.03} = 7.06 \times 10^{-3}\,m$$
$$= \underline{0.7\,cm}.$$

(ii) mercury (Capillary fall).

$$h = \frac{4 \times 0.0725 \times (-0.642)}{13600 \times 9.81 \times 0.03} = 4.65 \times 10^{-5}\,m$$
$$= \underline{4.65 \times 10^{-3}\,cm}.$$

Vapor Pressure :-

→ Consider a liquid which is confined in a closed vessel. Let the temp. of liquid is 20°C and pressure is atmospheric. This liquid will vaporise at 100°C.

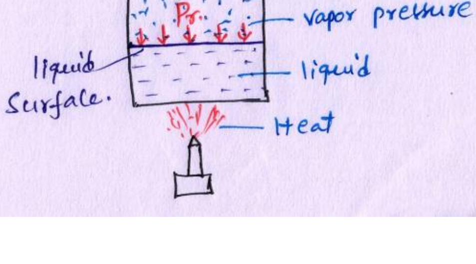
→ When vaporization takes place, the molecules escapes from liquid surface and moves to top portion in the vessel.

→ These accumlated vapors exert a pressure on the liquid surface.

→ This pressure is known as vapor pressure of liquid.

(or)

→ Simple definition, It is the pressure at which the liquid is converted into vapors.



Pressure Measurement :-

Let consider a small area 'dA' in large mass of fluid as shown in Figure 1. If the fluid is stationary, then the force exerted by the surrounding fluid on the area dA will always be $\perp^r$ to the surface dA.

→ Let 'dF' is the force acting on the area dA in the normal direction.

→ then the ratio of $\frac{dF}{dA}$ is known as the intensity of pressure or simply pressure. It is denoted by 'p'.

→ Hence, mathematically the pressure at a point in a fluid at rest is

$$P = \frac{dF}{dA}.$$

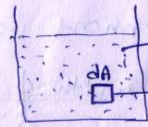


Fig. 1

→ If the force (F) is uniformly distributed over the area (A), then pressure at any point is given by $\boxed{P = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}}$.

∴ Force (or) pressure force = P×A.

<u>Units</u>:

MKS Units → $kg f/m^2$

SI Units → N/m^2 (or) N/mm^2

$1 N/m^2$ = 1 Pascal (or) 1 Pa.

Pressure Units ⟹

1 bar = 100 kPa = 100 kN/m^2 = 10^5 N/m^2
1 kpa = 1000 N/m^2.
1 atm. pr = 1.01325 bar = 101.325 kPa (or) kN/m^2 = 101325 N/m^2

Pascal's law :-

"It states that the pressure (or) Intensity of pressure at a point is a static fluid (fluid at rest), is equal in all directions."

→ Let consider small fluid element of wedge shape in the larger mass of the fluid. as shown in figure 2.

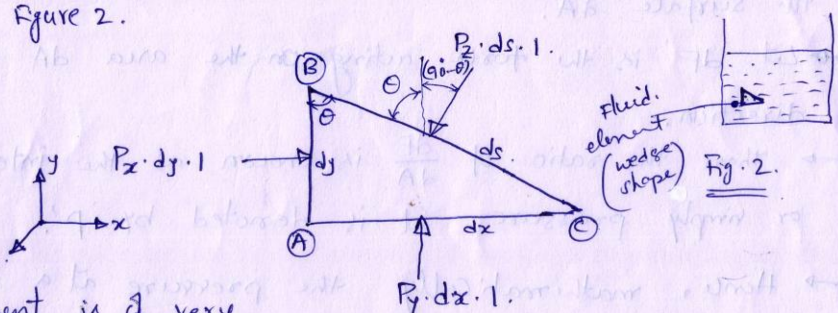


Fig. 2.

→ The fluid element is of very small dimensions i.e., dx, dy and ds.

→ Let the width of the element perpendicular to the plane of paper is unity and,

→ P_x, P_y and P_z are the pressures acting on the face AB, AC and BC.

→ Let $\angle ABC = \theta$. Then the forces acting on element are,

① pressure forces normal to the surfaces, and (F_x, F_y and F_z).

② weight of fluid element in the vertical direction.

→ Force on the face 'AB' $F_x = P_x \times$ Area of face AB.

$$F_x = P_x \times dx \times 1.$$

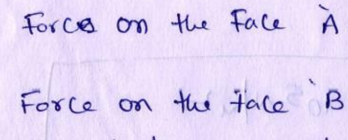
Forces on the Face 'AC'. $F_y = P_y \times dy \times 1$.

Force on the face 'BC'. $F_z = P_z \times ds \times 1$.

→ weight of the fluid element $=$ mass of element $\times g$

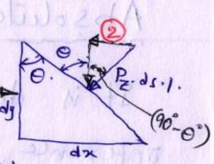
$$= (\rho \times \text{volume}) \times g.$$

Volume of the fluid element $= \frac{AB \times AC}{2} \times 1$.



Now, Resolving the forces in x-direction, we have

$$[①] - [②] = 0.$$



$$[P_x \cdot dy \cdot 1] - [(P_z \cdot ds \cdot 1)(\sin 90° - \theta°)] = 0.$$

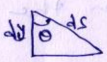
$$P_x \cdot dy \cdot 1 - [P_z \cdot ds \cdot \cos(\theta)] = 0.$$

$$\sin(90° - \theta°) = \frac{\text{Opp. side}}{\text{Hypo.}}$$

$$= \frac{②}{P_z \cdot ds \cdot 1}$$

$$② = \sin(90° - \theta°) \times (P_z \cdot ds \cdot 1)$$

But, from the fig,



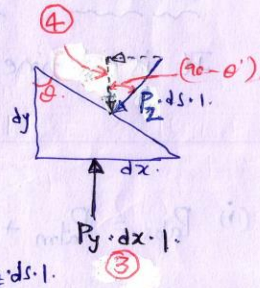
; $\cos\theta = \frac{dy.}{ds.} \Rightarrow ds \cdot \cos(\theta) = dy.$

$$P_x \cdot dy - P_z \cdot dy = 0.$$

$$\boxed{P_x = P_z.} \quad \text{— (i)}$$

Similarly, resolving forces in y-direction, we get

$$[③] - [④] - \left(\frac{dx \cdot dy}{2} \cdot 1 \times \rho \times g\right) = 0.$$



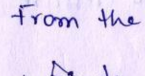
$$[P_y \cdot dx \cdot 1] - [(P_z \cdot ds \cdot 1) \cos(90 - \theta°)] - \left[\frac{dx \cdot dy}{2} \cdot 1 \times \rho \times g\right] = 0$$

$$[P_y \cdot dx] - [P_z \cdot ds \cdot \sin(\theta)] - \left[\frac{dx\,dy}{2} \times \rho \times g\right] = 0.$$

$$\cos(90 - \theta°) = \frac{\text{Adj. side}}{\text{Hypo.}} = \frac{④}{P_z \cdot ds \cdot 1.}$$

$$④ = \cos(90° - \theta°) \times P_z \cdot ds \cdot 1.$$

From the fig, $ds \cdot \sin(\theta) = dx$, and element is very small and hence weight is negligible.



; $\sin\theta = \frac{dx}{ds}$

$dx = ds \cdot \sin(\theta)$.

$\left\{ \frac{dx\,dy}{2} \times \rho \times g \text{ is neglected.} \right\}$.

$$(P_y \cdot dx) - (P_z \cdot dx) = 0.$$

$$\boxed{P_y = P_z} \quad \text{— (ii)}$$

Therefore, from the equation (i) & (ii), we have

$$\boxed{P_x = P_y = P_z} \quad \text{— (iii)}$$

The equation (iii) shows that the pressure in a fluid at any point in x, y and z directions is equal.

Absolute Pressure :-

It is defined as the pressure which is measured with reference to absolute vacuum pressure.
(zero)

Gauge Pressure :-

It is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum.

Vacuum Pressure :-

It is defined as the pressure below the atmospheric Pressure.

(i) $P_{abs} = P_{atm} + P_{gauge}$.

(ii) $P_{vac} = P_{atm} - P_{abs}$.

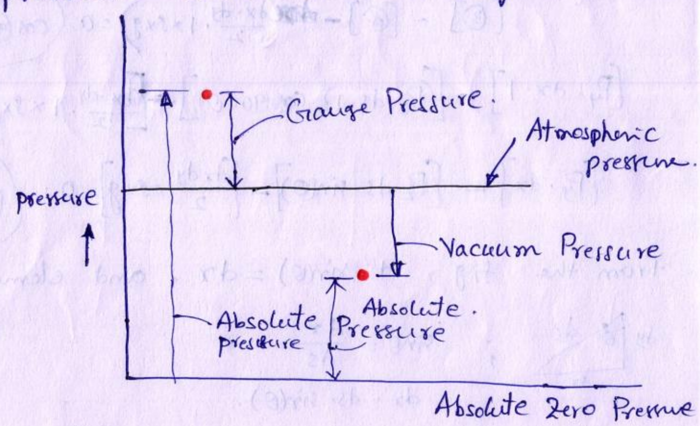


Fig: Relationship b/n Pressures.

1 atmospheric pressure = 760 mm of Hg

= 10.33 mm of water

Hydrostatic Law :-

"It states that the rate of increase of pressure in vertically downward direction must be equal to the specific weight of the fluid at that point."

→ Consider a small fluid element as shown in fig. 3.

Let ΔA = Cross-sectional area of element

ΔZ = Height of fluid element

Z = Distance of fluid element from free surface.

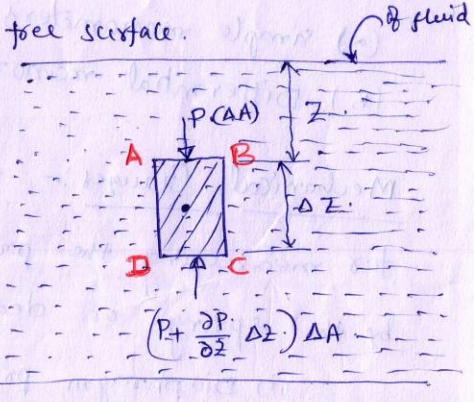


Fig:3 Forces on fluid element.

The forces acting on fluid element are,

1) Pressure force on face AB = $P \times \Delta A$. (downward direction)

2) Pressure force on face CD = $\left(P+\frac{\partial P}{\partial Z}\cdot\Delta Z\right)\times\Delta A$. (vertically upward) direction.

3) weight of fluid element = mass × g.

= ρ × Volume × g

= $\rho \times g \times (\Delta A\cdot\Delta Z)$.

4) pressure forces on surfaces AD and BC are equal and opposite.

$$P\cdot\Delta A - \left(P+\frac{\partial P}{\partial Z}\cdot\Delta Z\right)\Delta A + \left[\rho\times g\times(\Delta A\cdot\Delta Z)\right] = 0.$$

$$\cancel{P\Delta A} - \cancel{P\Delta A} - \frac{\partial P}{\partial Z}\cdot\Delta Z\cdot\Delta A + (\rho\cdot g\cdot\Delta A\cdot\Delta Z) = 0.$$

$\left\{\rho\times g = w = \text{weight density.}\right.$

$$\frac{\partial P}{\partial Z} = \frac{\rho\cdot g\cdot\Delta A\cdot\Delta Z}{\Delta A\cdot\Delta Z} = \rho\cdot g.$$

$$\boxed{\therefore \frac{\partial P}{\partial Z} = \rho\times g} \quad \text{— (iv)}$$

Equation (iv) states that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point. This is hydrostatic law.

By integrating eqn' (iv), for liquids, we get $\int dP = \int \rho g\cdot dZ$.

$$\boxed{P = \rho g Z} \Rightarrow \boxed{Z = \frac{P}{\rho g}}$$

↑ pressure head.

⇒ where, P → pressure above atm. pressure

Z → height of the point from free surface.

Z → also called pressure head.

Measurement Of Pressure :-

The pressure of a fluid is measured by the following devices.

1. Manometers.
2. ~~Measing~~ Mechanical gauges.

Manomiters :- These are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of ~~the~~ fluid*. They are classified as,

*[by same Or) other column of the fluid].

(a) simple manometers

(b) Differential manometers.

Mechanical Gauges :- These are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight.

(i) Diaphragm pressure gauge

(ii) Dead-weight pressure gauge

(iii) Bourdon tube pressure gauge

(iv) Bellows pressure gauge.

Simple Manometers :-

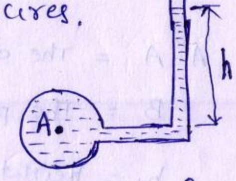
A simple manometer consists of a glass tube having one of its ends connected to a point where the pressure to be measured and other end remains open to atmosphere. Common types of simple manometers are,

1. Piezometer
2. U-Tube manometer
3. Single Column manometer.

Piezometer :-

→ It is used for measuring gauge pressures.

→ One end is connected to the point where pressure is to be measured



→ Otherend is open to the atmosphere as shown in figure.

· The rise of liquid gives the pressure head at that point.

If at a point 'A', the height of liquid say water is 'h' in piezometer tube, then pressure at A,

$$\boxed{P_A = \rho \times g \times h} \quad N/m^2$$

U-Tube Manometer :-

→ It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in figure.

→ The tube generally contains mercury (or) any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.

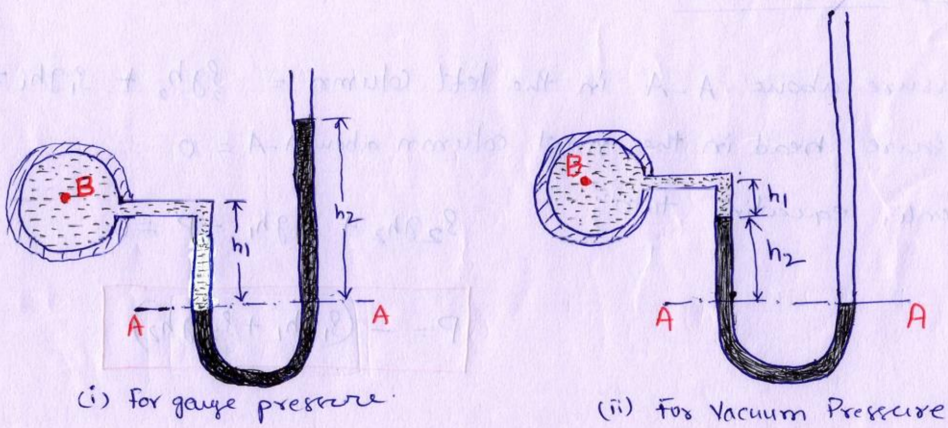


Fig: U-Tube Manometer.

(i) For gauge pressure :-

Let,

A-A = The datum line.

B = The point at which pressure is to be measured,

h_1 = Height of light liquid above the datum line.

h_2 = Height of heavy liquid above the datum line.

S_1 = Specific gravity of light liquid.

ρ_1 = Density of light liquid

S_2 = Specific gravity of heavy liquid

ρ_2 = Density of heavy liquid.

The pressure above the datum line A-A in the left column and in the right column of U-tube manometer should be same.

pressure above A-A in the left column $= (P) + (\rho_1 \times g \times h_1)$

pressure above A-A in the right column $= \rho_2 \times g \times h_2$

Hence, equating two ; $(P) + (\rho_1 \times g \times h_1) = \rho_2 \times g \times h_2$

$$\boxed{P = \rho_2 g h_2 - \rho_1 g h_1}$$

(ii) For Vacuum Pressure :

Pressure above A-A in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + P$

Pressure head in the right column above A-A = 0.

Hence, equating two ; $\rho_2 g h_2 + \rho_1 g h_1 + P = 0$.

$$\boxed{P = -(\rho_1 g h_1 + \rho_2 g h_2)}$$

Differential Manometers :-

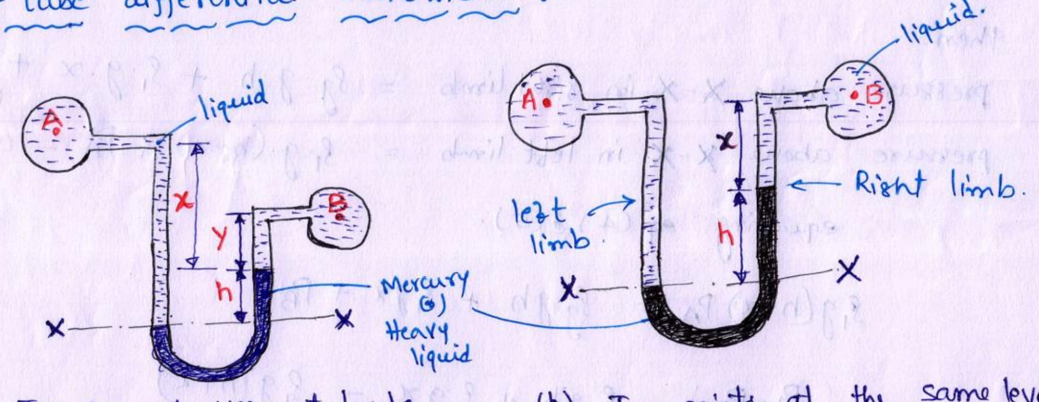
→ Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe (or) in two different pipes.

→ It consists of a U-tube, containing a heavy liquid.

→ Most commonly types of differential manometers are:

1. U-tube differential manometer
2. Inverted U-tube differential manometer.

U-tube differential manometer :-



(a) Two pipes at different levels. (A & B)

(b) Two points at the same level. (A & B)

Let, h = difference of mercury level in U-tube

y = distance of the centre of B, from the mercury level in right limb.

x = distance of the centre of A, from the mercury level in right limb.

ρ_1 = Density of liquid at A

ρ_2 = Density of liquid at B.

Taking datum line, X - X.

pressure above X-X in left limb = $\rho_1 g(h+x) + P_A$ —— (1)

pressure above X-X in right limb = $(\rho_g \cdot g \cdot h) + (\rho_2 \cdot g \cdot y) + P_B$ —— (2)

Equating equations (1) & (2),

$$\rho_1 g(h+x) + P_A = (\rho_g \cdot g \cdot h) + (\rho_2 \cdot g \cdot y) + P_B$$

$$P_A - P_B = (\rho_g \cdot g \cdot h) + (\rho_2 \cdot g \cdot y) - \rho_1 g h - \rho_1 g x.$$

$$P_A - P_B. = h \cdot g(\rho_g - \rho_1) + \rho_2 \cdot g \cdot y - \rho_1 g \cdot x.$$

∴ Two pipes at different level,

$$\boxed{P_A - P_B = h \cdot g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g \cdot x.} \quad \text{——— (3)}$$

∴ If two pipes at same level and contains the same liquid of density ρ_1,

then.

pressure above X-X in right limb $= \rho_g \cdot g \cdot h + \rho_1 \cdot g \cdot x + P_B$ — (4)

pressure above X-X in left limb $= \rho_1 \cdot g \cdot (h+x) + P_A$ — (5)

equating eq (4) & (5).

$$\rho_1 g(h+x) P_A = \rho_g g h + \rho_1 g x + P_B$$

$$P_A - P_B = \rho_g g h + \rho_1 g x - \rho_1 g(h+x)$$

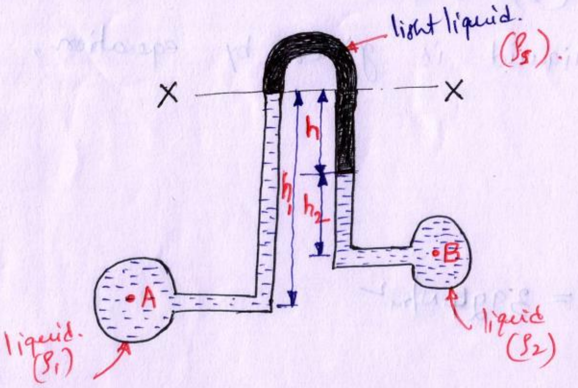
$$= \rho_g \cdot g \cdot h + \cancel{\rho_1 g x} - \rho_1 g h - \cancel{\rho_1 g x}$$

$$\boxed{P_A - P_B. = h \cdot g(\rho_g - \rho_1)} \quad \text{——— (6)}$$

Inverted U-tube differential Manometer :-

→ It consists of an inverted U-tube containing a light liquid.

→ It is used for measuring difference of low pressures.



Let, h_1 = Height of liquid in left limb below the datum line X-X

h_2 = Height of liquid in right limb

h = Difference of light liquid

ρ_1 = Density of liquid at A

ρ_2 = Density of liquid at B

ρ_s = Density of light liquid.

P_A = Pressure at A

P_B = Pressure at B. Taking X-X as datum line.

The pressure in the left limb below X-X, $= P_A - (\rho_1 \cdot g \cdot h_1)$. — (1)

pressure in the right limb below X-X, $= P_B - (\rho_2 g h_2) - (\rho_s \cdot g \cdot h)$ —(2)

equating (1) & (2),

$$P_A - (\rho_1 \cdot g \cdot h) = P_B - \rho_2 g h_2 - \rho_s \cdot g \cdot h.$$

$$\boxed{P_A - P_B = (\rho_1 \cdot g \cdot h_1) - (\rho_2 g h_2) - (\rho_s \cdot g \cdot h)} \quad \text{—(3)}$$

Problems on Pressure Measurement

① Calculate the pressure due to column of 0.4 m of (a) water (b) an oil of sp. gr. 0.8 and (c) mercury of sp. gr. 13.6. Take density of water, $\rho = 1000\ kg/m^3$.

Sol:- Height of liquid column $(Z) = 0.4\ m$

pressure at any point in liquid is given by equation,

$$P = \rho g Z.$$

(a) For water,

$$\rho = 1000\ kg/m^3$$

$$P = 1000 \times 9.81 \times 0.4 = 3924\ N/m^2$$

(b) For oil of sp. gravity, 0.8.

$$\rho_{oil} = \text{Sp. gravity}_{oil} \times \rho_{water}$$

$$= 0.8 \times 1000$$

$$= 800\ kg/m^3$$

$$P = 800 \times 9.81 \times 0.4 = 3139.2\ N/m^2$$

(c) For mercury,

$$\rho_{mercury} = \text{Sp. gr.}_{mercury} \times \rho_{water}$$

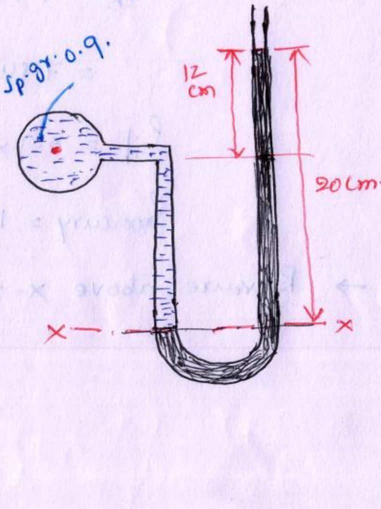
$$= 13.6 \times 1000$$

$$= 13600\ kg/m^3$$

$$P = 13600 \times 9.81 \times 0.4$$

$$= 53366.4\ N/m^2$$

(2) The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gr. 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.



Sol:- Given data,

→ Sp. gravity of fluid $= 0.9$

Density of fluid, $\rho_f = 0.9 \times \rho_w$

$= 0.9 \times 1000$

$\rho_1 = \rho_f = 900 \text{ kg/m}^3$

→ Sp. gravity of mercury $= 13.6$

Density of mercury, $\rho_m = 13.6 \times \rho_w$

$= 13.6 \times 1000$

$\rho_2 = \rho_m = 13600 \text{ kg/m}^3$

→ Height of mercury in right limb $h_2 = 20 \text{ cm} = 0.2 \text{ m}$.

→ Height of fluid in left limb $h_1 = 20 - 12 = 8 \text{ cm} = 0.08 \text{ m}$.

→ Let, P - Pressure of fluid in pipe

Equating the pressure above datum line X-X, we get

$$P + \rho_1 g h_1 = \rho_2 g h_2$$

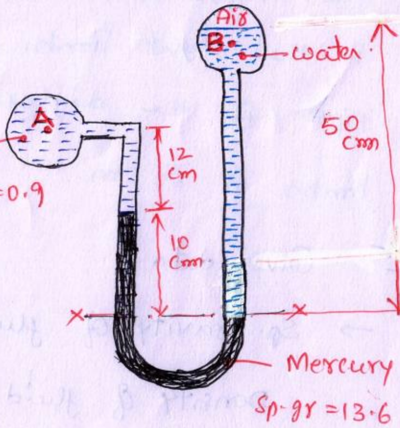
$$P = \rho_2 g h_2 - \rho_1 g h_1$$

$$= (13600 \times 9.81 \times 0.2) - (900 \times 9.81 \times 0.08)$$

$$= 26683.2 - 706.32$$

$$= 25976.88 \text{ N/m}^2$$

(3) A differential manometer is connected at the two points "A" and "B" as shown in fig. At 'B' air pressure is 7.848 N/cm² (abs), Find the absolute pressure at 'A'.



Sol:

Air pressure at 'B'

$$P_B = 7.848 \ N/cm^2$$

$$= 7.848 \times 10^4 \ N/m^2$$

$$\rho_{oil} = 0.9 \times 1000 = 900 \ kg/m^3$$

$$\rho_{mercury} = 13.6 \times 1000 = 13600 \ kg/m^3$$

→ Pressure above x-x in the right limb,

$$= P_B + (\rho_w \cdot g \cdot 0.5)$$

$$= (7.848 \times 10^4) + (1000 \times 9.81 \times 0.5)$$

$$= 83{,}385 \ N/m^2$$

→ Pressure above x-x in the left limb,

$$= P_A + [\rho_{oil} \cdot g \cdot (0.12)] + [\rho_{mercury} \times g \times 0.1]$$

$$= P_A + (900 \times 9.81 \times 0.12) + (13600 \times 9.81 \times 0.1)$$

$$= P_A + 14401.08$$

Equating two pressure heads,

$$P_A + 14401.08 = 83385$$

$$P_A = 83385 - 14401.08$$

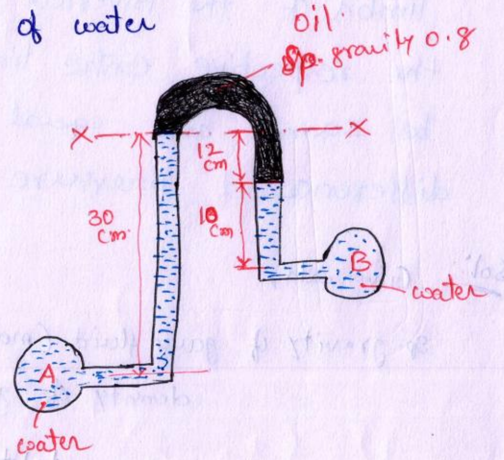
$$= 68983.92 \ N/m^2$$

$$P_A = \underline{\underline{6.898 \ N/cm^2}}$$

(4) Water is flowing through two different pipes to which an inverted differential manometer having an oil of Sp. gr. 0.8 is connected. The pressure head in the pipe "A" is 2m of water, find the pressure in the pipe "B" for the manometer readings as shown in Fig.

Sol:

pressure head at "A" = 2m of water



$P_A = \rho \cdot g \cdot Z$

$= 1000 \times 9.81 \times 2$

$P_A = 19620 \ N/m^2$

Pressure below datum line X-X in the left limb.

$= P_A - (\rho_w \cdot g \cdot 0.3)$

$= 19620 - (1000 \times 9.81 \times 0.3)$

$= 16677 \ N/m^2$

Pressure below datum line X-X in right limb

$\rho_{oil} = Sp.gr.oil \times \rho_{water}$

$\rho_{oil} = 0.8 \times 1000 = 800 \ N/m^3$

$= P_B - (\rho_w \cdot g \cdot 0.1) - (\rho_{oil} \times g \times 0.12)$

$= P_B - (1000 \times 9.81 \times 0.1) - (800 \times 9.81 \times 0.12)$

$= P_B - 1922.76.$

Equating two Pressures,

$P_B - 1922.76 = 16677$

$P_B = 18599.76 \ N/m^2$

$P_B = 1.8599 \ N/cm^2$

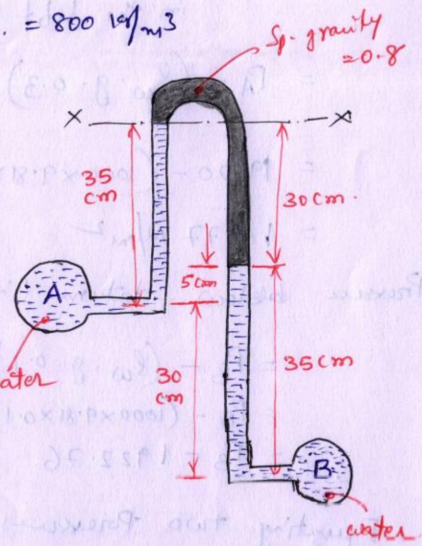
(5) An inverted U-tube manometer is connected to two horizontal pipes A & B, through which water is flowing. The vertical distance between the axes of these pipes is 30 cm. When an oil of sp. gravity 0.8 is used as a gauge fluid, the vertical heights of water columns in the two limbs of the inverted manometer (when measured from the respective centre lines of the pipes) are found to be same and equal to 35 cm. Determine the difference of pressure between the pipes. [Ans: 2354.4 N/m²]

Sol: Given data,

sp. gravity of gauge fluid (manometer fluid) = 0.8

density of gauge fluid = 0.8 × 1000

$\rho_{oil} = 800$ kg/m³



Pressure below x-x in the left limb,

$= P_A - (\rho_w \times g \times 0.35)$

$= P_A - (1000 \times 9.81 \times 0.35)$

$= P_A - 3433.5$

Pressure below x-x in the right limb,

$= P_B - (\rho_{oil} \times g \times 0.3) - (\rho_w \times g \times 0.35)$

$= P_B - (800 \times 9.81 \times 0.3) - 3433.5$

Equating two pressure heads,

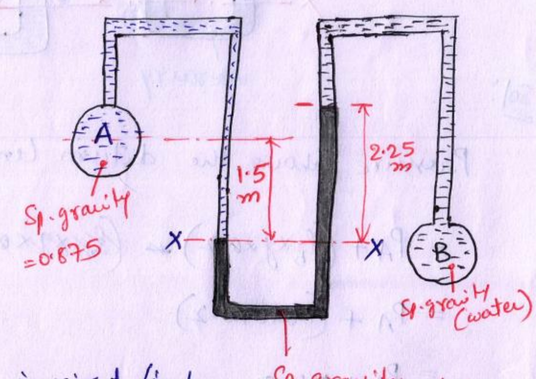
$\Delta P = P_B - P_A = (800 \times 9.81 \times 0.3)$

$= 2354.4$ N/m²

$= 0.2354$ N/cm².

⑥ Two pipes as shown in Figure convey toluence of sp. gravity 0.875 and water respectively. Both the liquids in the pipes are under pressure. The pipes are connected to a U-tube manometer and hoses connecting the pipes to the tubes are filled with the corresponding liquids. Find the difference of pressure in two pipes if the level of manometric liquid having sp. gravity 1.25 is 2.25 m higher in the right limb than the lower level of toluence in the left limb of the manometer.

Sol:



pressure head above datum line X–X in the left limb is

$= P_A + (0.875 \times 1000 \times 9.81 \times 1.5)$

$= P_A + 12875.625$

Pressure head above datum line X–X in right limb,

$= P_B - (2.25 \times 9.81 \times \rho_w) + (2.25 \times 9.81 \times 1.25 \times 1000)$

$= P_B - (2.25 \times 9.81 \times 1000) + (2.25 \times 9.81 \times 1.25 \times 1000)$

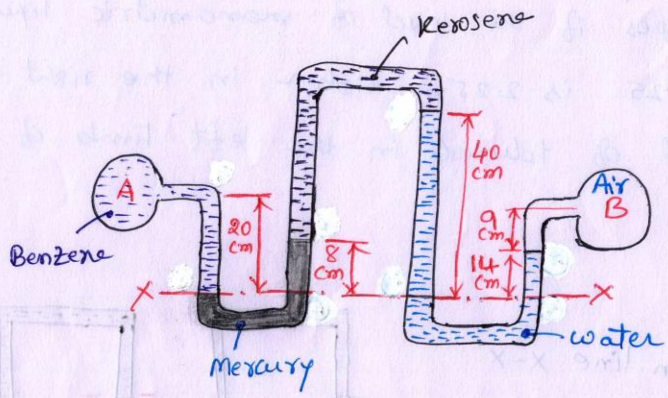
$= P_B - 27590.62 + 22072.5$

Equating two pressure heads,

$P_B - P_A = -27590.62 + 22072.5 - 12875.625$

$= 7357.5 \ N/m^2$

(7) In the Fig. below all fluids are at 20°C. Determine the pressure difference (Pa) between points A and B. Take the specific weights to be Benzene: $8640\ N/m^3$, Mercury $= 133100\ N/m^3$, Kerosene $7885\ N/m^3$, water $9790\ N/m^3$. (JNTUK - 2017) 10 Marks.



$\rho_{Benze} \times g = 8640\ N/m^3$

$\rho_{Kerosene} \times g = 7885\ N/m^3$

$\rho_{mercury} \times g = 133100\ N/m^3$

$\rho_{water} \times g = 9790\ N/m^3$

$\rho_{air} \times g = 1.2041 \times 9.81$

$= 11.81\ N/m^3$

Sol:

Pressure above the datum line X-X, in the left limb is,

$= P_A + (\rho_B \times g \times 0.2) \cancel{- (\rho_m \times g \times 0.08)}$

$= P_A + (8640 \times 0.2)$

$= P_A + 1728$

Pressure above the datum line X-X, in the Right limb is,

$= P_B + (\underbrace{\rho_{air} \times g}_{11.81} \times 0.9) + (\underbrace{\rho_w \times g}_{9790} \times 0.14) - (\underbrace{\rho_w \times g}_{9790} \times 0.4) + (\underbrace{\rho_{Kerosene} \times g}_{7885} \times (0.4 - 0.08))$

$+ (\underbrace{\rho_{mercury} \times g}_{133100} \times 0.08)$.

$= P_B + 10.629 + 1370.6 - 3916 + 2523.2 + 10648.$

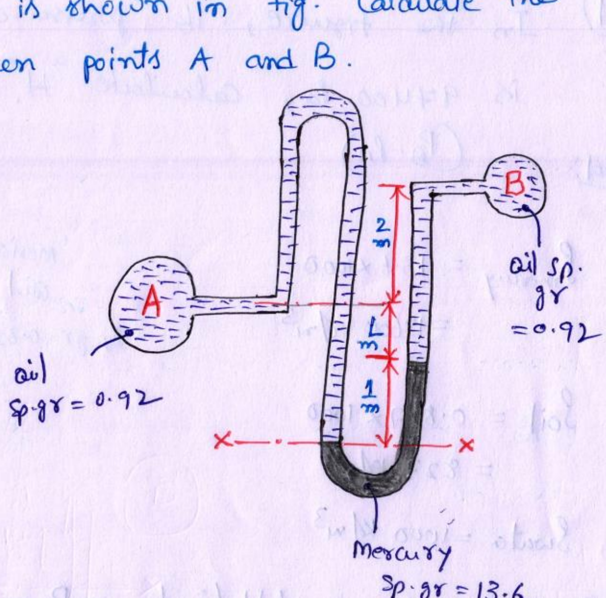
$= P_B + 10636.429$

Equating two pressure heads,

$P_A + 1728 = P_B + 10636.429 \Rightarrow P_A - P_B = 10636.429 - 1728$

$= \underline{\underline{8908.429\ N/m^2}}$

(8) A differential manometer is shown in fig. Calculate the pressure difference between points A and B.



Sol:-

$\rho_{oil} = 0.92 \times 1000 = 920 \ kg/m^3$

$\rho_{mercy} = 13.2 \times 1000 = 13600 \ kg/m^3$

Pressure head in left limb,

$= P_A + (\rho_{oil} \times g \times (L+1))$

$= P_A + [920 \times 9.81 \times (L+1)] = P_A + 9025.2 + (9025.2 \times L)$.

Pressure head in right limb,

$= P_B + [\rho_{oil} \times g \times (2+L)] + (\rho_m \times g \times 1)$

$= P_B + [920 \times 9.81 \times (2+L)] + (13600 \times 9.81 \times 1)$

$= P_B + 18050.4 + (9025.2 \times L) + 133416$.

Equating two pressure heads,

$$P_A + 9025.2 + (\cancel{9025.2 \times L}) = P_B + 18050.4 + (\cancel{9025.2 \times L}) + 133416$$

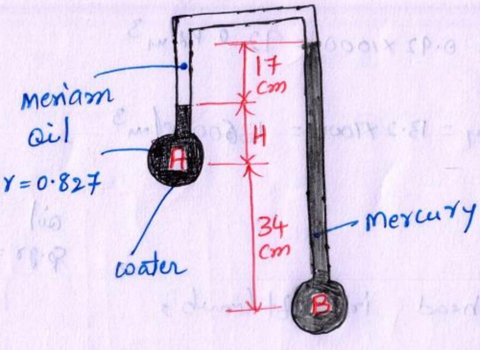
$$P_A - P_B = 18050.4 + 133416 - 9025.2$$

$$= 142441.2 \ N/m^2$$

$$= 14.24 \ N/cm^2$$

⑨ In the figure, if pressure difference between points A & B is 97400 Pa, calculate "H".

Sol:- $(P_B - P_A)$



$\rho_{mercury} = 13.6 \times 1000$

$= 13600 \ kg/m^3$

$\rho_{oil} = 0.827 \times 1000$

$= 827 \ kg/m^3$

$\rho_{water} = 1000 \ kg/m^3$

Pressure head in left limb, $= P_A - \left(1000 \times 9.81 \times \frac{H}{100}\right) - \left(827 \times 9.81 \times \frac{17}{100}\right)$.

H in cm

$\frac{H}{100}$ in m.

Pressure head in right limb, ~~P_B~~

$= P_B - \left[13600 \times 9.81 \times \left(\frac{34+H+17}{100}\right)\right]$

Equating,

$$P_A - P_B = \left(1000 \times 9.81 \times \frac{H}{100}\right) - \left(13600 \times 9.81 \times \frac{34+H+17}{100}\right) + 1379.18$$

$$-97,400 = (98.1 \times H) - 1334.16\,(51+H). + 1379.18$$

$$-97400 = (98.1 \times H) - 68042.16 - (1334.16 \times H) + 1379.18$$

$$-97400 = 98.1\,H - 68042.16 - 1334.16\,H + 1379.18$$

$$-97400 = -(H \cdot 1236.06) - 68042.16 + 1379.18$$

$$H \times 1236.06 = +97400 - 68042.16 + 1379.18$$

$$\cancel{H = \frac{164062.98}{1236.06}} \quad H = \frac{30737.02}{1236.06}$$

$$H = 24.866 \text{ cm.}$$

Buoyancy and floatation

UNIT-1
Part-B

Buoyancy :-

When a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body and is called the force of buoyancy or simply Buoyancy.

Centre of Buoyancy :-

It is defined as the point, through which the force of buoyancy is supposed to act. As the force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body, the centre of buoyancy will be the centre of gravity of the fluid displaced.

Meta Centre :-

→ It is defined as the point about which a body starts oscillating when the body is tilted by a small angle.

→ The meta-centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

→ Consider a body floating in a liquid as shown in Figure (a).
· Let the body is in equilibrium and "G" is the centre of gravity and "B" is the centre of buoyancy. For equilibrium, both the points lie on the normal axis, which is vertical.

Figure (a).

→ Let the body is given a small angular displacement in the clockwise direction as shown in figure (b).

→ The centre of buoyancy, which is the centre of gravity of the displaced liquid (or) centre of gravity of the portion of the body sub-merged in liquid, will now be shifted towards right from the normal axis.

→ Let it is at "B_1" as shown in figure (b); The line of action of the force of buoyancy in this new position, will intersect the normal axis of the body at the same point say "M." This point 'M' is called Meta-Centre.

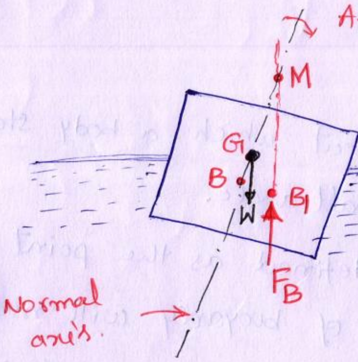


Figure (b): Meta-Centre.

F_B → Buoyancy force,
B → Centre of Buoyancy.
B_1 → Centre of Buoyancy new position.
G → Centre of gravity.
W → weight
M → meta centre.

Meta Centric Height :-

The distance MG, i.e., the distance between the meta-centre of a floating body and the centre of gravity of the body is called meta-centric height.

Calculation of Meta-Centre Height :-

(i) Analytical Method :-

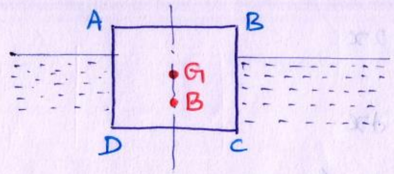


Fig: (a)

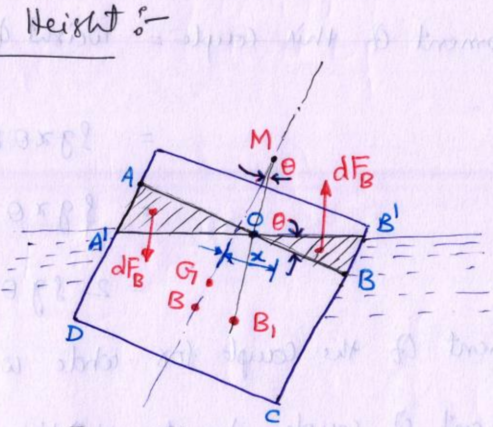


Fig: (b)

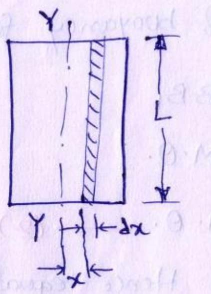


Fig: (c). plan of body at water line.

Consider towards the right of the axis is a small strip of thickness "dx" at a distance 'x' from 'O' as shown in Figure (b).

The height of strip, $x \cdot \angle BOB' = x \cdot \theta$.

Area of strip = Height × thickness $\{\angle BOB' = \angle AOA' = BMB_1' = \theta\}$

$= (x \cdot \theta) \times (dx)$.

Volume of strip = Area × Length

$= (x \cdot \theta \cdot dx) \times L$.

weight of the strip = $\rho \cdot g$ × volume = $\rho g\, \theta \cdot L \cdot x \cdot dx$.

similarly, if a small strip of thickness 'dx' at a distance 'x' from 'O' towards the left of the axis is considered, the weight of strip will be ~~towards the left of the axis is considered, the~~ $\rho g x \theta L dx$. The two weights are acting in the opposite direction and hence constitute a couple.

Moment of this couple = weight of each strip × distance b/w these two weights.

$$= \rho g x \theta L dx \, [x + x]$$

$$= \rho g x \theta L dx \; 2x$$

$$= 2 \rho g \theta \cdot L \cdot x^2 \cdot dx$$

Moment of the couple for whole wedge $= \int 2\rho g \theta L \cdot x^2 \cdot dx$ — (1)

Moment of couple due to shifting of centre of buoyancy from B to B_1.

$$= F_B \times BB_1$$

$$= F_B \cdot BM \cdot \theta.$$

$$= W \cdot BM \cdot \theta. \quad \text{— (2)}$$

But, these two couples are the same, Hence equating ① & ②

$$W \cdot BM \cdot \theta = \int 2 \rho g \theta L x^2 dx$$

$$W \cdot BM \cdot \cancel{\theta} = 2\rho g \cancel{\theta} \int L x^2 dx.$$

$$W \cdot BM = 2 \rho g \int x^2 L dx.$$

$$W \cdot BM = 2\rho g \int x^2 \cdot dA$$

$L dx$ = Element area on water line as shown in Fig.(c).

$L \cdot dx = dA$.

But, Fig(c), It is clear that $2\int x^2 dA$ is the second moment of area of the plan of the body at water surface about the axis Y–Y. Therefore

$$W \cdot BM = \rho g I. \quad \Rightarrow \quad \boxed{I = 2\int x^2 \cdot dA.}$$

$$BM = \frac{\rho g I}{W}.$$

substitute, $W = \rho g V$

$$= \frac{\rho g I}{\rho g \cdot V.} = \frac{I}{V.}$$

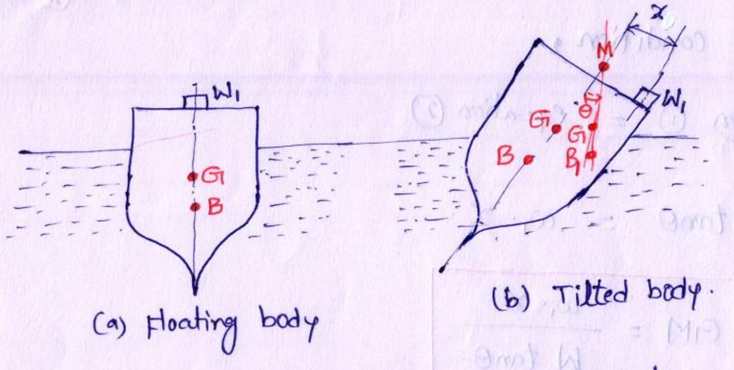
$$GM = BM - BG = \frac{I}{V} - BG$$

W = weight of body
= weight of fluid displaced by body
= ρg × volume of fluid displaced by the body
= ρg × volume of body-submerged in water

$\boxed{W = \rho g \times V}$

$$\boxed{\therefore \text{Meta-Centric Height } (GM) = \frac{I}{V} - BG}$$

(ii) Experimental Method :-



(a) Floating body

(b) Tilted body.

Fig: Meta-centric height.

W_1 – weight placed over the centre of the vessel as shown in Fig. (a).

G – Centre of gravity of the vessel

W – Weight of vessel including w_1.

B – Centre of buoyancy of the vessel.

→ The weight (w_1) is moved across the vessel towards right through a distance x as shown in Fig. (b). Then the vessel will be tilted.

→ The new centre of gravity of the vessel will shift to G_1 as the weight w_1 has been moved towards the right.

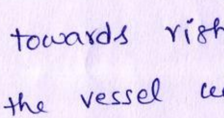
→ Also, centre of buoyancy will change to B_1.

→ Under equilibrium, the momentum caused by the movement of the load w_1 through a distance x must be equal to the moment caused by the shift of the centre of gravity from G to G_1. Thus,

The momentum due to change of gravity,

$$G = GG_1 \times W$$

$$G = W \times GM \cdot \tan\theta \quad \text{---} (1)$$



$$\tan\theta = \frac{GG_1}{GM}$$

The moment due to movement of $\omega_1 = \omega_1 \times x$. — (2)

Under equilibrium condition,

equation (1) = equation (2)

$$W \cdot GM \tan\theta = \omega_1 \cdot x.$$

$$\boxed{GM = \frac{\omega_1 \cdot x}{W \tan\theta}}$$

Stability of a sub-merged body :-

→ Stability :

A sub-merged (or) floating body is said to be stable if it comes back to its original position after a slight disturbance.

→ The relative position of centre of gravity (G) and centre of buoyancy (B) of a body determines the stability of a sub-merged body.

→ "The position of centre of gravity and centre of buoyancy in case of a completely sub-merged body are fixed".

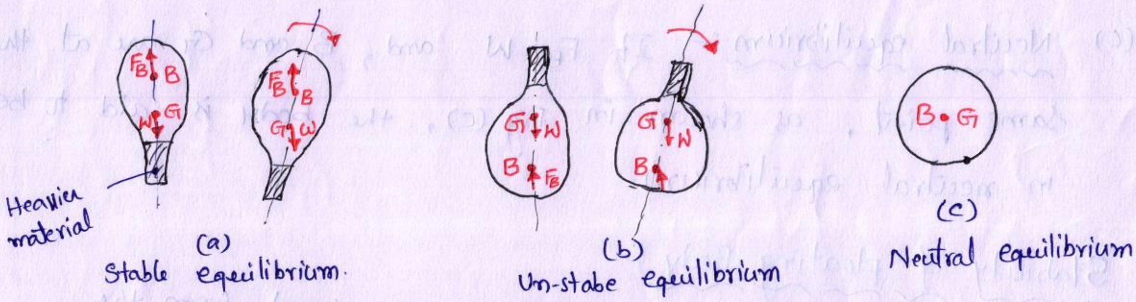


Figure: Stabilities of submerged bodies.

→ Consider a ballon, which is completely sub-merged in air. Let the lower portion of the ballon contains heavier material, so that its centre of gravity (G) is lower than its centre of buoyancy (B) as shown in Fig (a).

→ The weight of the balloon (W) is acting through G, vertically downward direction. The buoyant force (F_B) is acting vertically up, through B.

→ If $W = F_B$, the balloon will in equillibrium.

→ If the ballon is given an angular displacement in the clock-wise direction as shown in Fig (a), then the W and F_B constitute a couple acting in the anti-clock wise direction and brings the balloon in the original position. Thus the balloon is in stable equilibrium.

(a) Stable equilibrium :- When $W = F_B$ and point 'B' is above 'G', the body is said to be in stable equilibrium.

(b) Unstable equilibrium :- If $W = F_B$, but the centre of buoyancy (B) is below centre of gravity (G), the body is in unstable equilibrium as shown in Fig (b).

A slight angular displacement to the body, in the clock wise direction, gives the couple due to W and F_B also in the clockwise direction. Thus the body does not return to its original position and hence the body is in unstable equilibrium.

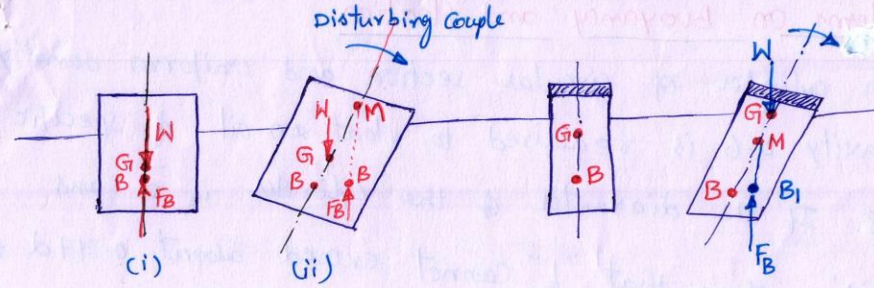
(c) Neutral equilibrium :- If $F_B = W$ and, B and G are at the same point, as shown in Fig (c), the body is said to be in neutral equilibrium.

Stability of Floating Body :-

The stability of a floating body is determined from the position of Meta-centre (M). In case of floating body, the weight of the body is equal to the weight of liquid displaced.

(a) Stable equilibrium. If the point M is above G, the floating body will be in stable equilibrium as shown in Fig (a). [next page].

If a slight angular displacement is given to the floating body in the clockwise direction, the centre of buoyancy shifts from B to B_1 such that the vertical line through B_1 cuts at M. Then buoyant force F_B through B_1 and weight W through G constitute a couple acting in the anti-clock wise direction and they bringing the floating body in the original position.



(a) Stable equilibrium M is above G (b) Unstable equilibrium M is below G.

Figure: Stability of floating bodies:

(b) Unstable equilibrium: If the point M is below G, the floating body will be in unstable equilibrium as shown in fig(b). The disturbing couple is acting in the clockwise direction. The couple due to buoyant force F_B and W is also acting in the clockwise direction and thus overturning the floating body.

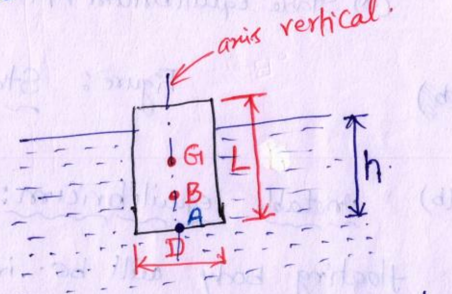
(c) Neutral equilibrium: If the point M is at the centre of gravity of the body, the floating body will be in neutral equilibrium.

Equilibrium conditions :-

Equilibrium	Floating body	Sub-merged body.
(i) Stable equilibrium	M is above G	B is above G
(ii) Unstable equilibrium	M is below G	B is below G
(iii) Neutral equilibrium	M and G Coincide	B and G Coincide.

Problems on Buoyancy and floation

(1) A wooden cylinder of circular section and uniform density, specific gravity 0.6 is required to float in oil of specific gravity 0.8. If the diameter of the cylinder is 'd' and length is 'L', show that "L" cannot exceed about 0.817d for cylinder to float with its longitudinal axis vertical.



Sol:- Cylinder dia (D) = D m.

Length of cylinder = L m.

Sp. gravity of cylinder, = 0.6

Sp. gravity of oil, = 0.8

h → Depth of cylinder immersed.

For the principle of Buoyancy,

weight of cylinder = weight of oil displaced.

$$\left(\rho_{cylinder} \times g \times \text{Volume of cylinder}\right) = \left(\rho_{oil} \times g \times \text{vol. of oil displaced by body.}\right)$$

$$\frac{\pi}{4}D^2 L \times g \times \rho_w \times \text{Sp.gr. of cylinder} = \rho_w \times \text{Sp.gr. oil} \times g \times \frac{\pi}{4}D^2 \times h.$$

$$\cancel{\left(\frac{\pi}{4}D^2\right)} L \times 9.81 \times 1000 \times 0.6 = 1000 \times 0.8 \times 9.81 \times \cancel{\left(\frac{\pi}{4}D^2\right)} \times h.$$

$$L \times 0.6 = 0.8\, h.$$

$$h = 0.75\,L.$$

The distance of centre of gravity "G" from "A", $AG = \frac{L}{2}$

The distance of centre of Buoyancy "B" from "A", $AB = \frac{h}{2}$

$$\therefore BG = AG - AB = \frac{L}{2} - \frac{h}{2} = \frac{L}{2} - \frac{0.75L}{2} = 0.125\,L$$

$$I = \frac{\pi}{64}D^4 \quad ; \quad \forall = \frac{\pi}{4}D^2 \times h.$$

$$\frac{I}{\forall} = \frac{\frac{\pi}{64}D^4}{\frac{\pi}{4}D^2 h} = \frac{1}{16}\frac{D^2}{h} = \frac{1}{16}\frac{D^2}{(0.75L)} = 0.0833\frac{D^2}{L}.$$

Meta-centric height $\therefore GM = \frac{I}{\forall} - BG = 0.0833\frac{D^2}{L} - 0.125L$

For stable equilibrium, GM should be "+ve" or

$GM > 0.$

$\left(0.0833 \frac{D^2}{L} - 0.125L\right) > 0.$

$0.0833 \frac{D^2}{L} > 0.125L$ (or) $0.125L > 0.$

$\frac{0.0833}{0.125} > \frac{L^2}{D^2}$

$\frac{L^2}{D^2} < \frac{0.0833}{0.125}$

$\frac{L}{D} < \sqrt{\frac{0.0833}{0.125}}$

$\frac{L}{D} < 0.8164.$

$\boxed{\therefore L < 0.816 D}$

(2) A ship of weight 32000 KN is floating in a sea water. The center of buoyancy is 1.6 m below its center of gravity. The momentum of inertia of the ship area at the water level is 8320 m^4 if the radius of gyration of the ship is 3.2 m. Find the its period of rolling. Sp. weight of sea water is 10100 N/m^3.

Sol:-

$I = 8320\ m^4$

$W = 32 \times 10^6\ N$

$K = 3.2m$

$BG = 1.6m$

$\forall$ = volume of water displaced

$= \frac{\text{weight of ship}}{\text{Sp. wt of sea water}} = \frac{32 \times 10^6}{10100} = 3168.31\ m^3$

$GM = \frac{I}{\forall} - BG = \frac{8320}{3168.31} - 1.6 = 1.026m.$

$\therefore$ Time period of oscillation $T = 2\pi \sqrt{\frac{K^2}{GM \cdot g}}$

$= 2 \times \pi \times \sqrt{\frac{(3.2)^2}{1.026 \times 9.81}}$

$= 6.39$ sec.

$T \approx 7$ sec.

25

UNIT-2
Part-A

Fluid Kinematics

→ Kinematics is defined as that branch of science which deals with motion of particles without considering the forces ~~acting~~ causing the motion.

→ Fluid mechanics: The velocity at any point in a flow field at any time is studied in this branch of Fluid mechanics.

→ The fluid motion is described by two methods.

(i) Lagrangian method
(ii) Eulerian method.

→ Lagrangian method :- a single fluid particle is followed during its motion and its velocity, acceleration, density, etc. are described.

→ Eulerian method :- The velocity, acceleration, pressure, density etc., are described at a point in a flow field. This method is commonly used in fluid mechanics.

⇒ Fluid flow types :-

(i) Steady and Unsteady flows
(ii) Uniform and non-uniform flows
(iii) Laminar and turbulent flows
(iv) Compressible and incompressible flows
(v) Rotational and irrotational flows, and.
(vi) One, two and three-dimentional flows.

(1) Steady and Unsteady flow :- The fluid characteristics like velocity, pressure, density, etc., at a point do not change with time. Mathematically,

$$\boxed{\left(\frac{\partial u}{\partial t}\right)_{(x_0,y_0,z_0)} = 0 \quad ; \quad \left(\frac{\partial P}{\partial t}\right)_{(x_0,y_0,z_0)} = 0 \quad ; \quad \left(\frac{\partial \rho}{\partial t}\right)_{(x_0,y_0,z_0)} = 0.}$$

where, (x_0, y_0, z_0) is a fixed point in fluid field.

Unsteady flow :- The velocity, pressure, density at a point changes with time. Mathematically,

$$\boxed{\left(\frac{\partial u}{\partial t}\right)_{(x_0,y_0,z_0)} \neq 0 \quad ; \quad \left(\frac{\partial P}{\partial t}\right)_{(x_0,y_0,z_0)} \neq 0 \quad ; \quad \left(\frac{\partial \rho}{\partial t}\right)_{(x_0,y_0,z_0)} \neq 0.}$$

(2) Uniform and Non-Uniform flow :-

⇒ Uniform flow :- The velocity at any given time does not change with respect to space (i.e. length of direction of the flow). Mathematically,

$$\boxed{\left(\frac{\partial u}{\partial s}\right)_{t=\text{constant}} = 0}$$

where, ∂u = change of velocity

∂s = Length of flow in the direction s.

⇒ Non-Uniform flow :- The velocity at any given time changes with respect to space. Mathematically,

$$\boxed{\left(\frac{\partial u}{\partial s}\right)_{t=\text{constant}} \neq 0.}$$

(3) Laminar and Turbulent flow :-

→ Laminar flow :- It is defined as that type of flow in which the fluid particles move along well-defined paths (or) stream line and all the stream-lines are straight and parallel. Thus the particles move in laminar (or) layers gliding smoothly over the adjacent layer. This type of flow is also called stream-line flow (or) viscous flow.

⇨ Turbulent flow :- It is the type of flow in which the fluid particles move in a zig-zag way. Due to the movement of fluid particles in a zig-zag way, the eddies formation takes place which are responsible for high energy loss.

→ For a pipe flow, the type of flow is determined by a non-dimensional number $\frac{UD}{\nu}$ called the Reynold number (Re).

→ $Re < 2000$, the flow is laminar

→ $2000 < Re < 4000$, the flow is transition

→ $Re > 4000$, the flow is turbulent.

where,
$U \rightarrow$ velocity of flow in pipe
$D \rightarrow$ pipe diameter (inner)
$\nu \rightarrow$ Kinematic viscosity of fluid.

(4) Compressible and incompressible flows :-

⇨ Compressible flow :- It is defined as that type of flow in which the density of the fluid changes from point to point (or) in other words the density is not constant for the fluid. Thus, mathematically, $\boxed{\rho \neq \text{constant}}$.

⇨ Incompressible flow :- It is defined as that type of flow in which the density is constant for the fluid flow. Liquids are generally incompressible while gases are compressible. Mathematically,

$$\boxed{\rho = \text{Constant}}$$

(5) Rotational and Irrotational flow :-

⇨ Rotational flow :- It is defined as that type of flow in which the fluid particles while flowing along stream-lines, also rotate about their own-axis. And if the fluid particles while flowing along stream-lines, do not rotate about their own axis then that type of flow is called irrotational flow.

(6) One, Two and Three - Dimensional Flow :-

⇒ **One - Dimensional flow (1-D) :** It is defined as that the type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only, say "x".

→ For a steady one-dimensional flow, the velocity is a function of one-space-co-ordinate only.

→ The variation of velocities in other two mutually perpendicular directions is assumed negligible.

Mathematically, For 1-D flow $\boxed{u = f(x), \quad v = 0 \text{ and } w = 0}$.

$u \rightarrow$ velocity in x - co-ordinate
$v \rightarrow$ velocity in y - co-ordinate
$w \rightarrow$ velocity in z - co-ordinate

⇒ **Two - Dimensional flow (2-D) :-** It is the type of flow in which the velocity is a function of time and two rectangular space co-ordinates say "x" and "y".

→ For a steady two-dimensional flow the velocity is a function of two-space co-ordinates only. The variation of velocity in third direction is negligible. Mathematically, for 2-D flow

$$\boxed{u = f_1(x, y), \quad v = f_2(x, y) \quad \text{and} \quad w = 0.}$$

⇒ **Three - Dimensional flow (3-D) :-** In which the velocity is a function of time and three mutually perpendicular directions. But for

→ a steady three-dimensional flow the velocity is function of three space co-ordinates (x, y and z) only. Mathematically, for 3-D flow

$$\boxed{\begin{aligned} u &= f_1(x, y, z) \\ v &= f_2(x, y, z) \\ w &= f_3(x, y, z). \end{aligned}}$$

Rate of flow (or) Discharge :-

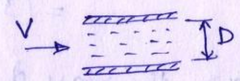
It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel.

→ For an incompressible fluid or (liquid) the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second.

→ For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section.

For liquids ⇒ Q – m^3/s (or) Lit/s

For gases ⇒ Q – kgf/s (or) N/s.



Consider a liquid flowing through a pipe, Then

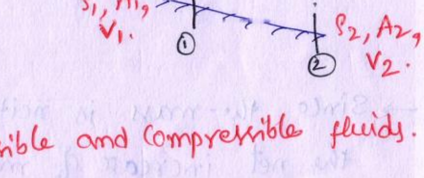
Discharge, $\boxed{Q = A \times V.}$ where, V → Average velocity of fluid across the section.

A → Cross-sectional area of pipe $(\frac{\pi}{4} D^2)$.

Continuity Equation :-

→ The equation based on the principle of conservation of mass is called continuity equation. (or) a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.

→ Consider two cross-sections of a pipe as shown in figure.



The rate of flow at section 1-1 = $\rho_1 A_1 V_1$.

The rate of flow at section 2-2 = $\rho_2 A_2 V_2$

According to law of conservation of mass,

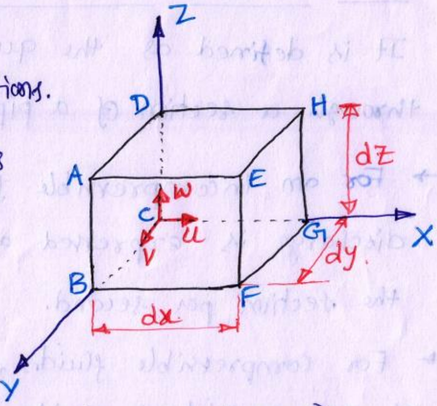
Rate of flow at 1-1 = Rate of flow at 2-2

Continuity equation ⇒ $\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$ – For incompressible and compressible fluids.

If the fluid is incompressible ($\rho_1 = \rho_2$), the $\boxed{A_1 V_1 = A_2 V_2}$

Continuity equation for One-Dimensional flow :-

- Consider a fluid element of lengths dx, dy and dz in x, y and z directions.
- Let u, v, w are the inlet velocity components in x, y, z directions respectively.



→ The mass of fluid entering the face ABCD per second

$$= (\rho) \times (\text{Velocity in } x\text{-direction}) \times (\text{Area of ABCD}).$$

$$= (\rho) \times (u) \times (dy \cdot dz).$$

→ Now, the mass of fluid leaving the face EFGH per second is

$$= (\rho u \, dy \cdot dz) + \frac{\partial}{\partial x}(\rho u \, dy dz)\, dx.$$

$\therefore$ Gain of mass in x-direction = Mass through ABCD − Mass through EFGH per second

$$= \cancel{\rho u \, dy dz} - \cancel{\rho u \, dy dz} - \frac{\partial}{\partial x}(\rho u \, dy dz) dx$$

$$= -\frac{\partial}{\partial x}(\rho u \, dy dz) dx$$

$$= -\frac{\partial}{\partial x}(\rho u)\, dx\, dy\, dz \text{ —— (1)}$$

Similarly, the net gain of mass in y-direction

$$= -\frac{\partial}{\partial y}(\rho v)\, dx\, dy\, dz \text{ —— (2)}$$

Similarly, the net gain of mass in z-direction

$$= -\frac{\partial}{\partial z}(\rho w)\, dx\, dy\, dz \text{ —— (3)}$$

→ Since the mass is neither created nor destroyed in the fluid element, the net increase of mass per unit time in the fluid element must be equal to the rate of increase of mass of fluid in the element.

But mass of fluid in the element is "$\rho \cdot dx dy dz$" and its rate of increase with time is $\frac{\partial}{\partial t}(\rho \cdot dx dy dz)$ —— (4)

$\Rightarrow$ From equations (1), (2) and (3), we have

$$\text{Net gain of masses} = -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx\,dy\,dz \quad \text{— (5)}$$

$\Rightarrow$ Now, equating equations (4) and (5),

$$-\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx\,dy\,dz = \frac{\partial \rho}{\partial t} \cdot dx\,dy\,dz$$

Cancelling "dxdydz" from both sides,

$$\Rightarrow \boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0.} \quad \text{— (6)}$$

← Continuity equation in Carterian - co-ordinates in general form.

$\Rightarrow$ This equation is applicable to

(i) steady and unsteady flow

(ii) Uniform and non-Uniform flow

(iii) Compressible and incompressible flow.

(i) For steady flow, $\frac{\partial \rho}{\partial t} = 0$. Hence equation (6) becomes,

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0. \quad \text{— (7)}$$

(ii) For incompressible and steady flow, (ρ is constant), $\frac{\partial \rho}{\partial t} = 0$. then eqn (6) becomes.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \quad \text{— (8)}$$

(iii) For 1-D, incompressible, steady flow $\left[\frac{\partial \rho}{\partial t} = 0,\ \rho = \text{Const},\ v = 0,\ w = 0\right]$

Hence equation (6) becomes.

$$\boxed{\frac{\partial u}{\partial x} = 0} \quad \text{— (9)}$$

The above equation (9) is the Continuity equation for One-dimensional flow.

Circulation and Vorticity :-

Circulation :- (Γ).

→ The flow along a closed curve is called circulation. (i.e. the flow in eddies and vortices). The mathematical concept of circulation is the line integral, taken completely around a closed curve, of the tangential component of the velocity vector.

→ • Consider a closed curve 'C' as shown in Figure (a).

• V – velocity at any point on the curve. (of flow of fluid)

• α – angle between a small element "ds" along the curve in the tangential direction and the velocity "V".

• Then the component of the velocity in the direction tangential to the curve is $V\cdot\cos\alpha$.

• By the definition the circulation around a closed curve is

$$\boxed{\Gamma = \int_c V\cdot\cos\alpha\cdot ds} \quad -(1)$$

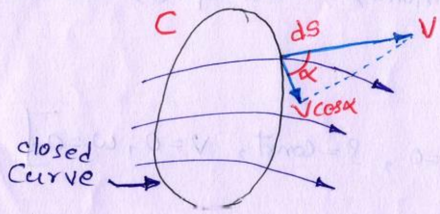


Figure (a): Circulation around a closed curve

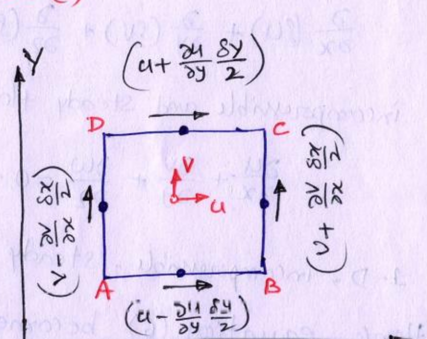


Figure (b): Circulation around an elementary rectangle in the plane of two dimensional steady flow field.

If u, v and w are the velocity components, and dx, dy and dz are the components of displacement ds, then the circulation can also be written as

$$\boxed{\Gamma = \int_c u\,dx + v\,dy + w\,dz} \quad -(2)$$

→ The circulation around an elementary rectangle with sides parallel to the axes x and y as shown in Figure (b) can be written as,

Circulation along $AB = \left(u - \frac{\partial u}{\partial y} \cdot \frac{\delta y}{2}\right) \delta x$

Circulation along $BC = \left(v + \frac{\partial v}{\partial x} \cdot \frac{\delta x}{2}\right) \delta y$

Circulation along $CD = -\left(u + \frac{\partial u}{\partial y} \frac{\delta y}{2}\right) \delta x$

Circulation along $DA = -\left(v - \frac{\partial v}{\partial x} \frac{\delta x}{2}\right) \delta y.$

→ The positive sense of integration is such that the enclosed surface is on left when viewed from the side of the outward normal.

→ The circulation around the periphery of the curve, must equal the sum of the circulation around the elementary surfaces. ~~of~~

~~which~~

$$\Gamma = \Gamma_{AB} + \Gamma_{BC} + \Gamma_{CD} + \Gamma_{DA}$$

$$= \left(u - \frac{\partial u}{\partial y} \frac{\delta y}{2}\right) \delta x + \left(v + \frac{\partial v}{\partial x} \frac{\delta x}{2}\right) \delta y - \left(u + \frac{\partial u}{\partial y} \frac{\delta y}{2}\right) \delta x - \left(v - \frac{\partial v}{\partial x} \frac{\delta x}{2}\right) \delta y$$

$$= (\cancel{u \delta x}) - \left(\frac{\partial u}{\partial y} \frac{\delta x \cdot \delta y}{2}\right) + \cancel{v \delta y} + \frac{\partial v}{\partial x} \frac{\delta x \cdot \delta y}{2} - \cancel{u \delta x} - \frac{\partial u}{\partial y} \frac{\delta x \, \delta y}{2} - \cancel{v \delta y} + \frac{\partial v}{\partial x} \frac{\delta x \delta y}{2}.$$

$$= -\frac{\partial u}{\partial y} \frac{\delta x \delta y}{2} + \frac{\partial v}{\partial x} \frac{\delta x \delta y}{2} - \frac{\partial u}{\partial y} \frac{\delta x \delta y}{2} + \frac{\partial v}{\partial x} \frac{\delta x \delta y}{2}$$

$$= \cancel{2} \frac{\partial v}{\partial x} \frac{\delta x \delta y}{\cancel{2}} - \cancel{2} \frac{\partial u}{\partial y} \frac{\delta x \delta y}{\cancel{2}}$$

$$= \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \delta x \cdot \delta y$$

$$\therefore \boxed{\Gamma = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \delta x \cdot \delta y} \quad \text{— (3)}$$

Vorticity :- (ζ).

→ The vorticity at any point is defined as the ratio of the circulation around an infinitesimal closed curve at that point to the area of the curve.

→ i.e. It is defined as circulation per unit area.

$$\zeta = \frac{\text{Circulation}}{\text{Area}} \qquad \{\text{Area} \rightarrow \delta x \cdot \delta y.$$

$$= \frac{\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \delta x \cdot \delta y}{\delta x \cdot \delta y}$$

$$\boxed{\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}} \quad \text{—(4)}$$

UNIT-2
PART-B

Fluid Dynamics

→ Fluid dynamics is the study of fluid motion with the forces causing flow. The dynamic behaviour of the fluid flow is analysed by the Newton's second law of motion, which relates the acceleration with the forces.

Newton's second law of motion :-

The net force F_x acting on a fluid element in the direction of x is equal to mass 'm' of the fluid element multiplied by the acceleration a_x in the x-direction.

$$F_x = m \cdot a_x .$$

In the fluid flow the following forces are present.

(i) F_g gravity force

(ii) F_p pressure force

(iii) F_v force due to viscosity

(iv) F_t force due to turbulence

(v) F_c force due to compressibility.

∴ Net force on fluid element is F_x, (in x-direction)

$$\boxed{F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x} \quad \text{——— (2)}$$

→ If 'F_c' is negligible, $F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$ ——— (3)

The above eqn (3) is called Reynold's equations of motion.

→ If "F_t" is negligible, $\boxed{F_x = (F_g)_x + (F_p)_x + (F_v)_x}$ ——— (4)

The above eqn (4) is called "Navier-Stokes equation."

→ If 'F_v' is zero (flow is ideal), $\boxed{F_x = (F_g)_x + (F_p)_x}$ ——— (5)

The above eqn (5) is called "Euler's equation."

Euler's Equation :-

→ This is equation of motion in which only gravitational force and pressure force are taken into consideration.

→ Consider a stream line in which flow is taking place in s-direction as shown in Fig(1). Consider a cylindrical element of cross-section "dA" and length "ds".

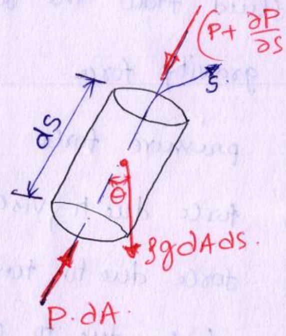
→ The forces acting on element are,

1). pressure force $(P.dA)$ in the direction of flow.

2). Pressure force $\left(P+\frac{\partial P}{\partial s}.ds\right)dA$ opposite to the direction of flow.

3). gravitational force (or) weight of fluid element $(\rho g\, dA ds)$.

→ The resultant force on fluid element = mass of fluid element × acceleration in s-direction.



Fig(1)

Fig(2)

$$P.dA-\left(P+\frac{\partial P}{\partial s}ds\right)dA-\rho g dA ds\cos\theta=\rho dA ds\times a_s. \quad —(1)$$

Now, $a_s=\frac{dV}{dt}$

$$=\frac{\partial V}{\partial s}.\frac{ds}{dt}+\frac{\partial V}{\partial t}$$

$$=\frac{V\partial V}{\partial s}+\frac{\partial V}{\partial t} \quad \left\{\text{if the flow is steady } \frac{\partial V}{\partial t}=0.\right.$$

$\therefore a_s=\frac{V\partial V}{\partial s}$. —(2) ; substitute "$a_s$" in eqn (1).

→ $-\frac{\partial P}{\partial s}dsdA-\rho g dAds\cos\theta=\rho dA ds\times\frac{V\partial V}{\partial s}$.

dividing by "$\rho ds dA$",

$$-\frac{\partial P}{\rho.\partial s}\cancel{dA}-g\cos\theta=\frac{V\partial V}{\partial s}$$

$$\frac{\partial P}{\rho\partial s}+g\cos\theta+V\frac{\partial V}{\partial s}=0.$$

From Fig (2), we have $\cos\theta = \frac{dz}{ds}$

$$\frac{1}{\rho}\frac{dP}{ds} + g\frac{dz}{ds} + \frac{VdV}{ds} = 0$$

$$\boxed{\frac{dP}{\rho} + gdz + VdV = 0} \quad \text{———— (3)}$$

This above equation (3) is known as Euler's equation of motion.

Bernoulli's equation :-

This equation is obtained by integrating the Euler's equation of motion. [eq. (3)],

$$\int \frac{dP}{\rho} + \int gdz + \int VdV = \text{Constant.}$$

$$\frac{P}{\rho} + gz + \frac{V^2}{2} = \text{Constant} \qquad \left\{ \begin{array}{l} \text{if flow is incompressible} \\ \rho = \text{Constant.} \end{array} \right.$$

(or)

$$\boxed{\frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{Constant.}} \quad \text{—— (4)}$$

← Bernoulli's Equation.

* $\frac{P}{\rho g}$ → Pressure energy per unit weight of fluid (or) pressure head.
* $\frac{V^2}{2g}$ → Kinetic energy per unit weight (or) Kinetic head.
* z → potential energy per unit weight (or) potential head.

Assumptions,

(i) The fluid is ideal (viscosity = 0)

(ii) Steady flow

(iii) Incompressible (ρ = constant) flow.

(iv) Irrotational flow.

Statement :-

"Bernoulli's equation states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant."

Total energy = Pr. energy + Kinetic energy + potential energy.

Bernoulli's Equation for Real fluid.

→ The equation (4) is valid for inviscid (Ideal fluid) fluid.

→ But all the real fluids are viscous and hence offer resistance to flow. thus there are always some losses in fluid flows and hence in the application of Bernoulli's equation, these losses have to be taken into consideration.

$$\boxed{\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L}$$

h_L → loss of energy (head loss) between points "1" & "2".

→ Total energy at point (1) $(E_1) = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1$

→ Total energy at point (2) $(E_2) = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$

→ $\boxed{\therefore \text{Loss of head } (h_L) = E_1 - E_2}$

Applications of Bernoulli's Equation :-

→ The Bernoulli's equation is applied in all problems of incompressible flow flow where energy considerations are involved.

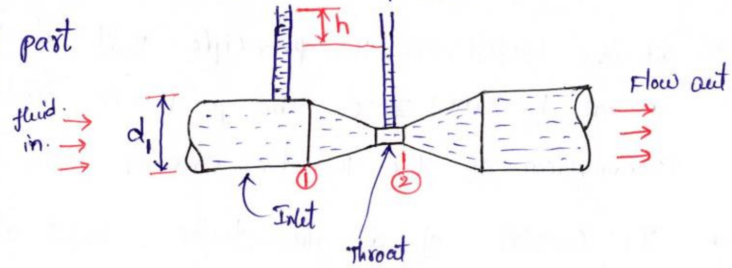
→ The ~~th~~ following measuring devices are application of this equation.

1) Venturimeter
2) Orifice meter
3) Pitot-tube.

(1) Venturimeter :- (For discharge)

It is a device used for measuring the rate of flow of a fluid flowing through a pipe. It consists of three parts :

(i) A short converging part
(ii) Throat
(iii) Diverging part



Theoretical dischage, $Q_{the} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$

Actual discharge, $Q_{act} = C_d \cdot \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$

⇒ C_d – co.effecient of venturimeter (or) co.efficient of discharge

⇒ $C_d < 1$. (always).

$$C_d = \frac{Q_{act}}{Q_{the}}$$

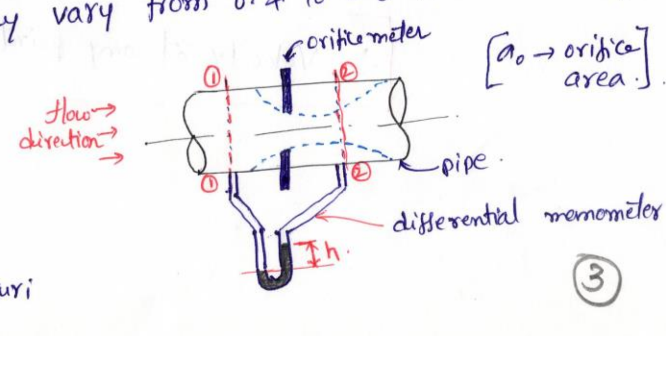
(2) Orifice Meter (or) Orifice plate :- (For discharge)

→ It is a device used for measuring the rate of flow of a fluid through a pipe. It consists of a flat circular plate which has a circular sharp edged hole called orifice, which is concentric with the pipe.

→ The orifice diameter is kept generally 0.5 times the diameter of the pipe, though it may vary from 0.4 to 0.8 times the pipe diameter.

$$\boxed{Q = \frac{C_d \cdot a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}}$$

[a_0 → orifice area.]



Always. ⇒ $C_{d\,orificemeter} < C_{d\,venturi}$

(3) Pitot - Tube :- (For velocity)

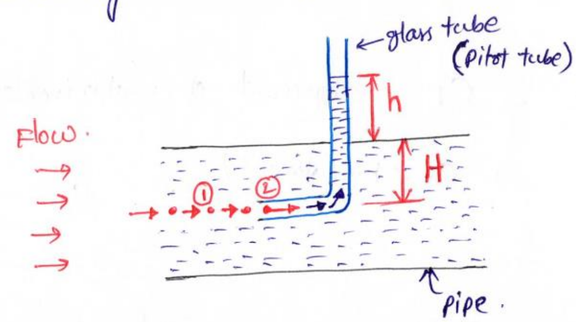
→ It is a device used for measuring the velocity of flow at any point in a pipe (or) channel.

→ It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy.

→ It consists of a glass-tube, bent at right angles as shown.

→ The liquid rises up in the glass-tube due to conversion of K.E into Pr. E.

→ The velocity is determined by measuring the rise of liquid in the tube.



P_1 — Intensity of Pressure at 1.

P_2 → " at 2

V_1 → velocity at 1

V_2 → velocity at 2.

H → Depth of tube in the liquid

h → rise of liquid in the tube above the free surface.

Theoretical Velocity $V_1 = \sqrt{2gh}$

Actual Velocity $(V_1)_{act} = C_v \cdot \sqrt{2gh}$

C_v → Co-efficient of pitot tube.

$$\boxed{\therefore \text{ Velocity at any point } = C_v\sqrt{2gh}}$$

The Momentum equation :-

→ It is based on law of conservation of momentum (or) on the momentum principle, which states that the net force acting on a fluid mass is equal to change in momentum of flow per unit time in that direction.

From Newton's second law of motion,

$$F = m \times a \qquad \left\{ a = \frac{dv}{dt} \right.$$

$$F = m \times \frac{dv}{dt}$$

$$= \frac{d(mv)}{dt} \qquad \left[m \text{ is constant and can be taken inside the differential} \right]$$

$$\boxed{F = \frac{d(mv)}{dt}} \leftarrow \text{momentum principle.} \text{ ——— (I)}$$

$$\therefore \boxed{F.dt = d(mv)} \leftarrow \text{Impulse-momentum equation.} \text{ ——— (II).}$$

It states that the impulse of a force (F) acting on a fluid of mass (m) in a short interval of time (dt) is equal to the change of momentum d(mv) in the direction of force.

Force exerted by a flowing fluid on a pipe bend :-

The impulse momentum equation is used to determine the resultant force exerted by a flowing fluid on a pipe bend.

Consider two sections (1) & (2), as shown in figure.

$V_1 \rightarrow$ velocity of flow at section 1.

$P_1 \rightarrow$ pressure intensity at section 1.

$A_1 \rightarrow$ C-S. area of pipe at section 1.

→ Let "F_x" and "F_y" be the components of the forces exerted by the flowing fluid on the bend in x and y directions respectively.

→ Refer Fig(1), Fig(2) and Fig(3).

→ ① The Force exerted by the bend on fluid in x & y directions will be equal to "F_x" and "F_y" but in opposite direction.

Force by bend on fluid in x- direction $= -F_x$

" " in y- direction $= -F_y$

→ ② Other external forces acting on the fluid are $P_1 A_1$ and $P_2 A_2$ at sections "1" and "2" respectively.

→ Then momentum equation in x- direction is,

⇒ Net force acting on fluid in x- direction $=$ Rate of change of momentum in x- direction.

$$P_1A_1 - P_2A_2\cos\theta - F_x = (\text{Mass per sec}) \times (\text{change of velocity in } x\text{-direction})$$

$$= \rho Q\,(\underset{\text{in } x\text{-dir}}{\text{Final velocity}} - \underset{\text{in } x\text{-direction}}{\text{Initial velocity}})$$

$$= \rho Q\,(V_2\cos\theta - V_1)$$

$$F_x = \rho Q\,(V_1 - V_2\cos\theta) + P_1A_1 - P_2A_2\cos\theta \quad \text{— ①}$$

⇒ Similarly, momentum equation in y- direction is,

$$0 - P_2A_2\sin\theta - F_y = \rho Q\,(V_2\sin\theta - 0)$$

$$F_y = \rho Q\,(-V_2\sin\theta) - P_2A_2\sin\theta$$

⇒ Now resultant force (F_R) acting on bend,

$$\boxed{F_R = \sqrt{F_x^2 + F_y^2}} \quad \text{— ③}$$

⇒ Angle made by "F_R" is,

$$\boxed{\tan\theta = \frac{F_y}{F_x}} \quad \text{— ④}$$

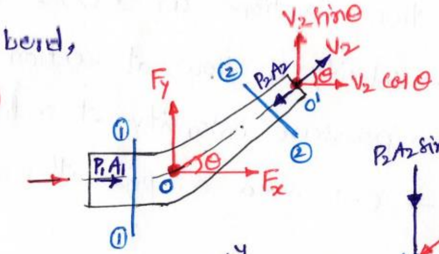


Fig ①.

Fig ②

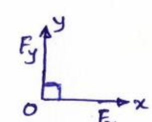
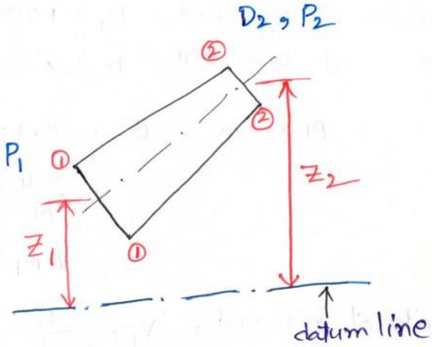


Fig ③

Problems on Bernollis Equation.

① Water is flowing through a pipe having diameter 300 mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm^2 and the pressure at the upper end is 9.81 N/cm^2. Determine the difference in datum head if the rate of flow through pipe is 40 lit/s.



Sol:-

→ at section ①,

$D_1 = 300\,mm = 0.3\,m$

$P_1 = 24.525\ N/cm^2$

$= 24.525 \times 10^4\ N/m^2$

→ at section ②,

$D_2 = 200\,mm = 0.2\,m$

$P_2 = 9.81\ N/cm^2 = 9.81 \times 10^4\ N/m^2$

→ Rate of flow $Q = 40$ lit/s.

$= 40 \times 10^{-3}\ m^3/s$

$= 0.04\ m^3/s$.

→ $A_1V_1 = A_2V_2 = Q$.

$$V_1 = \frac{Q}{A_1} = \frac{0.04}{\frac{\pi}{4}(D_1)^2} = \frac{0.04}{\frac{\pi}{4}(0.3)^2} = 0.5658\ m/s$$

$$V_2 = \frac{Q}{A_2} = \frac{0.04}{\frac{\pi}{4}(D_2)^2} = \frac{0.04}{\frac{\pi}{4}(0.2)^2} = 1.274\ m/s.$$

Apply Bernoulli's equation at ① & ②, $\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$

$$\frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{(0.5658)^2}{2 \times 9.81} + Z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.274)^2}{2 \times 9.81} + Z_2$$

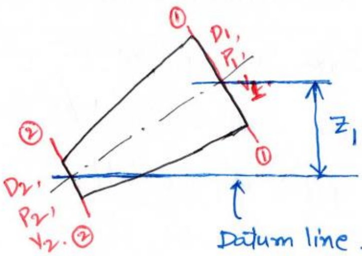
$$25 + 0.32 + Z_1 = 10 + 1.623 + Z_2$$

$$25.32 + Z_1 = 11.623 + Z_2$$

$$Z_2 - Z_1 = 13.697$$

∴ difference in datum head $= Z_2 - Z_1 = 13.697\ m$.

② A pipe line 300 m long has a slope of 1 in 100 and tapers from 1.2 m diameter at the high end to 0.6 m at the low end. The discharge through the pipe is 5.4 m^3/minute. If the pressure at the high end is 70 kPa, Find the pressure at the low end. Neglect the losses.



Sol:- Length of pipe $L = 300\,m = 0.3\,m$

Diameter at upper end, $D_1 = 1.2\,m$

Dia. at lower end, $D_2 = 0.6\,m$

$$\text{Discharge, } Q = 5.4\,m^3/min$$
$$= \frac{5.4}{60}\,m^3/s$$
$$= 0.09\,m^3/s.$$

velocity at high end, $V_1 = \frac{Q}{A_1} = \frac{0.09}{\frac{\pi}{4}(1.2)^2} = 0.0795\,m/s$

velocity at low end, $V_2 = \frac{Q}{A_2} = \frac{0.09}{\frac{\pi}{4}(0.6)^2} = 0.318\,m/s$

Let the datum line passes through the Centre of lower end, Then "$Z_2 = 0$".

As slope is 1 in 100 means, $Z_1 = \frac{1}{100} \times \text{Length}$
$$= \frac{1}{100} \times 300 = 3\,m.$$

Applying Bernoulli's equation at sections (1) & (2),

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{70 \times 10^3}{1000 \times 9.81} + \frac{(0.0795)^2}{2 \times 9.81} + 3 = \frac{P_2}{1000 \times 9.81} + \frac{(0.318)^2}{2 \times 9.81} + 0.$$

$$7.1355 + (3.22 \times 10^{-4}) + 3 = \frac{P_2}{9810} + (5.154 \times 10^{-3})$$

$$10.1358 = (1.019 \times 10^{-4})\,P_2 + (5.154 \times 10^{-3})$$

$$P_2 = 99417.5\,N/m^2$$

$$P_2 = 99.417\,kPa.$$

(3) A pipe line carrying oil of specific gravity 0.8, changes in diameter from 300 mm at a position A to 500 mm diameter to a position B which is 5 m at a higher level. If the pressures at A and B are 19.62 N/cm^2 and 14.91 N/cm^2 respectively, and the discharge is 150 Litres/s, determine the loss of head and direction of flow.

Sol:-

$\rho_{oil} = 0.8 \times 1000 = 800\ kg/m^3$

$D_A = 300\ mm = 0.3\ m$

$A_A = \frac{\pi}{4}(D_A)^2 = 0.07068\ m^2$

$D_B = 500\ mm = 0.5\ m$

$A_B = \frac{\pi}{4}(D_B)^2 = 0.196349\ m^2$

$P_A = 19.62\ N/cm^2 = 19.62 \times 10^4\ N/m^2$

$P_B = 14.91\ N/cm^2 = 14.91 \times 10^4\ N/m^2$

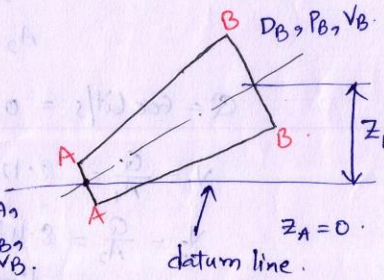
$Q = 150\ lit/s = 0.15\ m^3/s$.

$h_L = E_A - E_B = ?$

$\Rightarrow Q = V_A A_A = V_B A_B$.

$V_A = \frac{Q}{A_A} = \frac{0.15}{0.07068} = 2.122\ m/s$

$V_B = \frac{Q}{A_B} = \frac{0.15}{0.196349} = 0.7639\ m/s$



$\Rightarrow$ Total energy at "A", $E_A = \frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A$

$= \frac{19.62 \times 10^4}{800 \times 9.81} + \frac{(2.122)^2}{2 \times 9.81} + 0 = 25 + 0.2295 + 0$

$E_A = 25.229\ m$;

$\Rightarrow$ Total energy at "B", $E_B = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B$

$= \frac{14.91 \times 10^4}{800 \times 9.81} + \frac{(0.7639)^2}{2 \times 9.81} + 5 = 18.998 + 0.02974 + 5$

$E_B = 24.027\ m$.

$\Rightarrow$ (i) $\therefore$ The loss of head $= E_A - E_B = 25.229 - 24.027 = 1.209\ m$.

$\Rightarrow$ (ii) Direction of flow, as E_A is more than E_B and hence flow is taking from A to B.

Problems on momentum equation.

(4) A 45° reducing bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 600 mm and 300 mm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet to bend is 8.829 N/cm² and rate of flow of water is 600 litres/s.

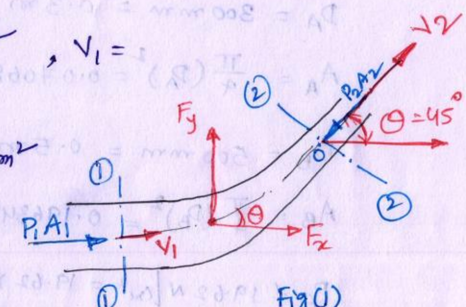
Sol:- Angle of bend, $\theta = 45^\circ$

Inlet diameter, $D_1 = 0.6\,m$

$A_1 = \frac{\pi}{4}(D_1)^2 = 0.2827\,m^2$, $V_1 =$

Outlet diameter, $D_2 = 0.3\,m$

$A_2 = \frac{\pi}{4}(D_2)^2 = 0.07068\,m^2$



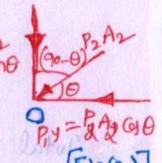
Fig(1)

$Q = 600\,lit/s = 0.6\,m^3/s$

$V_1 = \frac{Q}{A_1} = 2.122\,m/s$

$V_2 = \frac{Q}{A_2} = 8.488\,m/s.$

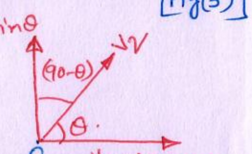
Applying Bernoulli's equation, $\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$



[Fig(3)]

$[z_1 = z_2]$ $\qquad \frac{8.829\times10^4}{1000\times9.81} + \frac{(2.122)^2}{2\times9.81} = \frac{P_2}{1000\times9.81} + \frac{(8.488)^2}{2\times9.81}$

$P_2 = 5.45\times10^4\,N/m^2$



[Fig(2)]

⇒ Forces on bend in x and y directions are given by,

$F_x = \rho Q(V_{1x} - V_{2x}) + P_{1x}A_1 + P_{2x}A_2$

→ $F_x = \underbrace{\rho Q(V_1 - V_2\cos\theta)}_{(Fig\,2)} + (P_1A_1) - \underbrace{(P_2A_2\cos\theta)}_{(Fig\,3)}$

$= 1000\times0.6\,[2.122 - 8.488\cos(45^\circ)] + 8.829\times10^4\times0.2827 - 5.45\times10^4\times0.7068 \times\cos45^\circ$

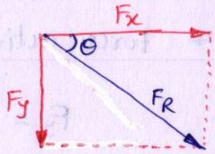
$= 19911.4\,N.$

→ $F_y = \rho Q(-V_2\sin\theta) - \overbrace{P_2A_2\sin\theta}^{(Fig\,3)}$

$= 1000\times0.6\,(-8.488\sin45^\circ) - (5.45\times10^4\times0.07068\times\sin45^\circ)$

$= -6322.2\,N.$ (-ve sign means F_y is acting in downward direction).

(i) ⇒ ∴ The Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$



$$= \sqrt{19911.4^2 + (-6322.2)^2}$$

$$F_R = 20890.9\,N.$$

(ii) ⇒ The angle made by resultant force with x-axis is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{6322.2}{19911.4} = 0.3175$$

$$\theta = \tan^{-1}(0.3175) = 17.61^\circ. = 17^\circ\,61'.$$

(5) 250 lit/s of water is flowing in a pipe having a diameter of 300 mm. If the pipe is bent by 135° (that is change from intial to final direction is 135°), Find the magnitude and direction of the resultant force on the bend. The pressure of water flowing is 39.24 N/cm². {JNTUK, NOV-2017} 8 marks

Sol:-

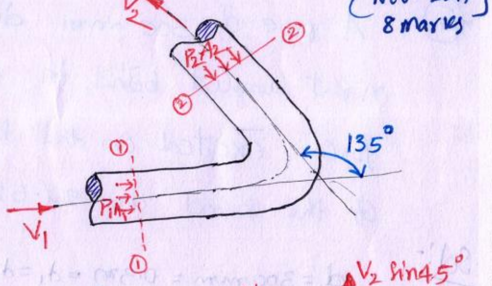


Fig ①

The pressure at inlet and outlet is same.

$$P_1 = P_2 = 39.24 \times 10^4\ N/m^2$$

$$Q = 250\ \text{lit/s} = 0.25\ m^3/s$$

Diameter at inlet and outlet is same.

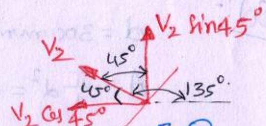


Fig ②

$$D_1 = D_2 = 0.3\,m.$$

$$A_1 = A_2 = \frac{\pi}{4}(D_1)^2 = 0.07068\,m^2$$

$$\text{Velocity},\ V_1 = V_2 = \frac{Q}{A_1} = \frac{0.25}{0.07068} = 3.537\ m/s.$$

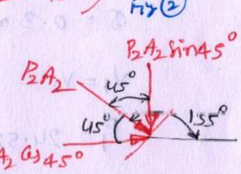


Fig ③

⇒ Forces acting on fluid in x- direction,

$$F_x = \rho Q (V_{1x} - V_{2x}) + (P_{1x} A_1) + (P_{2x} A_2)$$

$$= 1000 \times 0.25\left(3.537 - [-3.537 \times \cos 45^\circ]\right) + (39.24 \times 10^4 \times 0.07068) + (39.24 \times 10^4 \times 0.07068 \times \cos 45^\circ).$$

$$= 48855.4\,N.$$

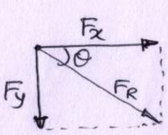
⇒ Forces acting on fluid in Y-direction,

$$F_y = \rho Q(V_{1y} - V_{2y}) + (P_1A_1)_y + P_{2y}A_2 .$$

$$= 1000\times 0.25\left(0 - [0.3537\times \cos 45^\circ]\right) + 0 + \left(-39.24\times 10^4\times 0.07068\times \cos 45^\circ\right)$$

$= -20236.3$ N. (-ve sign means, F_y is acting in downward direction).

∴ Resultant force on Bend, $F_R = \sqrt{F_x^2 + F_y^2} = \underline{52880.6\ N}$.

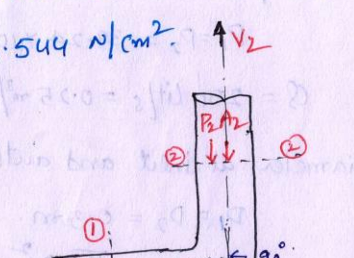


∴ The direction of resultant force is,

$$\tan\theta = \frac{F_y}{F_x} = \frac{20236.3}{48855.4} = 0.4142$$

$$\underline{\theta = 22^\circ 30'}.$$

⑥ A pipe of 300 mm diameter conveying 0.3 m³/s of water has a right angled bend in a horizontal plane. Find the resultant force exerted on the bend if the pressure at inlet and outlet of the bend are 24.525 N/cm² and 23.544 N/cm².



Sol:- $d = 300\ mm = 0.3\ m = d_1 = d_2$

$A = \frac{\pi}{4}d^2 = 0.07\ m^2 = A_1 = A_2$

$Q = 0.3\ m^3/s$.

$V_1 = V_2 = \frac{Q}{A} = \frac{0.3}{0.07} = 4.244\ m/s$.

$P_1 = 24.525\times 10^4\ N/m^2$

$P_2 = 23.544\ N/cm^2$

Force acting on bend along x-axis,

$$F_x = \rho Q(V_{1x} - V_{2x}) + (P_1A_1)_x + (P_2A_2)_x$$

as the bend is having Right angle ($\theta = 90^\circ$),

$V_{2x} = 0$ and $(P_2A_2)_x = 0$.

(F)

$\therefore F_x = \rho Q\,(V_{1x}) + (P_1A_1)_x$

$= 1000 \times 0.3\,(4.244) + (24.525 \times 10^4 \times 0.07068)$

$= 18607.5\,N$

Force acting on bend along y-axis,

$F_y = \rho Q\,(V_{1y} - V_{2y}) + (P_1A_1)_y + (P_2A_2)_y$

$V_{1y} = 0$ and $(P_1A_1)_y = 0.$

$V_{2y} = V_2$ and $(P_2A_2)_y = -(P_2A_2)_y.$

$F_y = \rho Q\,(-V_{2y}) + 0 - (P_2A_2)_y$

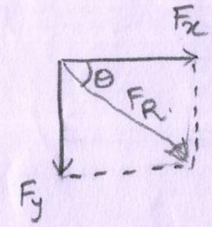
$= 1000 \times 0.3\,(-4.244) + 0 - (-23.544 \times 10^4 \times 0.07068)$

$= -17914.1\,N.$

$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{(18607.5)^2 + (17914.1)^2}$

$= 25829.3\,N$



$\tan\theta = \dfrac{F_y}{F_x}$

$= \dfrac{17914.1}{18607.5}$

$\tan\theta = 0.9627$

$\theta = \tan^{-1}(0.9627)$

$= 43^\circ\,54'.$

Closed Conduit Flow

UNIT-2, PART-C

Laminar Flow :- In laminar flow the fluid particles move along straight parallel path in layers (or) laminae, such that the paths of individual fluid particles do not cross those of neighbouring particles. Laminar flow is possible only at low velocities and high viscous.

Turbulent Flow :- When the velocity is increased (or) fluid is less viscous, the fluid particles do not move in straight paths. The fluid particles move in random manner resulting a general mixing of the particles. This type of flow is called turbulent flow.

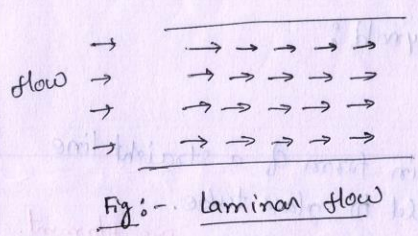


Fig :- Laminar flow

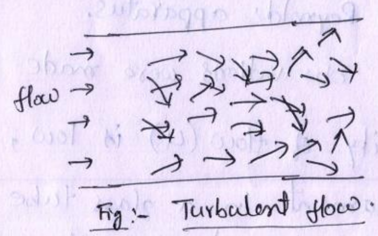


Fig :- Turbulent flow.

⇒ When the laminar flow changes to turbulent flow?

(i) Velocity is increased.

(ii) Diameter of a pipe is increased

(iii) Viscosity of fluid is decreased.

In case of circular pipe,

if $Re < 2000$, laminar flow

$Re > 4000$, Turbulent flow

$2000 < Re < 4000$, Transition [laminar to turbulent]

$$Re = \frac{\rho u D}{\mu}$$ ⇐ Reynold's Number.

Reynold's Experiment :-

⇒ In year 1883, O. Reynold was demonstrated the type of flow and developed a Relation (or) Number [Reynolds Number] by conducting an experiment. This is the Reynold's experiment as shown in Fig.

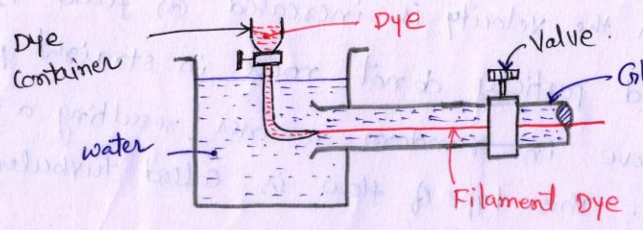


Fig: Reynolds apparatus.

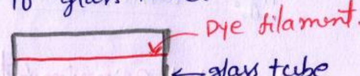
→ The following observations were made by Reynold:

(i) When Velocity of flow (u) is low,

→ The dye-filament in a glass tube was in form of a straight line

→ This straight line of filament was parallel to glass tube.

→ This is case of laminar flow.

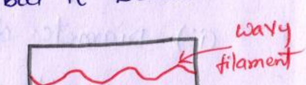


(i) laminar flow.

(ii) When velocity of flow is increased,

→ The dye-filament was no longer a straight line but it became a wavy one as shown in Figure (ii).

→ This shows that the flow is no longer laminar.



(ii) Transition

(iii) With Further increase of velocity,

→ The wavy dye-filament broke-up and finally diffused in water as shown in Figure (iii).

→ This means that the fluid particles of the dye at this higher velocity are moving in random fashion, which shows the case of turbulent flow.

(iii) Turbulent.

⇒ laminar flow, loss of Pressure head, $h_f \propto u$

⇒ Turbulent flow, loss of pressure head, $h_f \propto u^n$.

$$[n = 1.75 \text{ to } 2.0].$$

Frictional loss in Pipe flow (Darcy - Weisbach Equation).

Fritional loss :- When a liquid is flowing through a pipe, the velocity of the liquid layer adjacent to the pipe wall is zero. The velocity of liquid goes on increasing from the wall and thus velocity gradient and hence shear stresses are produced in the whole liquid due to viscosity. This viscous action causes loss of energy which is usually known as frictional loss.

For turbulent flow, frictional resistance (f_r)

(i) $f_r \propto u^n_{fluid}$

(ii) $f_r \propto \rho_{fluid}$

(iii) $f_r \propto A_{surface}$ in contact

(iv) independent of Pressure

(v) dependent on the nature of the surface in contact.

Expression for loss of head due to friction in pipes :- (Darcy - Weisbach Equation).

→ Consider a uniform horizontal pipe, having steady flow as show in Fig.

→ Let 1-1 and 2-2 are two sections of the pipe.

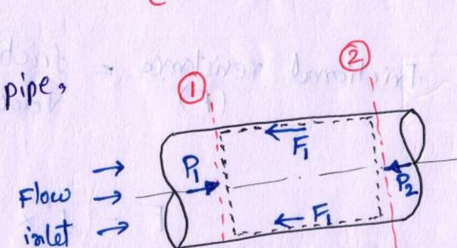


Fig: Uniform Horizontal pipe.

P_1, V_1 → Pressure & velocity at section 1.

P_2, V_2 → Pressure & velocity at section 2.

L → pipe length between sections ① & ②.

d = diameter of pipe

f' = frictional resistance per unit wetted area per unit velocity.

h_f → head loss due to friction.

→ Applying Bernoulli's equations between sections ①-① & ②-②.

Total head at 1-1 = Total head at 2-2 + loss of head due to friction between 1-1 & 2-2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_f \quad \text{—— ①}$$

→ pipe is horizontal, $Z_1 = Z_2$

dia of pipe uniform, $d_1 = d_2$, $A_1 = A_2$, $V_1 = V_2$

$$\therefore \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = P$$

$$h_f = \frac{P_1}{\rho g} - \frac{P_2}{\rho g}$$

$$P_1 - P_2 = \rho g h_f \cdot \quad \text{—— ②}$$

→ But, the intensity of pressure will be reduced in the direction of flow by frictional resistance.

Now,

Frictional resistance (F) = frictional resistance per unit area per unit velocity × wetted area × Velocity²

$$F = f' \times \pi d L \times V^2 \quad \text{—— ③}$$

⇒ wetted area — It is the cross sectional area of flow

→ The forces acting on the fluid between sections 1-1 & 2-2 are,

1) Pressure force at section 1-1 → $P_1 \cdot A$

2) Pressure force at section 2-2 → $P_2 A$

3) Frictional force as shown in Fig → F.

Resolving all forces in the horizontal direction, We have

$$P_1A - P_2A - F = 0.$$

$$\left\{ \begin{array}{l} \text{substitute } F = f' \times \pi d L \times V^2 \\ \text{from equation (3)} \end{array} \right.$$

$$(P_1 - P_2)A = f' \times \pi d L \times V^2$$

$$P_1 - P_2 = \frac{f' \times \pi d L \times V^2}{A}$$

Substitute $(P_1 - P_2) = \rho g h_f$ from equation (2),

$$\rho g h_f = \frac{f' \times \pi d L \times V^2}{A}$$

$$h_f = \frac{f' \times \pi d L \times V^2}{\rho g \times \frac{\pi}{4} d^2}$$

$$\boxed{h_f = \frac{4 f' L V^2}{\rho g d}}$$

By substituting $\frac{f'}{\rho} = \frac{f}{2}$; $f \rightarrow$ co-efficient of friction,

$$\boxed{h_f = \frac{4 f L V^2}{2 g d}} \quad \text{——— (4)}$$

→ Equation (4) is known as Darcy-Weisbach equation. This equation is commonly used for finding loss of head due to frictions in pipes.

$$f = \frac{16}{Re} \quad \text{for } Re < 2000 \quad \text{(viscous flow (or) laminar flow)}$$

$$f = \frac{0.079}{Re^{1/4}} \quad \text{for } Re > 4000 \quad \text{(Turbulent flow).}$$

***

NOTE:- Hydraulic Mean depth $(H.M.D) = \frac{\text{wetted area}}{\text{wetted perimeter}} = \frac{\text{Area}}{\text{Perimeter}} = \frac{A}{P}$.

(or) Hydraulic ratio (m or R).

Energy losses in pipes :-

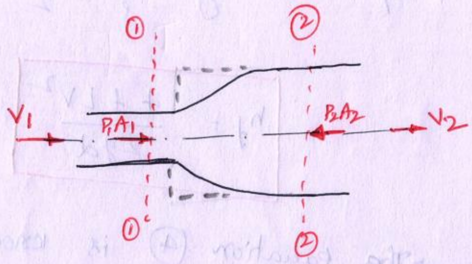
1. Major losses (due to friction)
2. Minor losses. (due to change of velocity of fluid in magnitude (or) direction).

Minor Energy (head) losses :-

1. Loss of head due to sudden enlargement
2. Loss of head due to sudden contraction
3. Loss of head at the entrance of a pipe
4. Loss of head at the exit of a pipe.
5. Loss of head due to an obstruction in a pipe.
6. Loss of head due to bend in a pipe
7. loss of head in various pipe fittings.

1 Sudden enlargement :-

$$h_e = \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) \quad \text{—(1)}$$



Force acting on liquid between 1-1 & 2-2,

$$F_x = P_1A_1 + P'(A_2-A_1) + (-P_2A_2) \quad \text{—(2)}$$

$(Z_1 = Z_2)$.

Change in momentum, $= \rho A_2V_2^2 - \rho A_1V_1^2$

$$= \rho A_2(V_2^2 - V_1V_2) \quad \text{—(3)}$$

$\left[A_1 = \frac{A_2V_2}{V_1}\right]$

eqn's (2) = (3) and $[P' = P_1]$

$\therefore \; F_x = \rho A_2(V_2^2 - V_1V_2)$

$(P_1 - P_2)A_2 = \rho A_2(V_2^2 - V_1V_2)$

$$\frac{(P_1 - P_2)}{\rho g} = \frac{V_2^2 - V_1V_2}{g} \quad \text{— (4)}$$

Substitute eqn (4) in eqn (1).

$$\boxed{h_e = \frac{(V_1 - V_2)^2}{2g}}$$

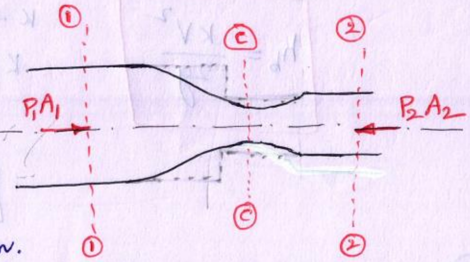
$h_e \to$ head loss due to sudden enlargement.

(2) Sudden Contraction :-

Section, C-C is Vena-Contracta.

1-1 to C-C → Contraction

C-C to 2-2 → Enlargement.



∴ head loss due to enlargement / contraction.

$$h_c = h_e = \frac{(V_c - V_2)^2}{2g} = \frac{V_2^2}{2g}\left(\frac{V_c}{V_2} - 1\right)$$

$$A_c V_c = A_2 V_2 \qquad \left\{ C_c = \frac{A_c}{A_2} \right\}$$

$$\frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{(A_c/A_2)}$$

$$\therefore \frac{V_c}{V_2} = \frac{1}{C_c}$$

where,

$C_c \to$ co-efficient of contraction

$$= \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)$$

$$\boxed{h_e = \frac{K V_2^2}{2g}} \qquad \left\{ K = \left(\frac{1}{C_c} - 1\right)^2 \right.$$

(3) At Entrance of a pipe :- (similar to sudden contraction).

$$\boxed{h_i = 0.5 \frac{V^2}{2g}}$$

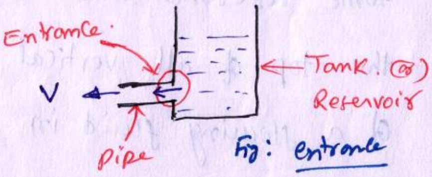


Fig: entrance

(4) At exit of a pipe :-

$$\boxed{h_o = \frac{V^2}{2g}}$$

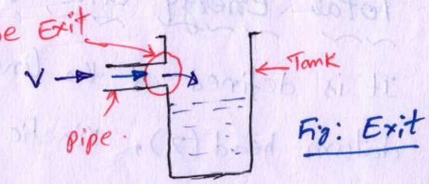
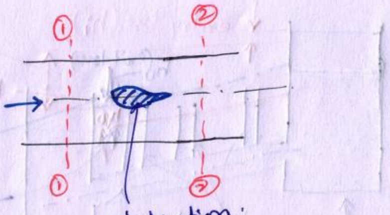


Fig: Exit

(5) An obstruction in pipe :-

$$h_{obstruction} = \frac{V^2}{2g}\left[\frac{A}{C_c(A-a)} - 1\right]^2$$



$a \to$ maximum area of obstruction

$C_c \to$ co-efficient of contraction

$A \to$ Area of pipe

$V \to$ velocity of liquid in a pipe.

⑥ Bend in Pipe :-

$$\boxed{h_b = \frac{KV^2}{2g}}$$

$K \to$ co-efficient of bend

K depends on

- Angle of bend
- Radius of Curvature of bend
- Diameter of pipe.

⑦ Various pipe fittings :- (Valves, Couplings, joints, etc.).

$$\boxed{h_{pf} = \frac{KV^2}{2g}}$$

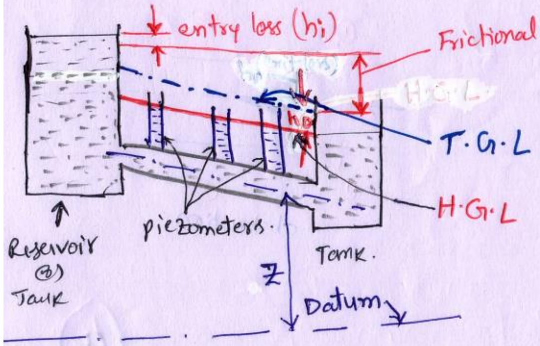
$K \to$ co-efficient of pipe fitting.

Hydraulic Gradient Line :- (H.G.L) [Refer figure] (or Pressure line)

It is defined as the line which gives the sum of pressure head $\left(\frac{P}{\rho g}\right)$, and datum head (z) of a flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the Pressure head $(P/\rho g)$ of a flowing fluid in a pipe from the Centre of a pipe.

Total Energy Line :- [Refer figure] (T.G.L) (or Energy line)

It is defined as the line which gives the sum of pressure head, $(P/\rho g)$ datum head (z), Kinetic head $\left(\frac{V^2}{2g}\right)$ of a flowing fluid in a pipe with respect to some reference line.

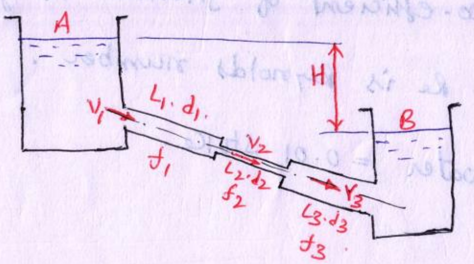


h_i = entry loss $= 0.5\frac{V^2}{2g}$

h_o = exit loss $= \frac{V^2}{2g}$

Figure: H.G.L and T.G.L

Flow through pipes in series (or) Flow through Compound pipes :-



Discharge through each Pipe is same.

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

$H \to$ sum of total head loss in pipes

$$H = (h_{inlet} + h_{friction_1}) + (h_{contraction} + h_{friction_2}) + (h_{enlargement} + h_{friction_3}) + h_{exit}$$

$$= h_i + h_{f_1} + h_c + h_{f_2} + h_e + h_{f_3} + h_o$$

$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{2d_1g} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{2gd_2} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{2gd_3} + \frac{V_3^2}{2g}.$$

if minor losses are neglected,

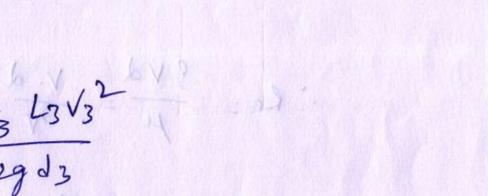
$$H = \frac{4f_1L_1V_1^2}{2gd_1} + \frac{4f_2L_2V_2^2}{2gd_2} + \frac{4f_3L_3V_3^2}{2gd_3}$$

if co-efficient of friction 'f' is same for all pipes,

$$f_1 = f_2 = f_3 = f.$$

$$\boxed{H = \frac{4f}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right]}$$

Flow through parallel pipes :-



$\to$ $Q = Q_1 + Q_2$

$\to$ $h_{f_1} = h_{f_2}$

$\to$ $\frac{4f_1L_1V_1^2}{2gd_1} = \frac{4f_2L_2V_2^2}{2gd_2} \Rightarrow \boxed{\frac{L_1V_1^2}{2gd_1} = \frac{L_2V_2^2}{2gd_2}}$ {if $f_1 = f_2$}.

Problems on Darcy's - weisbach equation.

① Water is flowing through a pipe of diameter 200 mm with a velocity of 3 m/s. Find the head lost due to friction for a length of 5 m if the co.efficient of friction is given by $f = 0.02 + \frac{0.09}{Re^{0.3}}$, where Re is Reynolds number. Take Kinematic viscosity of water = 0.01 stoke.

Sol:- dia. of pipe = 200 mm = 0.2 m

Velocity, $V = 3$ m/s

length, $L = 5$ m

Kinematic viscosity, $\nu = 0.01$ stoke 1 stoke = $\frac{1}{10^4}$ m²/s

$= 0.01 \times 10^{-4}$ m²/s

$$\therefore Re = \frac{\rho V d}{\mu} = \frac{V \cdot d}{\nu} = \frac{3 \times 0.2}{(0.01 \times 10^{-4})} = 6 \times 10^5$$

$$f = 0.02 + \frac{0.09}{Re^{0.3}} = 0.02 + \frac{0.09}{(6 \times 10^5)^{0.3}}$$

$$= 0.02 + 0.00166$$

$$= 0.02166.$$

$$\therefore \text{Heat loss due to friction, } h_f = \frac{4fLV^2}{2gd}$$

$$= \frac{4 \times 0.02166 \times 5 \times (3)^2}{2 \times 9.81 \times 0.2}$$

$$= 0.993 \text{ m of water.}$$

② Calculate the discharge through a pipe of diameter 200 m when the difference of pressure head between the two ends of a pipe 500 m apart is 4 m of water. Take the value of $f = 0.009$.

Sol:-

$d = 200m = 0.2$

$L = 500m$

$h_f = 4$ m of water

$f = 0.009$

$Q = ?$

$$h_f = \frac{4fLV^2}{2gd}$$

$$4 = \frac{4 \times 0.009 \times 500 \times V^2}{2 \times 9.81 \times 0.2}$$

$$4 = \frac{18}{3.924} V^2$$

$$V^2 = 0.872$$

$$V = \sqrt{0.872} = 0.9338 \text{ m/s}.$$

$$A = \frac{\pi}{4} d^2 = 0.03141 \text{ m}^2$$

$\therefore Q = A \cdot V$

$= \frac{\pi}{4} d^2 \times V$

$= 0.03141 \times 0.9338$

$= 0.0293 \text{ m}^3/s$

$= 29.3$ lit/s.

③ At a sudden enlargement of a water main from 240 mm to 480 mm diameter, the hydraulic gradient rises by 10 mm. Estimate the rate of flow. (JNTU-2002)

Sol:-

$D_1 = 240 mm = 0.24m$, $A_1 = \frac{\pi}{4} D_1^2$

$D_2 = 480mm = 0.48 m$, $A_2 = \frac{\pi}{4} D_2^2$

$Q = ?$

$\Rightarrow A_1V_1 = A_2V_2$

$V_1 = \frac{A_2}{A_1} V_2$

$= 4V_2$

→ Rise of hydraulic gradient, $\left(\frac{P_2}{\rho g} + Z_2\right) - \left(\frac{P_1}{\rho g} + Z_1\right) = 10 \text{ mm}$

$= 10 \times 10^{-3} m$

$= 0.01 m$.

→ $$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(4V_2 - V_2)^2}{2g} = \frac{9V_2^2}{2g}.$$

Applying Bernoulli's equation,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_e$$

$\left\{ h_e \to \text{head loss due to enlargement.} \right.$

$$\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_e = \left(\frac{P_2}{\rho g} + Z_2\right) - \left(\frac{P_1}{\rho g} + Z_1\right)$$

$$\frac{(4V_2)^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = 0.01$$

$$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = 0.01$$

$$\frac{6V_2^2}{2g} = 0.01$$

$$V_2 = \sqrt{\frac{0.01 \times 2g}{6}} = 0.1808 \text{ m/s}.$$

$\therefore$ Discharge, $Q = A_2 \times V_2 = \frac{\pi}{4} D_2^2 \times V_2$

$$= \frac{\pi}{4} (0.48)^2 \times 0.1808$$

$$= 0.03271 \text{ m}^3/\text{s} = 32.716 \text{ lit/s}.$$

(4) Determine the difference in the elevations between the water surfaces in the two tanks which one connected by a horizontal pipe of diameter 300 mm and length 400 m. The rate of flow of water through the pipe is 300 litres/s. Consider all losses and take the value of $f = 0.008$.

Sol:- $d = 300\text{ mm} = 0.3\text{ m}$

$L = 400$ m.

$Q = 300$ lit/s $= 0.3$ m^3/s.

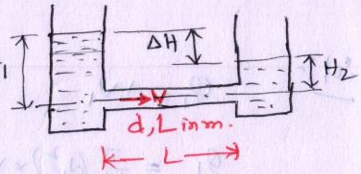
$f = 0.008$.

$\Rightarrow Q = AV$

$\Rightarrow V = \frac{Q}{A} = \frac{0.3}{\frac{\pi}{4}(0.3)^2}$

$V = 4.244$ m/s.

Applying Bernoulli's equation to the free surface of water in the two tanks,



$$H_1 = H_2 + \text{losses}.$$

$$H_1 = H_2 + h_i + h_f + h_o$$

$h_i \to$ loss at inlet

$h_f \to$ loss due to friction

$h_o \to$ loss at outlet.

$$\Delta H = H_1 - H_2 = h_i + h_f + h_o$$

$$= \frac{0.5V^2}{2g} + \frac{4fLV^2}{2gd} + \frac{V^2}{2g}.$$

$$= \frac{0.5 \times (4.244)^2}{2 \times g} + \frac{4 \times 0.008 \times 400 \times (4.244)^2}{2 \times 9.81 \times 0.3} + \frac{(4.244)^2}{2 \times 9.81}$$

$$= 0.459 + 39.16 + 0.918$$

$$\Delta H = H_1 - H_2 = 40.537\,m.$$

(5) Two pipes one of 10 cm diameter, 200 m long and another 15 cm diameter, 400 m long are connected in parallel. The friction factors are 0.0075 for the smaller pipe and 0.006 for the large pipe. The total discharge through the system is 50 lit/sec. Find the discharge and head loss in each pipe. Neglect minor losses. Calculate the equivalent length of a 20 cm diameter having $f = 0.005$.

$Q_1 = ?$ $h_{f_1} = ?$

$Q_2 = ?$ $h_{f_2} = ?$

Sol:-

$d_1 = 10\,cm = 0.1\,m$

$L_1 = 200\,m$

$d_2 = 15\,cm = 0.15\,m$

$L_2 = 400\,m.$

$f_1 = 0.0075$

$f_2 = 0.006$

$Q_{total} = 50\,lit/sec = 50 \times 10^{-3}\,m^3/s$

$= 0.05\,m^3/s.$

Sol:- $Q_1 = AV_1$

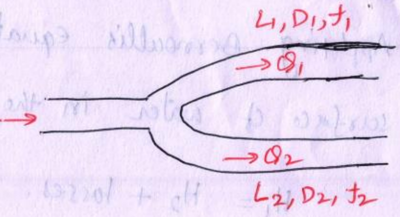
$Q_1 = \frac{\pi}{4}(d_1^2) \times V_1$

$\therefore \; V_1 = 127.33\, Q_1$

$Q_2 = AV_2$

$Q_2 = \frac{\pi}{4}(d_2^2) \times V_2$

$\therefore \; V_2 = 56.8\, Q_2$



$h_{f_1} = \frac{4 f_1 L_1 V_1^2}{2 g d_1}$ $\quad h_{f_2} = \frac{4 f_2 L_2 V_2^2}{2 g d_2}$

The loss of head in each branch pipe is same,

$h_{f_1} = h_{f_2}$

$$\frac{4 f_1 L_1 V_1^2}{2 g d_1} = \frac{4 f_2 L_2 V_2^2}{2 g d_2}$$

$$\frac{4 \times 0.0075 \times 200 \times (127.33\, Q_1)^2}{2 \times 9.81 \times 0.1} = \frac{4 \times 0.006 \times 400 \times (56.8\, Q_2)^2}{2 \times 9.81 \times 0.15}$$

$$243193.9\, Q_1^2 = 51619.84\, Q_2^2$$

$$\frac{Q_1}{Q_2} = 0.4607 \Rightarrow Q_1 = 0.4607\, Q_2 \quad \text{— (1)}$$

Total discharge, $Q_{total} = Q_1 + Q_2 = 0.05$ — (2).

$Q_2 + 0.4607\, Q_2 = 0.05$

$Q_2 = 0.0342\ m^3/s = \underline{34.2\ lit/s}$

$Q_1 = 0.0157\ m^3/s = \underline{15.7\ lit/s}$.

$\rightarrow V_1 = 127.33\, Q_1 = 1999.08\ m/s$.

$\rightarrow V_2 = 56.8\, Q_2 = 1.9425\ m/s$.

$\therefore h_{f_1} = \frac{4 f_1 L_1 V_1^2}{2 g d} = \underline{12.22\ m\ \text{of water}}$

$\therefore h_{f_2} = \frac{4 f_2 L_2 V_2^2}{2 g d} = \underline{12.3\ m\ \text{of water}}$

★ To equivalent length (a) diameters, we have

$$\frac{L_e}{D_e^5} = \left[\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \cdots \frac{L_n}{D_n^5}\right]$$

$$L_e = (0.2)^5 \left[\frac{200}{(0.1)^5} + \frac{400}{(0.15)^5}\right]$$

$$= 8085.5\ m.$$

$$L_e = \underline{8.085\ km}$$

$L_1 = 200\ m$
$D_1 = 0.1\ m$
$L_2 = 400\ m$
$D_2 = 0.15\ m$.
$D_e = 0.2\ m$.

⑥ A 60 cm diameter pipeline carries oil ($sp \cdot gr = 0.85$) at 82500 m^3/day. The friction head loss is 8.5 m per 1000 m of pipe. It is planned to place pumping stations every 20 km along the pipe. Make calculations for the pressure drop in kN/m^2 between pumping stations. (JNTUK-2017)

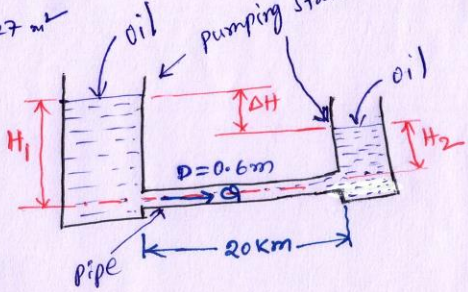
Sol:- $D = 60\,cm = 0.6\,m$, Area $= \frac{\pi}{4}D^2 = 0.2827\,m^2$

$Q = 82500\ m^3/day$

$= \frac{82500}{24 \times 3600}\ m^3/s$

$= 0.9548\ m^3/s$

$V = \frac{Q}{A} = \frac{0.9548}{0.2827} = 3.377\ m/s$



$h_f = 8.5\ m/1000\ m$ of pipe [i.e. 1 km = 1000 m]

But length of pipe (L) = 20 km

Total head loss.

$\{\Delta H = H_1 - H_2 = h_i + h_f + h_o\}$

$\therefore h_f = 8.5 \times 20\ m/20\,km$ of pipe

$h_f = 170\ m/20\,km$ of pipe

Applying Bernoulli's equation, we have

$$\frac{P_1}{\rho g} + \cancel{\frac{V_1^2}{2g}} + \cancel{Z_1} = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + \cancel{Z_2} + [h_i + h_f + h_o]$$

{as $V_1 = V_2$ and $Z_1 = Z_2$

$P_1 - P_2 = \rho g (h_i + h_f + h_o)$

$= (1000 \times 0.85) \times 9.81 \times (0.2906 + 170 + 0.581)$

{ $h_i = \frac{0.5V^2}{2g} = 0.2906$ m of oil.

$h_o = \frac{V^2}{2g} = 0.581$ m of oil.

$= 850 \times 9.81 \times 170.871$

$= 1424814.92\ N/m^2$

$\Delta P = P_1 - P_2 = \underline{1424.81\ kN/m^2}$

Boundary layer Theory

UNIT-3
Part-A

No slip condition :-

When a real fluid flows past a solid boundary (body (or) wall), the fluid particles adhere to the boundary due to viscosity. Thus, the velocity of fluid close to the boundary will be same as that of boundary. If the boundary (body (or) wall) surface is stationary, the velocity of fluid at the boundary will be zero, And there is no relative motion between the fluid and boundary. This condition is known as no-slip condition.

Boundary layer :-

→ When a real fluid flows past a solid stationary body, the fluid particles adhere to the boundary and condition no slip occurs. This means that the velocity of fluid at the boundary will be zero. Thus at the boundary surface the layer of fluid undergoes retardation. Farther away from the boundary, the velocity will be higher and as a result of this variation of velocity, the velocity gradient $\frac{du}{dy}$ will exist.

→ The velocity of fluid increases from zero velocity on the stationary boundary to free-stream velocity (U) of the fluid in the direction normal to the boundary. This variation of velocity takes place in a narrow region in the vicinity of solid boundary.

→ This narrow region of the fluid is called boundary layer.

→ The theory dealing with boundary layer flows is called boundary layer theory.

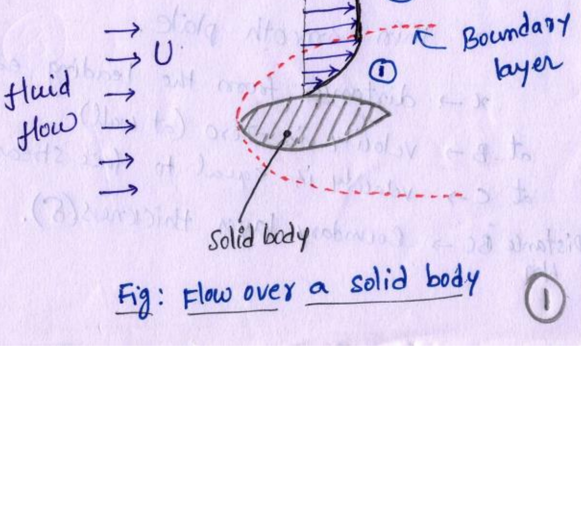


Fig: Flow over a solid body

→ Boundary layer has two regions, as shown in Fig.

① → $\tau = \mu \frac{du}{dy}$

② → The velocity outside B.L. is constant, and equal to free stream velocity. (no change in velocity).

$\therefore \frac{du}{dy} = 0$, $\tau = 0$.

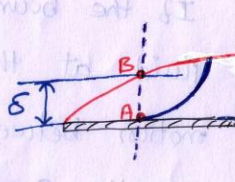
Boundary layer thickness :- (δ).

It is defined as the distance from the boundary of the solid body measured in Y-direction, to the point, where the velocity of the fluid is approximately equal to 0·99 times the free stream velocity (U) of the fluid. It is denoted by "deta" (δ). [AB = δ.]

δ_{lam} = Thickness of laminar boundary layer,

δ_{tur} = Thickness of turbulent boundary layer and

δ' = Thickness of laminar sub-layer.



Displacement thickness :- (δ^*).

It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in flow rate on account of boundary layer formation. It is denoted by "δ^*".

U → free stream velocity.

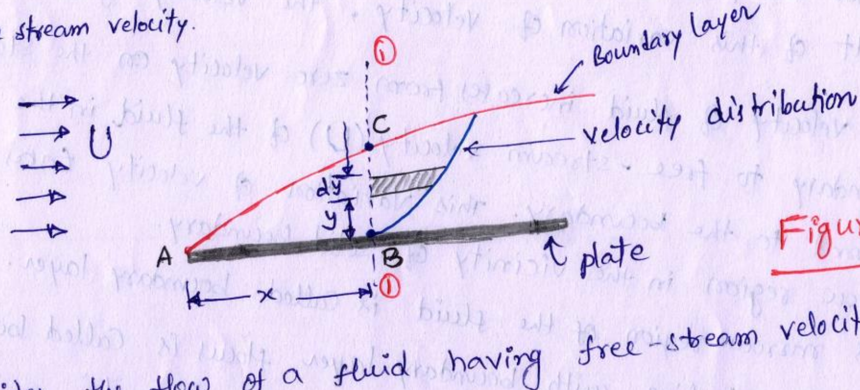


Figure: (1)

→ Consider the flow of a fluid having free-stream velocity (U) over a thin smooth plate.

x → distance from the leading edge to section ① - ①.

at B → velocity is zero (at wall)

at C → velocity is equal to free stream velocity (U).

Distance "BC" b/n → Boundary layer thickness (δ).

At section ① - ①, consider an elemental strip.

Let y = distance of elemental strip from the plate.

dy = thickness of elemental strip

u = velocity of fluid at strip

b = plate width.

→ Area of strip, $dA = b \times dy$.

→ mass flow rate flowing through strip, $= \rho \times$ velocity $\times$ Area of strip

$= \rho u dA$.

$= \rho u (b dy)$. —— (1)

→ If there had been no plate, which means fluid flow with free stream velocity through strip, then $U = u$.

∴ mass flow rate through strip $= \rho U (b dy)$. —— (2)

→ As U is more than u, hence due to presence of the plate and consequently due to formation of the boundary layer, there will be a reduction in mass flow rate through the strip.

⇒ ∴ Reduction in mass flow rate through strip $= \rho U (dy \cdot b) - \rho u (b dy)$.

$= \rho b (U - u) dy$. —— (3)

∴ Total reduction in mass flow rate through 'BC' due to plate $= \int_0^\delta \rho b (U - u) dy$

$= \rho b \int_0^\delta (U - u) dy$ [if incompressible fluid.]

→ Let the plate is displaced by a distance "δ^*" and velocity for the distance "δ^*" is equal to "U".

loss of mass flow rate through the distance $\delta^* = \rho \times U \times (\delta^* \cdot b)$ —— (4)

Equating equations ③ & ④, we have

$$\rho b \int_0^\delta (U-u)\, dy = \rho \times U \times (\delta^* \cdot b).$$

$$\rho b \int_0^\delta (U-u)\, dy = \rho b \times U \times \delta^*$$

$$\int_0^\delta (U-u)\, dy = U \times \delta^*$$

$$\delta^* = \frac{1}{U} \int_0^\delta (U-u)\, dy$$

$$= \int_0^\delta \frac{U-u}{U}\, dy$$ { as U is constant, can be taken inside the integral.

$$\boxed{\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right) dy}$$

Momentum Thickness :-

It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in momentum of the flowing fluid on account of boundary layer formation. It is denoted by "θ". Consider the Figure 1. (which is given in previous section).

→ ~~mass~~ of fluid through strip $= (\rho u\, b\, dy)\cdot u$

→ momentum of fluid (if there is no boundary layer) $= (\rho u b dy)\, U$

∴ loss of momentum through strip $= (\rho u b dy) U - (\rho u b dy) u$

$= \rho b u (U-u)\, dy$

∴ Total loss of momentum per sec through "BC" $= \int_0^\delta \rho b u (U-u)\, dy$. —— (1)

Let "θ" is the distance by which plate is displaced when the fluid is flowing with a constant velocity "U".

∴ loss of momentum per sec of fluid flowing through distance "θ" with a velocity "U".

$= \text{mass of fluid through "}\theta\text{"} \times \text{velocity}$

$= (\rho \times \text{area} \times \text{velocity}) \times \text{velocity}$

$= (\rho \times \theta \times b \times U) \times U$

$= \rho \theta b U^2$ —— ②

Equating equation's ② and ①,

$$\rho \theta b U^2 = \int_0^{\delta} \rho b u (U-u)\, dy$$

{ if the fluid is incompressible ρ = constant.

$$\rho b \theta U^2 = \rho b \int_0^{\delta} u (U-u)\, dy$$

{ $U \rightarrow$ constant.

$$\theta = \frac{1}{U^2} \int_0^{\delta} u (U-u)\, dy$$

$$= \int_0^{\delta} \frac{u(U-u)}{U^2}\, dy$$

$$\therefore \boxed{\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy}$$ ← Momentum thickness.

Energy thickness :- (δ^{**})

It is defined as the distance measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in kinetic energy of the flowing fluid on account of boundary layer formation. It is denoted by "δ^{**}".

Consider flow over a plate as shown in Fig (1).

$K.E$ of the fluid $= \frac{1}{2} m \times \text{velocity}^2$

$= \frac{1}{2} (\rho u b dy) u^2$

$K.E$ of the fluid (if no Boundary layer) $= \frac{1}{2} (\rho u b dy) U^2$

$\therefore$ Loss of $K.E$ through elemental strip $= \frac{1}{2} (\rho u b dy) U^2 - \frac{1}{2} (\rho u b dy) u^2$

$= \frac{1}{2} \rho u b [U^2 - u^2] dy.$

$\therefore$ Total loss of $K.E$ of fluid passing through "BC",

$$= \int_0^\delta \frac{1}{2} \rho u b (U^2 - u^2) dy \quad \text{——(1)}$$

Let $\delta^{**} \rightarrow$ distance by which the plate is displaced to compensate for the reduction in $K.E$.

$\therefore$ Loss of $K.E$ through δ^* of fluid flow with velocity U,

$= \frac{1}{2} (\rho \times \text{area} \times \text{velocity}) \times \text{velocity}^2$

$= \frac{1}{2} (\rho \times \delta^{**} \times b \times U) \times U^2$ $\quad [\text{Area} = b \times \delta^{**}]$

$= \frac{1}{2} \rho b \delta^{**} U^3$ — (2)

Equating (1) & (2),

$$\frac{1}{2} \rho b \delta^{**} U^3 = \frac{1}{2} \rho b \int_0^\delta u (U^2 - u^2) dy$$

$$\delta^{**} = \int_0^\delta \frac{u}{U^3} (U^2 - u^2) dy$$

$$\boxed{\delta^{**} = \int_0^\delta \frac{u}{U} \left(1 - \frac{u^2}{U^2}\right) dy} \leftarrow \text{Energy thickness.}$$

Momentum integral equation :-

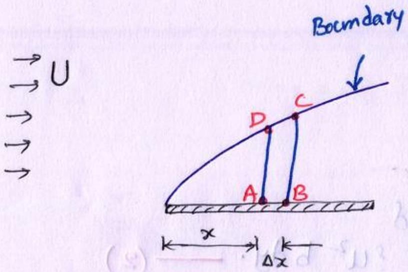


Fig :(1) Flow over flat plate

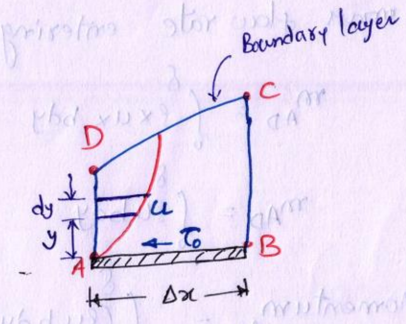


Fig :(2). Enlarged view of small length of plate.

→ Consider the flow of fluid having free-stream velocity (U), over a thin plate as shown in Fig (1).

→ The enlarged view of the small length of the plate is given in Fig (2).

→ The drag force on the plate can be determined if the velocity profile near the plate is known.

⇒ The drag force "F_D" (or) shear force on a small distance "Δx" given by,

$$\Delta F_D = \text{shear stress} \times \text{area}$$
$$= \tau_0 \times (\Delta x \times b)$$

{ area of small element, $A = \Delta x \times b$ ↓ width of plate.

⇒ The drag force (ΔF_D) = Rate of change of momentum over a distance "Δx".

$\therefore -\tau_0 \times (\Delta x \times b)$ = Momentum flux through BC − Momentum flux through AD − momentum flux through DC. —— (1).

↓ (opposite direction)

⇒ Momentum flux entering through "AD",

mass flow rate entering through the side AD,

$$m_{AD} = \int_0^{\delta} \rho \times u \times b\,dy$$

$$m_{AD} = \int_0^{\delta} \rho u\, b\,dy$$

$$\text{Momentum}_{AD} = \int_0^{\delta} (\rho u\, b\,dy) \times u = \int_0^{\delta} \rho u^2\, b\,dy. \quad \text{—— (2)}$$

⇒ Momentum flux leaving through the side "BC",

$$m_{BC} = m_{AD} + \left[\frac{\partial}{\partial x}(m_{AD}) \times \Delta x\right]$$

$$= \int_0^{\delta} \rho u\, b\,dy + \frac{\partial}{\partial x}\left[\int_0^{\delta} (\rho u\, b\,dy)\,\Delta x\right]$$

$$\text{Momentum}_{BC} = m_{BC} \times \text{velocity}$$

$$\text{Momentum}_{BC} = \int_0^{\delta} \rho u^2\, b\,dy + \frac{\partial}{\partial x}\left[\int_0^{\delta} \rho u^2\, b\,dy\right]\Delta x \quad \text{—— (3)}$$

⇒ Momentum flux entering the side "DC,"

$$m_{DC} + m_{AD} = m_{BC}.$$

$$m_{DC} = m_{BC} - m_{AD}$$

$$m_{DC} = \frac{\partial}{\partial x}\left[\int_0^{\delta} \rho \cdot u \cdot b\,dy\right]\Delta x.$$

$$\text{Momentum}_{DC} = m_{DC} \times \text{velocity}$$

$$= \frac{\partial}{\partial x}\left[\int_0^{\delta} \rho \cdot u \cdot b\,dy\right]\Delta x \times U$$

{ fluid is entering through 'DC' with a Uniform velocity "U"

$$\text{Momentum}_{DC.} = \frac{\partial}{\partial x}\left[\int_0^{\delta} \rho \cdot u \cdot U\, b\,dy\right]\Delta x. \quad \text{—— (4)}$$

Now, substitute equations, (2), (3) & (4) in equation (1),

Rate of change of momentum = Momentum$_{BC}$ − Momentum$_{AD}$ − Momentum$_{DC}$

$$= \int_0^\delta \cancel{\rho u^2 b\,dy} + \frac{\partial}{\partial x}\left[\int_0^\delta \rho u^2 b\,dy\right]\Delta x - \int_0^\delta \cancel{\rho u^2 b\,dy} - \frac{\partial}{\partial x}\left[\int_0^\delta \rho\cdot u\cdot U\,b\,dy\right]\Delta x$$

$$= \frac{\partial}{\partial x}\left[\int_0^\delta \rho u^2 b\,dy\right]\Delta x - \frac{\partial}{\partial x}\left[\int_0^\delta \rho u\cdot U\,b\,dy\right]\Delta x$$

$$= \frac{\partial}{\partial x}\left[\int_0^\delta \rho u^2 b\,dy - \int_0^\delta \rho\cdot u\cdot U\,b\,dy\right]\Delta x$$

$$= \frac{\partial}{\partial x}\left[\int_0^\delta (\rho u^2 b - \rho u U b)\,dy\right]\Delta x$$

$$= \rho b\,\frac{\partial}{\partial x}\left[\int_0^\delta (u^2 - uU)\right]\Delta x \quad \text{--- (5)}$$

Now substitute equation (5) in equation (1), we have

$$-\tau_0 \times \cancel{\Delta x} \times \cancel{b} = \rho \cancel{b}\,\frac{\partial}{\partial x}\left[\int_0^\delta (u^2 - uU)\,dy\right]\cancel{\Delta x}$$

$$-\tau_0 = \rho\,\frac{\partial}{\partial x}\left[\int_0^\delta (u^2 - uU)\,dy\right]$$

$$\tau_0 = \rho\,\frac{\partial}{\partial x}\left[\int_0^\delta (uU - u^2)\,dy\right]$$

Momentum thickness,

$$\left\{\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy\right.$$

$$= \rho\,\frac{\partial}{\partial x}\left[\int_0^\delta U^2\left(\frac{u}{U} - \frac{u^2}{U^2}\right)dy\right]$$

$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\left[\int_0^\delta \left(\frac{u}{U} - \frac{u^2}{U^2}\right)\right]dy = \frac{\partial}{\partial x}\left[\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)\right]dy$$

$$\Rightarrow \quad \boxed{\therefore \frac{\tau_0}{\rho U^2} = \frac{\partial \theta}{\partial x}}$$ ← Von-Karman momentum integral equation.

5

* The Von-Karman momentum integral equation is applied for,

(1) laminar boundary layers flows, (2) Transition B.L. flows. (3). Turbulent B.L. flows

⇒ The drag force on a small distance "Δx" of the plate,

$$\Delta F_D = \tau_0 \times \Delta x \times b.$$

∴ The total drag on the plate of Length "L",

$$F_D = \int_0^L \Delta F_D$$

$$\boxed{F_D = \int_0^L \tau_0 \times b \times dx.}$$

⇒ Local Co-efficient of Drag (C_D^*): It is defined as the ratio of the shear stress "τ_0" to the quantity "$\frac{1}{2}\rho U^2$".

$$\boxed{C_D^* = \frac{\tau_0}{\frac{1}{2}\rho U^2}}$$

⇒ Average Co-efficient of Drag (C_D): It is defined as the ratio of the total drag force to the quantity $\frac{1}{2}\rho \cdot A \cdot U^2$. It is also called Co-efficient of drag.

$$\boxed{C_D = \frac{F_D}{\frac{1}{2}\rho A U^2}}$$

$\rho \to$ fluid density
$A \to$ plate Area
$U \to$ Free stream velocity

⇒ Boundary Conditions for the velocity profiles:- The following Boundary Conditions must satisfy by any velocity profile, whether it is in L.B.L zone @) T.B.L. zone.

(1) @ $y=0$, $u=0$ and $\frac{du}{dy}$ has some finite value

(2) at $y=\delta$, $u=U$,

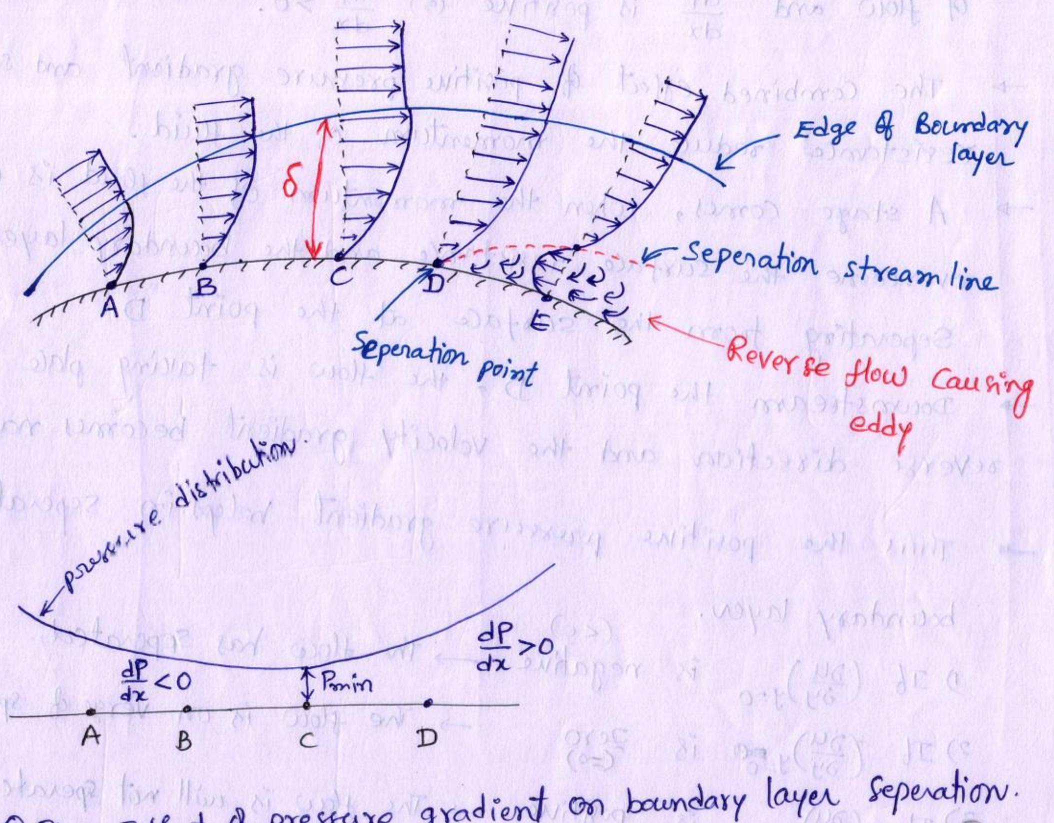
(3) at $y=\delta$, $\frac{du}{dy}=0$.

Seperation of Boundary layer;

When a fluid is flowing over a solid body, along the length of the solid body, at a certain point a stage may come. At this stage the boundary layer may not be able to keep sticking to the solid body if it cannot provide kinetic energy to overcome the resistance offered by the solid body. In other words, the boundary layer will be seperated from the surface. This phenomenon is called the boundary layer separation.

The point on the body at which the boundary layer is on the verge of seperation from the surface is called point of seperation.

Effect of pressure gradient on B.L. seperation :-



⇒ ① Fig :- Effect of pressure gradient on boundary layer seperation.

→ Consider flow over a curved surface "ABCDE" as shown in Fig (1).

⇒ In the region "ABC", the area of flow decreases and hence velocity increases. The flow gets accelerated in this region.

→ Due to increased velocity, the pressure decreases in the direction of flow and hence pressure gradient $\frac{dP}{dx}$ is negative in this region.

→ As long as $\frac{dP}{dx} < 0$, the entire boundary layer moves forward as shown in Fig.

⇒ In the region "CDE", The pressure is minimum at point C. the area of flow increases and hence velocity of fluid decreases.

→ Due to decreased velocity, the pressure increase in the direction of flow and $\frac{dP}{dx}$ is positive (or) $\frac{dP}{dx} > 0$.

→ The combined effect of positive pressure gradient and surface resistance reduce the momentum of the fluid.

→ A stage comes, when the momentum of the fluid is unable to overcome the surface resistance and the boundary layer starts seperating from the surface at the point "D".

→ Downstream the point 'D', the flow is taking place in reverse direction and the velocity gradient becomes nagative.

→ Thus the positive pressure gradient helps in seperating the boundary layer.

1) If $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is negative (<0) → The flow has seperated.

2) If $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is zero $(=0)$ → The flow is on verge of speration.

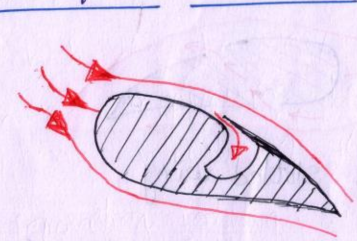
3) If $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is positive (>0) → The flow is will not sperate (or) flow will remain attached with surface.

Methods of Controlling of the Seperation of Boundary layer :-

When the boundary layer seperates from the Surface at point "D", a certain portion adjacent to the surface has a back flow and eddies are continuosly formed in this region and hence continuous loss of energy takes place. Therefore, this seperation can be avoided by various methods.

① Suction of the slow moving fluid by a suction solt. :-

In this method the slow moving fluid in boundary layer is removed by suction through slots or through a porous surface as shown in Figure.



So that, at point of suction a new boundary layer starts developing which is able to withstand an adverse pressure gradient and hence seperation is prevented.

②. supplying additional energy from a blower. :-

In this method some additional energy supplied to the fluid which is having very low velocity in the boundary layer. This may be achieved either by injecting fluid into boundary layer region from the interior of the body with help some suitable devices (Blower etc.)

A disadvantage of this method is that if the fluid is injected into laminar boundary layer, it undergoes a transition to turbulent boundary layer which results in an increased skin friction drag.

③. Providing a bypass in the slotted wing :-

This is similar to previous method (blower),
In this method, diverting a portion of the fluid of the main stream from the region of high pressure to the retarded region of boundary layer through a slot provided in the body as in the case of the slotted wing shown in Fig.

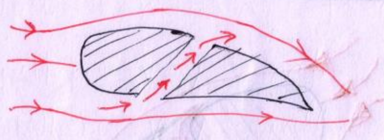
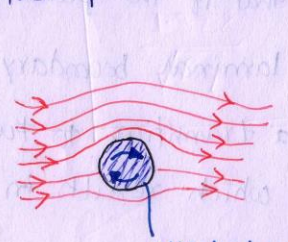


Fig: Slotted wing.

④ Rotating solid boundary in the direction of flow :-

→ The formation of boundary layer is due to the difference between the velocity of the flowing fluid and solid boundary. Therefore, it is possible to eliminate the formation of boundary layer by causing the solid boundary to move with the flowing fluid.

→ This can be achieved by rotating a circular cylinder which is in stream of fluid as shown in Figure. So that on the upper side of the cylinder, the fluid as well cylinder move in same direction, the boundary layer does not form and hence the seperation is completely eleminated. However on the lower side of the cylinder, where the fluid motion is opposite to that of cylinder, seperation would occur.



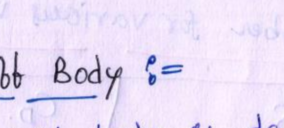
Stream - lined body :=

→ A stream-lined body is defined as that body whose surface coincides with the stream-lines, when the body is placed in a flow. In that case the seperation of flow will take place only at the trailing edge. Though the boundary layer will start at the leading edge, will become turbulent from laminar.

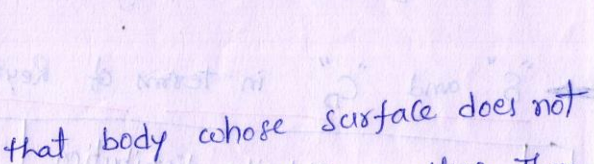
→ Thus, behind a stream-lined body, wake formation zone will be very small and consequently the pressure drag will be very small. Then the total drag on the stream-lined body will be due to friction (shear) only.

Blubb Body :=

- A bluff body is defined as that body whose surface does not coincide with the streamlines, when placed in a flow. Then the flow is seperated from the surface of the body at its trailing edge with the result of a very large wake formation zone. Then the drag due to pressure will be very large as compared to the drag due to friction on the body. Thus the bodies of such a shape in which the pressure drag is very large as compared to friction drag are called bluff bodies.



Stream lined Body



Blubb Body (i) & (ii)

Applications :-

1) Stream lined body :-

Sharks, Dolphins, Fishes, Airplanes, Bullets.

2) Bluff Body :-

→ Aerodynamic forces on cooling towers.

→ power station smokestacks (chimneys).

→ High rise buildings

→ Seashore tower.

→ Electronics Cooling etc.

→ "δ" and "C_D" in terms of Reynolds number for various velocity profiles.

	Velocity distribution	δ	C_D.
1.	$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$	$\frac{5.48 \cdot x}{\sqrt{Re_x}}$	$\frac{1.46}{\sqrt{Re_x}}$
2.	$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$	$\frac{4.64\, x}{\sqrt{Re_x}}$	$\frac{1.292}{\sqrt{Re_L}}$
3.	$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4$	$\frac{5.84\, x}{\sqrt{Re_x}}$	$\frac{1.36}{\sqrt{Re_L}}$
4.	$\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$	$\frac{4.79\, x}{\sqrt{Re_x}}$	$\frac{1.31}{\sqrt{Re_L}}$
5.	Blasius solution	$\frac{4.91\, x}{\sqrt{Re_x}}$	$\frac{1.328}{\sqrt{Re_L}}$

Problems on Boundary layer theory.

① Find the ratio of displacement thickness to momentum thickness and momentum thickness to energy thickness for the velocity distribution in the boundary layer given by

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

Sol:- velocity distribution, $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

(i) Displacement thickness $(\delta^*) = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$

$$\delta^* = \int_0^\delta \left\{1 - \left[2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2\right]\right\} dy$$

$$= \int_0^\delta \left[1 - 2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2\right] dy$$

$$= \int_0^\delta 1\, dy - \int_0^\delta 2\left(\frac{y}{\delta}\right) dy + \int_0^\delta \frac{y^2}{\delta^2}\, dy.$$

$$= [y]_0^\delta - \left[\frac{2y^2}{2\delta}\right]_0^\delta + \left[\frac{y^3}{3\delta^2}\right]_0^\delta$$

$$= \delta - \frac{\delta^2}{\delta} + \frac{\delta^3}{3\delta^2}$$

$$= \delta - \delta + \frac{\delta}{3}$$

$$\delta^* = \frac{\delta}{3}$$

(ii) Momentum thickness $(\theta) = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left[1 - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\right] dy$$

$$= \int_0^{\delta}\left(\frac{2y}{\delta}-\frac{y^2}{\delta^2}\right)\left[1-\frac{2y}{\delta}+\frac{y^2}{\delta^2}\right]dy$$

$$= \int_0^{\delta}\left[\frac{2y}{\delta}-\frac{4y^2}{\delta^2}+\frac{2y^3}{\delta^3}-\frac{y^2}{\delta^2}+\frac{2y^3}{\delta^3}-\frac{y^4}{\delta^4}\right]dy$$

$$= \int_0^{\delta}\left[\frac{2y}{\delta}-\frac{5y^2}{\delta^2}+\frac{4y^3}{\delta^3}-\frac{y^4}{\delta^4}\right]dy$$

$$= \left[\frac{2y^2}{2\delta}-\frac{5y^3}{3\delta^2}+\frac{4y^4}{4\delta^3}-\frac{y^5}{5\delta^4}\right]_0^{\delta}$$

$$= \left[\frac{\delta^2}{\delta}-\frac{5\delta^3}{3\delta^2}+\frac{\delta^4}{\delta^3}-\frac{\delta^5}{5\delta^4}\right]$$

$$= \delta-\frac{5\delta}{3}+\delta-\frac{\delta}{5}$$

$$\theta = \frac{2\delta}{15}.$$

(iii) Energy thickness, $\delta^{**} = \int_0^{\delta}\frac{u}{U}\left[1-\frac{u^2}{U^2}\right]dy$

$$= \int_0^{\delta}\left(\frac{2y}{\delta}-\frac{y^2}{\delta^2}\right)\left[1-\left(\frac{2y}{\delta}-\frac{y^2}{\delta^2}\right)^2\right]dy$$

$$= \int_0^{\delta}\left(\frac{2y}{\delta}-\frac{y^2}{\delta^2}\right)\left[1-\left[\frac{4y^2}{\delta^2}+\frac{y^4}{\delta^4}-\frac{4y^3}{\delta^3}\right]\right]dy$$

$$= \int_0^{\delta}\left(\frac{2y}{\delta}-\frac{y^2}{\delta^2}\right)\left[1-\frac{4y^2}{\delta^2}-\frac{y^4}{\delta^4}+\frac{4y^3}{\delta^3}\right]dy$$

$$= \int_0^{\delta}\left[\frac{2y}{\delta}-\frac{8y^3}{\delta^3}-\frac{2y^5}{\delta^5}+\frac{8y^4}{\delta^4}-\frac{y^2}{\delta^2}+\frac{4y^4}{\delta^4}+\frac{y^6}{\delta^6}-\frac{4y^5}{\delta^5}\right]dy$$

$$= \int_0^{\delta}\left[\frac{2y}{\delta}-\frac{y^2}{\delta^2}-\frac{8y^3}{\delta^3}+\frac{12y^4}{\delta^4}-\frac{6y^5}{\delta^5}+\frac{y^6}{\delta^6}\right]dy$$

$$= \left[\frac{2y^2}{2\delta}-\frac{y^3}{3\delta^2}-\frac{8y^4}{4\delta^3}+\frac{12y^5}{5\delta^4}-\frac{6y^6}{6\delta^5}+\frac{y^7}{7\delta^6}\right]_0^{\delta}$$

$$= \frac{\delta^2}{\delta} - \left[\frac{\delta^3}{3\delta^2} - \frac{2\delta^4}{\delta^3} + \frac{12\delta^5}{5\delta^4} - \frac{\delta^6}{\delta^5} + \frac{\delta^7}{7\delta^6}\right]$$

$$= \delta - \frac{\delta}{3} - 2\delta + \frac{12}{5}\delta - \delta + \frac{\delta}{7}$$

$$= -2\delta - \frac{\delta}{3} + \frac{12}{5}\delta + \frac{\delta}{7}$$

$$= \frac{-210\delta - 35\delta + 252\delta + 15\delta}{105}$$

$$\delta^{**} = \frac{22\delta}{105}.$$

$\Rightarrow$ Ratio of Displacement thickness to momentum thickness

$$\frac{\delta^*}{\theta} = \frac{\left(\frac{\delta}{3}\right)}{\left(\frac{2\delta}{15}\right)} = \frac{\delta}{3} \times \frac{15}{2\delta} = \underline{\underline{2.5}}$$

$\Rightarrow$ Ratio of Momentum thickness to energy thickness,

$$\frac{\theta}{\delta^{**}} = \frac{\left(\frac{2\delta}{15}\right)}{\left(\frac{22\delta}{105}\right)} = \frac{2\delta}{15} \times \frac{105}{22\delta} = \underline{\underline{\frac{7}{11}}}.$$

(2) For the velocity profile for laminar Boundary layer, $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$. Determine the boundary layer thickness, shear stress, drag force and co-efficient of drag in terms of Reynolds number.

Sol:- From the momentum integral equation, we have

$$\frac{\tau_0}{\rho U^2} = \frac{\partial \theta}{\partial x}$$

$$= \frac{\partial}{\partial x}\left[\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy\right]$$

$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\left[\int_0^\delta \left[\frac{3}{2}\left(\frac{y}{\delta}\right)-\frac{1}{2}\left(\frac{y}{\delta}\right)^3\right]\left[1-\left(\frac{3}{2}\left(\frac{y}{\delta}\right)-\frac{1}{2}\left(\frac{y}{\delta}\right)^3\right)\right]dy.\right.$$

$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\left[\int_0^\delta \left(\frac{3y}{2\delta}-\frac{y^3}{2\delta^3}\right)\left(1-\frac{3y}{2\delta}+\frac{y^3}{2\delta^3}\right)dy\right]$$

after integration we have,

$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\left(\frac{39\cdot\delta}{280.}\right)$$

$$\tau_0 = \frac{39}{280}\,\rho U^2\,\frac{\partial\delta}{\partial x}. \quad \text{———} (1)$$

and, $\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0}$.

$$= \mu\,\frac{3U}{2\delta}$$

$$\tau_0 = \frac{3}{2}\,\frac{\mu\cdot U}{\delta}. \quad \text{———} (2)$$

$$\left[\begin{aligned} u &= U\left[\frac{3}{2}\frac{y}{\delta}-\frac{y^3}{2\delta^3}\right] \\ \frac{du}{dy} &= U\left[\frac{3}{2\delta}-\frac{3y^2}{2\delta^3}\right] \\ \left(\frac{du}{dy}\right)_{y=0} &= \frac{3U}{2\delta}. \end{aligned}\right.$$

Equating equations (1) & (2),

$$\frac{39}{280}\,\rho U^2\,\frac{(\partial\delta)}{(\partial x)} = \frac{3}{2}\,\frac{\mu U}{\delta}.$$

$$\delta\cdot(\partial\delta) = \left(\frac{3}{2}\mu U\right)\times\frac{280}{39}\,\rho U^2\,\partial x.$$

Integrate both sides,

$$\int \delta\cdot(\partial\delta) = \int\left(\frac{3}{2}\times\frac{280}{39}\times\mu\times\rho\times U^3\right)\partial x$$

$$\frac{\delta^2}{2} = \frac{420}{39}\,\frac{\mu}{\rho U}\,x + C. \qquad \text{[where } C - \text{Constant]}$$

from the boundary condition,

at $x=0$, $\delta=0$, $\Rightarrow \therefore C=0$.

$$\therefore \quad \frac{\delta^2}{2} = \frac{420}{39} \cdot \frac{\mu}{\rho U} x \cdot$$

$$\delta = \sqrt{\frac{420 \times 2}{39} \frac{\mu}{\rho U} x}$$

$$= 4.64 \sqrt{\frac{\mu x}{\rho U}}$$

$$= 4.64 \sqrt{\frac{\mu x}{\rho U} \cdot \frac{x}{x}}$$

$$= 4.64 \sqrt{\frac{\mu}{\rho U x}} (x).$$

$$\left\{ R_e = \frac{\rho \cdot U \cdot x \cdot}{\mu \cdot} \right.$$

$$\therefore \quad \boxed{\delta \cdot = \frac{4.64\, x}{\sqrt{R_{ex}}}}$$

(i) shear stress, $\tau_0 = \frac{3}{2} \frac{\mu U}{\delta} = \frac{3}{2} \frac{\mu U}{\frac{4.64 x}{\sqrt{R_{ex}}}} = 0.323 \frac{\mu U}{x} \sqrt{R_{ex}} \cdot$

(ii) Drag force, $F_D = \int_0^L \tau_0 \times b \times dx$

$$= \int_0^L 0.323 \times \frac{\mu U}{x} \sqrt{R_{ex}} \times b \times dx$$

$$= \int_0^L 0.323 \times \frac{\mu U}{x} \sqrt{\frac{\rho U x}{\mu \cdot}} \times b \times dx$$

$$= \left[0.323 \times \mu U \times \sqrt{\frac{\rho U}{\mu}} \times b\right] \int_0^L \frac{\sqrt{x} \cdot}{x} \, dx \cdot$$

$$= 0.323 \times \mu U \times \sqrt{\frac{\rho U}{\mu}} \times b \int_0^L \frac{1}{\sqrt{x}} \cdot dx \cdot$$

$$= 0.323 \times \mu U \times \sqrt{\frac{\rho U}{\mu}} \times b \times \int_0^L x^{-\frac{1}{2}} dx \cdot$$

$$= 0.323 \times \mu U \times \sqrt{\frac{\rho U}{\mu}} \times b \times \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^L$$

$$\tau_0 = 0.323 \times 2 \times \mu U \times \sqrt{\frac{\rho U}{\mu}} \times b \times \sqrt{L}$$

$$\tau_0 = 0.646\, \mu U \sqrt{\frac{\rho U L}{\mu}} \times b.$$

$$\therefore \tau_0 = 0.646\, \mu U \sqrt{Re_L} \times b.$$

(iii) Drag Co-efficient (C_D)

$$C_D = \frac{F_D}{\frac{1}{2}\rho A U^2}$$

$$= \frac{0.646\, \mu U \sqrt{Re_L} \times b}{\frac{1}{2} \times \rho \times b \times L \times U^2}$$

$$= 0.646 \times 2 \times \frac{\mu}{\rho U L} \times \sqrt{\frac{\rho U L}{\mu}}$$

$$= \frac{1.292}{\sqrt{\frac{\rho U L}{\mu}}}$$

$$\boxed{C_D = \frac{1.292}{\sqrt{Re_L}}}$$

(3) A thin plate is moving in still atmospheric air at a velocity of 4 m/s. The length of the plate is 0.5 m and width 0.4 m. Calculate

(i) Thickness of the boundary at the end of the plate

(ii) Drag force on one side of the plate.

(iii) Take density of air as 1.25 kg/m³. and kinematic viscosity as 0.15 stokes.

JNTUK [NOV-2017]

Sol:- $U = 4\ m/s.$

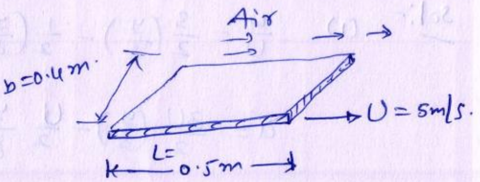
$L = 0.5\ m$

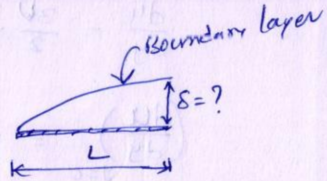
$b = 0.4\ m$

$\rho_{air} = 1.25\ kg/m^3$

$\nu = 0.15$ stokes

$= 0.15 \times 10^{-4}\ m^2/s.$





$$Re_L = \frac{U.L.}{\nu} = \frac{4 \times 0.5}{0.15 \times 10^{-4}} = 133333.3$$

From the Blasius Relation,

$$\delta = \frac{4.91\, x}{\sqrt{Re_x}} = \frac{4.91 \times L}{\sqrt{Re_L}} = \frac{4.91 \times 0.5}{\sqrt{133333.3}} = 6.72 \times 10^{-3}\ m$$

$$= 6.72\ mm.$$

$$C_D = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{133333.3}} = 3.636 \times 10^{-3}$$

$$\therefore F_D = C_D \times \frac{1}{2} \rho A U^2$$

$$= 3.636 \times 10^{-3} \times \frac{1}{2} \times 1.25 \times 0.5 \times 0.4 \times (4)^2$$

$$= 0.007215\ N.$$

(4) For the following velocity profiles, determine whether the flow has seperated or on the verge of seperation or will attach with the surface.

(i) $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

(ii) $\frac{u}{U} = 2\left(\frac{y}{\delta}\right)^2 - \left(\frac{y}{\delta}\right)^3$

(iii) $\frac{u}{U} = -2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2$

Sol:- (1) $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

$u = \frac{3U}{2}\left(\frac{y}{\delta}\right) - \frac{U}{2}\frac{y^3}{\delta^3}$

$\frac{du}{dy} = \frac{3U}{2} \cdot \frac{1}{\delta} - \frac{U}{2} \cdot \frac{3y^2}{\delta^3}$

$\left(\frac{du}{dy}\right)_{y=0} = \frac{3U}{2\delta} - 0 = \frac{3U}{2\delta}$.

$\therefore \left(\frac{du}{dy}\right)_{y=0} = \frac{3U}{2\delta} > 0$ (positive) ; Therefore the flow will not seperate (or) flow will remain attached with the surface.

(2) $\frac{u}{U} = 2\left(\frac{y}{\delta}\right)^2 - \left(\frac{y}{\delta}\right)^3$

$u = 2U \frac{y^2}{\delta^2} - \frac{y^3}{\delta^3}$

$\frac{du}{dy} = \frac{4Uy}{\delta^2} - \frac{3y^2}{\delta^3}$

$\left(\frac{du}{dy}\right)_{y=0} = 0 - 0 = 0$.

$\therefore \left(\frac{du}{dy}\right)_{y=0} = 0$, therefore the flow is on the verge seperation.

(3) $\frac{u}{U} = -2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2$

$u = -2U\left(\frac{y}{\delta}\right) + \frac{y^2}{\delta^2}$

$\left(\frac{du}{dy}\right) = -\frac{2U}{\delta} + \frac{2y}{\delta^2}$

$\left(\frac{du}{dy}\right)_{y=0} = -\frac{2U}{\delta} + 0 = -\frac{2U}{\delta}$ (negative)

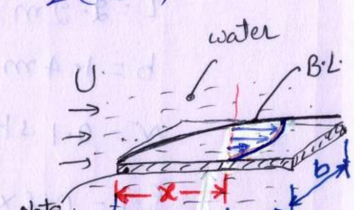
$\therefore \left(\frac{du}{dy}\right)_{y=0} = -\frac{2U}{\delta} < 0$,

The flow has seperated.

5. For the velocity profile in laminar Boundary layer as $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$. Find the thickness of the boundary layer and shear stress 1.8 m from the leading of a plate. The plate is 2.5 m long and 1.5 m wide and is placed in water which is moving with a velocity of 15 cm per second. Find the drag on one side of the plate if the viscosity of water = 0.01 poise.

Sol:- For given velocity profile, $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$,

the $\delta = \frac{4.64\,x}{\sqrt{Re_x}}$ and $C_D = \frac{1.292}{\sqrt{Re_L}}$



$L = 2.5\,m$

$b = 1.5\,m$

Area $= b \times L = 3.75\,m^2$

$U = 15\,cm/s = 0.15\,m/s$.

$\mu = 0.01$ poise $= 0.001\,Ns/m^2$

$\left[1\text{ poise} = \frac{1}{10}\,Ns/m^2\right]$

$\because x =$ Distance from the leading edge

$x = 1.8\,m$.

$\therefore Re_x = \frac{\rho \cdot U \cdot x}{\mu} = \frac{1000 \times 0.15 \times 1.8}{0.001}$

$Re_x = 2.7 \times 10^5$

$\therefore \delta = \frac{4.64 \times 1.8}{\sqrt{2.7 \times 10^5}} = 0.016\,m = \underline{1.6\,cm}$.

$Re_L = \frac{1000 \times 0.15 \times 2.5}{0.001}$

$= 3.75 \times 10^5$

$C_D = \frac{1.292}{\sqrt{Re_L}} = \frac{1.292}{\sqrt{3.75 \times 10^5}} = 2.109 \times 10^{-3}$

$\therefore F_D = C_D \times \frac{1}{2}\rho A U^2 = 2.109 \times 10^{-3} \times \frac{1}{2} \times 1000 \times 3.75 \times (0.15)^2 = \underline{0.089\,N}$

$\therefore \tau_0 = 0.323\,\frac{\mu U}{x}\sqrt{Re_x}$

$= 0.323 \times \frac{0.001 \times 0.15}{1.8}\sqrt{2.7 \times 10^5}$

$= \underline{0.0139\,N/m^2}$.

⑥ Oil with a free-stream velocity of 1.5 m/s flow over a thin plate 1.4 m wide and 2.2 m long. Calculate the boundary layer thickness and the shear stress at the trailing end point and determine the total surface resistance (or) drag of the plate. Take Sp. gravity of oil as 0.80 and kinematic viscosity as 0.1 stoke.

Sol:-

$U = 1.5 \, m/s$

$L = 2.2 \, m$

$b = 1.4 \, m$

$\nu = 0.1 \text{ stoke} = 0.1 \times 10^{-4} \, m^2/s$

$\rho_{oil} = 0.8 \times 1000 = 800 \, kg/m^3$.

$Re_x = \frac{U \cdot x}{\nu} = \frac{1.5 \times 2.2}{0.1 \times 10^{-4}} = 3.3 \times 10^5$

at $x = L$,

$\therefore Re_x = Re_L = 3.3 \times 10^5$

$\Rightarrow$ From the Blasius solution, $\delta = \frac{4.91 x}{\sqrt{Re_x}} = \frac{4.91 \times 2.2}{\sqrt{3.3 \times 10^5}} = 0.0188 \, m$

$= 1.88 \, cm$.

$\Rightarrow$ $\tau_0 = 0.332 \times \frac{\rho U^2}{\sqrt{Re_L}} = 0.332 \times \frac{800 \times (1.5)^2}{\sqrt{3.3 \times 10^5}} = 1.04 \, N/m^2$.

$\Rightarrow$ Surface Resistance (or) Drag force on one side of the plate is,

$C_D = \frac{1.328}{\sqrt{Re_L}}$.

$F_D = C_D \times \frac{1}{2} \rho A U^2$

$= 2.311 \times 10^{-3} \times \frac{1}{2} \times 800 \times 2.2 \times 1.4 \times 1.5^2$

$= 6.408 \, N$.

$C_D = \frac{1.328}{\sqrt{3.3 \times 10^5}}$

$= 2.311 \times 10^{-3}$

$\therefore$ Total resistance (or) Total drag force of the plate,

$F_{D_{total}} = 2 F_D$

$= 2 \times 6.408$

$= 12.816 \, N$.

⑦ Water is flowing over a thin smooth plate of length 4.5 m and width 2.5 m at a velocity of 0.9 m/s. If the boundary layer flow changes from laminar to turbulent at a $Re = 5\times10^5$, Find (i) the distance from the leading edge upto, which boundary layer is laminar, (ii) Thickness of the boundary layer at the transition point, (iii) Drag force on-one side of the plate at transition point. Take viscosity of water as 0.01 poise.

Sol:-

$L = 4.5\,m$

$b = 2.5\,m$

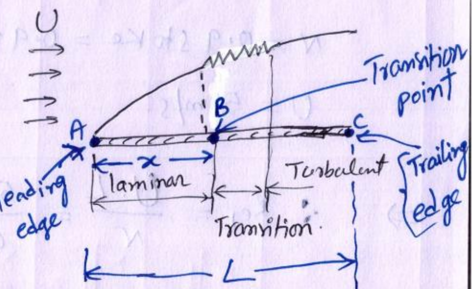
$U = 0.9\,m/s$

$\mu = 0.01$ poise $= 0.001\ Ns/m^2$

(i) $x = ?$

(ii) δ at transition point $= \dfrac{4.91x}{\sqrt{Re_x}}$

(iii) F_D



(i) $$Re = \frac{\rho U x}{\mu}$$

$$x = \frac{Re \cdot \mu}{\rho \cdot U} = \frac{5\times10^5 \times 0.001}{1000\times 0.9} = 0.555\,m$$

(ii) From Blasius solution,

$$\delta = \frac{4.91x}{\sqrt{Re_x}} = \frac{4.91\times 0.555}{\sqrt{5\times10^5}} = 3.85\,mm$$

(iii) $$C_D = \frac{1.328}{\sqrt{Re_x}} \qquad Re_L = \frac{\rho . U . x}{\mu} = \frac{1000\times 0.001\times 4.5}{0.001} = 40.5\times10^5$$

$$\therefore C_D = \frac{1.328}{\sqrt{40.5\times10^5}} = [illegible]\times10^{-3}$$

at transition, $F_D = C_D \times \frac{1}{2}\rho A U^2$

$$= 1.879\times10^{-3}\times\frac{1}{2}\times1000\times(2.5\times0.555)\times(0.9)^2$$

$$= 1.055\ N.$$

(8) Find the frictional drag on one side of the plate 200 mm wide and 500 mm long placed longitudinally in a stream of crude oil (sp. gravity = 0.925, kinematic viscosity = 0.9 stoke) flowing with undisturbed velocity of 5 m/s. Also find the thickness of boundary layer and the shear stress at the trailing edge of the plate.

Sol:-

$b = 200\ mm = 0.2\ m$

$L = 500\ mm = 0.5\ m$

$\rho = 0.925 \times 1000 = 925\ kg/m^3$

$\nu = 0.9\ stoke = 0.9 \times 10^{-4}\ m^2/s.$

$U = 5\ m/s.$

$$\Rightarrow \therefore Re_L = \frac{U \cdot L}{\nu} = \frac{5 \times 0.5}{0.9 \times 10^{-4}}$$

$$= 0.27 \times 10^5$$

From the Blasius solution,

$$\delta = \frac{4.91\,x}{\sqrt{Re_x}}$$

at trailing edge $x = L$,

$$\delta = \frac{4.91 \times L}{\sqrt{Re_L}}$$

$$\delta = \frac{4.91 \times 0.5}{\sqrt{0.27 \times 10^5}}$$

$$\delta = 0.01473\ m.$$

$$\delta = 14.73\ mm.$$

$$\Rightarrow \therefore \tau_0 = 0.332 \times \frac{\rho U^2}{\sqrt{Re_L}}$$

$$= 0.332 \times \frac{925 \times (5)^2}{\sqrt{0.27 \times 10^5}} = 46.72\ N/m^2$$

$$\Rightarrow C_D = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{0.27 \times 10^5}} = 8.0819 \times 10^{-3}$$

$$F_D = \frac{1}{2} \rho A U^2 \times C_D = \frac{1}{2} \times 925 \times 0.2 \times 0.5 \times 5^2 \times 8.0819 \times 10^{-3}$$

$$= 9.344\ N.$$

(9) A smooth plate of length 5 m and width 2 m is moving with a velocity of 4 m/s in stationary air of density as 1.25 kg/m³ and kinematic viscosity 1.5×10^{-5} m²/s. Determine thickness of the boundary layer at the trailing edge of the smooth plate. Find the total drag on one side of the plate assuming that the boundary layer is turbulent from the begining.

NOTE. **

For turbulent Boundary layer,

$$\delta = \frac{0.37\,x}{(Re_x)^{1/5}} \; ; \; C_D = \frac{0.072}{(Re_L)^{1/5}}$$

$$\tau_0 = 0.0225\,\rho U^2 \left(\frac{\mu}{\rho U \delta}\right)^{1/4}$$

Sol:- $L = 5m$

$b = 2m$

$U = 4\, m/s$

$\rho_{air} = 1.25\, kg/m^3$

$\nu = 1.5 \times 10^{-5}\, m^2/s$

at trailing edge $x = L$,

$$Re_L = Re_x = \frac{U.L}{\nu} = \frac{4 \times 5}{1.5 \times 10^{-5}} = 13.3 \times 10^5$$

$$\Rightarrow \delta = \frac{0.37 \times 5}{(13.3 \times 10^5)^{1/5}} = 0.1102\, m = \underline{110.2\, mm}.$$

$$\Rightarrow C_D = \frac{0.072}{(13.3 \times 10^5)^{1/5}} = 4.288 \times 10^{-3}$$

$$F_D = C_D \times \frac{1}{2} \rho A U^2$$

$$= 4.288 \times 10^{-3} \times \frac{1}{2} \times 1.25 \times (5 \times 2) \times 4^2$$

$$= \underline{0.428\, N}.$$

*** Important Relations for solving problems ***

S.NO:	Velocity distribution profile $\left(\frac{u}{U}\right)$	Boundary layer thickness (δ)	Drag Co-efficient (C_D)	Shear stress (τ_0)
1.	$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$	$\frac{5.48x}{\sqrt{Re_x}}$	$\frac{1.46}{\sqrt{Re_L}}$	$0.365 \frac{\mu U}{x} \cdot \sqrt{Re_x}$
2.	$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$	$\frac{4.64x}{\sqrt{Re_x}}$	$\frac{1.292}{\sqrt{Re_L}}$	$0.323 \frac{\mu U}{x} \sqrt{Re_x}$
3.	$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4$	$\frac{5.84x}{\sqrt{Re_x}}$	$\frac{1.36}{\sqrt{Re_L}}$	$0.34 \frac{\mu U}{x} \sqrt{Re_x}$
4.	$\frac{u}{U} = \sin\left(\frac{\pi}{2} \cdot \frac{y}{\delta}\right)$	$\frac{4.79x}{\sqrt{Re_x}}$	$\frac{1.31}{\sqrt{Re_L}}$	$0.327 \frac{\mu U}{x} \sqrt{Re_x}$
5.	Blasius's solution (if no profile is given)	$\frac{4.91x}{\sqrt{Re_x}}$	$\frac{1.328}{\sqrt{Re_L}}$	$0.332 \frac{\mu U}{x} \sqrt{Re_x}$
6.	Turbulent Boundary layer on flat plate, $[\because Re > 5 \times 10^5]$	$\frac{0.37x}{(Re_x)^{1/5}}$	$\frac{0.072}{(Re_L)^{1/5}}$	$0.225 \rho U^2 \left(\frac{\mu}{\rho U \delta}\right)^{1/4}$

⇒ $F_D = C_D \times \frac{1}{2} \rho A U^2$ ← For any kind of velocity distribution (on one side of the plate)

⇒ Total $F_D = 2 F_D$ ← Both sides of the plate.

-: DIMENSIONAL ANALYSIS :-

UNIT-3
PART-B

Dimensional analysis :-

It is a method of dimensions. It is a mathematical technique used in research work for design and for conducting model tests.

S.No.	physical quantity	Symbol	Dimensions.
	(a) Fundamental		
1.	Length	L	L
2.	Mass	M	M
3.	Time	T	T
	(b) Geometric		
4.	Area	A	L^2
5.	Volume	V	L^3
	(c) Kinematic Quantities		
6.	Velocity	u	LT^{-1}
7.	acceleration	a	LT^{-2}
8.	viscosity	ν	L^2T^{-1}
	(d) Dynamic Quantities		
9.	Force	F	MLT^{-2}
10.	weight	W	MLT^{-2}
11.	Density	ρ	ML^{-3}
12.	Dynamic viscosity	μ	$ML^{-1}T^{-1}$
13.	Surface tension	σ	$ML^0T^{-2} = MT^{-2}$

Methods of Model analysis :-

1. Rayleigh method.
2. Buckingham's π-theorem.

Model :- It is the small scale replica of the actual structure or machine. The actual structure or machine is called prototype.

Prototype :- The actual structure (or) machine is called prototype.

It is not necessary that the models should be smaller than the prototypes, they may be larger than the prototype.

Model analysis :- The study of models of actual machines is called model analysis. It is actually an experimental method of finding solutions of complex flow problems.

Advantages of the Dimensional and Model analysis :-

1. The performance of the hydraulic structure or hydraulic machine can be easily predicted in advance, from its model.
2. With the help of dimensional analysis, a relationship between the variables influencing a flow problem in terms of dimensionless parameters is obtained. This relationship helps in conducting tests on the model.
3. The merits of alternative designs can be predicted with the help of model testing. The most economical and safe design may be, finally adopted.
4. The tests performed on the models can be utilized for obtaining, in advance, useful information about the performance of the prototypes only if a complete similarity exists between the model and prototype.

Similitude :- It is defined as the similarity between the model and its prototype in every respect, which means that the model and prototype have similar properties or model and prototype are completely similar.

Three types of similarities must exist between the model and prototype. They are

(1.) **Geometric similarity :-**

It is said to be exist between the model and the prototype. The ratio of all corresponding linear dimensions in the model and prototype are equal.

Let, L_m = Length of the model, L_p = Length of the prototype.

b_m = Breadth " , b_m = Breadth "

D_m = Diameter " , D_m = Diameter "

A_m = Area " , A_m = Area "

V_m = Volume " , V_m = Volume "

For geometric similarity between model and prototype,

$$\boxed{\frac{L_p}{L_m} = \frac{b_p}{b_m} = \frac{D_p}{D_m} = \frac{V_p}{V_m} = L_r}$$

where, $L_r \rightarrow$ scale ratio.

For area's ratio $\Rightarrow \frac{A_p}{A_m} = \frac{L_p \times b_p}{L_m \times b_m} = L_r \times L_r = L_r^2$ $\quad \boxed{\therefore \frac{A_p}{A_m} = L_r^2}$

For volumes ratio $\Rightarrow \boxed{\frac{V_p}{V_m} = L_r^3}$

(2) Kinematic similarity :-

It is the similarity of motion between model and prototype.

Thus kinematic similiarity is said to exist between the model and the prototype if the ratios of the velocity and acceleration at the corresponding points in the model and at the corresponding points in the prototype are the same.

Let, V_{P_1} = Velocity of fluid at point 1 in prototype

V_{P_2} = " " point 2 "

a_{P_1} = Acceleration of fluid at point 1 in prototype

a_{P_2} = " " at point 2 "

$V_{m_1}, V_{m_2}, a_{m_1}, a_{m_2}$ = Corresponding values at corresponding points of fluid velocity and acceleration in the model.

For kinematic similarity, we must have

$$\boxed{\frac{V_{P_1}}{V_{m_1}} = \frac{V_{P_2}}{V_{m_2}} = V_r}$$ $V_r \rightarrow$ Velocity ratio.

$$\boxed{\frac{a_{P_1}}{a_{m_1}} = \frac{a_{P_2}}{a_{m_2}} = a_r}$$ $a_r \rightarrow$ acceleration ratio.

(3) Dynamic similarity :-

It is the similarity of forces between the model and prototype.

$$\boxed{\frac{(F_i)_P}{(F_i)_m} = \frac{(F_v)_P}{(F_v)_m} = \frac{(F_g)_P}{(F_g)_m} = F_r ,}$$ F_r = Force ratio.

$(F_i)_P \rightarrow$ Inertia force at a point in prototype

$(F_v)_P \rightarrow$ Viscous force "

$(F_g)_P \rightarrow$ Gravity force "

$(F_i)_m, (F_v)_m, (F_g)_m \Rightarrow$ Forces at a point in model.

Dimensionless Numbers :-

(1) Reynolds number (Re) :-

It is defined as the ratio of inertia force of a flowing fluid and the viscous force of the fluid.

$$\text{Inertia force } (F_i) = m \times \text{acceleration}$$

$$= (\rho \times \text{volume}) \times \frac{\text{Velocity}}{\text{time}}$$

$$= (\rho A V) \times V$$

$$= \rho A V^2$$

$\left\{ \frac{\text{Volume}}{\text{time}} = \text{discharge} = A \times V. \right.$

$V \rightarrow$ velocity m/s
$A \rightarrow$ area m^2

$$\text{Viscous force } (F_v) = \text{shear stress} \times \text{Area}$$

$$= \tau \times A$$

$$= \mu \left(\frac{du}{dy}\right) \times A$$

$$= \mu \frac{V}{L} \times A.$$

$$\frac{du}{dy} = \frac{\text{Velocity}}{\text{Length}} = \frac{V}{L}$$

$$\therefore Re = \frac{\rho \cancel{A} V^{\cancel{2}}}{\mu \frac{\cancel{V}}{L} \times \cancel{A}} = \frac{\rho V L}{\mu}.$$

Kinematic Viscosity
$\left\{ \nu = \frac{\mu}{\rho}. \right.$

$\rightarrow$ $$\boxed{Re = \frac{\rho \cdot V \cdot L}{\mu} \quad (\text{or}) \quad \frac{V \cdot L}{\nu}}$$

$\rightarrow$ In case of flow through a pipe,

$$\boxed{Re = \frac{\rho \cdot V \cdot d}{\mu} \quad (\text{or}) \quad \frac{V \cdot d}{\mu}}$$

where,
$\rho \rightarrow$ density of fluid kg/m^3
$V \rightarrow$ velocity of fluid m/s
$d \rightarrow$ diameter of pipe m.
$L \rightarrow$ Length of plate m
$\mu \rightarrow$ Dynamic viscosity Ns/m^2
$\nu \rightarrow$ Kinematic viscosity m^2/s

② Froude's number :- (F_e) (or) (F_r)

The froude's number is defined as the square root of the ratio of inertia force of a flowing fluid to the gravity force.

$$F_e \text{ (or) } F_r = \sqrt{\frac{F_i}{F_g}}$$

$\Rightarrow F_i$ = Inertia force
$= m \times$ acceleration
$= \rho A V^2$

$\Rightarrow F_g$ = Force due to gravity
= Mass × acceleration due to gravity
$= \rho \times \text{volume} \times g$
$= \rho \times L^3 \times g$
$= \rho \times L^2 \times L \times g$
$= \rho \times A \times L \times g$

$$\therefore F_e = \sqrt{\frac{\cancel{\rho A} V^2}{\cancel{\rho A} L g}}$$

$$= \sqrt{\frac{V^2}{Lg}}$$

$$\therefore \boxed{F_e = \frac{V}{\sqrt{Lg}}}$$

③ Euler's Number :- (E_u).

It is defined as the square root of the ratio of the inertia force of a flowing fluid to the pressure force.

F_P = Intensity of pressure × Area = $P \times A$.
$F_i = \rho A V^2$

$$E_u = \sqrt{\frac{F_i}{F_P}}$$

$$= \sqrt{\frac{\rho \cancel{A} V^2}{P \times \cancel{A}}}$$

$$\boxed{E_u = \frac{V}{\sqrt{\left(\frac{P}{\rho}\right)}}}$$

where,
$V \rightarrow$ velocity m/s.
$P \rightarrow$ pressure N/m²
$\rho \rightarrow$ Density kg/m³.

(4) Mach's Number (M) :-

It is defined as the square root of the ratio of the inertia force of a flowing fluid to the elastic force.

$$M = \sqrt{\frac{F_i}{F_e}}$$

F_e = Elastic stress × Area

$= K \times A.$

$= K \times L^2$

$\Rightarrow$ $K \rightarrow$ elastic stress.

$\Rightarrow$ $V \rightarrow$ velocity

$\Rightarrow$. $C \rightarrow$ velocity of sound.

$\Rightarrow$. $C = \sqrt{\frac{K}{\rho}}$.

$$= \sqrt{\frac{\rho A V^2}{K L^2}}$$

$$= \sqrt{\frac{\rho L^2 V^2}{K L^2}} = \sqrt{\frac{V^2}{(K/\rho)}} = \frac{V}{C}$$

$C = 331.2$ m/s. in dry air at 0°C.

$$\boxed{\therefore M = \frac{V}{C}.}$$

(5) Weber's Number (We) :-

It is defined as the square root of the ratio of the inertia force of a flowing fluid to the surface tension.

$$We = \sqrt{\frac{F_i}{F_s}}$$

$\Rightarrow$. F_s = Surface tension Force

= Surface tension per unit length × Length

$\therefore F_s = \sigma \times L.$

$$= \sqrt{\frac{\rho A V^2}{\sigma \times L.}}$$

$$= \sqrt{\frac{\rho L^2 V^2}{\sigma \times L}} = \sqrt{\frac{\rho \times L \times V.}{\sigma \times L}} = \sqrt{\frac{V^2}{\left(\frac{\sigma}{\rho L}\right)}} = \frac{V}{\sqrt{\left(\frac{\sigma}{\rho L}\right)}}.$$

$$\boxed{\therefore We = \frac{V}{\left(\frac{\sigma}{\rho L}\right)}}$$

where,

$V \rightarrow$ velocity m/s.

$\sigma \rightarrow$ surface tension N/m.

$\rho \rightarrow$ density kg/m³

$L \rightarrow$ Length m.

(UNIT- 4) (Complete)

Basics of Turbo machinery :- (Impact of jets)

→ The liquid comes out in the form of a jet from the nozzle, which is fitted to a pipe through which the liquid is flowing under pressure.

→ If some plate, which may be fixed or moving, is placed in the path of the jet, a force is exerted by the jet on the plate. This force is obtained from Newton's second law of motion or from impulse-momentum equation.

→ Thus impact of jet means the force exerted by the jet on a plate which may be stationary or moving.

Various cases we are going to discuss,

(1.) Force exerted by the jet on a stationary plate when

(a) plate is vertical to the jet

(b) plate is inclined to the jet

(c) plate is curved — at center / at tip

(2) Force exerted by the jet on the moving plate when

(a) plate is vertical to the jet,

(b) plate is inclined to the jet,

(c) plate is curved: — at center / at tip.

(3) Force exerted by the jet on a series of Radial curved vanes.

(1) Force exerted by the jet on a stationary vertical plate. :-

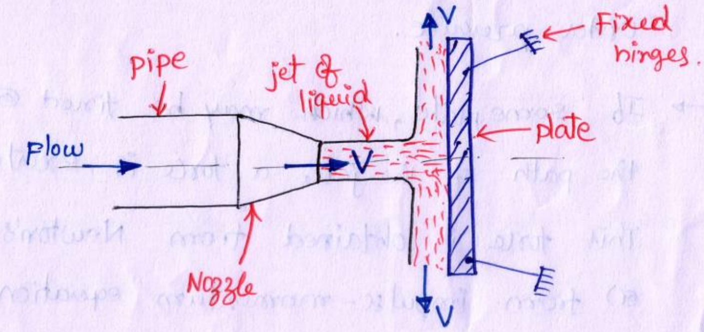
Let Consider a jet of liquid coming out from the nozzle, strikes a flat vertical plate as shown in Figure.

V – velocity of the jet

d – diameter of the jet

A – area of cross-section of the jet

$A = \frac{\pi}{4} d^2$



→ The jet striking the plate, will move along the plate. But the plate is at right angles to the jet. Hence the jet after striking, will get deflected through 90°. Hence the component of the velocity of jet, in the direction of jet, after striking will be zero.

∴ The force exerted by the jet on the plate in the direction of jet.

F_x = Rate of change of momentum in the direction of force

$$= \frac{(\text{Initial momentum}) - (\text{Final momentum})}{\text{Time}}$$

$$= \frac{(\text{Mass} \times \text{initial velocity of jet before striking}) - (\text{mass} \times \text{Final velocity of jet after striking})}{\text{Time}}$$

$$= \frac{\text{mass}}{\text{Time}} (\text{initial velocity} - \text{Final velocity}) \qquad \left\{ \begin{array}{l} V_1 = V \\ V_2 = 0. \end{array} \right\}$$

$$= \rho A \cdot V (V_1 - V_2)$$

$$= \rho A V (V - 0) = \rho A V^2$$

$$\therefore \boxed{F_x = \rho A V^2}$$

$\Rightarrow F_x \rightarrow$ Force exerted by jet on plate

NOTE: If to calculate Force exerted on jet, then the value of 'F_x' will be negative.

② Force exerted by a jet on stationary inclined flat plate :-

→ Let a jet of water coming out from nozzle, strikes an inclined flat plate as shown in Figure.

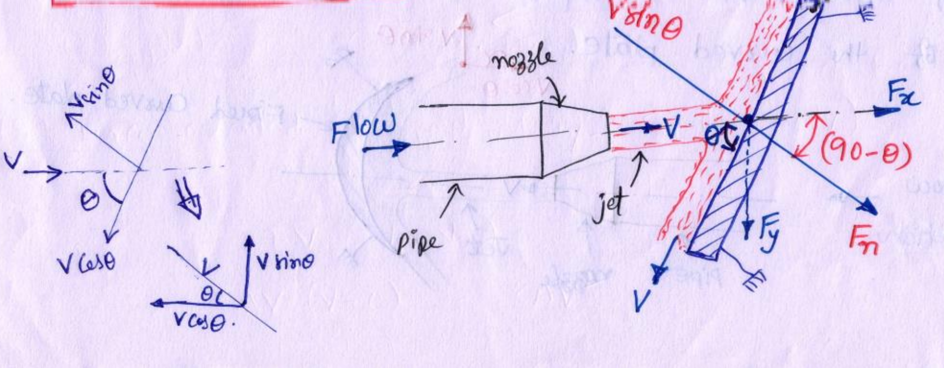
→ If the plate is smooth, then there is no loss of energy due to impact of the jet, then jet will move over the plate after striking with a velocity equal to initial velocity (V).

→ Force exerted by the jet on plate in the direction normal to plate.

$$F_n = \frac{\text{mass of jet striking}}{\text{sec}} \times \left(\begin{array}{l}\text{Initial velocity of jet}\\ \text{before striking}\\ \text{in direction of 'n'}\end{array} - \begin{array}{l}\text{Final velocity of}\\ \text{jet after}\\ \text{striking}\\ \text{in direction}\\ \text{of 'n'}\end{array}\right)$$

$$= \rho A V (V \sin\theta - 0)$$

$$\boxed{F_n = \rho A V^2 \sin\theta}$$



F_x = Component of 'F_n' in the direction of flow

$= F_n \cos(90^\circ - \theta)$

$= F_n \cdot \sin\theta$

$= \rho A V^2 \sin\theta \times \sin\theta$

$$\boxed{F_x = \rho A V^2 \sin^2\theta}$$

F_y = Component of 'F_n' perpendicular to flow.

$= F_n \sin(90^\circ - \theta)$

$= F_n \cos\theta$

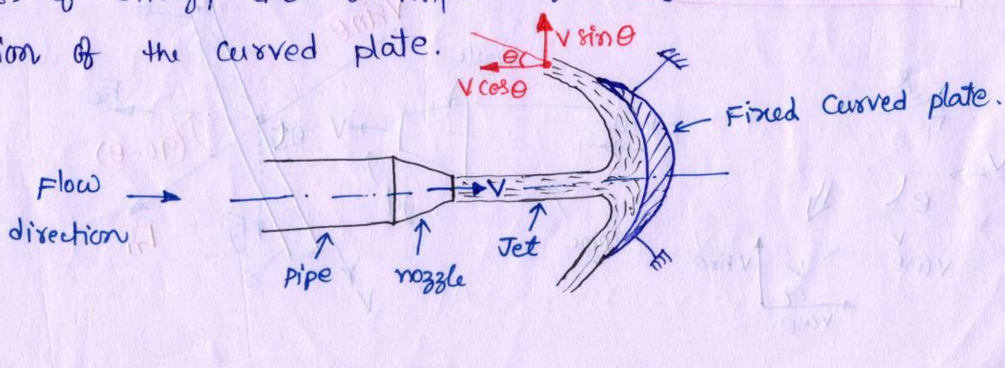
$$\boxed{F_y = \rho A V^2 \sin\theta \cdot \cos\theta}$$

③ Force exerted by a jet on stationary Curved plate :-

(a) Jet strikes the Curved plate at the Centre,

(a) Jet strikes the curved plate at the centre

→ Let a jet of liquid strikes a fixed curved plate at the centre as shown in Figure. The jet after striking the plate, comes out with the same velocity if the plate is smooth and there is no loss of energy due to impact of the jet, in the tangential direction of the curved plate.



The velocity at outlet of the plate can be resolved in two components, one in the direction of the jet and other perpendicular to the direction of the jet.

Velocity in direction of jet $(V_{2x}) = -V\cos\theta$

velocity perpendicular to jet $(V_{2y}) = V\sin\theta$.

∴ Force exerted by the jet in direction of jet,

$$F_x = \text{Mass per sec} \times [V_{1x} - V_{2x}] \qquad \left\{\begin{array}{l} V_{1x} = V \\ V_{2x} = -V\cos\theta \end{array}\right\}$$

$$= \rho AV[V - (-V\cos\theta)]$$

$$= \rho AV(V + V\cos\theta)$$

$$\boxed{F_x = \rho AV^2(1+\cos\theta)}$$

∴ Force exerted by the jet in perpendicular to direction of jet,

$$F_y = \text{Mass per sec} \times [V_{1y} - V_{2y}] \qquad \left\{\begin{array}{l} V_{1y} = 0 \\ V_{2y} = V\sin\theta \end{array}\right\}$$

$$= \rho AV(0 - V\sin\theta)$$

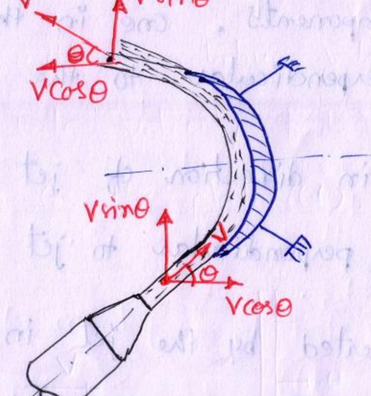
$$\boxed{F_y = -\rho AV^2\sin\theta}$$

(b) Jet strikes the curved plate at one end tangentially when the plate is symmertical. [MID-2]

→ Let the curved plate is symmetrical about x-axis.

→ Let the jet strikes the curved plate at one end tangentially as shown in Figure. [Next page]

The forces exerted by jet of liquid in x and y directions,



$$F_x = (mass/sec) \times (V_{1x} - V_{2x})$$

$$= \rho AV \left(V\cos\theta - (-V\cos\theta) \right)$$

$$= \rho AV \left(V\cos\theta + V\cos\theta \right)$$

$\rightarrow V_{1x} = V\cos\theta$

$\rightarrow V_{2x} = -V\cos\theta.$

$$\boxed{F_x = 2\rho AV^2 \cos\theta}$$

$\rightarrow V_{1y} = V\sin\theta$

$\rightarrow V_{2y} = V\sin\theta.$

$$F_y = (mass/sec) \times (V_{1y} - V_{2y})$$

$$= \rho AV \times (V\sin\theta - V\sin\theta)$$

$$\boxed{F_y = 0}$$

(c) Jet strikes the curved plate at one end tangentially when the plate unsymmetrical.

when the curved plate is unsymmetrical about x-axis, then angle made by the tangents drawn at the inlet and outlet tips of the plate with x-axis will be different.

let θ = angle made by tangent at inlet tip with x-axis,

ϕ = angle made by tangent at outlet tip with x-axis.

$V_{1x} = V\cos\theta$

$V_{2x} = -V\cos\phi$

$V_{1y} = V\sin\theta$

$V_{2y} = V\sin\phi$.

$\therefore F_x = (\text{mass/sec}) \times (V_{1x} - V_{2x})$

$= \rho AV \times [V\cos\theta - (-V\cos\phi)]$

$$\boxed{F_x = \rho AV^2(\cos\theta + \cos\phi)}$$

$\therefore F_y = (\text{mass/sec}) \times [V_{1y} - V_{2y}]$

$= \rho AV \times (V\sin\theta - V\sin\phi)$

$$\boxed{F_y = \rho AV^2(\sin\theta - \sin\phi)}$$

Force exerted by a jet on moving plates :-

(4) Force on flat vertical plate moving in direction of jet :-

Figure (Nextpage) shows a jet of water striking a flat vertical plate moving with a uniform velocity away from the jet.

V = Absolute velocity of jet

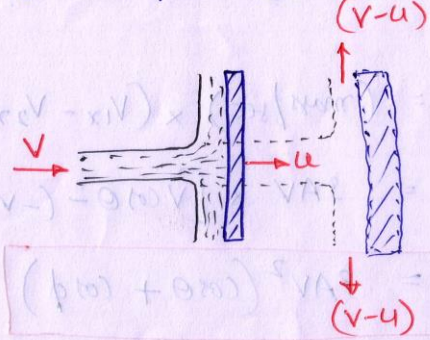
a = Area of cross-section of the jet

u = velocity of the flat plate.

In this case, the jet does not strike the plate with a velocity V, but it strikes with a relative velocity, which is equal to the absolute velocity of the jet of water (V) minus the velocity of the plate (u).

∴ Relative velocity of the jet,

$= (V-u)$.



∴ Mass of water striking the plate,

$m = \rho \times A \times (V-u)$

⇒ Force exerted by the jet on moving plate in direction of jet,

$F_x = \text{mass} \times [\text{Initial velocity with which water strikes} - \text{Final velocity}]$

$= \rho A (V-u) [(V-u) - 0]$

$$\boxed{F_x = \rho A (V-u)^2}$$

∴ Work done per second by jet on plate,

$W = \text{Force} \times \dfrac{\text{Distance in direction of force}}{\text{Time}}$

$= F_x \times u$

$$\boxed{W = \rho A (V-u)^2 \times u}$$ unit → Watt (or) N-m/s. (or) J/s.

∴ Efficiency of the jet $(\eta) = \dfrac{\text{output of jet per sec}}{\text{Input of jet per sec}} = \dfrac{W}{\frac{1}{2} m V^2}$

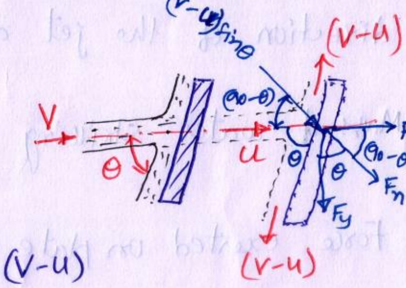
$= \dfrac{W}{\frac{1}{2} (\rho A V) V^2}$

$$\boxed{\eta = \frac{2W}{\rho A V^3}}$$

⑤ Force On Moving inclined plat plate :-

Let a jet of water strikes on inclined plate, which is moving with a uniform velocity in the direction of the jet as shown.

θ = Angle between jet and plate.



→ Relative velocity of jet of water $= (V-u)$

→ The velocity with which jet strikes $= (V-u)$

→ Mass of water striking per second,

$m = \rho \cdot A \cdot (V-u)$

→ The plate smooth and there is no loss of energy, There fore the jet of water will leave the plate with a velocity equal to $(V-u)$.

→ Force, in direction normal to the plate is,

F_n = Mass × [Initial velocity in normal direction − Final velocity]

$= \rho A (V-u) \left[(V-u) \sin\theta - 0 \right]$

$$\boxed{F_n = \rho A (V-u)^2 \sin\theta}$$

→ This "F_n" (normal force) resolved in two components (F_x and F_y).

$\Rightarrow F_x = F_n \sin\theta$ $\qquad \Rightarrow F_y = F_n \cos\theta$

$$\boxed{F_x = \rho A (V-u)^2 \sin^2\theta} \qquad \boxed{F_y = \rho A (V-u)^2 \sin\theta \cdot \cos\theta}$$

→ Work done per second by jet on plate, $W = F_x \times u$

$$\boxed{W = \rho A (V-u)^2 \cdot u \cdot \sin^2\theta} \qquad \boxed{\eta = \frac{2W}{\rho A V^3}}$$

⑥ Force on moving Curved plate :-

Let a jet of water strikes a curved plate at centre of the plate which is moving with a uniform velocity in direction of the jet as shown in Figure.

Mass of water striking the plate, $m = (V-u) \times \rho \times A$

∴ Force exerted on plate in direction of the jet,

$$F_x = \text{mass per sec} \times [V_{1x} - V_{2x}]$$

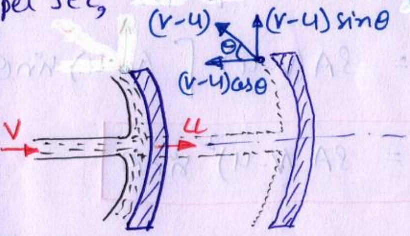
$$= \rho \times A \times (V-u) \times [(V-u) - (-(V-u)\cos\theta)]$$

$$\begin{cases} V_{1x} = (V-u) \\ V_{2x} = -(V-u)\cos\theta \end{cases}$$

$$\boxed{F_x = \rho A (V-u)^2 [1+\cos\theta]}$$

∴ Work done per sec by the jet on plate,

W = $F_x \times$ distance travelled per sec, in x-direction



$= F_x \times u$

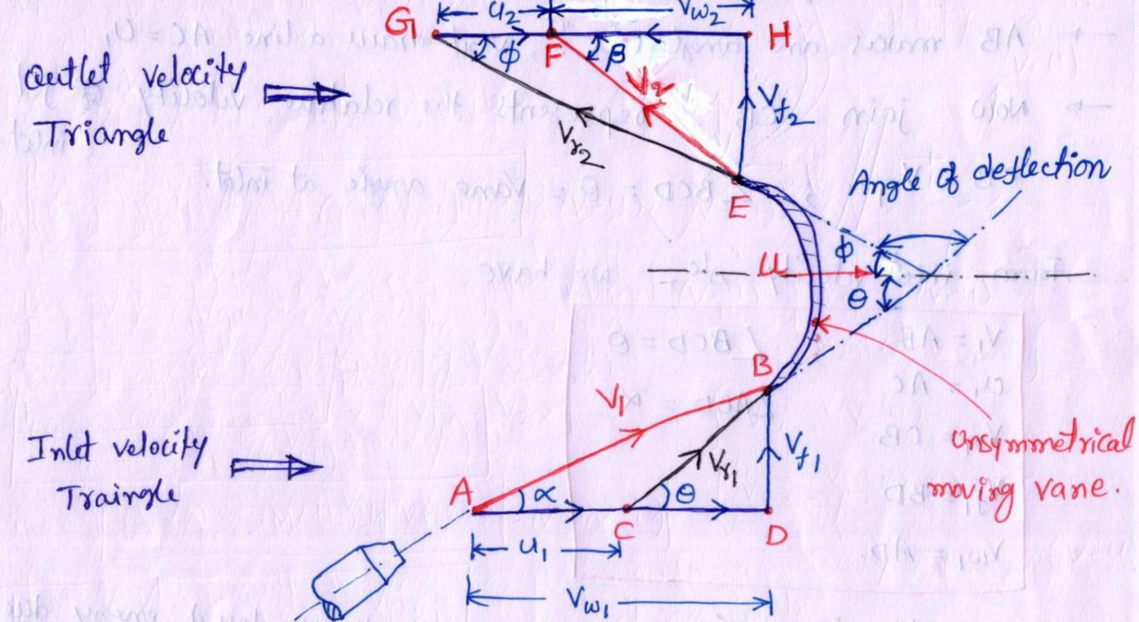
$$\boxed{W = \rho A (V-u)^2 \cdot u \cdot (1+\cos\theta)}$$

∴ Efficiency,

$$\eta = \frac{\text{Output of jet}}{\text{Input of jet}} = \frac{\text{Work done per sec}}{\text{K.E of jet per sec}} = \frac{W}{\frac{1}{2} m V^2}$$

$$\boxed{\therefore \eta = \frac{2W}{\rho A V^3}}$$

7. Force exerted by jet of water on an unsymmetrical moving curved plate when jet strikes Tangentially at one end of the tip of the curved plate.



Where,

V_1 & V_2 – velocity of jet at inlet and outlet

u_1 & u_2 – velocity of plate at inlet and outlet

V_{r_1} & V_{r_2} – Relative velocity of jet and plate at inlet and outlet

V_{w_1} & V_{w_2} → whirl velocity at inlet and outlet [whirl velocity is the tangential (horizontal) component of absolute velocity of jet (V_1)] either inlet (or) outlet.

V_{f_1} & V_{f_2} → Flow velocity at inlet and outlet.

α → gauide angle (Angle b/n the direction of jet and direction of motion of the plate).

θ → Vane angle at inlet (Angle made by relative velocity (V_{r_1}) with direction of motion at inlet).

β → Angle made by velocity "V_2" with the direction of motion of vane at outlet.

ϕ → Angle made by relative velocity "V_{r_2}" with direction of motion of the vane at outlet. (Vane angle at outlet)

Inlet velocity Δ^{le};

→ Draw line $AB = V_1$. and Draw vertical line "BD" $= V_{f1}$

→ Draw a horizontal line, up to D, Then we have 'AD' $= V_{w1}$.

→ AB makes an angle "α". Next draw a line $AC = u_1$

→ Now join CB, represents the relative velocity of jet at inlet.

$CB = V_{r1}$. ; $\angle BCD = \theta$ = Vane angle at inlet.

From inlet velocity Δ^{le}, we have

$V_1 = AB$	$\angle BCD = \theta$.
$u_1 = AC$	$\angle ABD = \alpha$.
$V_{r1} = CB$	
$V_{f1} = BD$	
$V_{w1} = AD$.	

Outlet velocity Δ^{le}; (If vane is very smooth, the loss of energy due to friction will be zero). Then $V_{r1} = V_{r2}$.

→ $V_{r1} = V_{r2}$

→ Draw a line in the tangential direction of vane at outlet and cut $EG = V_{r2}$.

→ From G draw a line upto F to form "$GF = u_2$" in the direction of vane at outlet.

→ Draw a vertical line from point "E". Now draw a horizontal line from "F" which meet ~~line E~~ vertical line from 'E'. Where these two lines are intersecting name as point "H".

→ Now $EH = V_{f2}$ and $FH = V_{w2}$. , Now join "F" and "E", $EF = V_2$

$EG = V_{r2}$	$\angle GEH = \phi$ = Vane outlet angle
$GF = u_2$	$\angle FHE = \beta$.
$EF = V_2$	
$EH = V_{f2}$	
$FH = V_{w2}$	

→ If the vane is smooth, then $u_1 = u_2$, $V_{r_1} = V_{r_2}$.

→ Mass of water per sec $m = \rho \cdot A \cdot V_{r_1}$.

⇒ $F_x = (\text{mass/sec}) \times \left[\text{Initial velocity of jet in direction of motion} - \text{Final velocity of jet in direction of motion} \right]$

$$\boxed{F_x = m\,(V_{r_{1x}} - V_{r_{2x}})} \quad \text{———} \quad (1)$$

Initial velocity with which jet strikes the vane $= V_{r_1}$.

The Component of this velocity in the direction of motion.

$CD = V_{r_{1x}} = V_{r_1} \cos\theta = (V_{w_1} - u_1)$. —(2)

similarly, at outlet, $GH = V_{r_2 x} = -(V_{r_2} \cos\phi) = -(u_2 + V_{w_2})$. —(3)

substitute (2) & (3) in equation (1),

$$F_x = \rho A V_{r_1} \left[(V_{w_1} - u_1) - \left(-(u_2 + V_{w_2})\right) \right]$$

$$= \rho A V_{r_1} \left[V_{w_1} - \cancel{u_1} + \cancel{u_2} + V_{w_2} \right]$$

$\left\{ \begin{array}{l} \text{as plate is smooth,} \\ u_1 = u_2 = u \end{array} \right.$

$$\boxed{F_x = \rho A V_{r_1} (V_{w_1} + V_{w_2})}$$

The above equation is truely valid for β is an ~~actute~~ Acute angle.

→ If $\beta = 90^\circ$, the $V_{w_2} = 0$. , Then $\boxed{F_x = \rho A V_{r_1} \cdot V_{w_1}}$

→ If β is obtuse angle, V_{w_2} will be negative, $\boxed{F_x = \rho A V_{r_1} (V_{w_1} - V_{w_2})}$

(1) ∴ $\boxed{F_x = \rho A V_{r_1} (V_{w_1} \pm V_{w_2})}$

$\left\{ \begin{array}{l} \beta = 90^\circ \text{ (Right angle)} \\ \beta = \text{Acute angle } (\beta < 90^\circ) \\ \beta = \text{obtuse angle } (\beta > 90^\circ). \end{array} \right.$

⇒ power $= F_x \times u$

(2) ∴ $\boxed{P = \rho A \cdot V_{r_1} \cdot u \cdot (V_{w_1} \pm V_{w_2})}$

NOTE :-
- Actue angle means → angle b/n 0° and 90°.
- Obtuse angle → angle b/n 90° & 180°.
- Straight angle → 180°.
- Right angle → 90°.

⇒ work done/sec per unit weight of fluid per second,

$$= \frac{P}{\text{weight of fluid striking per sec}}$$

$$= \frac{\rho A V_{r_1} u [V_{w_1} \pm V_{w_2}]}{\rho A V_{r_1} \times g}$$

(3) $$\boxed{\frac{P}{\text{weight}} = \frac{(V_{w_1} \pm V_{w_2}) \cdot u}{g} \quad Nm/N}$$

⇒ power per unit mass of fluid striking per sec,

$$\frac{P}{m} = \frac{\rho A V_{r_1} (V_{w_1} \pm V_{w_2}) \cdot u}{\rho A V_{r_1}}$$

(4) $$\boxed{\therefore \frac{P}{m} = (V_{w_1} \pm V_{w_2}) \cdot u \quad Nm/Kg}$$

⇒ Efficiency of jet, $\eta = \frac{\text{Output}}{\text{Input}}$

$$\eta = \frac{\text{Power}}{\text{Initial K.E. per sec}} = \frac{\rho A V_{r_1} \cdot u_1 \cdot (V_{w_1} \pm V_{w_2})}{\frac{1}{2} m V_1^2}$$

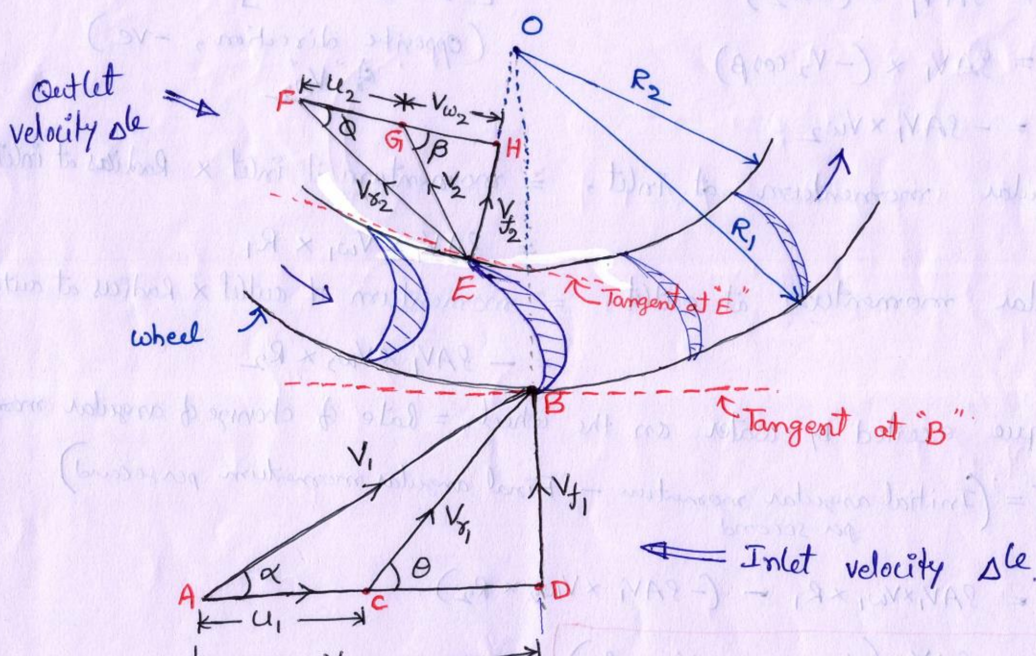
$$\eta = \frac{\rho A V_{r_1} \cdot u_1 \cdot (V_{w_1} \pm V_{w_2})}{\frac{1}{2} \rho A V_1 \cdot V_1^2} \qquad \{m = \rho A V_1\}$$

(5) $$\boxed{\eta = \frac{2 V_{r_1} \cdot u_1 \cdot (V_{w_1} \pm V_{w_2})}{V_1^3}}$$

(8) Force exerted on a series of Radial Curved Vanes :-

→ For a radial curved vane, the radius of the vane at inlet and outlet is different and hence the tangential velocities of the radial vane at inlet and outlet will not be equal.

→ Let consider a series of radial curved vanes mounted on a wheel as shown in Figure. The jet of water strikes the vanes and the wheel starts rotating at a constant angular speed.



R_1 = Radius of wheel at inlet of the vane.

R_2 = Radius of wheel at outlet of the vane.

ω = Angular speed of the wheel.

Then, velocity of the vane at inlet $u_1 = \omega R_1$

velocity of the vane at outlet $u_2 = \omega R_2$.

→ The mass of water striking per second for a series of vanes. = mass of coming out from nozzle.

$m = \rho \cdot A \cdot V_1$.

$A \rightarrow$ Jet area (or) nozzle exit area.

→ Momentum of water striking the vanes in tangential direction per second,

$= \rho A V_1 \times V_{w_1}$. $\quad [V_1 \cos\alpha = V_{w_1}]$

→ Momentum of water at outlet,

$= \rho A V_1 \times (V_{w_2})$. $\quad [V_2 \cos\beta = V_{w_2}]$

$= \rho A V_1 \times (-V_2 \cos\beta)$ $\quad$ (opposite direction, -ve). of "V_2"

$= -\rho A V_1 \times V_{w_2}$

→ Angular momentum at inlet, = momentum at inlet × Radius at inlet

$= \rho A V_1 \times V_{w_1} \times R_1$

→ Angular momentum at outlet, = momentum at outlet × Radius at outlet.

$= -\rho A V_1 \times V_{w_2} \times R_2$

→ Torque exerted by water on the wheel, = Rate of change of angular momentum

$T =$ (Initial angular momentum per second − Final angular momentum per second)

$= \rho A V_1 \times V_{w_1} \times R_1 - (-\rho A V_1 \times V_{w_2} \times R_2)$.

$$\boxed{T = \rho A V_1 (V_{w_1} \times R_1 + V_{w_2} \times R_2)}$$

Workdone per second on the wheel, = Torque × Angular velocity $\quad \{u_1 = \omega R_1,\ u_2 = \omega R_2\}$.

WD/sec = Power = $T \times \omega = \rho A V_1 (V_{w_1} \times R_1 + V_{w_2} \times R_2) \times \omega$.

$P = \rho A V_1 (V_{w_1} \times R_1 \cdot \omega + V_{w_2} \times R_2 \cdot \omega)$

$P = \rho A V_1 (u_1 \cdot V_{w_1} + u_2 V_{w_2})$.

$$\boxed{\therefore P = \rho A V_1 (u_1 V_{w_1} \pm u_2 V_{w_2})}$$

(i) if β is right angle ($\beta = 90°$), then $V_{w_2} = 0$.

$P = \rho A V_1 (u_1 V_{w_1})$.

(ii) if β is obtuse angle, then; ($\beta < 90°$).

$P = \rho A V_1 (u_1 V_{w_1} - u_2 V_{w_2})$

(iii) if β is Acute angle, then. ($\beta > 90°$)

$P = \rho A V_1 (u_1 V_{w_1} + u_2 V_{w_2})$.

→ Efficiency of the Radial curved vane,

$$\eta = \frac{\text{workdone per second}}{\text{K.E. per second}} = \frac{\rho A V_1 (u_1 V_{w_1} \pm u_2 V_{w_2})}{\frac{1}{2}(\rho A V_1) \times V_1^2}$$

$$\boxed{\therefore \eta = \frac{2(V_{w_1} u_1 + V_{w_2} u_2)}{V_1^2}}$$

→ The efficiency will be maximum when V_{w_2} is added to V_{w_1}.

- This is only possible, if β is an acute angle. $(\beta < 90^\circ)$.

→ Also for maximum efficiency V_{w_2} should be minimum.

- This is only possible, if β is right angle $(\beta = 0^\circ)$
- In this case $V_{w_2} = V_2 \cos\beta = V_2 \cos(0^\circ) = V_2$

$\therefore V_{w_2} = V_2$ and $\beta = 0$.

- But in actual practice ϕ cannot be zero. Hence for maximum efficiency, the angle ϕ should be minimum.

⇒ If there is no loss of energy, when water is flowing over the vanes,

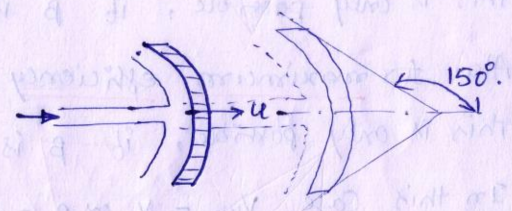
$$WD/\text{sec} = \Delta K.E \text{ per second}.$$
$$= (K.E_{inlet} - K.E_{out}) = \left(\frac{1}{2} m V_1^2\right) - \left(\frac{1}{2} m V_2^2\right) = \frac{1}{2} m (V_1^2 - V_2^2)$$

$$\eta = \frac{WD/\text{sec}}{K.E_{inlet}} = \frac{\frac{1}{2} m (V_1^2 - V_2^2)}{\frac{1}{2} m V_1^2} = \frac{V_1^2 - V_2^2}{V_1^2} = \left(1 - \frac{V_2^2}{V_1^2}\right).$$

$$\boxed{\therefore \eta = 1 - \frac{V_2^2}{V_1^2}}$$ ← if there is no loss of energy.

(1) A jet of water of diameter 100 mm strikes a curved plate at its centre with a velocity of 18 m/s. The curved plate is moving with a velocity of 9 m/s in the direction of the jet. The jet is deflected through an angle of 150°. Assuming the plate smooth. Find: (i) Force exerted on the plate in the direction of the jet (ii) power of the jet (iii) Efficiency.

Sol:- $d = 100\,mm = 0.1\,m$



Area $= \frac{\pi}{4} d^2 = 7.8539 \times 10^{-3}\,m^2$

plate velocity $(u) = 9\,m/s$.

Angle of deflection $= 150°$. , $V = 18\,m/s$.

(i) F_x

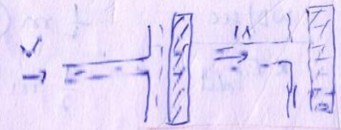
(ii) Power

(iii) η_{jet}

{ This problem is similar to problems No. (3). }

(2) A nozzle of 50 mm diameter delivers a stream of water at 20 m/s perpendicular ~~that~~ to a plate that moves away from the jet at 5 m/s. Find: (i) Force on the plate (ii) work done and (iii) η_{jet}.

Sol:- $d = 50\,mm = 0.05\,m$



$A = \frac{\pi}{4} d^2 = 0.001963\,m^2$

$V = 20\,m/s$.

$u = 5\,m/s$.

(i) $F_x = \rho A (v-u)^2 = 441.78\,N$

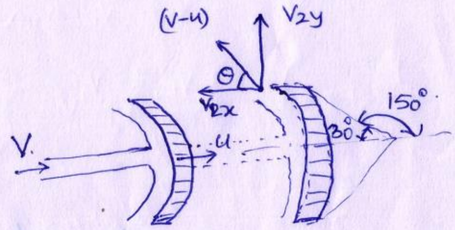
(ii) $W = F_x \times u = 2208.9\,Nm/s$.

(iii) $\eta = \dfrac{F_x \times u}{\frac{1}{2}(\rho AV) \times V^2} = 0.3377$

$= 33.77\%$.

(3) A jet of water of diameter 100 mm strikes a curved plate at its centre with a velocity of 15 m/s. The curved plate is moving with the velocity of 7 m/s in the direction of the jet. The jet is deflected through an angle of 150°. Assuming plate is smooth. Find: (i) Force exerted on the plate in the direction of the jet, (ii) power of the jet, (iii) Efficiency.

Sol:- diameter of jet (d) = 100 mm
= 0.1 m.



$\therefore$ Area of the jet (A) $= \frac{\pi}{4} d^2$

$= \frac{\pi}{4} (0.1)^2$

$= 7.853 \times 10^{-3} m^2$

$\therefore \theta = 30°$

Velocity of jet (V) = 15 m/s.

Velocity of plate (u) = 7 m/s.

Angle of deflection = 150°

$\therefore \theta = 180 - 150° = 30°$.

(i) Force exerted on plate in direction of jet

$F_x = \rho . A . (V-u)^2 (1 + \cos\theta)$

$= 1000 \times 7.853 \times 10^{-3} \times (15-7)^2 (1 + \cos 30°)$

$= 937.8 \, N$.

(ii) power of the jet,

$$P = F_x \times u$$

$$= 937.849 \times 7$$

$$= 6564.9 \, W$$

$$P = \underline{6.56 \, kW}.$$

(iii) Efficiency of the jet,

$$\eta = \frac{\text{Input of jet}}{\text{output of jet}}$$

$$= \frac{\text{Power}}{\frac{1}{2} m V^2}$$

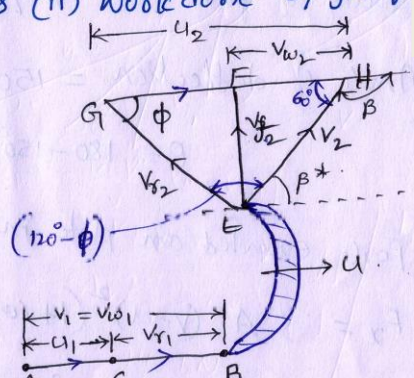
$$= \frac{2P}{\rho A V^3}$$

$$= \frac{2 \times 6564.9}{1000 \times 7.853 \times 10^{-3} \times (15)^3}$$

$$= 0.4953$$

$$= \underline{49.53\%}$$

(4) A 150 mm diameter jet moving at 30 m/s impinges on a curved vane moving at 15 m/s in the direction of the jet. The jet leaves the vanes at 60° with the direction of motion of the vanes. Calculate : (i) Force exerted by the jet in the direction of motion of vanes (ii) work done by jet per second.



Sol:- $d = 150 mm = 0.15 m$.

$$A = \frac{\pi}{4} d^2 = 0.01767 m^2$$

$$V_1 = 30 \, m/s$$

$$u = u_2 = u_1 = 15 \, m/s.$$

As jet and vane are moving in the same direction, $\alpha = 0°$, $\theta = 0°$

$\therefore \beta = 180 - \beta^* = 180 - 60° = 120°. > 90°$ ($\because \beta$ is obtuse angle).

$$u_1 = u_2 = u = 15 \, m/s.$$

$$V_{r_1} = V_{r_2} = 15 \, m/s.$$

$$V_{r_1} = AB - AC.$$

$$= V_1 - u_1$$

$$= 30 - 15 = \underline{15 \, m/s}.$$

apply sine rule to the Δ^{le} GEH, $\angle GEH = 180^\circ - (60^\circ + \phi) = 120^\circ - \phi$.

$$\frac{EG}{\sin 60^\circ} = \frac{GH}{\sin(120^\circ - \phi)} \Rightarrow \frac{V_{r_2}}{\sin 60^\circ} = \frac{u_2}{\sin(120^\circ - \phi)}$$

$$\frac{15}{\sin 60^\circ} = \frac{15}{\sin(120^\circ - \phi)}$$

$$\therefore 60^\circ = 120 - \phi$$

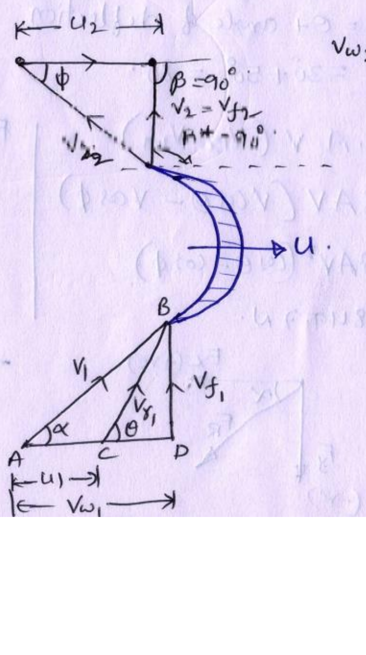
$$\phi = 60^\circ.$$

$\therefore V_{w_2} = HF = GH - \cancel{EH}\,GF = u_2 - V_{r_2}\cos\phi = 15 - (15 \times 60^\circ) = 7.5\ m/s.$

i) $F_x = \rho A V_{r_1}[V_{w_1} - V_{w_2}] = 1000 \times 0.01767 \times 15 \times [30 - 7.5] = 5964.11\ N.$

(ii) Power $= F_x \times u = 5964.1 \times 15 = 89461.7$ Watts $= 89.461$ KW.

(5) A jet of water having a velocity of 40 m/s strikes a curved vane, which is moving with a velocity of 20 m/s. The jet makes an angle of 30° with the direction of motion of vane at inlet and leaves at an angle of 90° to the direction of motion of vane at outlet. Draw the velocity triangles at inlet and outlet and determine the vane angles at inlet and outlet so that the water enters and leaves the vane without shock.



Sol:- $V_1 = 40$ m/s

$u_1 = 20$ m/s $- u_2 = u$

$\alpha = 30^\circ$

$\beta^* = 90^\circ$

$\beta = 180 - 90^\circ = 90^\circ$.

$\therefore V_2 = V_{f_2}$

$V_{w_2} = 0$.

From ΔBCD, $\quad \tan\theta = \dfrac{BD}{CD} = \dfrac{BD}{AD - AC} = \dfrac{V_{f_1}}{V_{w_1} - u_1}$

$V_{f_1} = V_1 \cdot \sin\alpha = 40 \times \sin 30^\circ = 20\, m/s.$

$V_{w_1} = V_1 \cos\alpha = 40 \times \cos 30^\circ = 34.64\, m/s.$

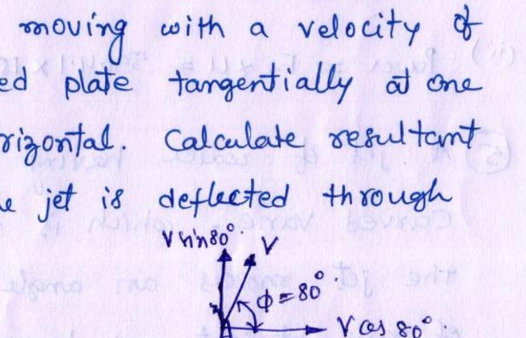
$\therefore\ \theta = \tan^{-1}\left(\dfrac{V_{f_1}}{V_{w_1} - u_1}\right) = \tan^{-1}\left(\dfrac{20}{34.64 - 20}\right) = \underline{53.79^\circ}.$

From ΔBCD,

$\sin\theta = \dfrac{V_{f_1}}{V_{r_1}} \Rightarrow V_{r_1} = \dfrac{V_{f_1}}{\sin\theta} = \dfrac{20}{\sin(53.79)} = 24.78\, m/s.$

From ΔEFG, $\left[\cos\phi = \dfrac{u_2}{V_2}\right] = \dfrac{20}{24.78} = 0.8071 \Rightarrow \phi = \underline{36.18^\circ}.$

6) A jet of water of diameter 50 mm moving with a velocity of 25 m/s impinges on a fixed curved plate tangentially at one end at an angle of 30° to the horizontal. Calculate resultant force of the jet on the plate if the jet is deflected through an angle of 50°. Take $g = 10\, m/s^2$.



Sol:- $d = 0.05\, m$
$V = 25\, m/s.$

→ $\theta = 30^\circ$. (Angle made by jet at inlet).

→ Angle of deflection $= 50^\circ$.

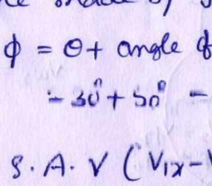
→ Angle made by jet at outlet is ϕ,

$\phi = \theta +$ angle of deflection
$= 30^\circ + 50^\circ = 80^\circ.$

$F_x = \rho \cdot A \cdot V\,(V_{1x} - V_{2x})$
$= \rho A V\,(V\cos\theta - V\cos\phi)$
$= \rho A V^2 (\cos\theta - \cos\phi)$
$= 849.7\, N.$

$F_y = \rho A V\,[V_{1y} - V_{2y}]$
$= \rho A V\,[V\sin\theta - V\sin\phi]$
$= \rho A V^2\,[\sin\theta - \sin\phi]$
$= -594.9\, N.$

$F_R = \sqrt{F_x^2 + F_y^2}$
$= \underline{1037\, N}.$



-ve sign shows that force "F_y" is acting in the downward direction,

$\tan\alpha = \dfrac{F_y}{F_x} \Rightarrow \alpha = \tan^{-1}\left(\dfrac{F_y}{F_x}\right) = \underline{35^\circ}.$

⑦ A jet of water having a velocity of 35 m/s impinges on a series of vanes moving with a velocity of 20 m/s. The jet makes an angle of 30° to the direction of motion of vanes when entering and leaves at an angle of 120°. Draw the velocity triangles at inlet and outlet and find:

(a) The angles of vanes tips so that water enters and leaves without shock.

(b) The work done per unit weight of water entering the vanes,

(c) The efficiency.

Sol:- $V_1 = 35\ m/s$

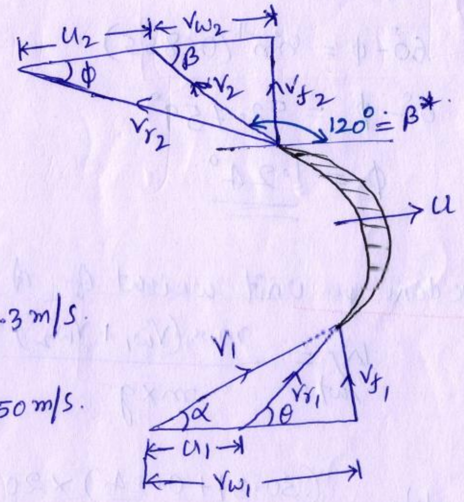
Velocity of vane $u_1 = u_2 = u = 20\ m/s$.

Angle of jet at inlet $\alpha = 30°$.

Angle made by jet at outlet with the direction of motion of vanes, $\beta^* = 120°$.

$(\beta = 180 - \beta^*)$.

$\therefore \beta = 180 - 120° = 60°$. (Acute angle)



ⓐ Angle of vanes tips,

From Inlet velocity Dle,

$V_{w_1} = V_1 \cos\alpha = 35 \times \cos(30°) = 30.3\ m/s$.

$V_{f_1} = V_1 \sin\alpha = 35 \times \sin(30°) = 17.50\ m/s$.

$$\theta = \tan^{-1}\left[\frac{(V_{f_1})}{(V_{w_1} - u_1)}\right]$$

$$= \tan^{-1}\left(\frac{17.50}{30.3 - 20}\right) \quad \therefore \theta = 60°.$$

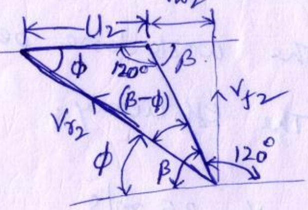
By sine rule,

$$\frac{V_{r_1}}{\sin 90^\circ} = \frac{V_{f_1}}{\sin\theta}.$$

$$V_{r_1} = \sin 90^\circ \times \frac{V_{f_1}}{\sin\theta} \Rightarrow V_{r_1} = 1 \times \frac{17.50}{\sin(60^\circ)} = 20.25\ m/s.$$

as plate is smooth, $V_{r_1} = V_{r_2} = 20.25\ m/s.$

From outlet velocity Δle, by sine rule,



$$\frac{V_{r_2}}{\sin 120^\circ} = \frac{U_2}{\sin(\beta-\phi)}$$

$$\sin(\beta-\phi) = \sin(120^\circ) \times \frac{U_2}{V_{r_2}}$$

$\beta = 180 - 120^\circ = 60^\circ.$

$$\sin(60^\circ-\phi) = \sin(120^\circ) \times \frac{20}{20.25}$$

$u_1 = u_2 = u = 20\ m/s.$

$\sin(60^\circ-\phi) = 0.855$

$60^\circ - \phi = \sin^{-1}(0.855)$

$60^\circ - \phi = 58.759^\circ$

$\phi = 1.24^\circ.$

$V_{w_2} = (V_{r_2}\cos\phi - U_2)$

$= (20.25 \times \cos(1.24^\circ)) - (20)$

$= 0.24\ m/s.$

(b) Work done per unit weight of of water entering the vanes, $= \frac{\text{Power}}{m \times g}.$

$$W/wt. = \frac{m \times (V_{w_1} + V_{w_2}) \times u}{m \times g} = \frac{(V_{w_1} + V_{w_2}) \times u}{g}.$$

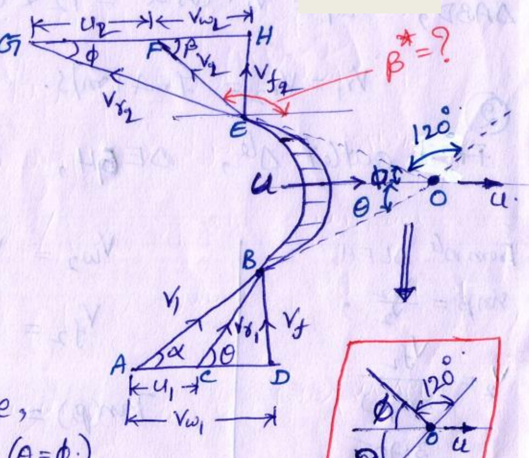
$$\frac{W}{wt} = \frac{(30.31 + 0.24) \times 20}{9.81} = 62.28\ \frac{Nm}{N}.$$

(c) $$\eta = \frac{2 \times P}{\rho A V_1^3} = \frac{2 \times V_{r_1} \times u_1 (V_{w_1} + V_{w_2})}{V_1^3} = \frac{2 \times 20.25 \times 20 \times (30.31 + 0.24)}{(35)^3} = 0.9974$$

$= 99.74\%$

A jet of water moving at 12 m/s impinges on vane shaped

(8) to deflect the jet through 120° when stationary. If the vane is moving at 5 m/s, Find the angle of the jet so that there is no shock at inlet. What is the absolute velocity of the jet at exit in magnitude and direction and the workdone per second per unit weight of water striking per second? Assume that the vane is smooth.



Sol:-

$V_1 = 12$ m/s

$u_1 = u_2 = u = 5$ m/s.

Angle of deflection $= 120°$

$\theta + \phi = 180° - 120° = 60°.$

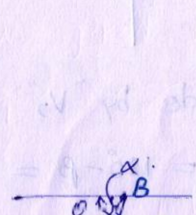
In problem, the plate is symmetrical (or) unsymmetrical not given. Therefore, Let consider the plate is symmetrical. ($\theta = \phi$).

$\therefore \theta = \phi = 30°.$

$$\left\{\begin{aligned} \theta + \phi &= 60° \\ \theta + \theta &= 60° \\ 2\theta &= 60° \\ \theta &= 30° \end{aligned}\right.$$

(1)

From Inlet Δ^{le}, by sine rule, $\angle ABC$.



$$\frac{V_1}{\sin(180-\theta)} = \frac{u_1}{\sin(\theta - \alpha)}$$

$$\sin(\theta - \alpha) = \frac{u_1}{V_1} \times \sin(180° - \theta)$$

$$= \frac{5}{12} \times \sin(180° - 30°)$$

$$\sin(\theta - \alpha) = 0.2083$$

$\theta - \alpha = \sin^{-1}(0.2083)$

$\alpha = \theta^\circ - \sin^{-1}(0.2083)$

$\alpha = 17.98^\circ$.

Apply sine rule, to $\angle ABC$.

$$\frac{V_1}{\sin(180-\theta)} = \frac{V_{r_1}}{\sin\alpha} \Rightarrow V_{r_1} = 7.41 \text{ m/s}.$$

ΔABD; $V_{w_1} = V_1 \cdot \cos\alpha = 12 \times \cos(17.98^\circ) = 11.41$ m/s.

$V_{r_1} = V_{r_2} = 7.41$ m/s.

(2)

From outlet Δ^{le}, ΔEGH, $V_{r_2}\cos\phi = u_2 + V_{w_2}$

$V_{w_2} = V_{r_2}\cos\phi - u_2 = 7.41(\cos 30^\circ) - 5 = 1.417$ m/s.

$V_{f_2} = V_{r_2}\sin 30^\circ = 7.41\sin(30^\circ) = 3.705$ m/s.

$\tan(\beta) = \dfrac{V_{f_2}}{V_{w_2}} \Rightarrow \beta = \tan^{-1}\left(\dfrac{V_{f_2}}{V_{w_2}}\right)$.

$= \tan^{-1}\left(\dfrac{3.705}{1.417}\right)$.

$\beta = 69.07^\circ$. (Acute angle).

From Δ^{le}. ΔEFH,

$\sin\beta = \dfrac{V_{f_2}}{V_2}$,

$V_2 = \dfrac{V_{f_2}}{\sin(\beta)}$

$= \dfrac{3.705}{\sin(69.07)}$

$V_2 = 3.96$ m/s.

$\therefore$ Angle made by "V_2" at outlet with the direction of motion of vane,

$\beta^* = 180^\circ - \beta = 110.93^\circ$.

(3)

Work done per sec per unit weight of water striking per second,

$$\frac{\text{Power}}{(\text{wt/sec})} = \frac{(V_{w_1} + V_{w_2})}{g} \times u. \quad \text{in } \frac{\text{Nm/s}}{\text{N/s}}$$

$$= \frac{(11.41 + 1.417)}{9.81} \times 5 \quad \frac{\text{Nm}}{\text{N}}.$$

$= 6.537$ Nm/N.

$\Rightarrow$ weight of water per second $= m \times g$ in N/s. $\{m = \rho A V_{r_1}\}$

$\Rightarrow$ work done per sec $=$ power in Nm/s

$= m \times u \times (V_{w_1} + V_{w_2})$.

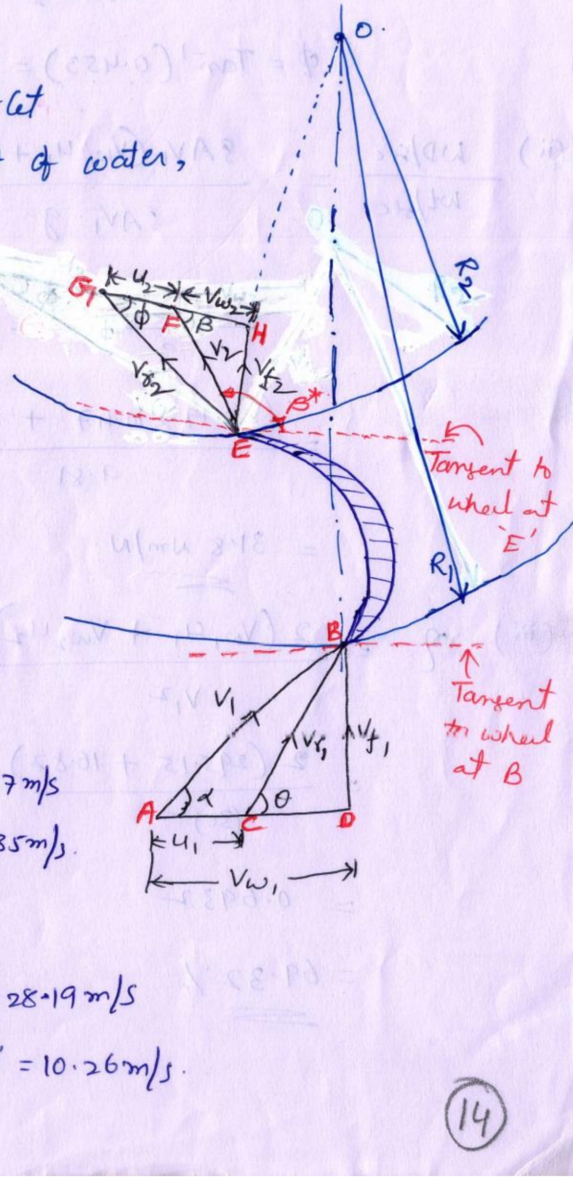
$\{m = \rho \cdot A \cdot V_{r_1}\}$.

A jet of water having a velocity of 30 m/s strikes a series
(9) of radial curved vanes mounted on a wheel ~~which~~ which
is rotating at 200 r.p.m. The jet makes an angle of 20°
with the tangent to the wheel at inlet and leaves the
wheel with a velocity of 5 m/s at an angle of 130° to
the tangent to the wheel at outlet. Water is flowing
from outward in a radial direction. The outer and
inner radii of the wheel are 0.5 m and 0.25 m
respectively. Determine :

(i) Vane angles at inlet and outlet

(ii) Work done per unit weight of water,

(iii) Efficiency of the wheel.



Sol:-

$V_1 = 30\ m/s$

$N = 200\ rpm$

$\omega = \frac{2\pi N}{60} = 20.94\ rad/s.$

$\alpha = 20°$

$V_2 = 5\ m/s.$

$\beta^* = 130°$

$\beta = 180 - 130 = 50°.$

$u_1 = \omega \times R_1 = 20.94 \times 0.5 = 10.47\ m/s$

$u_2 = \omega \times R_2 = 20.94 \times 0.25 = 5.235\ m/s.$

(i)

From ΔABD,

$V_{w_1} = V_1 \cos\alpha = 30 \cos 20° = 28.19\ m/s$

$V_{f_1} = V_1 \sin\alpha = 30 \sin 20° = 10.26\ m/s.$

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{10.26}{28.19 - 10.47} = 0.579$$

$$\theta = \tan^{-1}(0.579) = \underline{\underline{30.07^\circ}}$$

Δ^{le} EFH, $\quad V_{w_2} = V_2 \cos\beta = 5 \times \cos 50^\circ = 3.214 \text{ m/s}$

$$V_{f_2} = V_2 \sin\beta = 5 \times \sin 50^\circ = 3.83 \text{ m/s}$$

Δ^{le} EGH, $\quad \tan\phi = \dfrac{V_{f_2}}{u_2 + V_{w_2}} = \dfrac{3.83}{5.235 + 3.214} = 0.453$

$$\phi = \tan^{-1}(0.453) = \underline{\underline{24.385^\circ}}$$

(ii) $\dfrac{WD/sec}{wt/sec} = \dfrac{\rho A V_1 (V_{w_1} \cdot u_1 + V_{w_2} u_2)}{\rho A V_1 \cdot g}$ $\left\{\begin{matrix}\beta < 90^\circ \\ \text{Acute angle} \\ (+ve)\end{matrix}\right\}$

$$= \frac{(V_{w_1} u_1 + V_{w_2} u_2)}{g}$$

$$= \frac{(28.19 \times 10.47 + 3.214 \times 5.235)}{9.81}$$

$$= \underline{\underline{31.8 \text{ Nm/N}}}$$

(iii) $\eta = \dfrac{2 (V_{w_1} u_1 + V_{w_2} u_2)}{V_1^2}$

$$= \frac{2 (295.15 + 16.82)}{(30)^2}$$

$$= 0.6932$$

$$= \underline{\underline{69.32\%}}$$

Centrifugal pumps :-

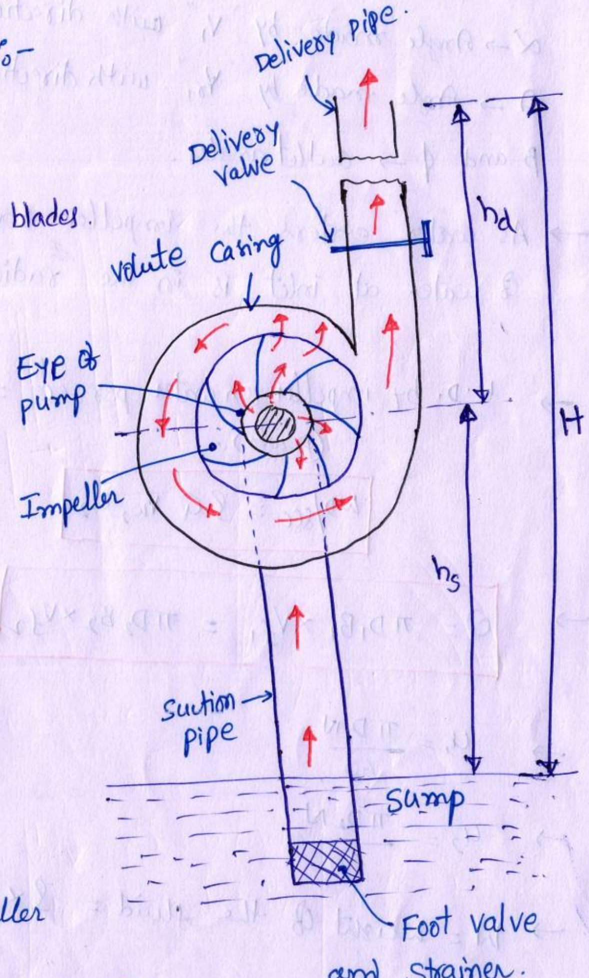
pump :- The hydraulic machines which convert mechanical energy into hydraulic energy are called pumps. The hydraulic energy is in the form of pressure energy.

Centrifugal pump :- If the mechanical energy is converted into pressure energy by means of centrifugal force acting on the fluid, the hydraulic machine is called centrifugal pump.

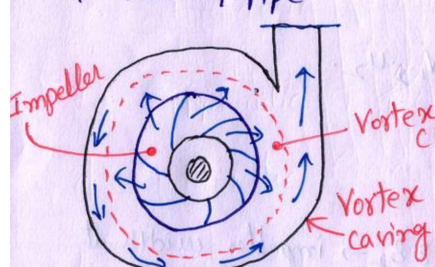
→ It is acted as a reverse of an inward radial flow reaction turbine.

Main parts of Centrifugal pump :-

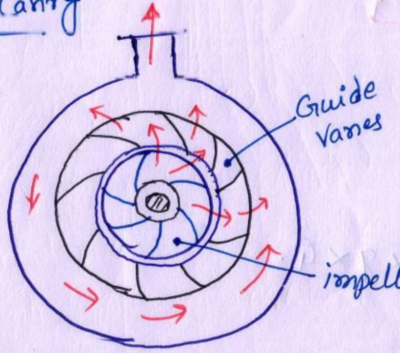
1. Impeller
2. Casing — Volute casing / Vortex casing / Casing with guide blades.
3. Suction pipe
4. Delivery pipe



① Volute Casing pump.

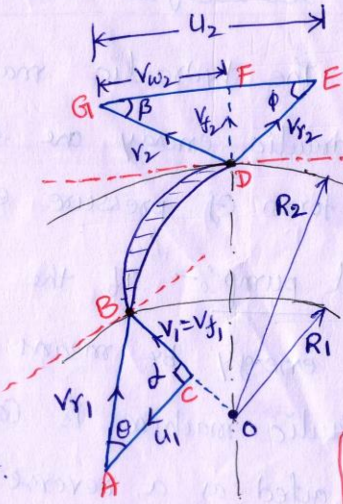


② Vortex Casing



③ Casing with Guide Blades.

Work done by centrifugal pump :-



$$\begin{bmatrix} \alpha = 90^\circ \\ V_{w_1} = 0 \end{bmatrix}$$

$N \rightarrow$ Speed of impeller

$D_1 \rightarrow$ impeller dia. at inlet

$D_2 \rightarrow$ impeller dia. at outlet

$u_1 \rightarrow$ Tangential velocity at inlet

$u_2 \rightarrow$ Tangential velocity of impeller at outlet.

$V_1 \rightarrow$ absolute vel. of water at inlet

$V_2 \rightarrow$ absolute vel. of water at outlet.

V_{r_1} & $V_{r_2} \rightarrow$ relative vel. of water at inlet & outlet.

$\alpha \rightarrow$ Angle made by "V_1" with direction of vane motion at inlet

$\theta \rightarrow$ Angle made by "V_{r_1}" with direction of vane motion at inlet.

β and $\phi \rightarrow$ outlet angles.

$\rightarrow$ As water enters the impeller radially which means absolute velocity of water at inlet is in the radial direction and hence $\alpha = 90^\circ$, $V_{w_1} = 0$.

$\rightarrow$ W.D. by impeller on water per sec (power).. $= \rho \times Q \times V_{w_2} \cdot u_2$.

$$\boxed{WD/sec = \rho Q V_{w_2} u_2}$$

$\rightarrow$ $\boxed{Q = \pi D_1 B_1 \times V_{f_1} = \pi D_2 B_2 \times V_{f_2}}$

B_1 & $B_2 \rightarrow$ impeller width at inlet & outlet.

$\rightarrow$ $u_1 = \dfrac{\pi D_1 N}{60}$

$\rightarrow$ $u_2 = \dfrac{\pi D_2 N}{60}$

$\rightarrow$ W = weight of the fluid $= \rho \times g \times Q$.

Multistage Centrifugal pumps :-

→ If a Centrifugal pump consists of two (or) more impellers, the pump is Called a multistage Centrifugal pump.

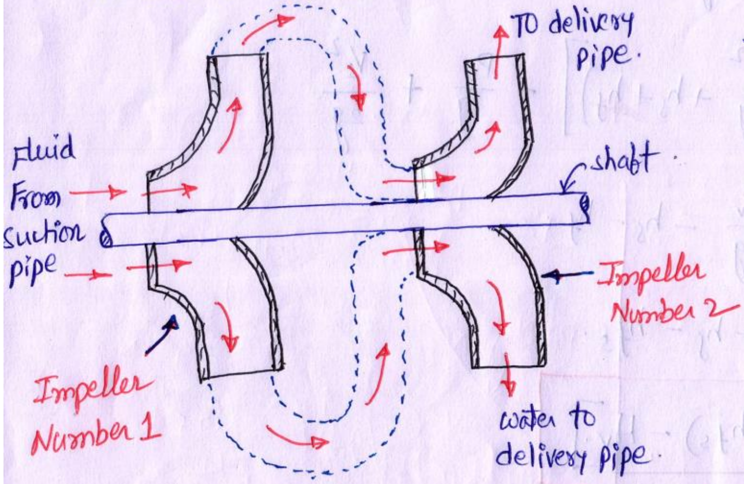
→ It is having two important functions.

(1) To produce a high head

(2) To discharge a large quantity of liquid.

① Pumps in Series (For high heads) :-

For developing a high head, a number of impellers are mounted in series (or) on the same shaft.



→ Dischare is Same in each impeller.

→ Total head developed. (H_{total})

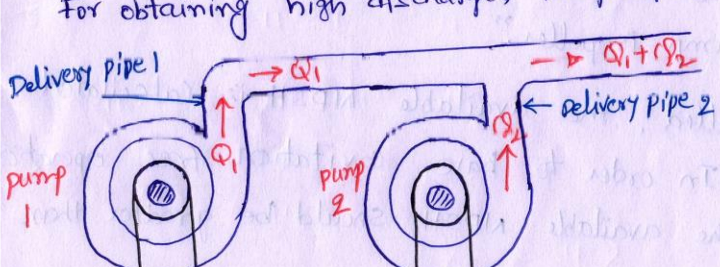
$$\boxed{H_{total} = n \times H_m}$$

$n \to$ no. of impellers

$H_m \to$ Head developed by each impeller.

② Pumps in Parallel (For high dischare) :-

For obtaining high discharge, the pumps should be Connected in parallel.



→ Head is same in each pump.

→ Total discharge (Q_{total}).

$$\boxed{Q_{total} = n \times Q}$$

$n \to$ no. of pumps

$Q \to$ Discharge from each pump.

Net positive suction Head (NPSH) :-

→ The term NPSH is very commonly used in the pump industry.

→ The net positive suction head (NPSH) is defined as the absolute pressure head at the inlet to the pump, minus the vapor pressure head plus the velocity head.

∴ NPSH = (Absolute pressure head at inlet of pump) − (Vapour pressure head) + (Velocity head).

$$NPSH = \left(\frac{P_1}{\rho g}\right) - \left(\frac{P_v}{\rho g}\right) + \left(\frac{V_s^2}{2g}\right)$$

$$\therefore NPSH = \left[\frac{P_a}{\rho g} - \left(\frac{V_s^2}{2g} + h_s + h_f\right)\right] - \frac{P_v}{\rho g} + \frac{V_s^2}{2g}$$

$$= \frac{P_a}{\rho g} - \frac{P_v}{\rho g} - h_s - h_f.$$

$$= H_a - H_v - h_s - h_{fs}.$$

$$\boxed{NPSH = \left[\underbrace{(H_a - h_s - h_{fs})}_{\text{Total suction head.}} - H_v\right]}$$

"The total head required to make the liquid flow through the suction pipe to the pump impeller."

→ when the pump is installed, the available NPSH is calculated from above equation. In order to have cavitation free operation of centrifugal pump, the available NPSH should be greater than the required NPSH.

Specific Speed of a Centrifugal pump :-

Definition :- The Specific speed of a centrifugal pump is defined as the speed of a geometrically similar pump which would deliver $1 m^3/s$ of liquid against a head of 1 m. It is denoted by "N_s".

Expression :- Let consider 'Q' is the discharge for a Centrifugal pump,

$$Q = \text{Area} \times \text{Velocity of flow}$$

$$= (\pi D \times B) \times (V_f).$$

$D \rightarrow$ Dia. of impeller of pump

$B \rightarrow$ width of the impeller.

$\rightarrow$ $Q \propto D \times B \times V_f$. ——(i).

$\rightarrow$ as we know that, $B \propto D$.,

$$\therefore Q \propto D \times D \times V_f$$

$$Q \propto D^2 \times V_f \text{ ——— (ii)}$$

Now, "u" and "V_f" are related to "H_m" as

Tangential velocity,

$$\left\{ \begin{array}{l} u = \frac{\pi D N}{60} \\ u \propto DN. \end{array} \right.$$

$$u \propto V_f \propto \sqrt{H_m}$$

$$u \propto \sqrt{H_m} \; ; \; V_f \propto \sqrt{H_m}$$

$$DN \propto \sqrt{H_m} \text{ ——— (iii)}$$

$$\left\{ \begin{array}{l} DN \propto \sqrt{H_m} \\ D \propto \frac{\sqrt{H_m}}{N} \end{array} \right.$$

Substitute (iii) in (ii),

$$Q \propto \left(\frac{\sqrt{H_m}}{N}\right)^2 \times V_f.$$

$$Q \propto \frac{H_m}{N^2} \times \sqrt{H_m}$$

$$\left\{ V_f \propto \sqrt{H_m} \right.$$

$$Q \propto \frac{H_m^{3/2}}{N^2}$$

$$Q = K \cdot \frac{H_m^{3/2}}{N^2} \text{ —(iv).}$$

K is Constant of proportionality.

From the Specific Speed of pump definition,

if $H_m = 1m$.

$Q = 1 m^3/s$

The Speed N becomes Specific Speed N_s.

$$N = N_s,$$

From equation (iv), substitute $H_m = 1m$, $Q = 1 m^3/s$, $N = N_s$.

$$1 = K \frac{1^{3/2}}{N_s^2}$$

$$K = N_s^2 \quad \text{——(v)}$$

Now, substitute 'K' in eqn (iv),

$$Q = N_s^2 \frac{H_m^{3/2}}{N^2}$$

$$N_s^2 = \frac{N^2 Q}{H_m^{3/2}}$$

$$\boxed{N_s = \frac{N \cdot \sqrt{Q}}{H_m^{3/4}}}$$

Priming of a centrifugal pump :-

It is defined as the operation in which the suction pipe, casing of the pump and portion of the delivery pipe upto the delivery valve is completely filled up from outside with liquid to be raised by the pump before starting the pump. Thus the air from these parts of the pump is removed and these parts are filled with the liquid to be pumped.

Characteristic Curves of a Centrifugal pumps :-

→ Characteristic curves of centrifugal pumps are defined as those curves which are plotted from the results of a number of tests on the centrifugal pump.

→ These ~~pumps~~ curves are necessary to predict the behaviour and performance of the pump when the pump is working under different flow rate, head and speed.

→ The important characteristic curves for pumps are,

1. Main characteristic curves
2. Operating characteristic curves
3. Constant efficiency (or) Muschel curves.

① Main characteristic curves :-

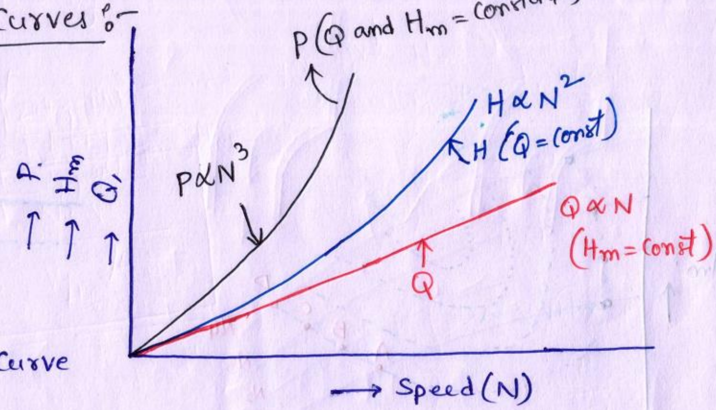
→ $\sqrt{H_m} \propto DN$

$\frac{\sqrt{H_m}}{DN}$ = constant.

$D \to$ constant

$\sqrt{H_m} \propto N$.

$\boxed{H_m \propto N^2}$ → parabolic curve



→ $\frac{P}{D^5 N^3}$ = constant

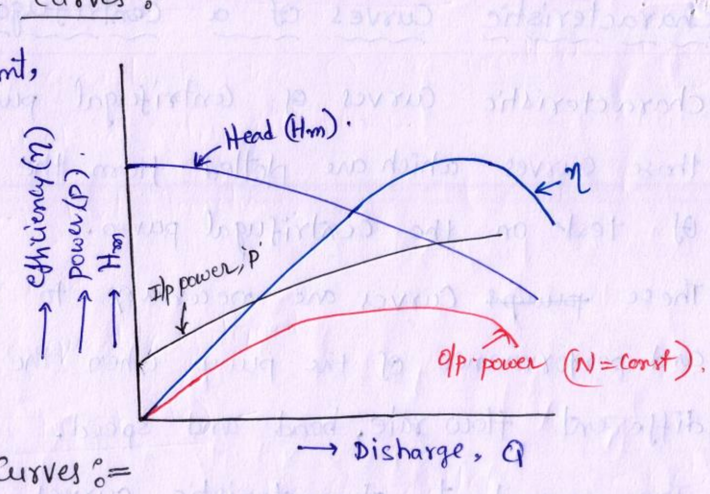
$\boxed{P \propto N^3}$ → cubic curve.

→ $\frac{Q}{D^3 N}$ = constant

$\boxed{Q \propto N}$ → straight line.

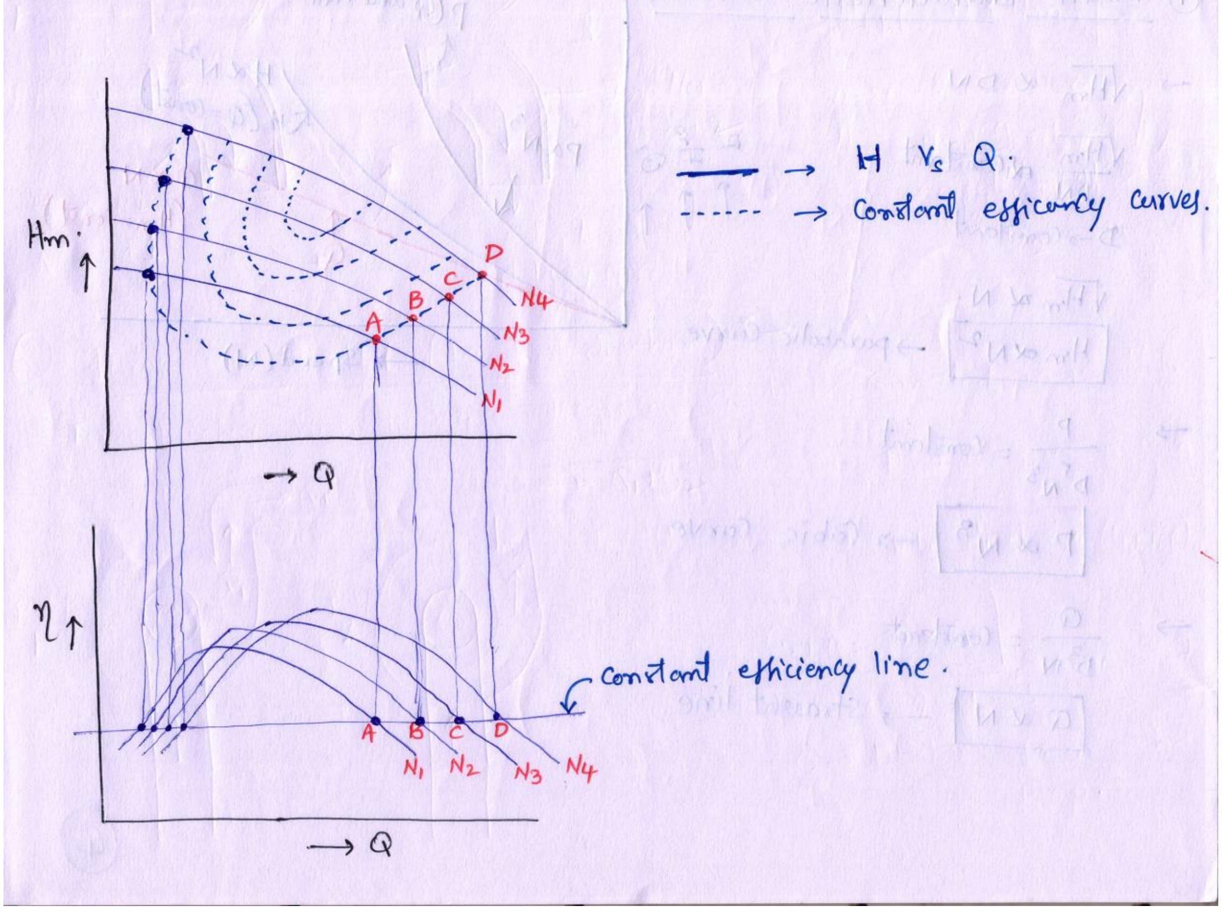
② Operating characteristic Curves :-

If the speed is constant, the variation of H_m, P and η with respect to discharge gives operating characteristics of pump.



③ Constant Efficiency Curves :-

For obtaining constant efficiency curves for a pump, the head versus discharge curves and efficiency versus discharge curves for different speed are used.



Suction head (h_s):- Vertical height of the centre line of the pump above the water surface in sump.

Delivery head (h_d):- Vertical distance b/w the centre line of pump and the water surface in tank to which water is delivered.

Static head (H_s):- $\boxed{H_s = h_s + h_d}$

Manometric head (H_m):-

(1) $H_m = h_s + h_d + h_{fs} + h_{fd} + \frac{V_d^2}{2g}$.

Efficiencies of pump:-

① $\eta_{man} = \frac{\text{Manometric head}}{\text{Head imparted by impeller to water}}$ $\boxed{\eta_{man} = \frac{g H_m}{V_{w_2} u_2}}$

② $\eta_m = \frac{\text{Power at impeller}}{\text{Power at shaft}}$ $\boxed{\eta_m = \frac{\rho \cdot Q \cdot V_{w_2} u_2 \text{ in watts}}{\text{shaft power in watts}}}$

③ $\eta_o =$ weight of water lifted $\times H_m$ $\boxed{\eta_o = \rho g Q \times H_m}$

$\boxed{\eta_o = \eta_{man} \times \eta_m}$

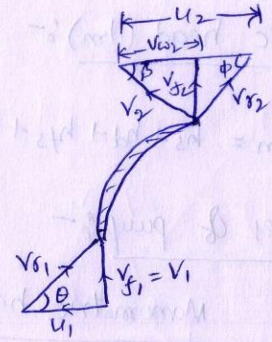
$\eta_{man} \rightarrow$ Manometric efficiency

$\eta_m \rightarrow$ mechanical efficiency

$\eta_o \rightarrow$ Overall efficiency.

Problems on Centrifugal pumps:-

① A Centrifugal pump is running at 1000 r.p.m. The outlet Vane angle of the impeller is 45° and velocity of flow at outlet is 2.5 m/s. The discharge through the pump is 200 lit/sec when the pump is working against a total head of 20 m. If the $\eta_{man} = 80\%$. Determine (i) Diameter of impeller (ii) width of impeller at outlet.



Sol:- $N = 1000$ rpm

$\phi = 45°$

$V_{f2} = 2.5\, m/s$

$Q = 200$ lit/sec $= 0.2\, m^3/se$.

$H_m = 20\,m$.

$\eta_{man} = 0.8$.

$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}}$$

$$V_{w2} = u_2 - \frac{V_{f2}}{\tan\phi}$$

$$V_{w2} = u_2 - 2.5 \quad \text{—①}$$

$$\eta_{man} = \frac{gH_m}{V_{w2}u_2} \Rightarrow V_{w2}u_2 = 245.25$$

Now substitute $V_{w2} = \frac{245.25}{u_2}$ in equation ①, then we have,

$$u_2^2 - 2.5u_2 - 245.25 = 0.$$

$$u_2 = \frac{-(-2.5) \pm \sqrt{(-2.5)^2 + 4\times1\times(-245.25)}}{2\times1}$$

$= 16.96$ (or) -14.46

$u_2 = 16.96$. $\Big\{$ -ve value is not possible.

① $u_2 = \frac{\pi D_2 N}{60}$

$$D_2 = \frac{60\times16.96}{\pi\times1000}$$

$= 0.324\,m$

$= 324\,mm$.

② $Q = \pi D_2 B_2 V_{f2}$

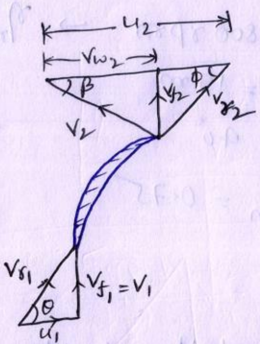
$$B_2 = \frac{Q}{\pi D_2 V_{f2}} = \frac{0.2}{\pi\times0.324\times2.5} = 0.0786\,m = 78.6\,mm.$$

② A Centrifugal pump having outer diameter equal to two times the inner diameter and running at 1000 r.p.m. works against a total head of 40m. The velocity of flow through the impeller is constant and equal to 2.5 m/s. The vanes are set back at an angle of 40° at outlet. If the outer diameter of the impeller is 500 mm and width at outlet is 50mm, determine :

(i) Vane angle at inlet

(ii) workdone by impeller on water per sec

(iii) Manometric efficiency.



Sol:- Speed $N = 1000$ rpm

Head $H_m = 40$ m.

flow velocity $V_{f_1} = V_{f_2} = 2.5$ m/s.

Vane angle at outlet $\phi = 40°$

impeller outer dia. $D_2 = 500mm = 0.5m$

impeller inner dia. $D_1 = \frac{D_2}{2} = 0.25m$.

impeller width at outlet $B_2 = 50mm = 0.05m$.

→ $u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1000}{60} = 13.09$ m/s

→ $u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 1000}{60} = 26.18$ m/s.

→ $Q = \pi D_2 B_2 V_{f_2} = \pi \times 0.5 \times 0.05 \times 2.5$
$= 0.1963$ m³/s.

(i) Vane angle at inlet (θ),

$\tan\theta = \frac{V_{f_1}}{u_1}$

$\theta = \tan^{-1}\left(\frac{V_{f_1}}{u_1}\right)$

$= \tan^{-1}\left(\frac{2.5}{13.09}\right)$

$\theta = 10.81°$.

(ii) WD/sec.

$WD/sec = \rho \times Q \times V_{w_2} \times u_2$

$= 1000 \times 0.1963 \times 23.2 \times 26.18$

$= 119227.9$ Watts.

$= 119.227$ KW.

$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$

$V_{w_2} = u_2 - \frac{V_{f_2}}{\tan\phi}$

$= 26.18 - \frac{2.5}{(\tan 40°)}$

$V_{w_2} = 23.2$ m/s.

(iii) η_{man}

$\eta_{man} = \frac{g H_m}{V_{w_2} u_2}$

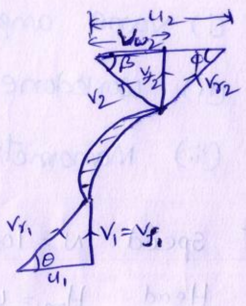
$= \frac{9.81 \times 40}{23.2 \times 26.18}$

$= 0.646$

$= 64.6\%$

⑥

③ The outer diameter of an impeller of a centrifugal pump is 400 m.m and outlet width is 50mm. The pump is running at 800 r.p.m. and is working against a total head of 15m. The vanes angle at outlet is 40° and manometric efficiency is 75%. Determine. (i) velocity of flow at inlet (ii) velocity of water leaving the vane (iii) angle made by the absolute velocity at outlet with the direction of motion at outlet. and (iv) Discharge.



Sol:- $D_2 = 0.4m$ $\rightarrow u_2 = \frac{\pi D_2 N}{60} = 16.75 m/s.$

$B_2 = 0.05m$

$N = 800\ rpm$ $\rightarrow \eta_{man} = \frac{g H_m}{V_{w2} u_2}$

$H_m = 15m$

$\phi = 40°$ $\quad V_{w2} = \frac{g H_m}{\eta_{man} u_2}$

$\eta_{man} = 0.75$

$= \frac{9.81 \times 15}{0.75 \times 16.75}$

$= 11.71\ m/s.$

(i) flow velocity at inlet (V_{f1}),

$Tan\phi = \frac{V_{f2}}{u_2 - V_{w2}}$

$V_{f1} = V_{f2} = Tan\phi \times (u_2 - V_{w2})$

$= 4.23 m/s.$

(ii) Velocity of Water leaving the vane(V_2)

$V_2^2 = V_{w2}^2 + V_{f2}^2$

$V_2 = \sqrt{V_{w2}^2 + V_{f2}^2}$

$= 12.45\ m/s.$

(iii) Angle made by "V_2" (β).

$Tan\ \beta = \frac{V_{f2}}{V_{w2}}$

$\beta = Tan^{-1}\left(\frac{V_{f2}}{V_{w2}}\right)$

$= 19.8°.$

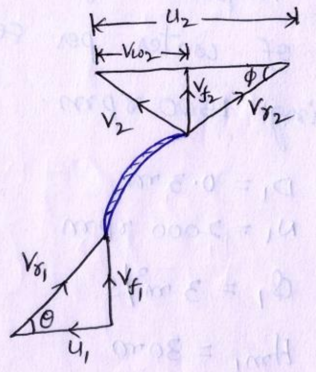
(iv) Discharge (Q),

$Q = \pi D_2 B_2 \times V_{f2}$

$= \pi \times 0.4 \times 0.05 \times 4.23$

$= 0.265\ m^3/s.$

(4) A centrifugal pump is to discharge 0.118 m³/s at a speed of 1450 r.p.m. against a head of 25 m. The impeller diameter is 250 mm its width at outlet is 50 mm and manometric efficiency is 75%. Determine the vane angle at the outer periphery of the impeller.



Sol:- Discharge, $Q = 0.118\ m^3/s$

Speed, $N = 1450$ r.p.m.

Head, $H = 25$ m.

Impeller diameter at outlet $D_2 = 250$ mm $= 0.25$ m.

width of impeller at outlet $B_2 = 50$ mm $= 0.05$ m.

Manometric efficiency $\eta_{man} = 75\% = 0.75$

Vane angle at outlet $(\phi) = ?$

$\rightarrow u_2 = \dfrac{\pi D_2 N}{60} = \dfrac{\pi \times 0.25 \times 1450}{60} = 18.98\ m/s.$

$\rightarrow Q = \pi D_2 B_2 \times V_{f_2}$

$V_{f_2} = \dfrac{Q}{\pi D_2 B_2} = \dfrac{0.118}{\pi \times 0.25 \times 0.05} = 3\ m/s.$

$\rightarrow \eta_{man} = \dfrac{g H_m}{V_{w_2} u_2}$

$V_{w_2} = \dfrac{g H_m}{\eta_{man} \times u_2} = \dfrac{9.81 \times 25}{0.75 \times 18.98}$

$= 17.23\ m/s.$

$\tan\phi = \dfrac{V_{f_2}}{(u_2 - V_{w_2})}$

$\phi = \tan^{-1}\left(\dfrac{V_{f_2}}{u_2 - V_{w_2}}\right)$

$\phi = \tan^{-1}\left(\dfrac{3}{18.98 - 17.23}\right)$

$\phi = 59.74°$

(5) A single-stage centrifugal pump with impeller dia. of 30 cm rotates at 2000 rpm. and lifts 3 m^3 of water per second to a height of 30 m with an efficiency of 75%. Find the number of stages and diameter of each impeller of a similar multistage pump to lift 5 m^3 of water per second to a height of 200 m. when running 1500 rpm.

Sol:-

single stage

$D_1 = 0.3\,m$

$N_1 = 2000$ rpm

$Q_1 = 3\,m^3/s$

$H_{m_1} = 30\,m$

$\eta_{man} = 0.75$

multistage

$Q_2 = 5\,m^3/s$

Total height $= 200\,m$

Height per stage $= H_{m_2}$

$N_2 = 1500$ rpm

D_2 = Dia of each impeller

→ From "N_s" equation,

$$\left(\frac{N\sqrt{Q}}{H_m^{3/4}}\right)_{\text{stage }1} = \left(\frac{N\sqrt{Q}}{H_m^{3/4}}\right)_{\text{stage }2}$$

$$\frac{N_1\sqrt{Q_1}}{H_{m_1}^{3/4}} = \frac{N_2\sqrt{Q_2}}{H_{m_2}^{3/4}}$$

$$H_{m_2}^{3/4} = 12.411$$

$$H_{m_2} = (12.411)^{4/3} = 28.71\,m.$$

$$\therefore \text{Number of stages} = \frac{\text{Total head}}{\text{Head per stage}} = \frac{200}{28.71} = 6.96 \simeq \underline{7.}$$

$$\Rightarrow \frac{\sqrt{H_{m_1}}}{D_1 N_1} = \frac{\sqrt{H_{m_2}}}{D_2 N_2}$$

$$D_2 = 0.3913\,m = \underline{391.3\,mm}$$

(6) A Centrifugal pump is running at 1000 r.p.m. The outlet vane angle of the impeller is 30° and velocity of flow at outlet is 3 m/s. The pump is working against a total head of 30 m and the discharge through the pump is 0.3 m³/s. If $\eta_{man} = 75\%$. Determine (i) the diameter of the impeller and (ii) width of the impeller.

Sol:-

$\phi = 30^\circ$

$N = 1000$ rpm

$V_{f2} = 3$ m/s.

$H_m = 30$ m

$Q = 0.3\ m^3/s$.

$\eta_{man} = 75\%$.

$$\eta_{man} = \frac{g H_m}{V_{w2} u_2}$$

$$V_{w2} u_2 = \frac{9.81 \times 30}{0.75}$$

$$V_{w2} u_2 = 392.4$$

$$V_{w2} = \frac{392.4}{u_2}$$

$$\tan\phi = \frac{V_{f2}}{(u_2 - V_{w2})}$$

$$u_2 - V_{w2} = \frac{V_{f2}}{\tan\phi}$$

$$u_2 = V_{w2} + \frac{V_{f2}}{\tan\phi}$$

$$= \frac{392.4}{u_2} + \frac{3}{\tan(30)}$$

$$u_2^2 = 392.4 + 5.196 u_2$$

$$u_2^2 - 5.196 u_2 - 392.4 = 0.$$

$$\therefore u_2 = \frac{-(-5.196) \pm \sqrt{(-5.196)^2 - (4 \times 1 \times (-392.4))}}{2 \times 1}$$

$$= 22.57\ m/s.$$

(1) $$u_2 = \frac{\pi D_2 N}{60}$$

$$D_2 = \frac{60 \times u_2}{\pi N} = \frac{60 \times 22.57}{\pi \times 1000}$$

$$= 0.4311\ m$$

$$D_2 = 431.1\ mm$$

(2) $$Q = \pi D_2 B_2 V_{f2}$$

$$B_2 = \frac{Q}{\pi D_2 V_{f2}}$$

$$B_2 = \frac{0.3}{\pi \times 0.4311 \times 3}$$

$$B_2 = 0.07382\ m$$

$$B_2 = 73.82\ mm$$

(8)

Reciprocating Pumps

[UNIT-5 Part-B]

Def:- The mechanical energy is converted into hydraulic energy or (Pressure energy) by sucking the liquid into a cylinder in which a piston is reciprocating (moving back and front), which exerts the thrust on the liquid and increases its hydraulic energy (Pressure energy), this kind of pump is known as reciprocating pump.

Main parts and working of reciprocating pump:-

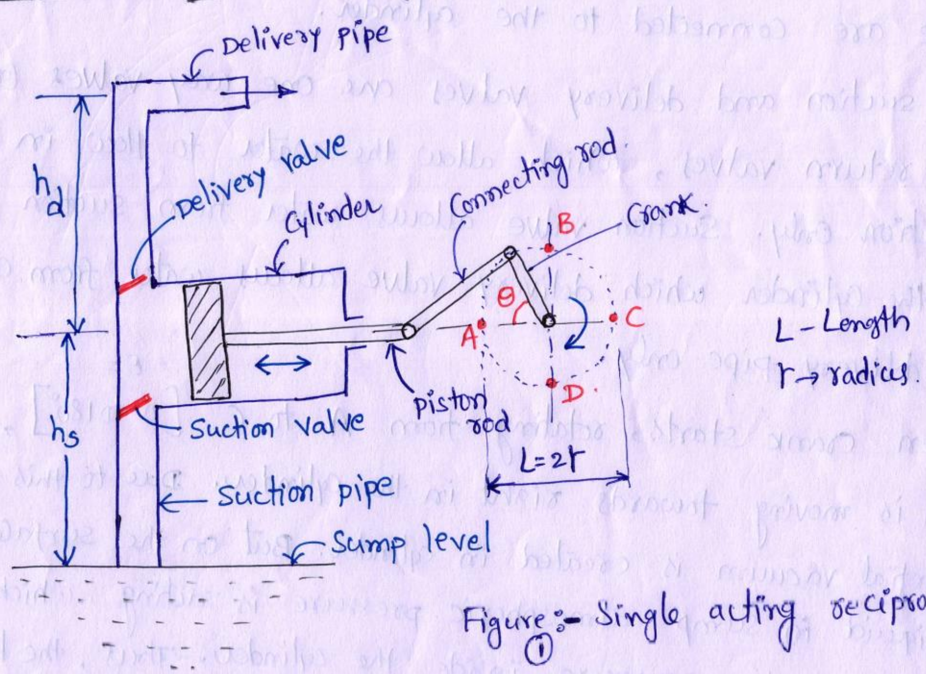


Figure ①:- Single acting reciprocating pump.

1. Suction pipe
2. Suction valve
3. Delivery pipe
4. Delivery valve
5. A cylinder with a piston, piston rod, Connecting rod and a crank.

Working :-

→ The single acting reciprocating pump, which ~~is~~ consists of a piston which moves forward and backwards in close fitting cylinder.

→ The movement of piston is obtained by connecting the piston rod to crank by means of a connecting rod. The crank is rotated by means of an electric motor.

→ Suction and delivery pipes with suction valve and delivery valve are connected to the cylinder.

→ The suction and delivery valves are one way valves (or) non-return valves, which allow the water to flow in one direction only. Suction valve allows water from suction pipe to the cylinder which delivery valve allows water from cylinder to delivery pipe only.

→ When crank starts, rotating from A to C [$\theta = 0^\circ$ to 180°], the piston is moving towards right in the cylinder. Due to this movement a partial vacuum is created in cylinder. But on the surface of the liquid in sump atmospheric pressure is acting, which is more than the pressure inside the cylinder. Thus, the liquid is forced in suction pipe from sump. This liquid opens the suction value and enters the cylinder.

→ when crank is rotating from C to A [$\theta = 180^\circ$ to 360°], the piston moves from right to left in cylinder. The movement of piston towards left increases the pressure of liquid inside cylinder more than atmospheric pressure. Hence suction value closes and delivery value opens. The liquid is forced into delivery pipe and is raised to a required height.

Discharge through a reciprocating pump :- [Single-acting pump]

Consider a single-acting reciprocating pump as shown in Fig(1).

→ Volume of water delivered in one revolution (or) discharge of water in one revolution = Area × Length of stroke

$= A \times L$

Number of revolution per second $= \frac{N}{60}$.

∴ Discharge of pump per second, (Q),

$Q =$ Discharge in one revolution × no. of revolution per sec.

$= A \times L \times \frac{N}{60}$.

$$\boxed{\therefore Q = A \times L \times \frac{N}{60}}$$

→ weight of water delivered per second, (W),

$$W = \rho \times g \times Q \Rightarrow \boxed{W = \frac{\rho g A L N}{60}}$$

Workdone by pump :- (WD/sec = Power).

Power = wt of water lifted per sec × Total height

$= W \times (h_s + h_d)$. $\left\{ \begin{aligned} h_s + h_d &= \text{Total height} \\ &= \text{Total head} \end{aligned} \right\}$.

$P = \frac{\rho g A L N}{60} (h_s + h_d)$. $\left\{ W = \frac{\rho g A L N}{60} \right\}$.

$$\boxed{\therefore P = \frac{\rho g A L N (h_s + h_d)}{60}}$$ in watts.

D → Diameter of cylinder

A → Area of cylinder $= \frac{\pi}{4} D^2$

r → radius of crank

N → r.p.m of crank

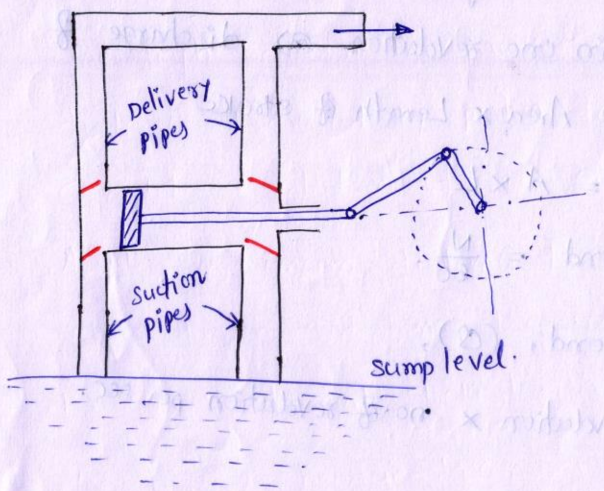
L → stroke length $= 2 \times r$

h_s = height of axis of cylinder from water surface in sump. (suction head).

h_d = Height of delivery outlet above the cylinder axis (delivery head)

Discharge, Workdone and power required to a Double acting pump:-

(double-acting pump).



D = Diameter of piston.

d = diameter of piston rod.

→ Area on one side of piston,

$$A = \frac{\pi}{4} D^2.$$

→ Area on the other side of the piston, where piston rod is connected to piston,

$$A_1 = \frac{\pi}{4} D^2 - \frac{\pi}{4} d^2$$

$$= \frac{\pi}{4}(D^2 - d^2).$$

Volume of water delivered in one revolution of crank,

$$V = (A \times L) + (A_1 \times L)$$

$$= AL + A_1 L = (A + A_1) L = \left[\frac{\pi}{4} D^2 + \frac{\pi}{4}(D^2 - d^2)\right] L$$

$\therefore$ Discharge per second $=$ Volume $\times \frac{N}{60}$.

$$= \left[\frac{\pi}{4} D^2 + \frac{\pi}{4}(D^2 - d^2)\right] \times L \times \frac{N}{60}.$$

if 'd' is diamete of piston rod, and it is very small can be neglect, $(D >> d)$.

$$\therefore Q = 2 \times \frac{\pi}{4} D^2 \times \frac{L \times N}{60} \Rightarrow \boxed{Q = \frac{2ALN}{60}}$$

Workdone, per sec,

$$P = \rho g \times Q \times \text{Total height}$$

$$\boxed{P = 2\rho g \frac{ALN}{60}(h_s + h_d).}$$

Slip of Reciprocating pump :-

Slip of a pump is defined as the difference between the theoretical discharge and actual discharge of the pump.

$$\text{slip} = Q_{the} - Q_{act}$$

slip is mostly expressed in percentage slip,

$$\%\ \text{slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 = \left(1 - \frac{Q_{act}}{Q_{th}}\right) \times 100$$

$$\boxed{\%\ \text{slip} = (1 - C_d) \times 100} \qquad \left\{ C_d = \frac{Q_{act}}{Q_{th}} \right\}.$$

$C_d \rightarrow$ co-efficient of discharge.

Negative slip :-

→ if Q_{act} is more than Q_{th}, the slip of the pump will become "-ve". In that case, the slip of pump known as negative slip.

→ Negative slip occurs when delivery pipe is short, suction pipe is long and pump runing at high speed.

Classification of Reciprocating pumps :-

→ According to contact of water with side of piston.

(i) single acting (ii) Double acting.

→ According to the number of cylinder provided,

(i) Single cylinder pump

(ii) Double cylinder pump

(iii) Triple cylinder pump

(iv) Duplex double acting pump.

(v) Quintruplex pump.

Indicator diagrams :-

→ The indicator diagram for a reciprocating pump is defined as the graph between the pressure head in the cylinder and the distance travelled by piston from inner dead centre for one complete revolution of the crank.

→ As the maximum distance travelled by the piston is equal to stroke length and hence the indicator diagram is graph b/n pressure head and stroke length of piston for one complete revolution.

→ The pressure head is taken as vertical axis (ordinate) and stroke length as abscissa (horizontal axis).

1) Ideal indicator diagram
2). Effect of acceleration in suction and delivery pipes on indicator diagrams.
3). Effect of friction in suction and delivery pipes on indicator diagram.
4) Effect of acceleration and friction in suction and delivery pipes on indicator diagrams.

Problems on Reciprocating pumps :-

① A single-acting reciprocating pump, running at 50 r.p.m. delivers 0.01 m^3/s of water. The diameter of the piston is 200 mm and stroke length 400 mm. Determine :

(i) The theoretical discharge of the pump

(ii) Co-efficient of discharge

(iii) Slip and the percentage slip of the pump.

Sol:- $N = 50$ r.p.m.

$Q_{act} = 0.01\ m^3/s$

$D = 200\ mm = 0.2\ m$ Area, $A = \frac{\pi}{4}D^2 = 0.031416\ m^2$

$L = 400\ mm = 0.4\ m$.

(i) $Q_{th} = \frac{A\ L\ N}{60} = \frac{0.031416 \times 0.4 \times 50}{60} = 0.01047\ m^3/s$.

(ii) $C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.01}{0.01047} = 0.955$

(iii) (b) $\%\ slip = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 = 4.489\%$.

(a) $Slip = Q_{th} - Q_{act} = 0.00047\ m^3/s$.

② A double-acting reciprocating pump, runing at 40 r.p.m., is discharging 1.0 m^3 of water per minute. The pump has a stroke of 400 mm. The diameter of the piston is 200 mm. The delivery and suction head are 20m and 5m respectively. Find the slip of the pump and power required to drive the pump.

Sol:- $N = 40\ rpm$

$Q_{act} = 1\ m^3/min = 0.01666\ m^3/s$

$L = 0.4m$

$D = 0.2m$ $\quad A = 0.031416\ m^2$

$h_s = 5m$

$h_d = 20m$

$$Q_{th} = \frac{2ALN}{60} = 0.1675\ m^3/s.$$

(i) $slip = Q_{th} - Q_{act} = 0.00009\ m^3/s.$

(ii) $$power = \frac{2 \times \rho g \times ALN \times (h_s + h_d)}{60} = 4109\,W.$$

$power = 4.109\,KW.$

③ A double acting reciprocating pump having piston area $0.1m^2$ has a stroke of 0.3m long. The pump is discharging $2.4m^3$ of water per minute at 45 rpm through a height of 10m. Find the slip of the pump and power required to drive the pump.

Sol:- $A = 0.1m^2$ $\quad$ $$Q_{th} = \frac{2ALN}{60} = \frac{2 \times \cancel{7.8539 \times 10^{-3}}\ 0.1 \times 0.3 \times 45}{60} = 0.045\ m^3/s$$ $= \cancel{3.53 \times 10^{-3}}\ m^3/s.$

$L = 0.3m.$

$Q_{act} = 2.4m^3/min.$ (i) $slip = Q_{th} - Q_{act} = \cancel{3.53 \times 10^{-3}} - (2.4/60) = \cancel{0.036}\ 0.005\ m^3/s.$

$N = 45\ rpm.$

$h_s + h_d = 10\ m.$ (ii) $$power = \frac{2 \times \rho g \times ALN(h_s + h_d)}{60} = 4415\,W$$

$= 4.415\,KW.$

$\cancel{Area = \frac{\pi}{4}D^2}$

$= \cancel{7.8539 \times 10^{-3}\ m^2}$

$= \cancel{346.7\,W}.$

$= \cancel{0.344\,KW}.$

④ A single acting reciprocating pump has a plunger of diameter 250 mm and stroke of 350 mm. If the speed of the pump is 60 r.p.m. and it delivers 16.5 litres per second of water against a suction head of 5 m and a delivery head of 20 m, Find the theoretical discharge, coefficient of discharge, the slip, the % of slip of the pump and the power required to drive the pump.

Sol:-

$D = 250\,mm = 0.25\,m$

$L = 350\,mm = 0.35\,m$

$N = 60\,rpm$

$Q_{act} = 16.5\,lit/sec = 0.0165\,m^3/sec$

$h_s = 5\,m$

$h_d = 20\,m$

(i) $Q_{th} = \frac{ALN}{60}$

$= \frac{\pi}{4} D^2 \times L \times \frac{N}{60}$

$= 0.0172\,m^3/s$

(ii) $C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.0165}{0.0172} = 0.96$

(iii) $slip = Q_{th} - Q_{act} = 0.7 \times 10^{-3}\,m^3/s$

(iv) $\%\ slip = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 = 4.07\,\%$

(v) $P = \frac{\rho g ALN (h_s + h_d)}{60} = \frac{1000 \times 9.81 \times \frac{\pi}{4} (0.25)^2 \times 0.35 \times 60 (5 + 20)}{60}$

$= 4218.3\,W = 4.2183\,kW$

⑤ A single acting reciprocating pump has piston of diameter 150 mm and stroke of length 250 mm. The piston makes 50 double strokes per minute. The suction and delivery head are 5 m and 15 m respectively. Find (i) Discharge capacity of the pump in litres per minute, (ii) Force required to work the piston during the suction and delivery strokes if the efficiency of suction and delivery strokes are 60% and 75% respectively and (iii) power required to operate the pump.

Sol:- $D = 0.15\,m$ (i) $Q_{th} = \dfrac{ALN}{60} = 0.00368\ m^3/s$

$L = 0.25\,m$ $= 3.68\ lit/s$

$N = 50\ rpm$

$h_s = 5\,m$ $= 221\ lit/min$

$h_d = 15\,m$

(i) Q_{th} in lit/min

(ii) Force (F_s) & (F_D) if $\eta_s = 0.6$ and $\eta_d = 0.75$

(iii) Power

(ii) F_s & F_D ⇒ $F_S = \dfrac{\rho g h_s \cdot A}{\eta_s} = \dfrac{9.81 \times 1000 \times 5 \times \frac{\pi}{4}(0.15)^2}{0.6}$

$= 1444.64\,N$

$F_D = \dfrac{\rho g h_d \cdot A}{\eta_s} = \dfrac{9.81 \times 1000 \times 15 \times \frac{\pi}{4}(0.15)^2}{0.75}$

$= 3467.14\,N$

(iii) Power ⇒ $P = \dfrac{\rho g\, ALN\,(h_s + h_d)}{60}$

$= \dfrac{1000 \times 9.81 \times \frac{\pi}{4}(0.15)^2 \times 0.25 \times 50 \times \left(\frac{5}{0.6} + \frac{15}{0.75}\right)}{60}$

$= 1023\,W$

$= 1.023\,KW$

⑥ A double acting reciprocating pump has piston of dia. 250 mm and piston rod of dia. 50 mm which is on one side only. Length of piston stroke is 350 mm and speed of crank moving that piston is 60 rpm. The suction and delivery heads are 4.5 m and 18 m respectively. Determine the discharge capacity of the pump and the power required to operate the pump.

Sol:-

$D = 0.25\,m$

$d_p = 0.05\,m$

$L = 0.35\,m$

$N = 60\,rpm$

$h_s = 4.5\,m$

$h_d = 18\,m$

$$Q_{th} = \frac{(2A - A_p)LN}{60}$$

$$A = \frac{\pi}{4}D^2 = 0.04909\,m^2$$

$$A_p = \frac{\pi}{4}d_p^2 = 0.00196\,m^2$$

$$\therefore Q_{th} = \frac{(2\times0.04909 - 0.00196)\times 0.35\times 60}{60} = 0.0337\,m^3/s = 33.7\,lit/s.$$

$$\therefore \text{Power} = \frac{\rho g\, Q_{th}(h_s + h_d)}{1000} \text{ in KW}.$$

$$= \frac{1000\times9.81\times0.0337\times(4.5+18)}{1000}$$

$$= \underline{\underline{7.438\ KW}}$$

UNIT-6 PART- A and B

Hydraulic Turbines

→ Turbines:- The hydraulic machines, which converts the hydraulic energy into mechanical energy, are called turbines.

→ pumps:- The hydraulic machines, which converts the mechanical energy into hydraulic energy, are called pumps.

* The reverse of hydraulic turbine is called hydraulic pump (or) simply called as pump.

→ Hydroelectric power:- The mechanical energy, which is developed by turbine is converted into electrical energy. The electric power which is obtained from the hydraulic energy is known as hydroelectric power. At present, this is the cheapest as compared by power generated by other sources such as oil, coal etc.

Classification of hydraulic turbines:-

1). According to type of energy at inlet:

(a) Impulse turbine (b) Reaction turbine

2). According to the direction of flow through runner:

(a) Tangential flow turbine (b) Radial flow turbine
(c) Axial flow turbine (d) Mixed flow turbine

3). According to the head at the inlet of turbine:

(a) High head turbine (b) Medium head turbine (c) Low head turbine

4) According to the specific speed of the turbine:

(a) Low specific speed turbine
(b) Medium specific speed turbine
(c) High specific speed turbine

① Differentiate between Impulse Turbine and Reaction Turbine :-

[MID-2]

S.No.	Impulse Turbine	Reaction Turbine
1.	The entire available energy of the water is first converted to K.E.	The available energy of the water is not converted from one form to another.
2.	The water flows through the nozzles and impinges on the buckets, which are fixed to the outer periphery of the wheel.	The water is guided by the guide blades to flow over the moving vane.
3.	The water impinges on the buckets with K.E.	The water glides over the moving vanes with Pressure energy.
4.	The pressure of the flowing water remains unchanged and is equal to the atmospheric pressure.	The pressure of the flowing water is reduced after gliding over the vane.
5.	It is not essential that the wheel should run full.	It is essential that the wheel should always run full and kept full of water
6.	It is possible to regulate the flow without loss.	It is not possible to regulate the flow without loss.
7.	Impulse turbine has more (η_h) Hydraulic efficiency.	Reaction turbine relatively less (η_h) Hydraulic efficiency
8.	It operates at high water heads.	It operates at low and medium heads.
9.	Example : pelton wheel.	Examples : Francis turbine, kaplan turbine, propeller turbine, Deriaz turbine, tubular turbine, etc.

② What is Tangential flow, Radial flow, Axial flow and Mixed flow Turbine.

(i) Tangential flow turbine :- In this type of turbine the water strikes the runner in the direction tangent to the wheel.

Ex:- pelton wheel.

(ii) Radial flow turbine :- In this type of turbine the water strikes in the radial direction. It is further classified as,

(a) Inward flow turbine :- The flow is inward from periphery to the centre (centripetal type). Ex :- Old francis turbine.

(b) Outward flow turbine :- The flow is outward from the centre to periphery (centrifugal type). Ex :- Fourneyron turbine.

(iii) Axial flow turbine :- The flow of water in the direction parallel to the axis of the shaft. Ex :- Kaplan turbine and propeller turbine.

(iv) Mixed flow turbine :- The water enters the runner in the radial direction and leaves in axial direction.

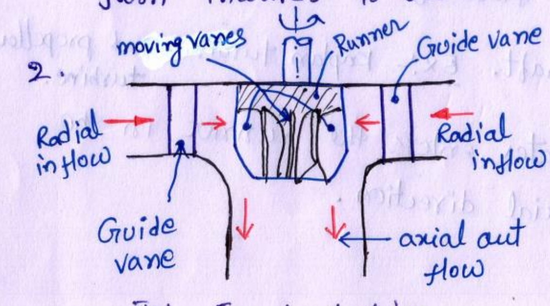
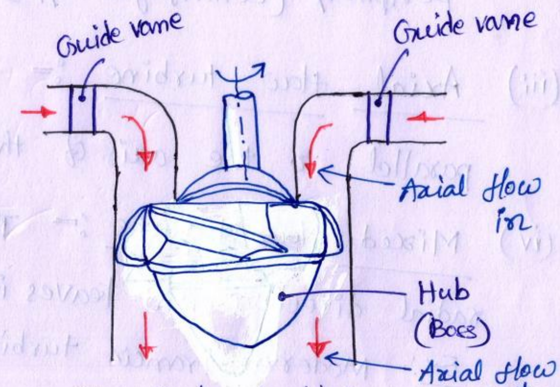
Ex :- Modern Francis turbine.

③ What is the difference between Turbine and pump.

S.No.	Turbine	pump
1.	Input of hydraulic energy (water under pressure)	Input of mechanical energy. (By shaft)
2.	pressure head decreases with increasing flow	pressure head increases with increasing flow.
3.	Ex :- Impulse, Reaction, etc.	Ex : Centrifugal, Reciprocating etc.

(4) Differentiate between Radial and Axial flow turbine.

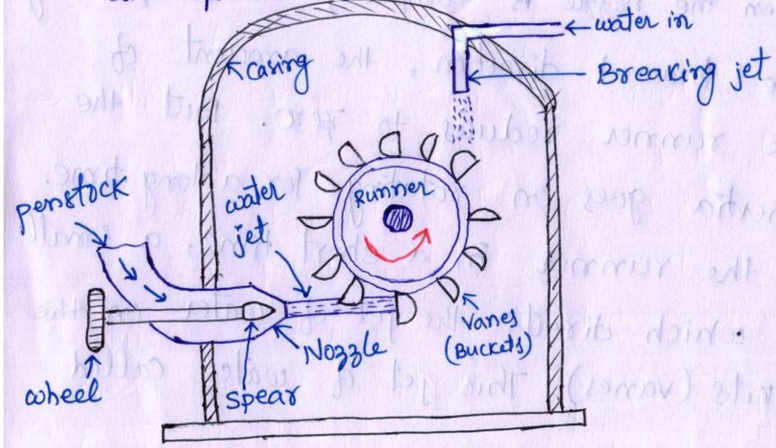
[MID-2]

S.No.	Radial turbine	Axial flow turbine
1.	If the water flows in radial direction, the turbine is called radial flow turbine. The water may flow radically from outwards to inwards (or) from inwards to outwards.	If the water flows parallel to axis of the rotation of the shaft, the turbine is called axial flow turbine
2.	 Fig: Francis turbine	 Fig: Kaplan turbine.
3.	The shaft of turbine is vertical. The lower end of shaft is called runner.	The shaft of turbine is vertical. The lower end of the shaft is made larger which is known as hub.
4.	The water at inlet possesses K.E as well as pressure energy.	The water at inlet posses K.E as well as pressure energy.
5.	Examples: Francis turbine.	Examples: propeller turbine (vanes are fixed not adjustable) and Kaplan turbine (vanes on hub are adjustable)

Pelton Wheel :- (Pelton turbine).

→ It is a tangential flow impulse turbine. This turbine used for high heads.

→ The water strikes the bucket along the tangent of the runner. The energy available at the inlet of the turbine is only kinetic energy. The pressure at inlet and outlet of the turbine is atmospheric. (pressure is constant).



Main parts;

1. Nozzle
2. Spear (Flow regulator)
3. Runner with buckets
4. Breaking jet
5. Casing.

Flow regulating arrangement :- The spear in the nozzle is used to control the water striking the buckets of runner. The spear is a conical needle which is operated by hand wheel or automatically in an axial direction depending upon the size of the unit.

→ When the spear is pushed forward into the nozzle the amount of water striking the runner is reduced.

→ when the spear is pushed back, the amount of water striking the runner is increases.

Runner with buckets :- The runner consists of circular disc on which a number of buckets are fixed. The shape of the buckets is a double "hemispherical cup or bowl." Each bucket is divided into two symmetrical parts by a dividing wall is called splitter.

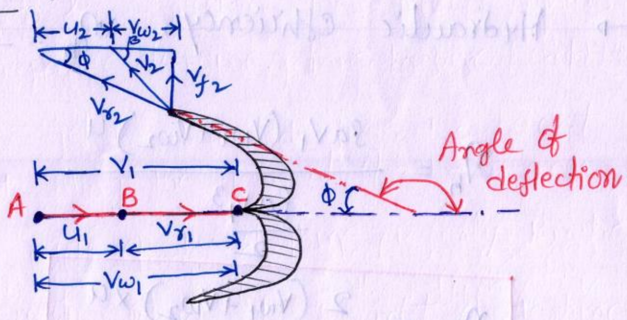
→ The jet of water strikes on the splitter. The splitter divides the jet into two equal parts and the jet comes out at the edge of the bucket. The jet is deflected from the bucket with 160° and 170°.

→ The buckets are made of Cast Iron, cast steel bronze (or) stainless steel depending on the head at the inlet of turbine.

Breaking jet :- When the nozzle is completely closed by moving the spear in the forward direction, the amount of water striking the runner reduces to zero. But the runner due to inertia goes on rotating for a long time. Therefore, to stop the runner in a short time, a small nozzle is provided which directs the jet of water on the back of the buckets (Vanes). This jet of water called breaking jet.

Casing :- The function of casing is to prevent the splashing of the water and to discharge water to tailrace. It also acts as safegaurd against accidents. It is made of Cast Iron (or) fabricated steel plates. It does not perform any hydraulic function.

Workdone on pelton wheel :-



→ Gross head, H_g.

→ head loss, $h_f = \frac{4fLV^2}{2gD^*}$

→ ∴ Net head acting on wheel,

$$\boxed{H = H_g - h_f}$$

D^* = Dia. of penstock

D = Dia. of pelton wheel

N = Speed of wheel

d = dia. of jet.

$u = u_1 = u_2 = \frac{\pi D N}{60}$.

a = area of jet $= \frac{\pi}{4} d^2$.

→ Velocity of jet at inlet $\boxed{V_1 = \sqrt{2gH}}$

→ $\left.\begin{array}{l} V_{r_1} = V_1 - u_1 \\ * \ V_1 = V_{w_1} \\ \alpha = 0^\circ \\ \theta = 0^\circ \end{array}\right\}$ From inlet velocity Δ^{le}

→ $\left.\begin{array}{l} V_{r_2} = V_{r_1} \\ * \ V_{w_1} = V_{r_2}\cos\phi - u_2 \end{array}\right\}$ From outlet velocity Δ^{le}.

→ Force exerted by jet of water in direction of motion,

$$\boxed{F_x = \rho a V_1 [V_{w_1} \pm V_{w_2}]} \text{ in N}$$

→ Workdone by jet on runner per second, (or) power

$W = F_x \times u \Rightarrow \boxed{W = \rho a V_1 (V_{w_1} \pm V_{w_2}) \times u}$ in N·m/sec.

→ power per unit weight of water striking per sec.

$$\frac{P}{wt} = \frac{\rho a V_1 (V_{w_1} \pm V_{w_2}) u}{\rho a V_1 \times g}$$

$$\boxed{\frac{P}{wt} = \frac{(V_{w_1} \pm V_{w_2}) u}{g}} \text{ in N·m/N.}$$

(i) if $\beta < 90^\circ$ acute angle then V_{w_2} is +ve.

(ii) if $\beta > 90^\circ$ obtuse angle then V_{w_2} is −ve.

→ Kinetic energy of jet per sec $= \frac{1}{2}(\rho a V_1) \times V_1^2$

$$\boxed{K.E = \frac{\rho a V_1^3}{2}} \text{ in N·m/s (or) watt.}$$

→ Hydraulic efficiency, $\eta_h = \dfrac{WD \text{ per second}}{K.E \text{ of jet per second}}$

$$\eta_h = \frac{\rho a V_1 (V_{w1}+V_{w2})u}{\frac{\rho a V_1^3}{2}}$$

$$\boxed{\eta_h = \frac{2(V_{w1}+V_{w2})\times u}{V_1^2}} \qquad \boxed{Max.\ \eta_h = \frac{1+\cos\phi}{2}}$$

→ jet ratio, (m^*),

$$m^* = \frac{\text{pitch diameter of wheel}}{\text{diameter of jet}} = \frac{D}{d}. \quad [\approx 12 \text{ for most cases}]$$

$$\boxed{m^* = \frac{D}{d}}$$

→ Number of jets, (n).

$$n = \frac{\text{Total rate of flow through turbine}}{\text{rate of flow of water through a single jet}}$$

$$\boxed{n = \frac{\text{mass flow out turbine}}{\text{mass flow per jet}}}$$

*

Overall efficiency,

$$\eta_o = \frac{\text{Shaft power}}{\text{water power}}$$

$$= \frac{\text{shaft power in watts}}{\text{water power in watts}}$$

$$\boxed{\eta_o = \frac{\text{shaft Power in watts}}{\rho g Q H \text{ in watts.}}}$$

→ Number of buckets on runner, (Z)

$$Z = 15 + \frac{D}{2d}$$

$$\boxed{Z = 15 + 0.5 m^*}$$

→ Speed ratio, (ϕ^*)

$$\boxed{u = \phi^* \sqrt{2gH}} \qquad \boxed{V_1 = C_v \sqrt{2gH}}$$

→ Coefficient velocity (C_v) $= 0.98$ (or) 0.99.

→ Speed ratio, (ϕ^*)
$\phi^* = 0.43$ to 0.48.

① The penstock supplied water from a reservoir to the pelton wheel with a gross head of 500 m. One third of the gross head is lost in friction in the penstock. The rate of flow of water through the nozzle fitted at the end of the penstock is 2.0 m^3/s. The angle of deflection of the jet is $165°$. Determine the power given by the water to the runner and also hydraulic efficiency of the pelton wheel. Take speed ratio = 0.45, $C_v = 1.0$.

Sol:- $H_g = 500\ m$

$h_f = \frac{H_g}{3} = \frac{500}{3} = 166.7\ m$

$\therefore$ Net head, $H = H_g - h_f = 333.30\ m$.

$Q = 2\ m^3/s$,

$\phi = 180 - 165 = 15°$.

Speed ratio = 0.45

$C_v = 1$.

From inlet sle

$V_{r_1} = V_1 - u_1 = 44.473\ m/s$.

$V_{w_1} = V_1 = 80.86\ m/s$.

From outlet sle,

$V_{r_2} = V_{r_1} = 44.473\ m/s$

$V_{w_2} + u_2 = V_{r_2}\cos\phi$.

$V_{w_2} = V_{r_2}\cos\phi - u_2$

$= 44.47 \times \cos(15°) - 36.38$

$= 6.57\ m/s$.

$V_1 = C_v\sqrt{2gH}$

$= 80.86\ m/s$.

$u = u_1 = u_2 = \text{Speed ratio}\sqrt{2gH}$

$= 36.387\ m/s$.

$\text{power} = \rho Q (V_{w_1} + V_{w_2}) \times u$

$= 1000 \times 2 (80.86 + 6.57) \times 36.38$

$= 6362630\ W$.

$= 6362.63\ kW$.

$\eta_h = \frac{2(V_{w_1} + V_{w_2})u}{V_1^2}$

$= 0.9731$

$= 97.31\%$.

5

② A 137 mm diameter jet of water issuing from a nozzle impinges on the buckets of a pelton wheel and jet is deflected through an angle of 165° by the buckets. The head available at the nozzle is 400 m. Assuming co-efficient of velocity as 0.97, speed ratio as 0.46 and reduction in relative velocity while passing through buckets as 15%. Find:

(i) The force exerted by the jet on buckets in tangential direction.

(ii) The power developed. (iii) Overall efficiency if shaft power is 4000 kW.

Sol:- jet diameter $(d) = 0.137 m$, Area of jet $(a) = \frac{\pi}{4} d^2 = 0.01474 \, m^2$

$\phi = 180° - 165° = 15°$.

$H = 400 \, m$,

$C_V = 0.97$

Speed ratio $= 0.46$

$V_{r_2} = \left(1 - \frac{15}{100}\right) \times V_1$

$V_{r_2} = 0.85 \times V_1$

jet velocity, $V_1 = C_V \sqrt{2gH}$

$= 0.97 \sqrt{2 \times 9.81 \times 400}$

$= 85.93 \, m/s$.

$u_1 = \text{Speed ratio} \times \sqrt{2gH}$

$= 0.46 \times \sqrt{2 \times 9.81 \times 400}$

$u_1 = u_2 = u = 40.75 \, m/s$.

$V_{r_1} = V_1 - u_1 = 45.18 \, m/s$

$V_{r_2} = 0.85 \, V_{r_1} = 38.4 \, m/s$.

$V_{r_2} \cos\phi = 37.092$

"$V_{r_2} \cos\phi$" is less than u_2, Hence β is an obtuse angle.

(i) $F_x = \rho a V_1 [V_{w_1} - V_{w_2}]$

$= 104206 \, N$.

(ii) power $= \frac{F_x \times u}{1000} = \frac{104206 \times 40.75}{1000}$

$= 4246.4 \, kW$

[V_{w_1} & V_{w_2} are in same direction. Therefore V_{w_2} is -ve].

(iii) $\eta_o = \frac{\text{Shaft Power}}{\text{water power}} = \frac{S.P}{\rho g Q H} = \frac{4000 \times 10^3}{1000 \times 9.81 \times (0.01474 \times 85.93) \times 400} = 0.8043$

$\{Q = A \times V_1\}$.

$\eta_o = 80.43\%$

③ The pelton wheel is to be designed for the following specifications. shaft power = 11,772 KW, Head = 380 m, Speed = 750 rpm, Overall efficiency = 86%, jet diameter is not to exceed One-sixth of the wheel diameter. Determine :

(i) The wheel diameter

(ii) The number of jets required

(iii) Diameter of the jet. Take $K_{v_1} = 0.985$ and $K_{u_1} = 0.45$

Sol:- → Shaft power, $S.P = 11,772$ KW

Head, $H = 380 m$

Speed, $N = 750$ rpm.

Overall efficiency, $\eta_o = 86\% = 0.86$

→ Ratio of jet diameter to wheel diameter $= \frac{d}{D} = \frac{1}{6}$

$\{D = 6d\}$

→ Co-efficient of Velocity $K_{v_1} = C_v = 0.985$, $V_1 = C_v\sqrt{2gH} = 85.05$ m/s.

→ ϕ = Speed ratio $= K_{u_1} = 0.45$.

→ Velocity of wheel $u = u_1 = u_2 = \phi\sqrt{2gH} = 0.45 \times \sqrt{2 \times 9.81 \times 380} = 38.85$ m/s.

$u = \frac{\pi D N}{60} \Rightarrow D = \frac{60 \times 38.85}{\pi \times 750} = 0.989 m$.

→ Diameter of jet, $d = \frac{1}{6} \times D = 0.165 m$.

→ $\eta_o = \frac{S.P}{\text{Wheel power}} \Rightarrow W.P = \frac{S.P}{\eta_o} \Rightarrow \rho g Q H = \frac{S.P}{\eta_o}$.

$Q_{total} = \frac{S.P}{\eta_o \cdot \rho g H} = \frac{11772 \times 10^3}{0.86 \times 1000 \times 9.81 \times 380}$

$= 3.672 m^3/s$

→ Number of jets $= \frac{Q_{total}}{Q_{per\,jet}} = \frac{3.672}{1.818}$

$= 2$ jets.

$Q_{per\,jet} = \frac{\pi}{4} d^2 \times V_1$.

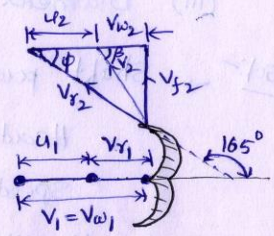
$= \frac{\pi}{4}(0.165)^2 \times 85.05$

$= 1.818 m^3/s$.

④ A pelton wheel has a mean bucket speed of 10 m/s with a jet of water flowing at the rate of 700 litres/s under a head of 30 m. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and hydraulic efficiency of the turbine. Assume co-efficient of velocity as 0.98.

Sol:- Bucket speed (or) velocity $u = u_1 = u_2 = 10\ m/s$

flow rate (or) Discharge $Q = 700$ litres/s $= 0.7\ m^3/s$.



Total head, $H = 30\ m$.

Angle of deflection $= 160°$ $\Rightarrow \phi = 180 - 160° = 20°$.

Co-efficient velocity $C_V = 0.98$

$\therefore$ The velocity of jet $V_1 = C_V\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 30} = 23.77\ m/s$.

Inlet Δ^{le}, $\rightarrow$ $V_{r_1} = V_1 - u_1 = 23.77 - 10 = 13.77\ m/s$.

$\rightarrow$ $V_{w_1} = V_1 = 23.77\ m/s$.

Outlet Δ^{le}, $\rightarrow$ $V_{r_2} = V_{r_1} = 13.77\ m/s$.

$V_{w_2} = V_{r_2}\cos\phi - u_2 = 13.77 \times \cos(20°) - 10 = 2.94\ m/s$.

WD/sec (or) power $= \rho a V_1 [V_{w_1} + V_{w_2}] \times u$. $\quad [Q = a V_1 = 0.7\ m^3/s]$

$= 1000 \times 0.7 \times [23.77 + 2.94] \times 10$

$= 186970\ W$.

$= 186.97\ KW$.

Hydraulic efficiency $\eta_h = \dfrac{2(V_{w_1} + V_{w_2})u}{V_1^2}$

$= \dfrac{2(23.77 + 2.94) \times 10}{(23.77)^2} = 0.9454$

$\eta_h = 94.54\%$.

(5) A pelton wheel is to be designed for a head of 60 m when running at 200 r.p.m. The pelton wheel develops 95.6475 KW shaft power. The velocity of the buckets = 0.45 times the velocity of the jet, overall efficiency = 0.85 and co-efficient of the velocity is equal to 0.98.

Sol:-

Head, $H = 60\,m$

Speed, $N = 200$ rpm

Shaft power, $S.P. = 95.647\,KW = 95647\,W$.

Velocity of bucket $u = u_1 = u_2 = 0.45\,V_1$.

Overall efficiency $\eta_o = 0.85$

$C_V = 0.98$.

** Design of pelton wheel means to find (i) jet diameter (d)
(ii) Wheel diameter (D)
(iii) Width and depth of buckets.
(iv) Number of buckets.

$$V_1 = C_V\sqrt{2gH} = 0.98\times\sqrt{2\times 9.81\times 60} = 33.62\,m/s$$

$$u = u_1 = u_2 = 0.45\,V_1 = 0.45\times 33.62 = 15.13\,m/s.$$

(i) wheel dia (D)

$$u = \frac{\pi D N}{60} \Rightarrow D = \frac{60\times u}{\pi N} = \frac{60\times 15.13}{\pi\times 200} = 1.44\,m.$$

(ii) jet dia (d);

Q = Area of jet × velocity of jet

$= a\times V_1$

$= \frac{\pi}{4}d^2\times V_1$

$$d = \sqrt{\frac{4Q}{\pi V_1}} = \sqrt{\frac{4\times 0.1912}{\pi\times 33.62}} = 0.085\,m = 85\,mm$$

$$\eta_o = \frac{S.P}{W.P}$$

$$\eta_o = \frac{S.P}{\rho g Q H}$$

$$Q = \frac{S.P}{\rho g \eta_o H}$$

$$= 0.1912\,m^3/s.$$

(iii) width of buckets $= 5\times d = 5\times 85 = 425\,mm$

Depth of buckets $= 1.2\times d = 1.2\times 85 = 102\,mm$

(iv) Number of buckets on wheel,

$$Z = 15 + \left(0.5\frac{D}{d}\right) = 15 + \left(0.5\times\frac{1.44}{0.085}\right) = 23.5 \text{ Say } 24.$$

"D & d → in m".

⑥ Determine the power given by the jet of water to the runner of a ~~pelt~~ pelton wheel which is having tangential velocity as 20 m/s. The net head on the turbine is 50 m and discharge through the jet water is 0.03 m^3/s. The side clearance angle is 15° and take $C_v = 0.975$.

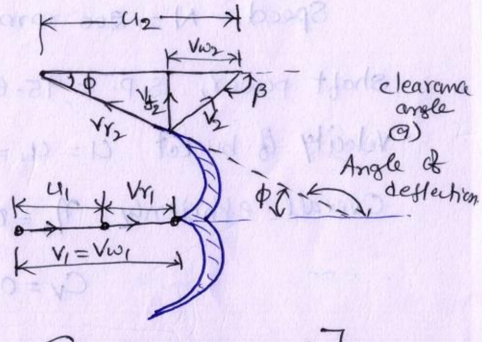
Sol:- Tangential velocity of wheel, $u = u_1 = u_2 = 20$ m/s.

Net head, $H = 50$ m.

$Q = 0.03\ m^3/s$.

Side clearance angle, $\phi = 15°$.

$C_v = 0.975$.



inlet

$V_1 = \sqrt{2gH} \times C_v = 30.54$ m/s.

$V_{w_1} = V_1 = 30.54$ m/s.

$V_{r_1} = V_{w_1} - u_1 = 10.54$ m/s.

[$\beta > 90°$, obtuse angle].

Outlet

$V_{r_1} = V_{r_2} = 10.54$ m/s

$V_{r_2} \cos\phi = 10.54 \times \cos(15°) = 10.18$ m/s $< u_2$.

* The value of "$V_{r_2} \cos\phi$" is less than "u_2", then 'β' will be obtuse angle.

$\therefore$ power $= \rho Q (V_{w_1} - V_{w_2}) \times u$

$V_{w_2} = u_2 - V_{r_2}\cos\phi$

$= 9.82$ m/s.

$= 1000 \times 0.03\ (30.54 - 9.82) \times 20$

$= 12432$ W.

$= 12.432$ kW

Problems on Pelton Wheel (or) Pelton Turbine :-

(7) A pelton wheel is working under a gross head of 400m. The water is supplied through penstock of diameter 1m and length 4m from reservoir to the pelton wheel. The co-efficient of friction for the penstock is given as 0.008. The jet of water of diameter 150mm strikes the buckets of the wheel and gets deflected through an angle of 165°. The relative velocity of water at outlet is reduced by 15% due to friction between inside surface of the bucket and water. If the velocity of the bucket is 0.45 times the jet velocity at inlet and mechanical efficiency as 85%. Determine :

(i) power given to runner

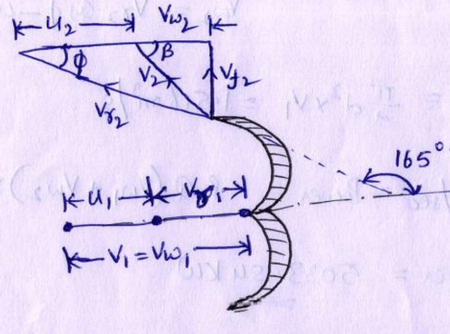
(ii) shaft power

(iii) Hydraulic efficiency and overall efficiency.

Sol:-

$H_g = 400m$

$h_f = \dfrac{4 L V_1^2 f}{2 g D^*}$



jet dia, $d = 150mm$

penstock dia, $D^* = 1m$

penstock length, $L = 4m$

friction factor $f = 0.008$

Angle of deflection $= 165°$

$V_{r_2} = 0.85\, V_{r_1}$

$u = 0.45 \times$ jet velocity

$u = 0.45 \times V_1$

$\eta_{mech} = 0.85$

$Q^*_{penstock} = Q_{jet}$

$V^* \times$ Area of penstock $=$ Area of jet $\times V_1$.

$\frac{\pi}{4} D^{*2} \times V^* = \frac{\pi}{4} d^2 \times V_1$

$V^* = \dfrac{d^2}{D^{*2}} \times V_1$

$= \dfrac{0.15^2}{1.0^2} \times V_1$

$V^* = 0.0225\, V_1$

$$H_g = H_{net} + h_f.$$

$$400 = \frac{4fLV^{*2}}{2gD} + \frac{V_1^2}{2g}.$$

[Substitute $V^* = 0.0225\,V_1$]

$$400 = \frac{4 \times 0.008 \times 4000}{2 \times 9.81} \times (0.0225 V_1)^2 + \frac{V_1^2}{2g}$$

$$V_1 = 85.83 \text{ m/s}.$$

~~flow~~ velocity of Bucket, $u_1 = 0.45\,V_1 = 38.62$ m/s.

$$V_{r_1} = V_1 - u_1 = 47.21 \text{ m/s}.$$

$$V_{w_1} = V_1 = 85.83 \text{ m/s}.$$

$$V_{r_2} = 0.85 \times V_{r_1} = 40.13 \text{ m/s}. \quad [u_1 = u_2]$$

$$V_{w_2} = V_{r_2}\cos\phi - u_2 = 0.143 \text{ m/s}.$$

$\rightarrow$ $Q = \frac{\pi}{4} d^2 \times V_1 = 1.516 \text{ m}^3/\text{s}$

$\rightarrow$ WD/sec = Power = $\rho Q (V_{w_1} + V_{w_2}) \times u = 5033540$ W.

(i) Power = $\underline{5033.54 \text{ kW}}$

(ii) $\eta_m = \frac{S.P}{P} \Rightarrow$ Shaft power $= \eta_m \times$ Power $= 4.278$ kW.

(iii) $\eta_h = \frac{2(V_{w_1} + V_{w_2})u}{V_1^2} = \underline{90.14\%}.$

$\eta_{overall} = \eta_m \times \eta_h = \underline{76.62\%}.$

Francis Turbine :- (Inward flow reaction turbine) :-

→ The inward flow reaction turbine having radial discharge at outlet is known as Francis turbine.

→ In the modern Francis turbine, the water enters the runner of the turbine in the radial direction at the inlet of the runner, and leaves in the axial direction at outlet.

→ Thus the modern Francis turbine is a mixed flow type turbine.

Work done :—

As in case of Francis turbine, the discharge is radial at outlet, the velocity of whirl at outlet (V_{w_2}) will be zero.

∴ Workdone by water on runner per second (power),

$$\boxed{P = \rho Q [V_{w_1} u_1]}$$

[$Q = \pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$]

∴ workdone per sec per unit weight of water striking per sec,

$$\boxed{\frac{P}{(wt/sec)} = \frac{V_{w_1} u_1}{g}}$$

∴ Hydraulic efficiency, $\boxed{\eta_h = \frac{V_{w_1} \cdot u_1}{gH}}$

$$\boxed{\therefore \text{ Flow ratio} = \frac{V_{f_1}}{\sqrt{2gH}}}$$

[Flow ratio = 0.15 to 0.3]

∴ Speed ratio $\boxed{\phi^* = \frac{u_1}{\sqrt{2gH}}}$

[$\phi^* = 0.6$ to 0.9]

∴ $\boxed{n = \frac{\text{width of wheel}}{\text{diameter of wheel}} = \frac{B_1}{D_1}}$

Kaplan turbine :- (Axial flow reaction turbine)

→ If the water flows parallel to the axis of the rotation of the shaft, the turbine is known as axial flow reaction turbine.

→ The head at inlet of this turbine is sum of pressure energy and kinetic energy and during the flow of water through runner a part of pressure energy is converted into kinetic energy.

→ The turbine shaft is vertical and lower end of the shaft is made larger which is called "hub" (or) "Boss". The vanes are fixed on hub and hence hub acts as runner of this turbine.

→ There are two types of axial flow reaction turbine,
(i) Propeller turbine (ii) Kaplan turbine.

Propeller turbine :- When the vanes are fixed to the hub and they are not adjustable, the turbine is called propeller turbine.

Kaplan turbine :- If the vanes ~~are~~ on the hub are adjustable, the turbine is called kaplan turbine. It is suitable for large Quantity of water at low head is available.

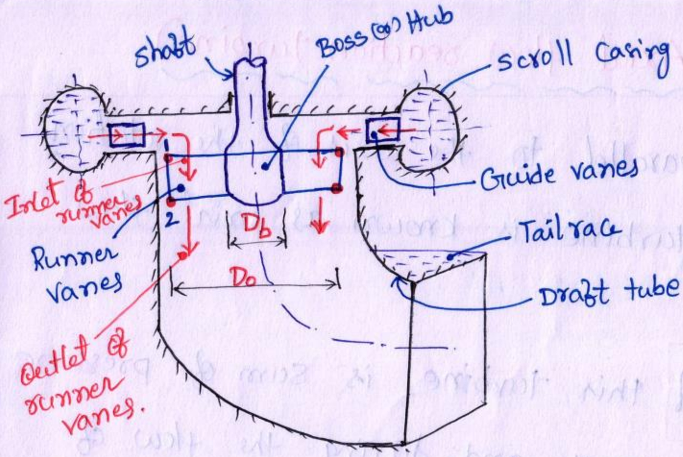


Figure: Kaplan turbine.

Main parts :-

1. Scroll Casing
2. Guide vanes mechanism
3. Hub with vanes (or) runner of the turbine.
4. Draft tube.

→ power developed = $\rho Q V_{w_1} u$ $\boxed{P = \rho Q V_{w_1} u}$ in watts. $[V_{w_2} = 0]$

→ Flow ratio, $\boxed{FR = \frac{V_{f_1}}{\sqrt{2gH}}}$ $\boxed{P = \frac{\rho Q V_{w_1} u}{1000}}$ in KW.

→ Speed ratio $\boxed{\phi^* = u/\sqrt{2gH}}$

→ Discharge, $\boxed{Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1}}$

D_o → outer dia of runner.
D_b → Dia. of hub (or) boss
V_{f_1} → flow velocity at inlet.

→ peripheral velocity at inlet and outlet, are equal. $\boxed{u_1 = u_2 = \frac{\pi D_o N}{60}}$;

→ $\boxed{V_{f_1} = V_{f_2}}$

→ Flow area at inlet = Flow area at outlet = $\frac{\pi}{4}\left(D_o^2 - D_b^2\right)$.

→ water power, $\boxed{P_w = \frac{\rho g Q H}{1000}}$ in KW.

→ Hydraulic eff; $\boxed{\eta_h = \frac{V_{w_1} u_1}{gH}}$

→ Overall efficiency, $\boxed{\eta_o = \frac{\text{Shaft power}}{\text{Water power}}}$ $\boxed{\eta_o = \frac{S.P}{\rho g H Q}}$ in watts / in watts.

DRAFT TUBE :-

→ It is a long pipe gradually increasing area which connects the outlet of the runner to the tail race.

→ It is used for discharging water from exit of the turbine to the tail race.

→ One end of the draft tube is connected to the outlet of the runner while the other end is sub-merged below the level of water in the tail race.

Main functions :-

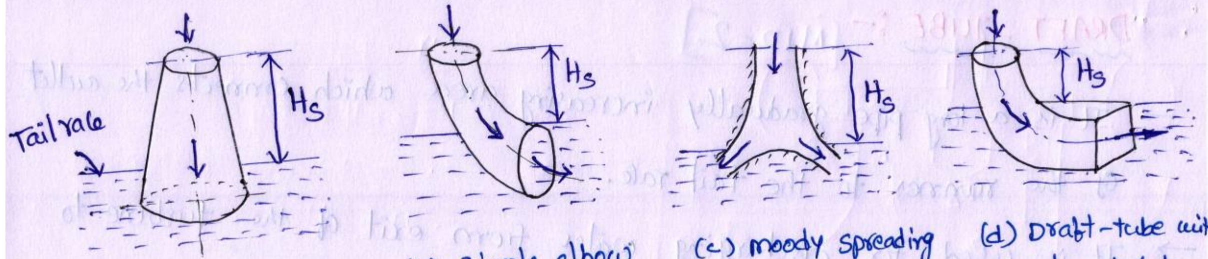
1. It permits a negative head to be established at the outlet of the runner and thereby increase the net head on the turbine.

2. It converts a large portion of the K.E. $\left(\frac{V_2^2}{2g}\right)$ rejected at outlet of the turbine into usefull pressure energy.

Hence, by using draft tube, the net head on the turbine increases. The turbine develops more power and also the efficiency of the turbine increases.

Types of draft tubes :-

1. Conical draft tube
2. Simple elbow tubes
3. Moody Spreading tubes
4. Elbow draft-tubes with circular inlet and rectangular outlet.



(a) Conical draft-tube (b) Simple elbow tube (c) moody Spreading tube (d) Draft-tube with Circular inlet and rectangular at outlet.

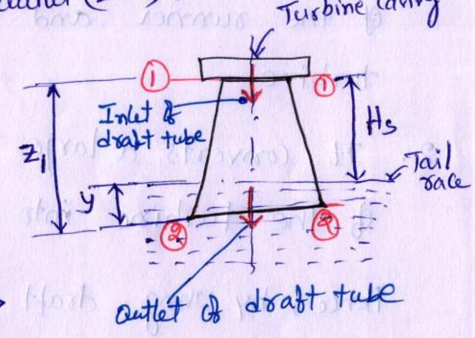
→ The conical draft tubes and moody spreading draft tubes are most efficient.

→ The simple elbow and draft-tube with circular inlet and rectangular outlet requires less space as compared to other draft-tubes.

Draft-tube theory:-

Apply Bernoullis equation at inlet (1-1) and outlet (2-2).

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+\underbrace{(H_s+y)}_{(Z_1)}=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+\underbrace{0}_{(Z_2=0)}+h_f$$



$$\boxed{\frac{P_1}{\rho g}=\frac{P_a}{\rho g}-H_s-\left(\frac{V_1^2}{2g}-\frac{V_2^2}{2g}-h_f\right)}\quad \left\{\frac{P_1}{\rho g}=\frac{P_a}{\rho g}+y\right\}$$

$\frac{P_a}{\rho g}$ → Atmospheric pressure head.

H_s → Vertical distance of draft tube above tail race.

Efficiency of draft tube:-

$$\eta_d=\frac{\text{Actual Conversion of kinetic head into pressure head}}{\text{Kinetic head at inlet of the draft tube.}}$$

$$\boxed{\eta_d=\frac{\left(\frac{V_1^2}{2g}-\frac{V_2^2}{2g}\right)-h_f}{\frac{V_1^2}{2g}}}$$

V_1 → water velocity at inlet of draft-tube

V_2 → " at outlet "

h_f → loss of head in draft tube.

problems on Kaplan turbine and Francis Turbine.

① A Kaplan turbine working under a head of 15m develops 7357.5 kW shaft power. The outer diameter of the runner is 4m and hub diameter is 2m. The guide blade angle at the extreme edge of the runner is 30°. The hydraulic and overall efficiencies of the turbine are 90% and 85% respectively. If the velocity of whirl is zero at outlet, Determine: (i) runner vane angles at inlet and outlet at the extreme edge of the runner and (ii) Speed of the turbine.

Sol:-

$H = 15m$

Shaft power = 7357.5 kW

Outer dia, $D_o = 4m$

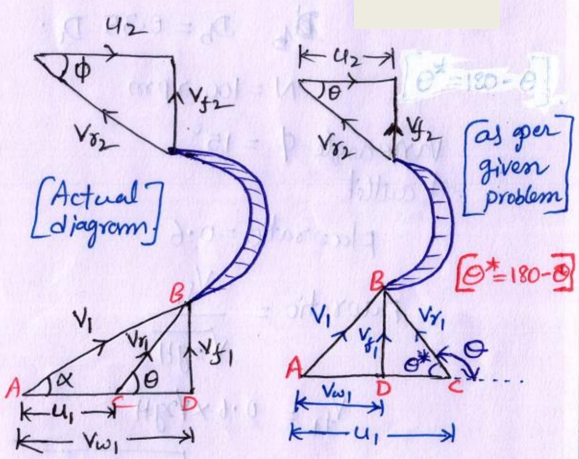
Hub dia, $D_b = 2m$

Guide blade angle $\alpha = 30°$

$\eta_h = 90\%$

$\eta_o = 85\%$

$V_{w_2} = 0$.



$\rightarrow \eta_o = \dfrac{S.P}{W.P} = \dfrac{\text{Shaft power in watts}}{\rho g Q H \text{ in watts}}$

$0.85 = \dfrac{7357500}{1000 \times 9.81 \times Q \times 15}$

$Q = 58.82\ m^3/s$.

$\rightarrow Q = \dfrac{\pi}{4}(D_o^2 - D_b^2) \times V_{f_1}$

$= \dfrac{\pi}{4}(4^2 - 2^2) \times V_{f_1}$

$V_{f_1} = 6.24\ m/s$.

From inlet Δ^{le}, $\tan\alpha = \dfrac{V_{f_1}}{V_{w_1}}$.

$V_{w_1} = \dfrac{V_{f_1}}{\tan\alpha} = 10.8\ m/s$.

$\eta_h = \dfrac{\rho Q (V_{w_1} + V_{w_2}) u}{\rho g Q H}$.

$\eta_h = \dfrac{V_{w_1} u_1}{gH} \Rightarrow u_1 = 12.25\ m/s$.

(i) Runner vane angle at inlet and outlet,

$\rightarrow \tan\theta^* = \tan(180-\theta) = \dfrac{V_{f_1}}{u_1 - V_{w_1}}$

$\theta = 180° - \tan^{-1}\left(\dfrac{V_{f_1}}{u_1 - V_{w_1}}\right) = \theta = 103.08°$.

$\tan\phi = \dfrac{V_{f_2}}{u_2} = \dfrac{V_{f_1}}{u_1}$

$\phi = 26.9°$.

$\rightarrow$ (ii) $u = \dfrac{\pi D N}{60} \Rightarrow N = 58.48\ rpm$

② The hub diameter of a Kaplan turbine, working under a head of 12m, is 0.35 times the diameter of the runner. The turbine is running at 100 rpm. If the vane angle of the extreme edge of the runner at outlet is 15° and flow ratio is 0.6. Find: (i) Diameter of the runner (ii) Diameter of the boss (hub) (iii) Discharge through runner. The velocity of whirl at outlet is given as zero.

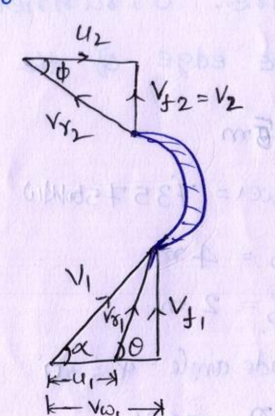
Sol:- head $H = 12\,m$

$D_b = 0.35\ D_o$

$N = 100$ rpm

Vane angle $\phi = 15°$ at outlet

Flow ratio $= 0.6$



$$\text{Flow ratio} = \frac{V_{f_1}}{\sqrt{2gH}}$$

$$V_{f_1} = 0.6 \times \sqrt{2gH}$$

$$= 0.6 \times \sqrt{2 \times 9.81 \times 12}$$

$$= 9.2\ m/s$$

$V_{w_2} = 0$ $\quad [V_{f_1} = V_{f_2}]$

$$\tan\phi = \frac{V_{f_2}}{u_2} = \frac{V_{f_1}}{u_2}$$

$$\because \tan\phi = \frac{V_{f_1}}{u_2} \Rightarrow u_2 = \frac{V_{f_1}}{\tan\phi}$$

$$= \frac{9.2}{\tan(15°)}$$

$$u = u_1 = u_2 = 34.33\ m/s$$

(i) $u_1 = \dfrac{\pi D_o N}{60} \Rightarrow D_o = \dfrac{60 \times u}{\pi N}$

$$= \frac{60 \times 34.33}{\pi \times 100}$$

$$= \underline{6.55\,m}.$$

(ii) $D_b = 0.35 \times D_o = \underline{2.3\ m}$

(iii) $Q = \dfrac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1} = \dfrac{\pi}{4}\left(6.55^2 - 2.3^2\right) \times 9.2 = \underline{271.77\ m^3/s}.$

③ A kaplan turbine runner is to be designed to develop 9100 kW. The net available head is 5.6m. If speed ratio is 2.09, flow ratio is 0.68, overall efficiency is 86% and the diameter of the boss is 1/3 the diameter of the runner. Find the diameter of the runner, its speed and the specific speed of the turbine.

Sol:-

Power $P = 9100\ kW$

Head $H = 5.6\ m$

Speed ratio $= 2.09$

Flow ratio $= 0.68$

Overall efficiency $\eta_o = 0.86$

$$D_b = \frac{1}{3} D_o$$

(i) $D_o = ?$

(ii) $N = ?$

(iii) $N_s = ?$

$\rightarrow$ Speed ratio, $= \frac{u_1}{\sqrt{2gH}}$

$u_1 = \text{speed ratio} \times \sqrt{2gH}$

$= 2.09 \times \sqrt{2 \times 9.81 \times 5.6}$

$u_1 = 21.95\ m/s$

$\rightarrow$ Flow ratio $= \frac{V_{f_1}}{\sqrt{2gH}}$

$V_{f_1} = \text{Flow ratio} \times \sqrt{2gH}$

$= 0.68 \times \sqrt{2 \times 9.81 \times 5.6}$

$V_{f_1} = 7.12\ m/s.$

$\rightarrow$ $\eta_o = \frac{S.P}{W.P} = \frac{\text{shaft power}}{\rho g Q H}$

$Q = \frac{\text{shaft power in watts}}{\rho \times g \times \eta_o \times H}$

$= \frac{9100 \times 1000}{1000 \times 9.81 \times 0.86 \times 5.6}$

$= 192.5\ m^3/s.$

(i) Runner diameter "D_o",

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) V_{f_1}$$

$$192.5 = \frac{\pi}{4}\left[D_o^2 - \left(\frac{D_o}{3}\right)^2\right] \times 7.12$$

$$D_o = 6.21\ m$$

(ii) $u = \frac{\pi D_o N}{60} \Rightarrow N = \frac{60 \times u}{\pi D_o} = 67.5\ rpm.$

(iii) $N_s = \frac{N\sqrt{P}}{(H)^{5/4}} = \frac{67.5 \times \sqrt{9100}}{(5.6)^{5/4}}$

$= 74.6.$

④ A Francis turbine with an overall efficiency of 75% is required to produce 148.25 KW power. It is working under a head of 7.62m. The speed ratio and velocity ratio are 0.26 and 0.96 respectively. The wheel runs at 150 rpm and the hydraulic losses in the turbine are 22% of the available energy. Assuming radial discharge, Determine:

(i) The guide blade angle

(ii) The wheel vane angle at inlet

(iii) Diameter of wheel at inlet

(iv) width of the wheel at inlet.

Sol: $\eta_o = 75\% = 0.75$, Hydraulic losses = 22% of available energy

$P = 148.25$ KW. $N = 150$ rpm.

$H = 7.62m$

$u = u_1 = u_2 = \text{Speed ratio} \times \sqrt{2gH}$

$= 0.26 \times \sqrt{2 \times 9.8 \times 7.62}$

$= 3.179$ m/s.

$V_{f_1} = \text{velocity ratio} \times \sqrt{2gH}$

$= 0.96 \times \sqrt{2 \times 9.81 \times 7.62}$

$= 11.738$ m/s.

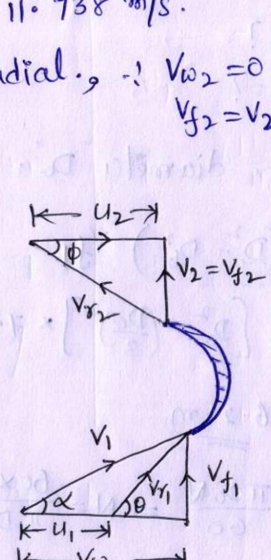
Given Condition, discharge at outlet is radial., ∴ $V_{w_2} = 0$; $V_{f_2} = V_2$

$\rightarrow \eta_h = \dfrac{H - H_{loss}}{H} = \dfrac{H - 0.22H}{H} = 0.78.$

$\eta_h = \dfrac{V_{w_1} u_1}{gH} \Rightarrow V_{w_1} = \dfrac{\eta_h \times gH}{u_1}$

$V_{w_1} = \dfrac{0.78 \times 9.81 \times 7.62}{3.179}$

$V_{w_1} = 18.34$ m/s.



$\begin{cases} V_{w_2} = 0 \\ V_{f_2} = V_2 \end{cases}$

(i) Vane angles at inlet,

$\rightarrow \tan\alpha = \dfrac{V_{f_1}}{V_{w_1}}$

$\alpha = \tan^{-1}\left(\dfrac{V_{f_1}}{V_{w_1}}\right)$

$\alpha = 32.619°$

$\tan\theta = \dfrac{V_{f_1}}{V_{w_1} - u_1}$

$\theta = \tan^{-1}\left(\dfrac{V_{f_1}}{V_{w_1} - u_1}\right)$

$= 37.74°$

(iii) Diameter of wheel at inlet (D_1).

$$u_1 = \frac{\pi D_1 N}{60} \Rightarrow D_1 = \frac{60 \times u_1}{\pi \times N} = 0.4047\,m$$

(iv) width of wheel at inlet (B_1),

$$\eta_o = \frac{S.P}{\text{water power}} = \frac{S.P}{\rho g Q H}$$

$$Q = \frac{S.P}{\rho \times g \times \eta_o \times H}$$

$$= \frac{148.25 \times 10^3}{1000 \times 9.81 \times 0.75 \times 7.62}$$

$$= 2.644\ m^3/s.$$

$$\therefore Q = \pi \times D_1 \times B_1 \times V_{f1}$$

$$B_1 = \frac{Q}{\pi \times D_1 \times V_{f1}}$$

$$= \frac{2.644}{\pi \times 0.4047 \times 11.738}$$

$$= 0.177\,m.$$

(5) A turbine is to operate under a head of 25 m at 200 rpm. The discharge is 9 m^3/s. If the turbine efficiency is 90%. Determine (i) Specific speed of the turbine (ii) power generated (iii) performance under a head of 20 m. Also state the type of the turbine.

Sol:- Head = $H = 25m$.

Speed $N = 200$ rpm

Discharge $Q_1 = 9\ m^3/s$.

Overall efficiency $\eta_0 = 90\% = 0.9$.

(i) $\eta_0 = \frac{\text{Power developed}}{\text{Water power}}$

$\eta_0 = \frac{P_1}{\rho g Q H}$

$P_1 = \eta_0 \times \rho g Q H$

$= 0.9 \times 1000 \times 9.81 \times 9 \times 25 =$

$= 1986500\ W$.

$P_1 = 1986.5\ KW$

(ii) Specific Speed N_S

$$N_S = \frac{N\sqrt{P}}{H^{5/4}} = \frac{200 \times \sqrt{1986.5}}{(25)^{5/4}}$$

$= 159.46$ rpm

Specific speed is lies between 51 and 255. Therefore the turbine is Francis turbine.

~~(iii)~~

(iii) If head $H_2 = 20\ m$.

→ What is performance of turbine?

→ It means $N_2, Q_2, Power_2 = ?$

$$\Rightarrow \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$N_2 = \sqrt{\frac{H_2}{H_1}} \times N_1$$

$$= \sqrt{\frac{20}{25}} \times 200$$

$N_2 = 178.88$ rpm.

$$\Rightarrow \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$$

$$Q_2 = \sqrt{\frac{H_2}{H_1}} \times Q_1$$

$$= \sqrt{\frac{20}{25}} \times 9$$

$= 8.05\ m^3/s$.

$$\frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$$

$$P_2 = \left(\frac{H_2}{H_1}\right)^{3/2} \times P_1$$

$$= \left(\frac{20}{25}\right)^{3/2} \times 1986500$$

$= 1421420\ W$

$= 1421.42\ KW$.

(iv).

The speed of the turbine lies between 51 and 255. Therefore the turbine is Francis turbine.

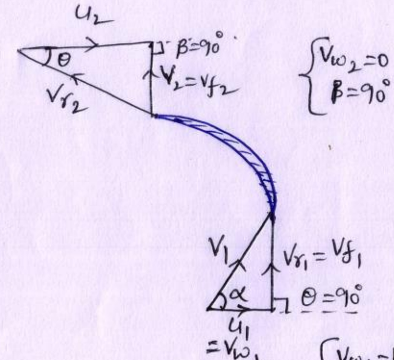
Problems on inward Reaction turbine.

⑥ The external and internal diameters of an inward flow reaction turbines are 1·20m and 0·6m respectively. The head on the turbine is 22m and velocity of flow through the runner is constant and equal to 2·5 m/s. The guide blade angle is given as 10° and the runner vanes are radial at inlet. If the discharge at outlet is radial. determine,

(i) The Speed of the turbine

(ii) The vane angle at outlet of the runner

(iii) Hydraulic efficiency.



$\begin{cases} V_{w_2}=0. \\ \beta=90°. \end{cases}$

$\begin{cases} V_{w_1}=u_1 \\ \theta=90°. \end{cases}$

Sol:- External dia $D_1 = 1.20m$

Internal dia $D_2 = 0.6m$

$H = 22m$

$V_{f_1} = V_{f_2} = 2.5 m/s$

Guide blade angle $\alpha = 10°$.

→ The runner vanes are radial at inlet, means $\theta = 90°$,

$V_{w_1} = u_1$

$V_{r_1} = V_{f_1}$

→ Discharge is radial,

$V_{w_2} = 0$,

$V_2 = V_{f_2} = 2.5 m/s$.

$\tan\alpha = \frac{V_{f_1}}{u_1}$.

$u_1 = \frac{V_{f_1}}{\tan\alpha} = \frac{2.5}{\tan(10°)} = 14.178 m/s$

$\therefore V_{w_1} = u_1 = 14.178 m/s$.

(i) Speed (N),

$u_1 = \frac{\pi D_1 N}{60} \Rightarrow N = \frac{60 \times u_1}{\pi \times D_1} = 225.65 rpm$

$u_2 = \frac{\pi D_2 N}{60} = 7.09 m/s$.

(ii) Vane angle at outlet (ϕ),

$\tan\phi = \frac{V_{f_2}}{u_2}$

$\phi = \tan^{-1}\left(\frac{V_{f_2}}{u_2}\right)$

$= 19.42°$.

(iii) $\eta_h = \frac{V_{w_1} u_1}{gH}$

$= \frac{14.178 \times 14.178}{9.81 \times 22}$

$= 0.9314$

$= 93.14\%$

Performance of Hydraulic Turbines :-

Characteristic Curves of hydraulic turbines :-

The behaviour and performance of the turbine under different working conditions, can be estimated by using characteristic curves.

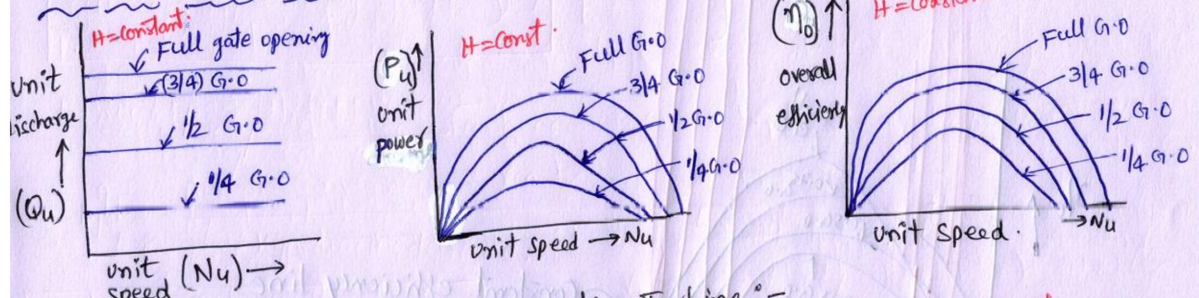
→ The important parameters which are varied during a test on turbine.

(1) Speed (N)
(2) Head (H)
(3) Discharge (Q)
(4) Power (P)
(5) Overall efficiency (η_o)
(6) Gate opening.

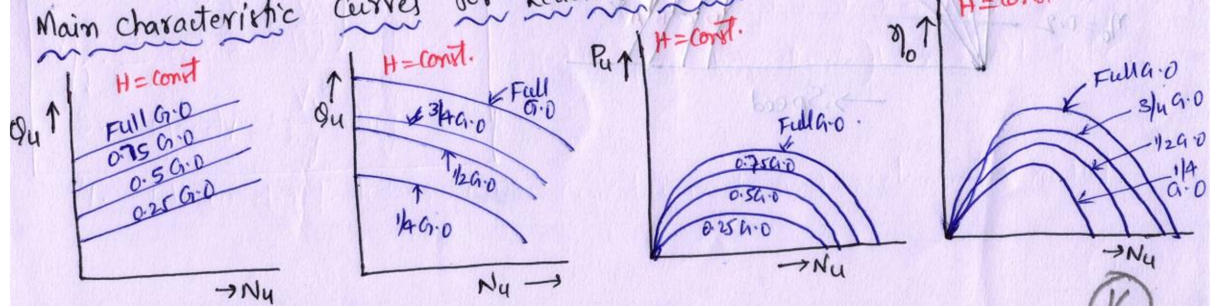
→ Important characteristic curves,

1. Main characteristic Curves (a) Constant head Curves
2. Operating characteristic curves (a) Constant Speed Curves
3. Muschel Curves (a) Constant efficiency Curves.

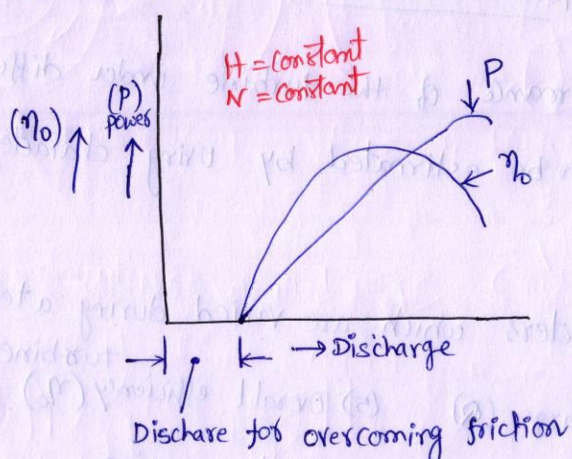
Main characteristic Curves for a pelton wheel :- (Impulse Turbine)



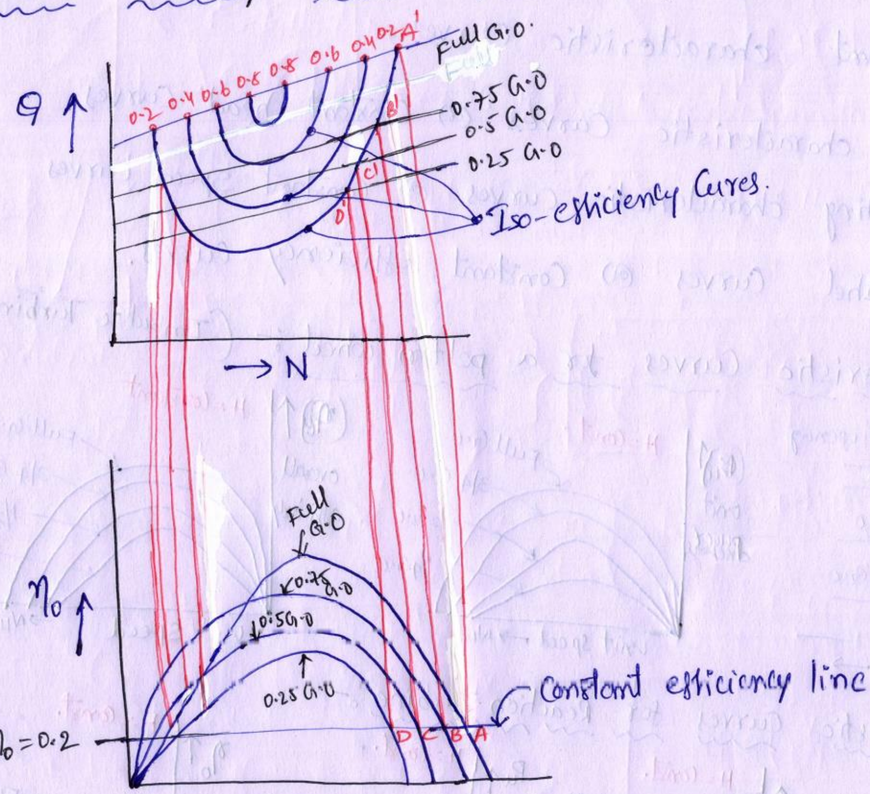
Main characteristic Curves for Reaction Turbine :-



Operating characteristic (or) Constant Speed Curves :-



Constant efficiency curves (or) Iso-efficiency Curves :-



Specific Speed of the turbine,

$\Rightarrow$ It is defined as at what speed, the turbine develop unit power when working under unit head. It is denoted by Symbol "N_s".

$\Rightarrow$ Derivation :- $\eta_0 = \dfrac{\text{shaft power}}{\text{water power}} = \dfrac{\text{Power developed in KW}}{\dfrac{\rho \times g \times Q \times H}{1000} \text{ in KW}}$

$\therefore P = \eta_0 \times \dfrac{\rho \times g \times Q \times H}{1000}$

$\Rightarrow P \propto Q \times H$.

$\rightarrow Q$ = Area × velocity.

$Q \propto B \times D \times V$. $\{B \propto D\}$

$Q \propto D^2 \times \sqrt{H}$. $\{V \propto \sqrt{H}$

$u \propto V$. where, $V \propto \sqrt{H}$

$\frac{\pi D N}{60} \propto \sqrt{H}$

$DN \propto \sqrt{H}$.

$\boxed{D \propto \frac{\sqrt{H}}{N}}$

$\therefore P \propto D^2 \times \sqrt{H} \times H$.

$P \propto \left(\frac{\sqrt{H}}{N}\right)^2 \times H^{3/2}$

$P \propto \frac{H^{5/2}}{N}$

$\Rightarrow P = K \frac{H^{5/2}}{N^2}$

$Q \propto D^2 \times \sqrt{H}$

$\propto \left(\frac{\sqrt{H}}{N}\right)^2 \times \sqrt{H}$

$\propto \frac{H}{N^2} \times \sqrt{H}$

$\boxed{Q \propto \frac{H^{3/2}}{N^2}}$

if $P = 1$, $H = 1$, $N = N_s$.

$\therefore K = N_s^2$.

$P = N_s^2 \cdot \frac{H^{5/2}}{N^2}$

$N_s^2 = \frac{N^2 P}{H^{5/2}}$

$$\boxed{N_s = \frac{N\sqrt{P}}{H^{5/4}}}$$

where, $N \rightarrow$ Speed in rpm.
$P \rightarrow$ shaft power in KW.
$H \rightarrow$ Head in m.

S.No.	Specific Speed	Type of turbine
1.	8.5 to 30	pelton wheel with single jet
2.	30 to 51	pelton wheel with two (or) more jets
3.	51 to 225	Francis turbine
4.	225 to 860	Kaplan (or) propeller turbine

UNIT QUANTITIES :-

→ To predict the behaviour of turbine working under varying conditions of head, speed, output and gate opening, the results are expressed in terms of quantities when head is 1m.

→ The following three important unit quantities are,

① **UNIT SPEED :-** It is defined as the speed of a turbine working under a unit head (1m). It is denoted by "N_u".

$u = \frac{\pi D N}{60}$ $\quad u \propto V \quad V \propto \sqrt{H}$

$\propto \sqrt{H}$

as $D \to$ Constant,

$N \propto u$

$\propto \sqrt{H}$

$N = K_1 \sqrt{H}$

$N = N_u \sqrt{H}$

$$\boxed{N_u = \frac{N}{\sqrt{H}}}$$

Now, if $H = 1\,m$, $\quad N = N_u$

$\boxed{N_u = K_1}$

$$\boxed{\therefore N_u = \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}}$$

② **UNIT DISCHRGE :-** It is defined as the discharge passing through a turbine, which is working under a unit head (i.e. 1m). Denoted by "Q_u".

$Q = A \times$ velocity

$A \to$ Constant $\rightarrow$ $Q \propto V$

$\propto \sqrt{H}$

if $H = 1m$, $\quad Q = Q_u$

$\therefore K_2 = Q_u$

$Q = K_2 \sqrt{H}$

$Q = Q_u \sqrt{H} \Rightarrow \boxed{Q_u = \frac{Q}{\sqrt{H}}}$

$$\boxed{Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}}$$

③ **UNIT POWER :-** It is defined as the power developed by a turbine, working under a unit head.

$$\eta_o = \frac{\text{Power developed}}{\text{water power}} = \frac{P}{\left(\frac{\rho g Q H}{1000}\right)} \Rightarrow P = \eta_o \times \frac{\rho g Q H}{1000}$$

$\propto Q \times H \quad \{Q = \sqrt{H}\}$.

$\propto \sqrt{H} \times H$

$\propto H^{3/2}$

$P = K H^{3/2}$

$P = P_u H^{3/2}$ $\quad \therefore P_u = \frac{P}{H^{3/2}}$

$\therefore$ if $H = 1m$, $\quad P = P_u$

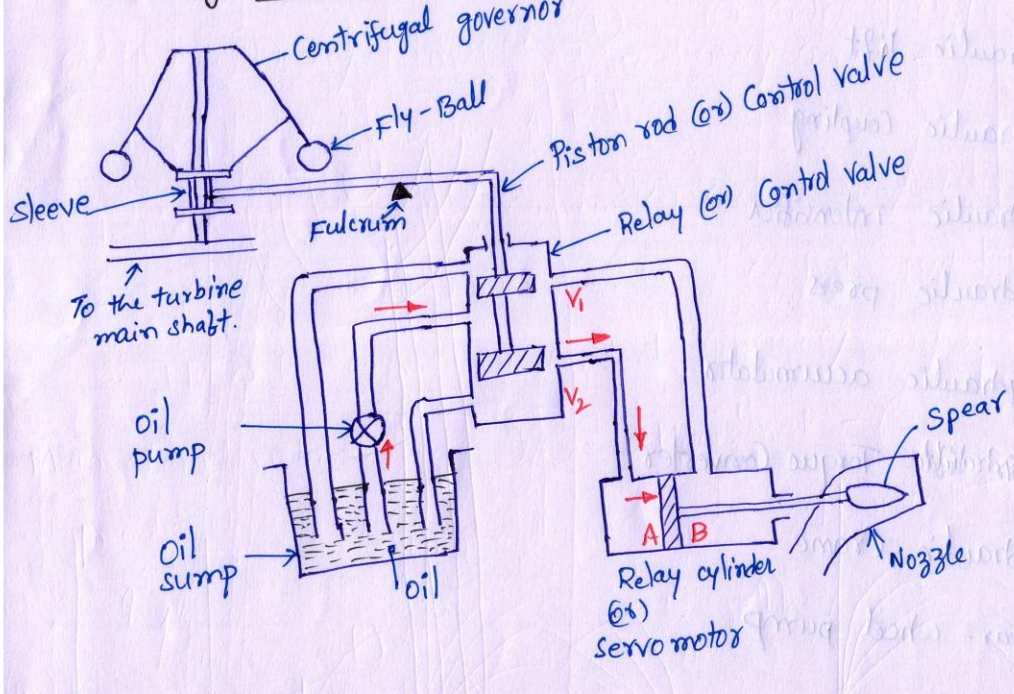
$P_u = K$

$$\boxed{\therefore P_u = \frac{P}{H^{3/2}}} \qquad \boxed{P_u = \frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}}$$

Governing of turbines :-

It is defined as the operation by which the speed of the turbine is kept Constant under all Conditions of working. It is done automatically by means of a governor, which regulates the rate of flow through the turbines according to the changing load conditions on the turbine.

Governing of impulse turbine. (Pelton turbine).



1) Oil sump.
2) Gear pump (oil pump), which is driven by power obtained from turbine shaft.
3) The servomotor (Relay Cylinder)
4) The Control value (or) distribution valve (or) relay valve
5) The Centrifugal governor (or) pendulum operated by gear from turbine shaft
6) The spear rod (or) needle.

Hydraulic system :-

→ It is defined as the device in which power is transmitted with the help of a fluid which may be liquid (water or oil) under pressure.

→ Most of these devices are operated on the pinciples of fluid statics and fluid kinematics.

Types :-

1. Hydraulic ram
2. Hydraulic lift
3. Hydraulic Coupling
4. Hydraulic intensifier
5. Hydraulic press
6. Hydraulic accumulator
7. Hydraulic Torque Converter.
8. Hydraulic Crane
9. Gear- wheel pump