ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

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ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

Syllabus

UNIT–I	Analog Ammeter and Voltmeters
UNIT–II	Analog Watt meters and Power Factor Meters
UNIT–III	Measurements of Electrical parameters
UNIT–IV	Transducers
UNIT–V	Digital meters

Text Books:

- Electrical & Electronic Measurement & Instruments by A.K.Sawhney Dhanpat Rai & Co.Publications
- Electrical and Electronic Measurements and instrumentation by R.K.Rajput, S.Chand
- Electrical Measurements and measuring Instruments by E.W. Golding and F.C.Widdis, fifth Edition, Wheeler Publishing
- Electrical Measurements by Buckingham and Price, Prentice – Hall

ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

UNIT-I

Analog Ammeter and Voltmeters

UNIT–I

Analog Ammeter and Voltmeters

Topics:

- Classification
- Deflecting, control and damping torques
- PMMC, moving iron type and electrostatic instruments
  - Construction, Torque equation, Range extension
  - Effect of temperature, Errors and compensations
  - advantages and disadvantages
- Instrument transformers: Current Transformer and Potential Transformer
  - construction, theory, errors
- Numerical problems

MEASURING INSTRUMENTS

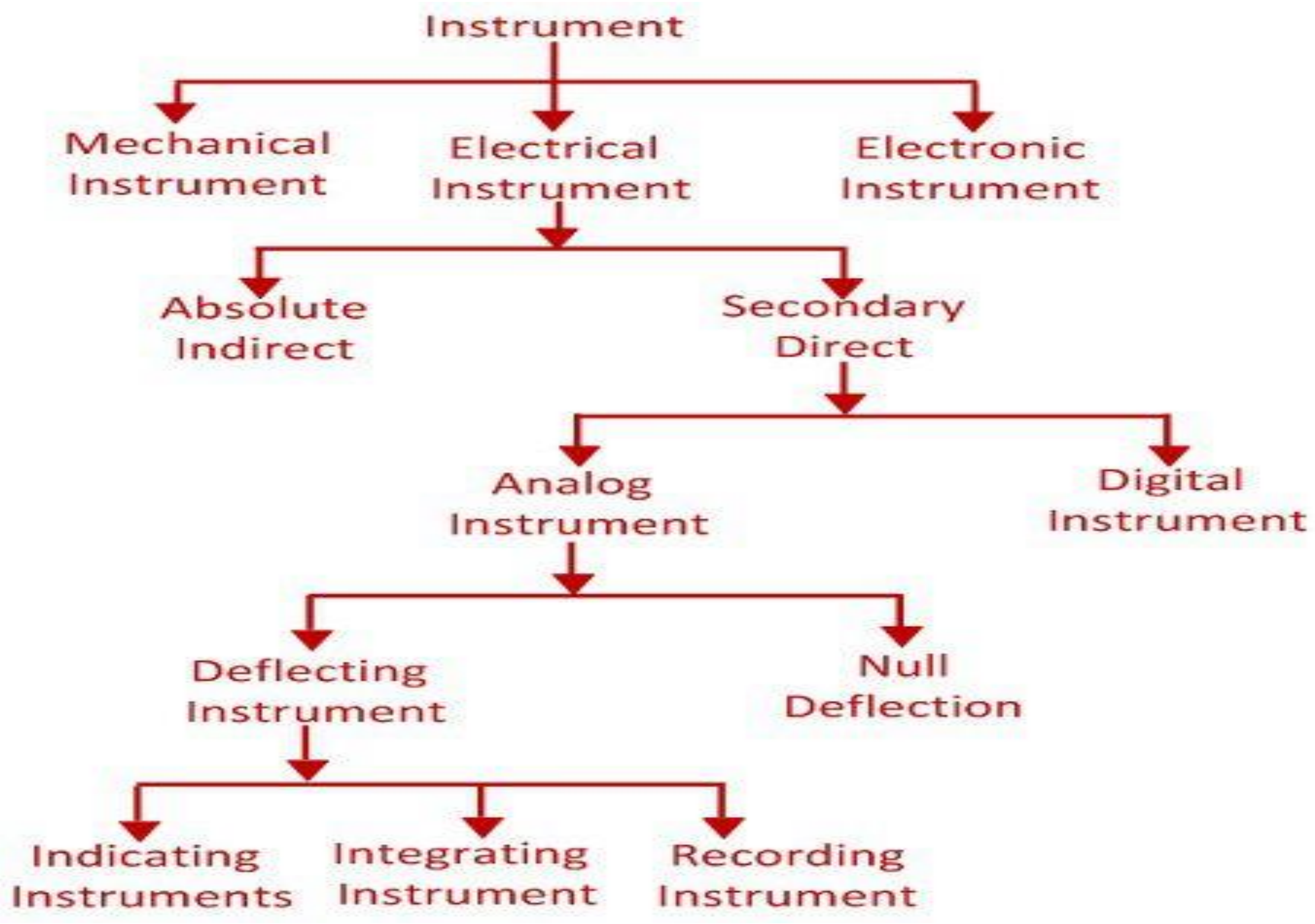
Introduction

- An instrument is a device in which we can determine the magnitude or value of the quantity to be measured.
- The measurement of a given quantity is the result of comparison between the quantity to be measured and a definite standard.
- The instruments which are used for such measurements are called measuring instruments.

The necessary requirements for any measuring instruments are :

1) With the introduction of the instrument in the circuit, the circuit conditions should not be altered. Thus the quantity to be measured should not get affected due to the instrument used.
2) The power consumed by the instruments for their operation should be as small as possible

CLASSIFICATION OF MEASURING INSTRUMENTS



- The measuring instrument categorised into three types;
  - Electrical Instrument
  - Electronic Instrument
  - Mechanical Instrument
- The mechanical instrument uses for measuring the physical quantities. This instrument is suitable for measuring the static and stable condition because the instrument is unable to give the response to the dynamic condition.
- The electronic instrument has quick response time. The instrument provides the quick response as compared to the electrical and mechanical instrument.
- The electrical instrument is used for measuring electrical quantities likes current, voltage, power, etc. The ammeter, voltmeter, wattmeter are the examples of the electrical measuring instrument. The ammeter measures the current in amps; voltmeter measures voltage and Wattmeter are used for measuring the power. The classification of the electric instruments depends on the methods of representing the output reading.

➢ Absolute Instrument

- The absolute instrument gives the value of measures quantities regarding the physical constant. The physical constant means the angle of deflection, degree and meter constant. The mathematical calculation requires for knowing the value of a physical constant. Such instruments do not require comparison with any other standard.
- The tangent galvanometer is the examples of the absolute instruments. In tangent galvanometer, the magnitude of current passes through the coil determines by the tangent of the angle of deflection of their coil, the horizontal component of the earth magnetic field, radius and the number of turns of wire used. The most common applications of this type of instrument are found in laboratories.

➢ Secondary Instrument

- In the secondary instrument, the deflection shows the magnitude of the measurable quantities. The calibration of the instruments with the standard instrument is essential for the measurement. The output of this type of device is directly obtained, and no mathematical calculation requires for knowing their value.
- These instruments are generally used in practice.

➢ Digital Instruments :

- The digital instrument gives the output in the numeric form. The instrument is more accurate as compared to the analogue instrument because no human error occurs in the reading.

➢ Analog Instruments

- The instrument whose output varies continuously is known as the analogue instrument. The analogue instrument has the pointer which shows the magnitude of the measurable quantities.

✓ The analogue device classifies into two types.

○ Null Type Instrument

- In this instrument, the zero or null deflection indicates the magnitude of the measured quantity. The instrument has high accuracy and sensitivity.
- In null deflection instrument, the one known and one unknown quantity use. When the value of the known and the unknown measuring quantities are equal, the pointer shows the zero or null deflection. The null deflection instrument is used in the potentiometer and in galvanometer for obtaining the null point.

Deflection Type Instruments

- The instrument in which the value of measuring quantity is determined through the deflection of the pointer is known as the deflection type instrument.
- The measuring quantity deflects the pointer of the moving system of the instrument which is fixed on the calibrated scale. Thus, the magnitude of the measured quantity is known.

✓ **The deflection type instruments(**Secondary instruments)**are further sub-classified into three types.**

- Indicating Instrument – The instrument which indicates the magnitude of the measured quantity is known as the indicating instrument. The indicating instrument has the dial which moves on the graduated dial. The voltmeter, ammeter, power factor meter are the examples of the indicating instrument.
- Integrating Instrument – The instrument which measures the total energy supplied at a particular interval of time is known as the integrating instrument. The total energy measured by the instrument is the product of the time and the measures electrical quantities. The energy meter, watt-hour meter are the examples of integrating instrument.
- Recording Instrument – The instrument records the circuit condition at a particular interval of time is known as the recording instrument. The moving system of the recording instrument carries a pen which lightly touches on the paper sheet. The movement of the coil is traced on the paper sheet. The curve drawn on the paper shows the variation in the measurement of the electrical quantities. Examples are ECG ,seismograph and X-Y recorder etc.

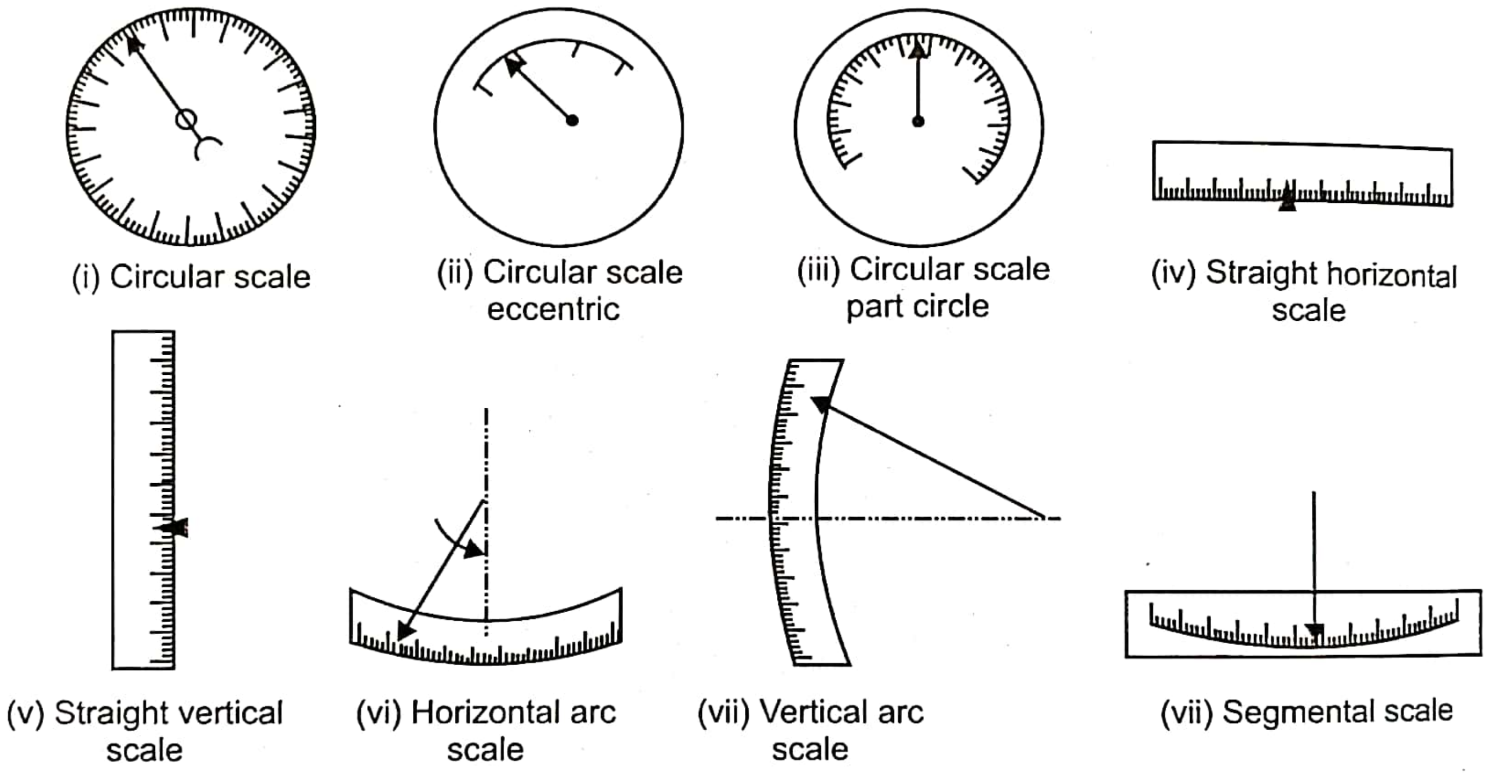


Fig. 4.2 (a). Fixed-scale and movable-pointer indicators.

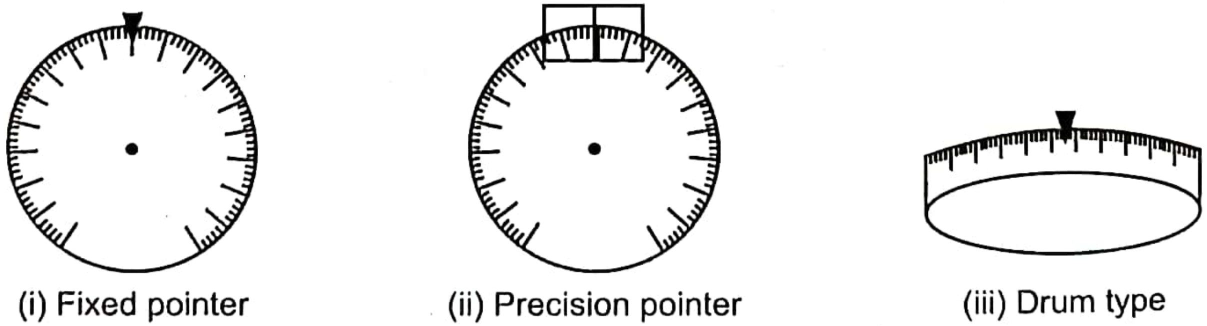


Fig. 4.2 (b). Fixed-pointer and movable-scale indicators.

- **Operating principles:**
  - Comparison and balancing methods-unknown quantity is compared with known quantity.
- **Various forces/torques required in measuring instruments:**
  - In case of measuring instruments, the effect of unknown quantity is converted into a mechanical force which is transmitted to the pointer which moves over a calibrated scale. The moving system of such instrument is mounted on a pivoted spindle.
  - For satisfactory operation of any indicating instrument, following systems must be present in an instrument.

1) Deflecting system producing deflecting torque (T_d)
2) Controlling system producing controlling torque(T_c)
3) Damping system producing damping torque.

➢ Deflecting System

- In most of the indicating instruments the mechanical force proportional to the quantity to be measured is generated.
- It is the torque which deflects the pointer on a calibrated scale according to the electrical quantity passing through the instrument
- The system which produces such a deflecting torque is called deflecting system and the torque is denoted as T_d

The deflecting torque overcomes,

1) The inertia of the moving system
2) The controlling torque provided by controlling system
3) The damping torque provided by damping system.

The deflecting system uses one of the following effects produced by current or voltage, to produce deflecting torque.

1) Magnetic Effect:

- When a current carrying conductor is placed in uniform magnetic field, it experiences a force which causes to move it.
- This effect is mostly used in many instruments like permanent magnet moving coil instrument,moving iron instrument etc;

2) Thermal Effect:

- The current to be measured is passed through a small element (platinum iridium wire) , the property of the element is, it expanses when the temperature increase.
- Due to the current flowing through the element, the temperature of the element increases, due to the elasticity property the moving system of the instrument moves from the zero position.

3) Electrostatic Effects:

- When two charged plates are kept with a small distance , there is a attraction or repulsion force experience between the two plates, this effect is called Electrostatic effect. This force is used to move the pointer of the instrument.
- This effect is used in electrostatic instruments which are normally voltmeters

4) Induction Effects :

- This type of instrument works on the principle of induction motor. This instruments are used to measure only A.C quantities.
- When a non-magnetic conducting disc is placed in a magnetic field produced by electromagnets which are excited by alternating currents, an emf is induced in it.
- If a closed path is provided, there is a flow of current in the disc.
- The interaction between induced currents and the alternating magnetic fields exerts a force on the disc which causes to move it.
- This interaction is called an induction effect. This principle is mainly used in energy meters.

5) Hall Effect:

- If a bar of semiconducting material is placed in uniform magnetic field and if the bar carries current, then an e.m.f. is produced between two edges of conductor.
- The magnitude of this e.m.f. depends on flux density of magnetic field, current passing through the conducting bar and hall effect co-efficient which is constant for a given semiconductor.
- This effect is mainly used in flux-meters. Thus the deflecting system provides the deflecting torque or operating torque for movement of pointer from its zero position.
- It acts as the prime mover for the deflection of pointer

➢ Controlling System:

- This system should provide a force so that current or any other electrical quantity will produce deflection of the pointer proportional to its magnitude.

The important functions of this system are ,

1) It produces a force equal and opposite to the deflecting force in order to make the deflection of pointer at a definite magnitude. If this system is absent, then the Pointer will swing beyond its final steady position for the given magnitude and deflection will become indefinite.

2) It brings the moving system back to zero position when the force which causes the movement of the moving system is removed. It will never come back to its zero position in the absence of controlling system.

➢ There are two methods to provide controlling torque

1) Springs control
2) Gravity control

❑ Gravity Control

- This type of control consists of a small weight attached to the moving system whose position is adjustable.
- This weight produces a controlling torque due to gravity. This weight is called control weight.
- The Fig. 1.1 shows the gravity control system. At the zero position of the pointer, the controlling torque is zero. This position is shown as position A of the weight in the Fig. 1.2
- If the system deflects, the weight position also changes, as shown in the Fig. 1.2
- The system deflects through an angle ϴ. The control weight acts at a distance l from the center.
- The component W sin ϴ of this weight tries to restore the pointer back to the zero position. This is nothing but the controlling torque (T_c)

Thus

Controlling torque $T_c = W \sin \Theta \times l$

$= K \sin \Theta$

Here K=Wl

=Gravity control

Now generally all meters are current sensing meters where

Deflecting torque $T_d = K_t I$

Where K_t = another constant

In equilibrium position $T_d = T_c$

$K_t I = K \sin \Theta$

$I \propto \sin \Theta$

Thus the deflection is proportional to current i.e,quantity to be measured.

But as it is a function of sin ϴ,the scale for the instrument using gravity control is not uniform.

Its advantages are :

1) Its performance is not time dependent.
2) It is simple and cheap.
3) The controlling torque can be varied by adjusting the position of the control weight
4) Its performance is not temperature dependent.

Its disadvantages are:

1) The scale is non uniform causing problems to record accurate readings.
2) The system must be used in vertical position only and must be properly levelled. Otherwise it may cause serious errors in the measurements
3) As delicate and proper levelling required, in general it is not used for indicating instruments and portable instruments.

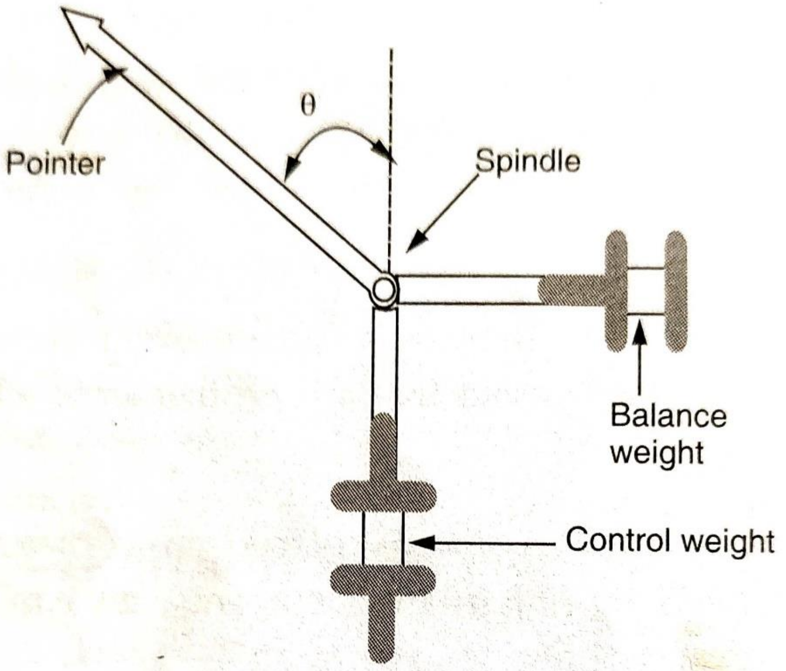


Fig. 1.1 Gravity control

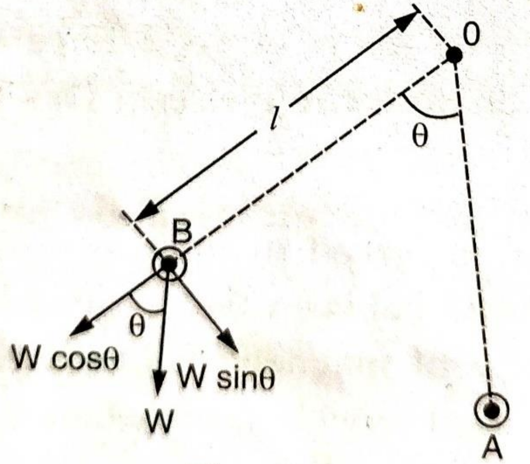


Fig. 1.2

❑ Spring Control

- Two hair springs are attached to the moving system which exerts controlling torque.
- To employ spring control to an instrument, following requirements are essential.

1) The spring should be non-magnetic.
2) The spring should be free from mechanical stress.
3) The spring should have a small resistance, sufficient cross-sectional area.
4) It should have low resistance temperature co-efficient

- The arrangement of the springs is shown in the Fig. 1.3.
- The springs are made up of non-magnetic materials like silicon bronze, hard rolled Silver or copper, platinum silver and germanium silver.
- For most of the instruments, phosphor bronze spiral springs are provided. Flat spiral springs are used in almost all indicating instruments.

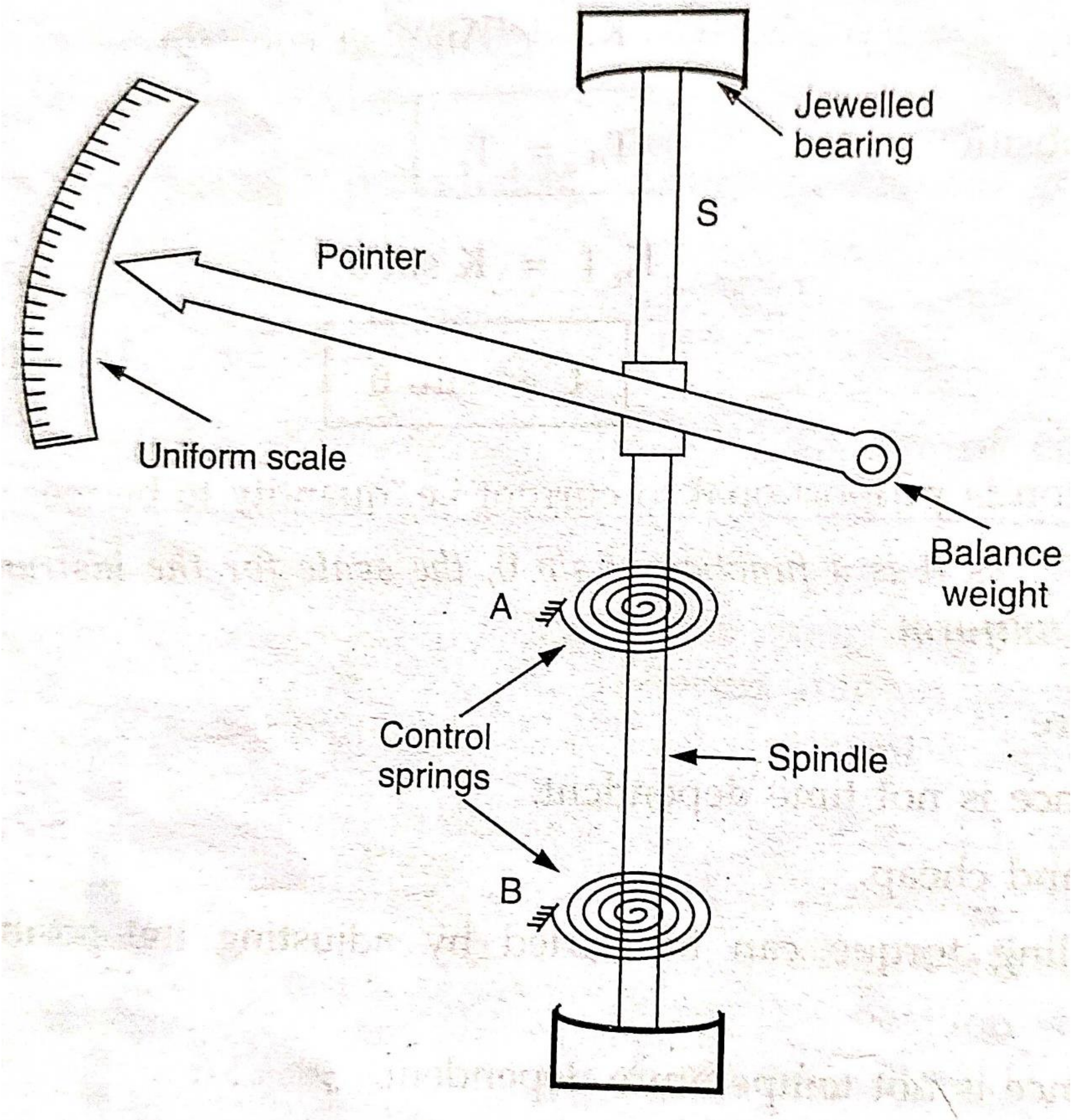


Fig. 1.3 Spring control

- The inner end of the spring is attached to the spindle while the outer end is attached to a lever or arm which is actuated by a set of screw mounted at the front of the instrument. So zero setting can be easily done.
- The controlling torque provided by the instrument is directly proportional to the angular deflection of the pointer.
- So the controlling torque produced by spiral spring is given by,

The controlling torque produced by spiral spring is given by,

$$\boxed{T_c = \frac{E\,b\,t^3}{12\,L}\theta = K_s\theta}$$

where E = Young's modulus of spring material in N/m^2

t = thickness in metres

b = depth in metres

L = length in metres

K_s = spring constant = $\frac{Ebt^3}{12L}$

$$\therefore \quad \boxed{T_c \propto \theta}$$

Now deflecting torque is proportional to current.

$$\therefore \quad \boxed{T_d \propto I}$$

At equilibrium, $T_d = T_c$

$$\therefore \quad \boxed{I \propto \theta}$$

- ✓ Thus the deflection is proportional to the current. Hence the sale of the Instrument using spring control is uniform.
- ✓ When the current is removed, due to spring force the pointer comes back to initial position.
- ✓ The spring control is very popular and is used in almost all indication instruments.

Comparison of controlling systems

Sr. No.	Gravity Control	Spring Control
1.	Adjustable small weight is used which produces the controlling torque.	Two hair springs are used which exert controlling torque.
2.	Controlling torque can be varied.	Controlling torque is fixed.
3.	The performance is not temperature dependent.	The performance is temperature dependent.
4.	The scale is nonuniform.	The scale is uniform.
5.	The controlling torque is proportional to sin θ.	The controlling torque is proportional to θ.
6.	The readings can not be taken accurately.	The readings can be taken very accurately.
7.	The system must be used in vertical position only.	The system need not be necessarily in vertical position.
8.	Proper levelling is required as gravity control.	The levelling is not required.
9.	Simple, cheap but delicate.	Simple, rigid but costlier compared to gravity control.
10.	Rarely used for indicating and portable instruments.	Very popularly used in most of the instruments.

Example 4.1. *The length, width and thickness of the control spring of an instrument are 390 mm, 0.52 mm and 0.075 mm. The Young's modulus of the spring material is 110 GN/m^2. Determine the torque exerted by the spring when it is turned through 80°.*

Solution. *Given* : l = 390 mm; b = 0.52 mm; t = 0.075 mm; T_c = 110 GN/m^2;

$$\theta = 80° = 80 \times \frac{\pi}{180} \text{ rad. or } 1.396 \text{ rad.}$$

We know that, controlling torque,

$$T_c = \frac{Ebt^3}{12l} \cdot \theta \qquad \text{[Eqn. (4.1)]}$$

$$= \frac{(110 \times 10^9) \times (0.52 \times 10^{-3}) \times (0.075 \times 10^{-3})^3}{12 \times (390 \times 10^{-3})} \times 1.396$$

$$= \mathbf{5.156 \times 10^{-6} \text{ Nm}} \quad \text{(Ans.)}$$

Example 4.2. *The toque of an ammeter varies as the square of the current through it. If a current of 5A produces a deflection of 90°, what deflection will occur for a current of 3A when the instrument is :*

(*i*) *Spring-controlled;* (*ii*) *Gravity controlled.* **(Jadavpur University)**

Solution. Since deflecting torque varies as (current)^2, we have $T_d \propto I^2$

For spring control : $T_c \propto \theta$ $\therefore$ $\theta \propto I^2$

For gravity control : $T_c \propto \sin\theta$ $\therefore$ $\sin\theta \propto I^2$

(*i*) *For spring control :* $90° \propto 5^2$ and $\theta° \propto 3^2$

$$\theta = 90° \times \frac{3^2}{5^2} = \mathbf{32.4°} \qquad \textbf{(Ans.)}$$

(*ii*) *For gravity control :* $\sin 90° \propto 5^2$, and $\sin\theta° \propto 3^2$

$\therefore$
$$\frac{\sin\theta}{\sin 90°} = \frac{3^2}{5^2} = \frac{9}{25}$$

or,
$$\sin\theta = \frac{9}{25}$$

$\therefore$
$$\theta = \sin^{-1}(9/25) = \mathbf{21.2°} \qquad \textbf{(Ans.)}$$

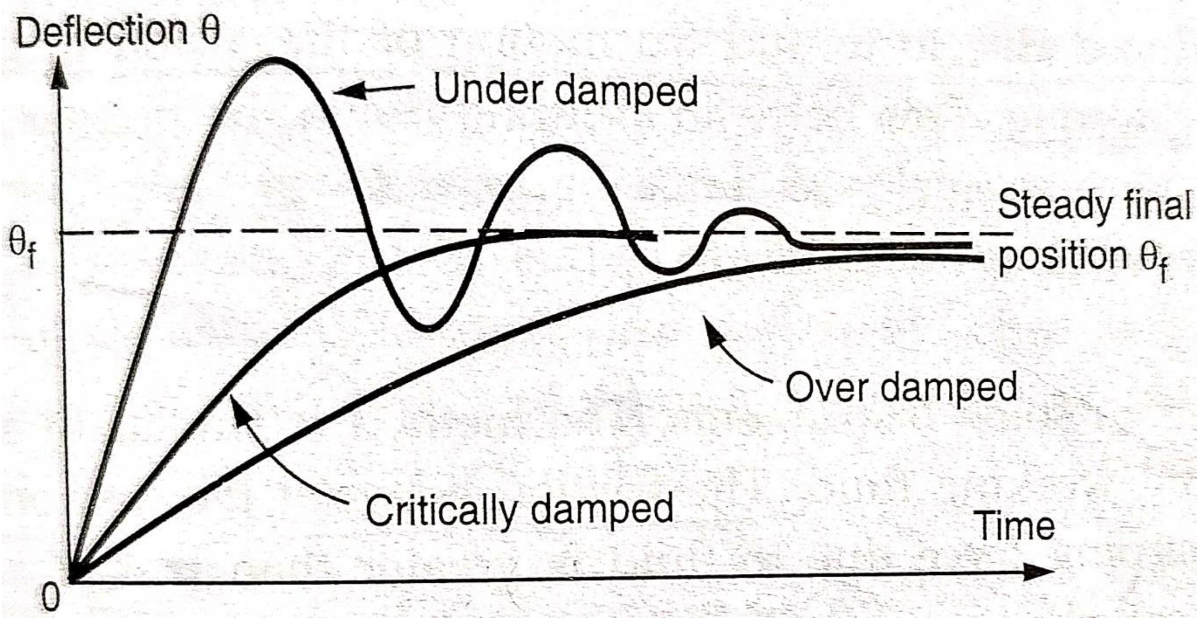
➢ Damping System

- Damping system producing damping torque/force.
- A damping force is required to act in a direction opposite to the movement of the moving system.
- This brings the moving system to rest at the deflected position reasonably quickly without any oscillation or very small oscillation.

  This is provided by

  1. Air friction damping
  2. Fluid friction damping
  3. Eddy current damping

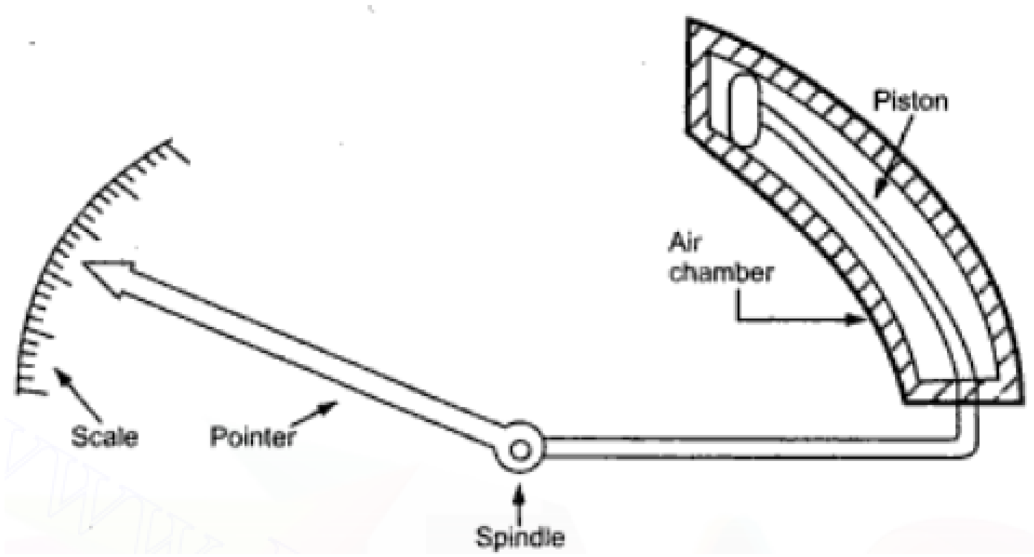
- Depending upon the degree of damping introduced in the moving system, the instrument may have any one of the following conditions as depicted in Fig.



1.Under damped condition: The response is oscillatory
2.Over damped condition: The response is sluggish and it rises very slowly from its zero position to final position.
3.Critically damped condition: When the response settles quickly without any oscillation, this system is said to be critically damped. In practice, the best response is slightly obtained when the damping is below the critical value the instrument is slightly under damped.

1. Air Friction Damping:

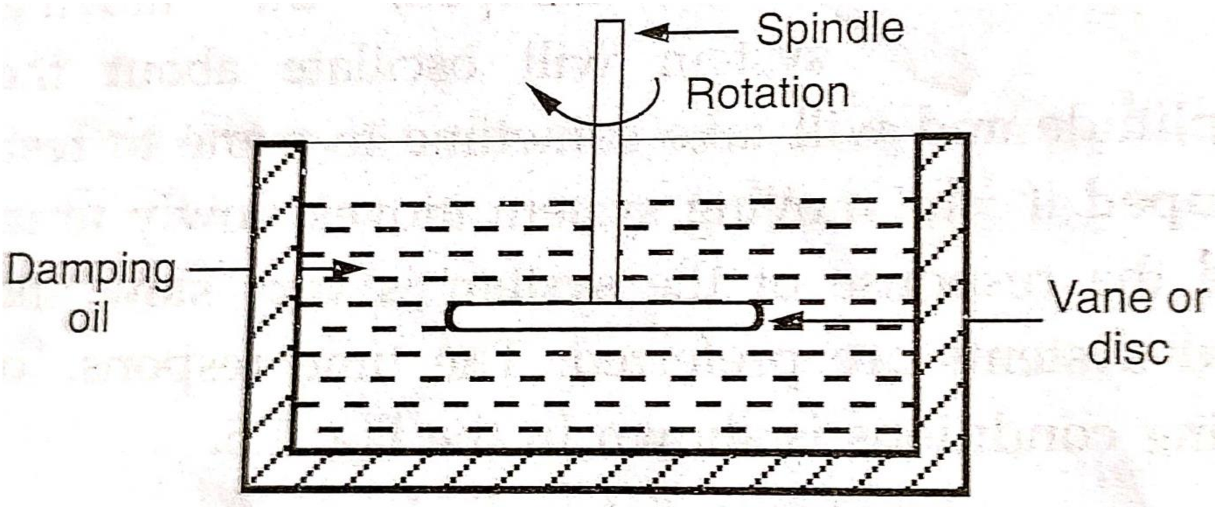
- The arrangement of Air Friction Damping is shown in Fig. consists of a light aluminium piston which is attached to the moving system.
- This piston moves in a fixed chamber which is closed at one end. Either circular or rectangular chamber may be used.
- The clearance (or gap) between the piston and chamber walls should be uniform through out and as small as possible.
- When the piston moves rapidly into the chamber the air in the closed space is compressed and the pressure of air thus developed opposes the motion of the piston and thereby the whole moving system.
- If the piston is moving out of the chamber, rapidly, the pressure in the closed space falls and the pressure on the open side of the piston is greater than that on the opposite side. Motion is thus again opposed.



- With this damping system care must be taken to ensure that the arm carrying the piston should not through the sides of the chamber during its movement.
- The friction which otherwise would occur may introduce a serious error in the deflection.
- The air friction damping is very simple and cheap. But care must be taken to ensure that the piston is not bent or twisted. This method is used in moving iron and hot wire instruments.

2. Fluid Friction Damping:

- ✓ This form is damping is similar to air friction damping. The action is the same as in the air friction damping. Mineral oil is used in place of air and as the viscosity of oil is greater, the damping force is also much greater. The vane attached to the spindle is arranged to move in the damping oil.
- It is rarely used in commercial type instruments.
- The oil used must fulfill the following requirements.
- It should not evaporate quickly
- It should not have any corrosive effect on metals.
- Its viscosity should not change appreciably with temperature
- It should be good insulator.



- The arrangements of fluid damping are shown in Fig.
- As shown In Figure a disc attached to the moving system is immersed in the fluid (damping oil).
- When the moving system moves the disc moves in oil and a frictional drag is produced.
- For minimizing the surface tension affect, the suspension stem of the disc should be cylindrical and of small diameter.

❖ Advantages of Fluid Friction Damping:

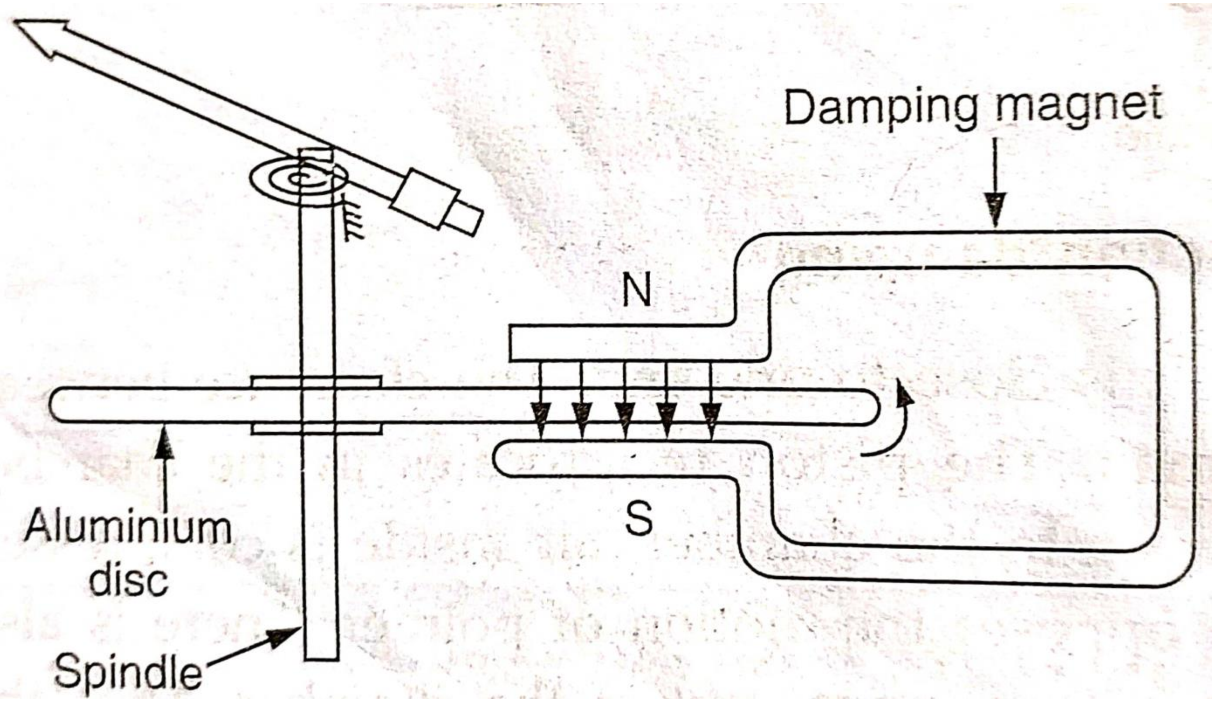
1. The oil used for damping can also be used for insulation purpose in some forms of instruments which are submerged in oil.
2. The clearance between the vanes and oil chamber is not as critical as with the air friction clamping system.
3. This method is suitable for use with instruments such as electrostatic type where the movement is suspended rather than pivoted.
4. Due to the up thrust of oil, the loads on bearings or suspension system is reduced thereby the reducing the frictional errors.

❖ Disadvantages of Fluid Friction Damping:

1. The instruments with this type of damping must be kept always in a vertical position.
2. It is difficult to keep the instrument clean due to leakage of oil.
3. It is not suitable for portable instruments.

3. Eddy Current Damping:

- This is the most effective way of providing damping. It is based on the Faraday's law and Lenz's law.
- When a conductor moves in a magnetic field cutting the flux, e mf. gets induced in it. And direction of this emf is so as to oppose the cause producing it.
- In this method, an aluminium disc is connected to the spindle. The arrangement of disc is such that when it rotates, it cuts the magnetic flux lines of a permanent magnet. The arrangement is shown in the figure. When the pointer oscillates, aluminium disc rotates under the influence of magnetic field of damping magnet.



- So disc cuts the flux which causes an induced emf in the disc.
- The disc is a closed path hence induced e.m.f. circulates current through the disc called eddy current.
- The direction of such eddy current is so as oppose the cause producing it.
- The cause is relative motion between disc and field.
- Thus it produces an opposing torque so as to reduce the oscillations of pointer.
- This brings pointer to rest quickly. This is most effective and efficient method of damping.

D'Arsonval Galvanometer

D'Arsonval Galvanometer

- A galvanometer is an instrument used for detecting presence of small currents or voltages in a closed circuit or for measuring their magnitudes or to indicate zero current in applications like bridge circuits. Thus galvanometer has to be very much sensitive.
- The great majority of the galvanometers in use now are the permanent-magnet moving coil type, normally referred as "D'Arsonval type". This is D.C. galvanometer.

➢ **It consists of the following parts,**

1. Moving coil :

- The moving coil is rectangular or circular in cross-section, carrying number of turns of fine wire.
- It carries the current proportional to the quantity to be measured.
- It is suspended in the air gap between the poles of a permanent magnet and iron core.
- It is free to turn about its vertical axis. The pole faces are of particular shape such that the magnetic field is radial.

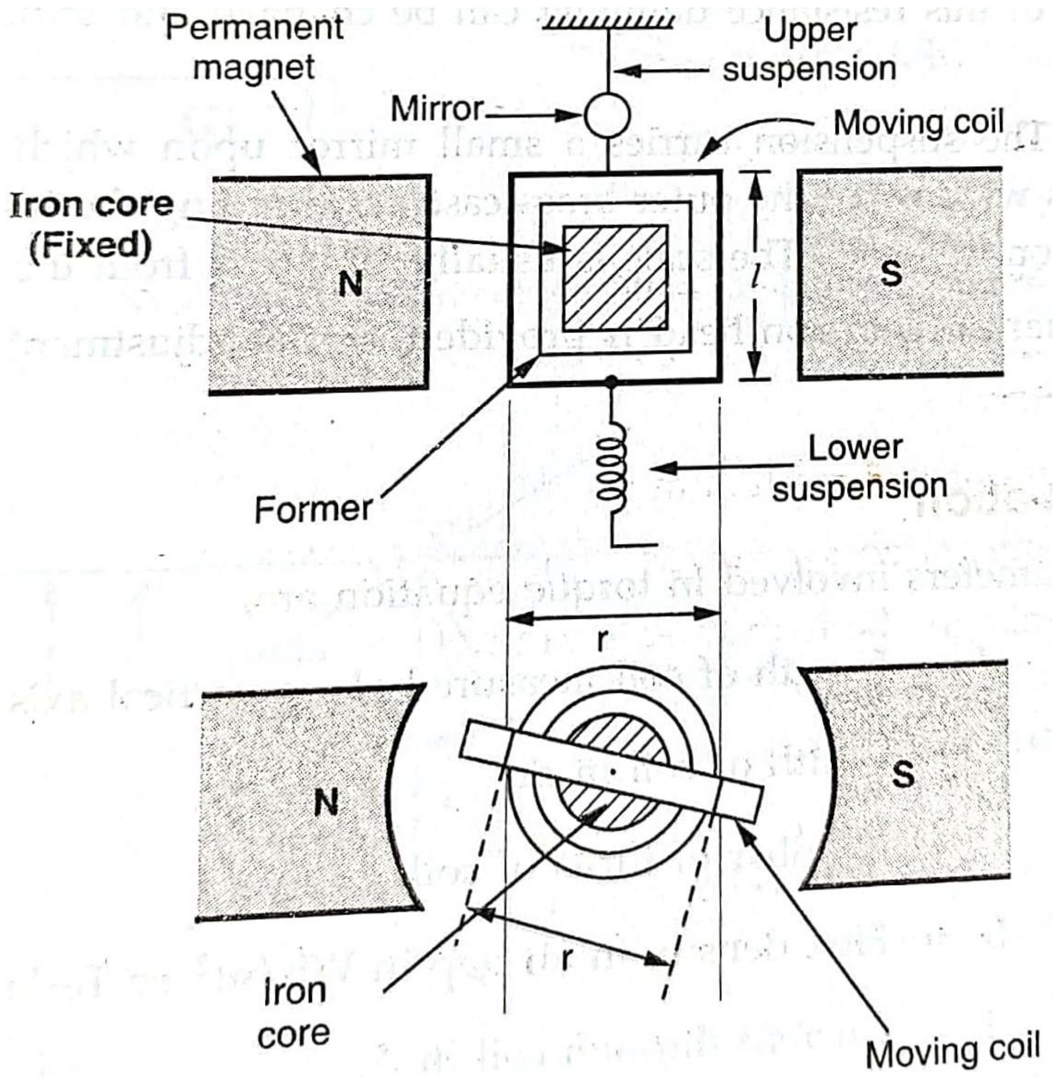


Fig.The construction of D'Arsonval galvanometer

2. Iron core :

- It is spherical if coil is circular and cylindrical if coil is rectangular. It is basically used to provide low reluctance path to the magnetic flux and to produce strong magnetic field This ensures higher deflecting torque and better sensitivity of the galvanometer. The air gap is about 1/16 inches Le, about 15 mm If small moment of inertia is necessary, the iron core can be omitted but it decreases the sensitivity.

3. Suspension :

- The suspension is a single fine strip of phosphor-bronze and serves as one lead of the coil The other lead takes the form of a loosely coiled spiral of fine wire leading downwards from the bottom of the coil. This is lower suspension. This type of galvanometer requires a perfect leveling so that the suspension coils remain straight and in central position without rubbing the poles or iron core.
- In galvanometers which do not require the perfect leveling, taut suspensions with straight flat strips are used, which are kept under tension from both sides.

4. Damping :

- The damping is eddy current damping. The eddy currents developed in the metal former on which coil is mounted, are responsible to produce damping torque.
- For effective damping a low resistance is connected across the galvanometer terminals. By adjusting the value of this resistance damping can be changed and critical damping can be achieved

5. Indication :

- The suspension carries a small mirror upon which a beam of light is cast through a glass window in the outer brass case surrounding the instrument. The beam of light is reflected on the scale. The scale is usually 1m away from the mirror.

6. Zero adjustment :

- A torsion head is provided for the adjustment of the coil position and zero setting.

➢ Working/Operation :

- When current flows through the coil, a deflecting force proportional to the flux density, the coil current and dimensions of the coil makes the coil to rotate on its vertical axis.
- The "deflecting force is opposed by the restraining force" of the suspension filament so that coil does not continue to rotate as in a motor but turns until the deflecting torque is balanced by the restraining force.
- Since the deflecting torque is directly proportional to the coil current, therefore, the amount of deflection of the coil indicates the magnitude of current flowing in the coil.
- The amount of deflection and hence the magnitude of curent flowing through the coil may be indicated by a pointer attached to the moving element and moving over a calibrated scale. For the adjustment of coil and also for zero adjustment, a "torsion head" is provided.

✓ Galvanometers find their principal applications in bridge and potentiometer measurements.

✓ Therefore, a galvanometer in addition to being sensitive, should have :

  - A stable zero;
  - A short periodic time;
  - Nearly critical damping.

- **Other types of galvanometers are :**

- Ballistic galvanometer:
  - It is used to measure the quantity of electricity (charge) passed through it.
  - Uniform scale may be attained.
  - These are the features that have caused it to be adopted, in a modified form, for D.C. ammeters and voltmeters.

- Vibration galvanometer:
  - These galvanometers are most widely used "tuned detectors".
    These galvanometers are of the following types :
    (i) Moving magnet type.
    (ii) Moving coil type.
  - The moving coil type galvanometer is more generally used, the moving magnet having the disadvantage of being seriously affected by magnetic fields of the resonance frequency, unless adequately screened.

Torque equation

Let

N = Number of turns in coil,
B =Flux density, Wb/m^2,
I =Current through the coil,
L= Length of coil side (vertical), m
b= Width of coil, m,
Ө=Deflection of pointer, rad., and
C = Spring (or controlling or restoring) constant, Nm/rad.

- Force on each side of coil = BILnewtons (here Field is radial)
- Since the coil has N turns and each turn has two sides, therefore,
  Total force=2NBILnewtons
  This force acts at a radius b/2
  ∴ Deflecting torque, T_d= Force x radius

or, $$T_d = 2NBIL \times \frac{b}{2} = NBILb = NBIA \text{ Nm}$$

Where, A=Area of coil(=L×b), m^2

N,B,A are constants for a galvanometer

∴ Deflecting torque, T_d= GI

G is called the displacement constant (= NBA) of galvanometer.

Controlling torqque exerted by the suspension at deflection Ө,

$$T_c = C\Theta$$

For *steady deflection*,
$T_d = T_C$

GI=C Ө

$\Theta = \frac{GI}{C} = \frac{NBIA}{C}$

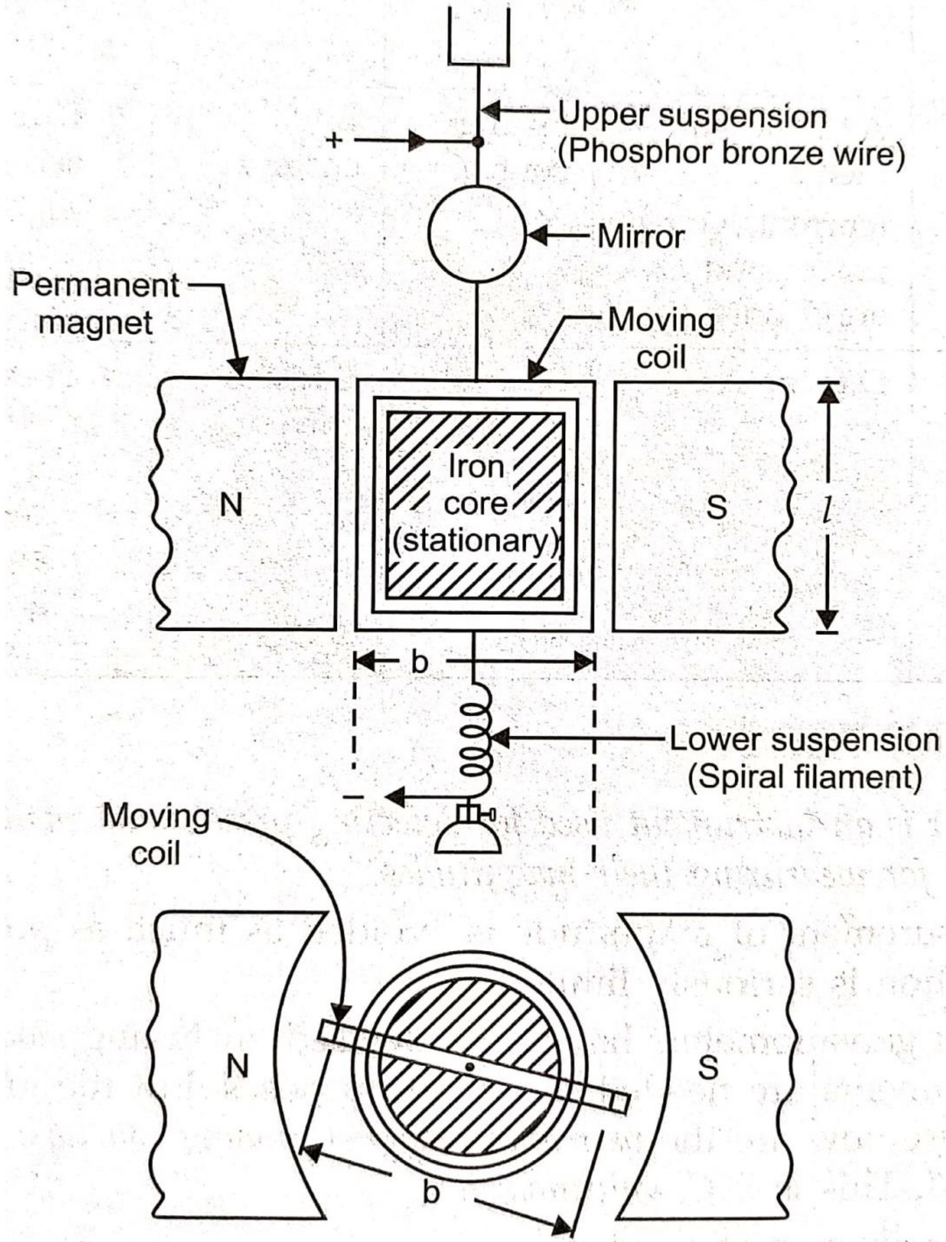


Fig.D'Arsonval galvanometer

Galvanometer Constants:

The galvanometer has four constants known as "Intrinsic constants" These are

1. Displacement constant
2 Inertia constant.
3. Damping constant
4. Control or restoration constant.

1.Displacement constant:

- The deflection torque is given by T_d = GI where G is the displacement constant of the galvanometer and is equal to NBLb. The units of G are Nm/A.

2. Inertia constant:

- Owing to inertia of moving system, a retarding torque is produced.
- This torque (T_j) is dependent upon the moment of inertia of moving system and the angular acceleration.

$$T_j = J\frac{d^2\theta}{dt^2}$$

where, J = Moment of inertia of moving system about the axis of rotation, kg-m, and
θ = Deflection at any time t, rad.

3. Damping constant:

- Damping is provided by friction due to motion of the coil air and also by induced electrical effects if a closed circuit is provided.
- Damping torque (T_d) is usually assumed to be proportional to the angular velocity of the system and may be expressed as

$$T_D = D\frac{d\theta}{dt}$$

Where D = Damping constant

4. Control or restoration constant:

A controlling torque (T_c) is produced due to elasticity of the system which tries to restore the moving system back to its original position, and may be expressed as

$$T_c = C\Theta$$

where, C = Control constant, Nm/rad.

- **Dynamic response**

The deflecting torque T_d(=GI) causes the motion, while the inertia torque $\left(T_j = J \cdot \frac{d^2\Theta}{dt^2}\right)$,damping torque $\left(T_D = D\frac{d\Theta}{dt}\right)$ and controlling torque (T_c = CΘ) opposite the motion.

The "equation of motion" is given as:

$$T_j + T_D + T_c = T_d$$

or,

$J \cdot \frac{d^2\Theta}{dt^2}$	+	$D\frac{d\Theta}{dt}$	+	$C\Theta$	=	GI
Inertia torque		Damping torque		Controlling torque		Deflecting torque

✓ Note:

- Several times the deflection is measured on a scale kept at a distance r metres from a mirror. A beam of light is reflected on the scale by a small mirror which is mounted on the moving system (Fig. 4.11).
- The distance r is usually kept as 1 metre.
- If the moving system rotates through angle Θ, the light beam is turned through angle 2Θ.
- Then, Deflection oa i.e., d = (r) x (2Θ) metres.

Example 4.4. *Determine the angle through which coil turns when a deflection of 48 mm is observed on a scale of a galvanometer placed at a distance of 650 mm from the mirror.*

Solution. Refer to Fig. 4.11

Given: d = 48 mm or 0.048 m; r = 650 mm or 0.65 m.

The angle turned, θ :

The angle through which the mirror or coil turns may be found from the following expression :

$$d = r \times 2\theta$$

or,
$$\theta = \frac{d}{2r} = \frac{0.048}{2 \times 0.65} = 0.0369 \text{ rad. or } \mathbf{2.11^\circ} \quad \textbf{(Ans.)}$$

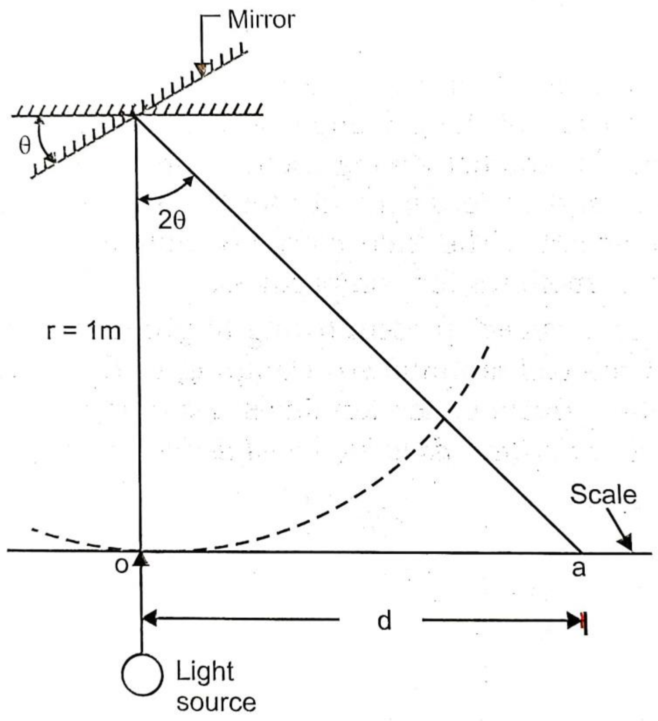


Fig. 4.11. Measurement of deflection using mirror and scale.

Example 4.5. *Determine the displacement constant of a suspended coil in a galvanometer of a moving coil vibrating type having 30 turns, 4.5 mm × 2.5 mm mean area and situated in a magnetic field of 0.8 tesla.*

Solution. *Given:* $N = 30$; $A = 4.5 \text{ mm} \times 2.5 \text{ mm} = 11.25 \text{ mm}^2$ or $11.25 \times 10^{-6} \text{ m}^2$; $B = 0.8 \text{ Wb/m}^2$

Displacement constant, G :

Displacement constant is given by :

$$G = NBA$$
$$= 30 \times 0.8 \times 11.25 \times 10^{-6} = \mathbf{270 \times 10^{-6} \text{ Nm/A}} \quad \text{(Ans.)}$$

Example 4.6. *The suspended coil of a galvanometer has a 5 mm × 2.5 mm mean area and is situated in a magnetic field of 1.1 tesla. The moment of inertia of moving parts is 0.25×10^{-6} kg-m and the control string constant is 35×10^{-6} Nm/radian. If a current of 12 mA produces a deflection of 110°, calculate the number of turns of the suspended coil.*

Solution. *Given* : $A = 5 \text{ mm} \times 2.5 \text{ mm} = 5 \times 2.5 \times 10^{-6} \text{ m}^2$ or $12.5 \times 10^{-6} \text{ m}^2$; $B = 1.1 \text{ Wb/m}^2$; $J = 0.25 \times 10^{-6}$ kg-m; $C = 35 \times 10^{-6}$ Nm/rad.; $\theta = 110°$ or 1.92 radians.

Number of turns of the suspended coil, N :

We know that for steady deflection,

$$GI = C\theta \qquad \text{[Eqn. (4.11)]}$$

But,
$$G = NBA$$

$\therefore$
$$NBAI = C\theta$$

or,
$$N = \frac{C\theta}{BAI} = \frac{35 \times 10^{-6} \times 1.92}{1.1 \times 12.5 \times 10^{-6} \times 0.012} = \mathbf{407} \quad \text{(Ans.)}$$

Example 4.19. *The Ayrton universal shunt has a total resistance of 9000 Ω and galvanometer has a resistance 3000 Ω. Determine the multiplying power of shunt for 1000 Ω, 3000 Ω, 6000 Ω tapping.*

Solution. Total shunt resistance, $R_{sh} = 9000\ \Omega$

Galvanometer resistance, $R_g = 3000\ \Omega$

$$\text{Multiplying power of shunt, } N = \frac{I}{I_g} = \frac{R_{sh} + R_g}{R_1}$$

$$\therefore \quad (i) \qquad N = \frac{R_{sh} + R_g}{R_1} = \frac{9000 + 3000}{1000} = \mathbf{12} \quad \textbf{(Ans.)}$$

...when $R_1 = 1000\ \Omega$

$$(ii) \qquad N = \frac{9000 + 3000}{3000} = \mathbf{4} \quad \textbf{(Ans.)}$$

...when $R_1 = 3000\ \Omega$

$$(iii) \qquad N = \frac{9000 + 3000}{6000} = \mathbf{2} \quad \textbf{(Ans.)}$$

...when $R_1 = 6000\ \Omega$

ANALOG AMMETERS, VOLTMETERS

- Analogue ammeters and voltmeters are classified together, since there is no basic difference in their operating principles.
- The action of all ammeters and voltmeters, except those of the electrostatic variety, depends upon a deflecting torque principle by an electric current.
- In an ammeter, this torque is produced by the current to be measured, or by a definite fraction of it.
- In a voltmeter, it is produced by a current that is proportional to the voltage to be measured.
- Hence, both voltmeters and ammeters are essential current measuring devices.
- The "ammeter" has a low resistance so that when it is connected in series with any circuit, it does not change the current. Consequently, there is a small voltage drop and small power is absorbed.
- The "voltmeter" has a high resistance and it is so designed that when connected in parallel to the circuit for measuring voltages it does not take appreciable current, consequently power consumed is small.
- An ammeter of low range can be used as a voltmeter by connecting an external resistance in series with it.

Permanent Magnet Moving Coil (PMMC) Instruments

➤ Permanent Magnet Moving Coil (PMMC) Instruments

- The permanent magnet moving coil instruments are most accurate type for d.c. measurements.
- The action of these instruments is based on the motoring principle.
- When a current carrying coil is placed in the magnetic field produced by permanent magnet, the coil experiences a force and moves.
- As the coil is moving and the magnet is permanent, the instrument is called permanent magnet moving coil instrument.
- This basic principle is called D'Arsonval principle. The amount of force experienced by the coil is proportional to the current passing through the coil.
  The PMMC instrument is shown in figure.

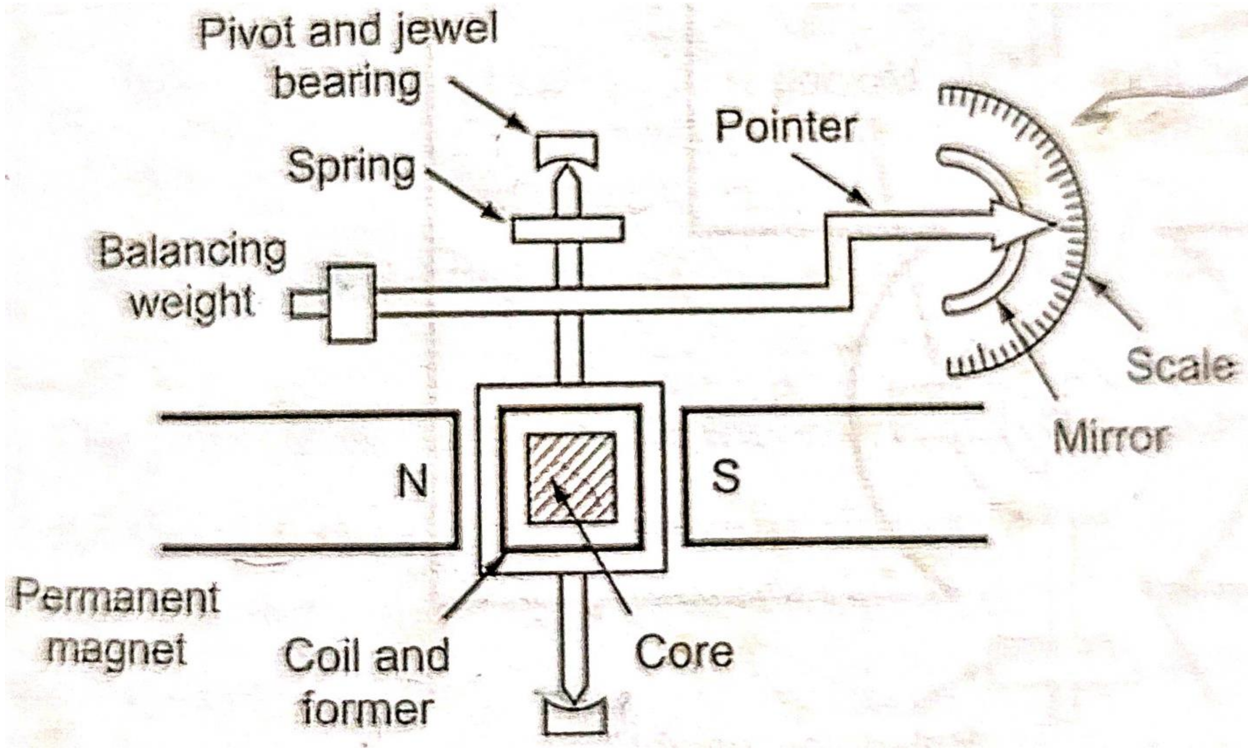


Fig. Construction of PMMC instrument

➢ Construction & working of PMMC instrument

- The moving coil either rectangular or circular in shape.
- It has number of turns of fine wire.
- The coil is suspended so that it is free to turn about its vertical axis.
- The coil is placed in uniform, horizontal and radial magnetic field of a permanent magnet in the shape of a horse-shoe.
- The iron core is spherical if coil is circular and is cylindrical if the coil is rectangular.
- Due to iron core, the deflecting torque increases, increasing the sensitivity of the instrument.
- The controlling torque is provided by two phosphor bronze hair springs.
- The damping torque is provided by eddy current damping. It is obtained by movement of the aluminium former, moving in the magnetic field of the permanent magnet.
- The pointer is carried by the spindle and it moves over a graduated scale.
- The pointer has light weight so that it can deflect rapidly.
- The mirror is placed below the pointer to get the accurate reading by removing the parallax.
- The weight of the instrument is normally counter balanced by the weights situated diametrically opposite and rigidly connected to it.
- The scale markings of the basic D.C. PMMC instruments are usually linearly spaced as the deflecting torque and hence the pointer deflection are directly proportional to the current passing through the coil.

- The top view of PMMC instrument is shown in the Figure.
- In a practical PMMC instrument, a Y shaped member is attached to the fixed end of the front control spring.
- An eccentric pin through the instrument case engages the Y shaped member so that the zero position of the pointer can be adjusted from outside.

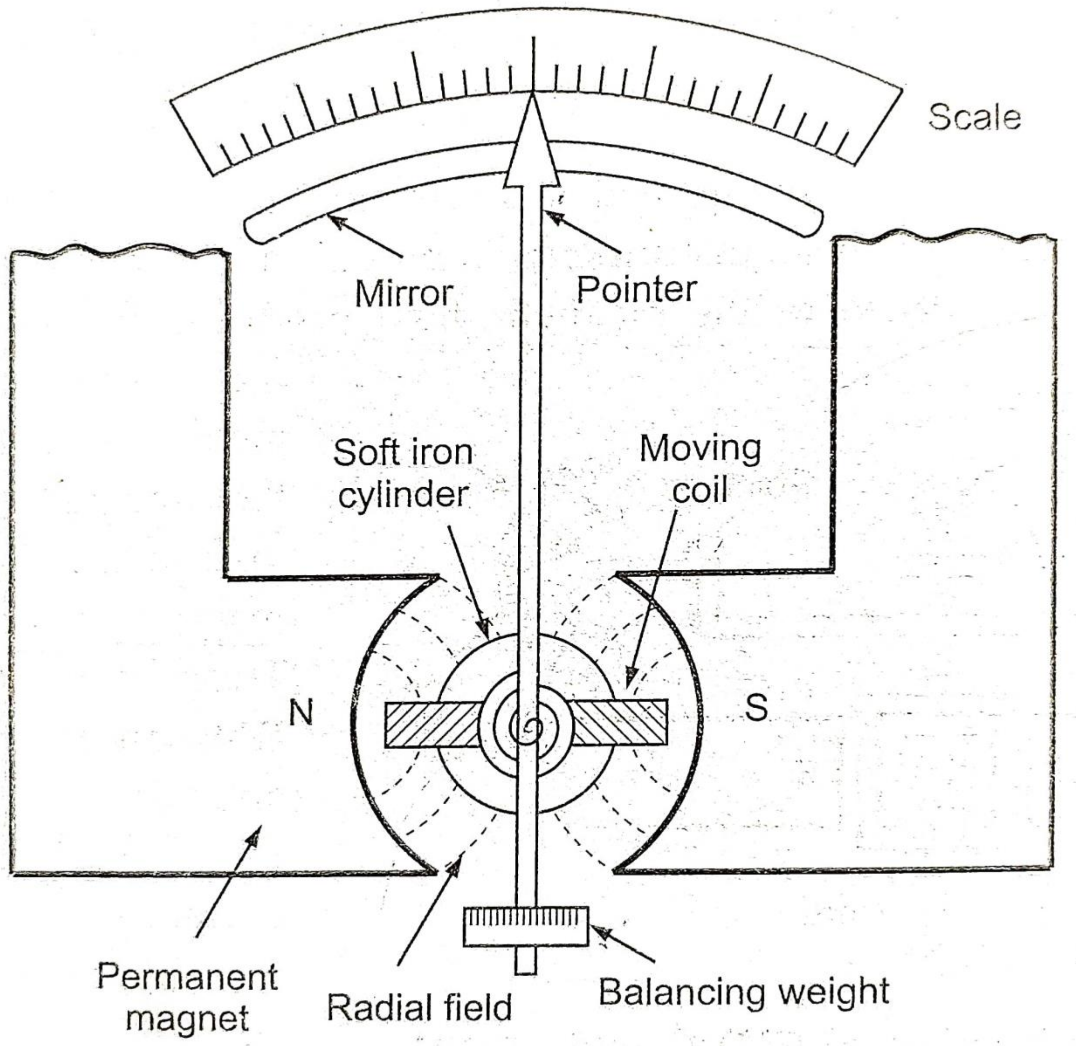


Fig.The top view of PMMC instrument

➢ Torque equation of PMMC instrument

The equation for the developed torque can be electromagnetic torque.
The deflecting torque is given by,

$$\boxed{T_d = NBAI}$$

where T_d = deflecting torque in N-m

B = flux density in air gap, Wb/m^2

N = number of turns of the coil

A = effective coil area m^2

I = Current in the moving coil, amperes

$\therefore \quad T_d = GI$

where $G = NBA$ = constant

The controlling torque is provided by the springs and is proportional to the angular deflection of the pointer.

$$T_c = K\theta$$

where T_c = controlling torque

K = spring constant, Nm/rad or Nm/deg

θ = angular deflection

For the final steady state position,

$$T_d = T_c$$

$$\therefore \quad GI = K\theta$$

$$\therefore \quad \theta = \left(\frac{G}{K}\right) I$$

or

$$\boxed{I = \left(\frac{K}{G}\right)\theta}$$

Thus the deflection is directly proportional to the current passing through the coil.

- The pointer deflection can therefore be used to measure current.
- As the direction of the current through the coil changes, the direction of the deflection of the pointer also changes. Hence such instruments are well suited for the D.C. measurements.
- In the micro ammeters and milli ammeters upto about 20 mA, the entire current to be measured is passed through the coil.
- The springs carry current to the coil. Thus the current carrying capacity of the springs, limits the current which can be safely carried.
- For higher currents, the moving coil is shunted by sufficient resistance. While the yolt meters having high ranges use a moving coil together with sufficient series resistance, to limit the instrument current.
- Most d.c. voltmeters are designed to produce full scale deflection with a current of 20, 10, 5 or 1 mA.
- The power requirement of PMMC instrument is very small, typically of the order of 25 μW to 200 μW.
- Accuracy is generally of the order of 2 to 5 % of the full scale reading.

Advantages of PMMC instruments are,

1) It has uniform scale.
2) With a powerful magnet, its torque to weight ratio is very high. So operating current is small.
3) The sensitivity is high.
4) The eddy currents induced in the metallic former over which coil is wound, provide effective damping,
5) It consumes low power, of the order of 25 μW to 200 μW.
6) It has high accuracy.
7) Instrument is free from hysteresis error.
8) Extension of instrument range is possible.
9) Not affected by external magnetic fields called stray magnetic fields.

Disadvantages:

1) Suitable for DC measurements only,
2) Ageing of permanent magnet and the control springs introduces the errors.
3) The cost is high due to delicate construction and accurate machining.
4) The friction due to jewel-pivot suspension.

➢ Errors in PMMC Instrument

- ➢ The basic sources of errors in PMMC instruments are
    1) friction,
    2) temperature and
    3) aging of various parts.
- ❑ To reduce the frictional errors ratio of torque to weight is made very high.

- ❑ The most serious errors are produced by the heat generated or by changes in the temperature. This changes the resistance of the working coil, causing large errors. In case of voltmeters, a large series resistance of very low temperature coefficient is used. This reduces the temperature errors.

- ❑ The aging of permanent magnet and control springs also cause errors. The weakening of magnet and springs cause opposite errors. The weakening of magnet cause less deflection while weakening of the control springs cause large deflection, for a particular value of current. The proper use of material and pre ageing during manufacturing can reduce the errors due to weakening of the control springs.

Instrument used to eliminate friction

Taut Band Instrument

- The friction due to jewel-pivot suspension can be eliminated by using taut band movement.
- The working principle of taut band instrument is same based on D'Arsonval's principle.
- The main difference is the method of mounting the coil.
- In the taut band instrument the movable coil is suspended by means of two torsion ribbons.
- The ribbons are placed under sufficient tension to eliminate any sag.
  This tension is provided by the tension string.
- The coil is mounted in cradle and surrounded by ring bar magnet.
- The construction is shown in the Figure.

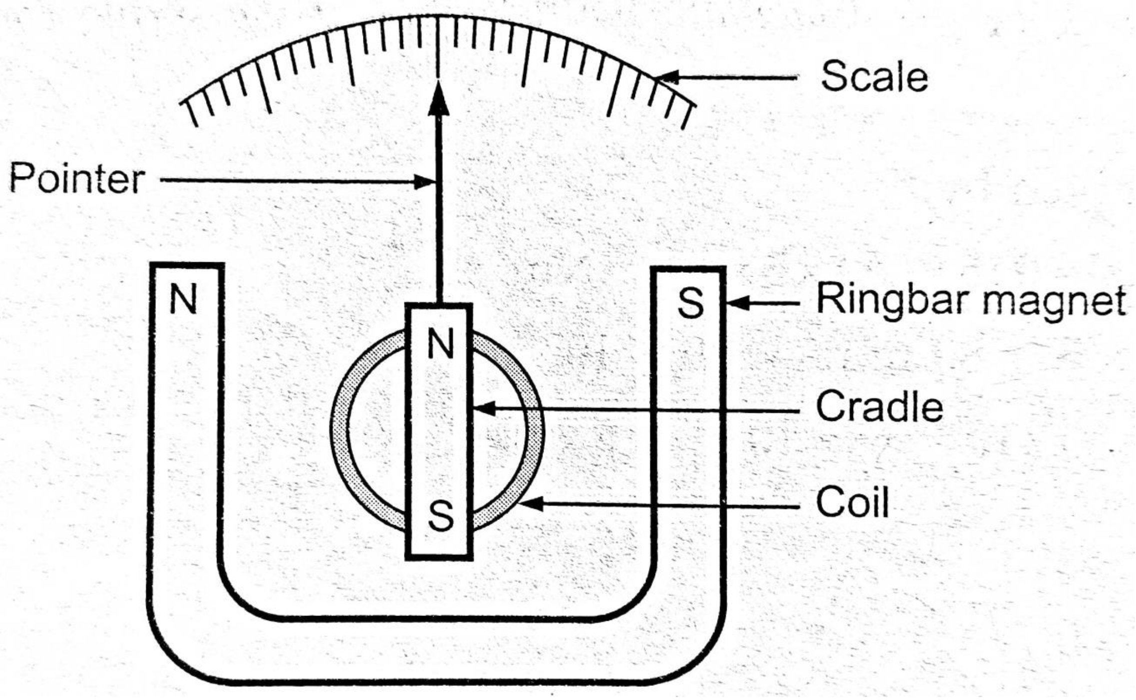


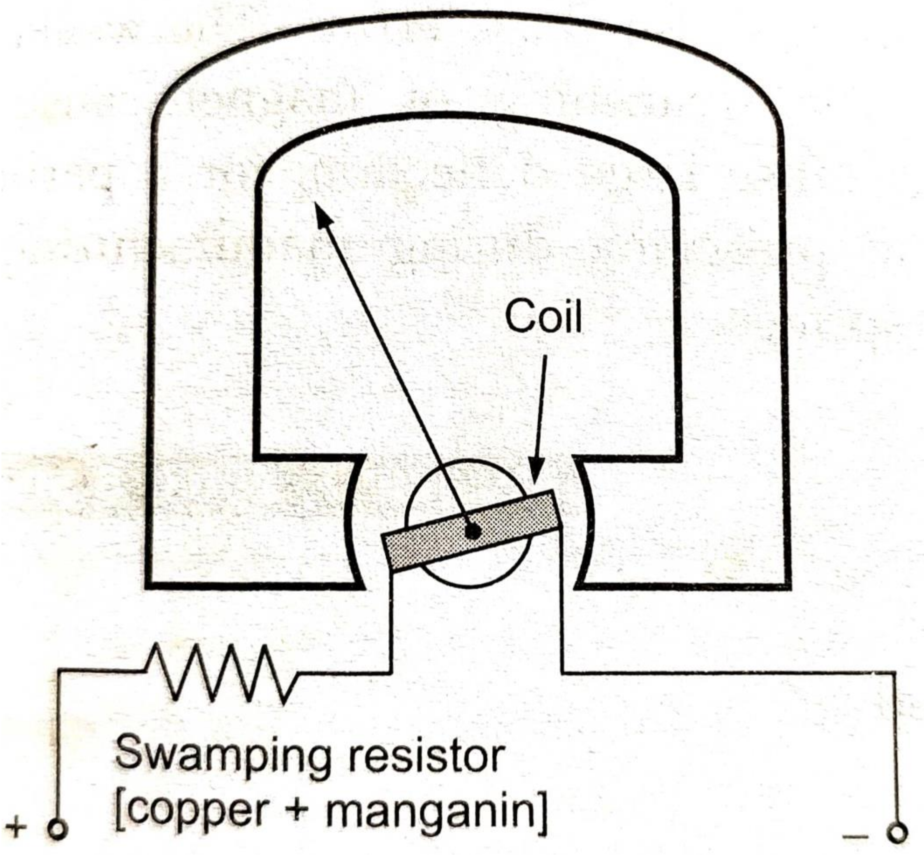
Fig. Taut band instrument

- The taut band instrument can be used in any position while Jewel-pivot instrument should be used vertically.
- The sensitivity of the tout band instruments is higher than jewel-pivot instruments.
- The taut band instruments are relatively insensitive to shocks and temperature and are capable of withstanding overloads.

Instrument used to Temperature Compensation

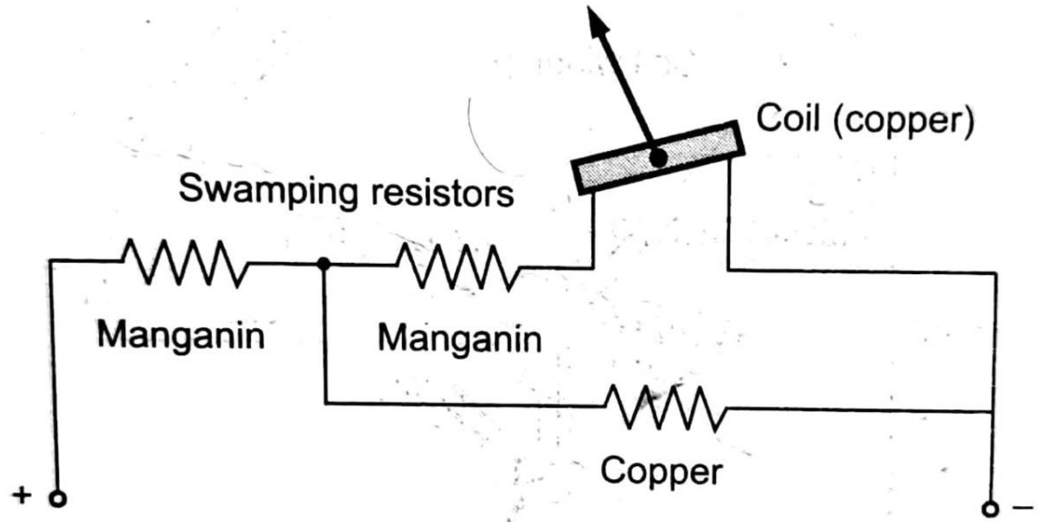
➢ Temperature Compensation

- The basic PMMC instrument is sensitive to the temperature.
- The magnetic field strength and spring tension decrease with increase in temperature.
- The coil resistance increases with increase in the temperature. Thus pointer reads low for a given current.
- The meter tends to read low by approximately 0.2% per °C rise in the temperature.
- Hence the Coil temperature compensation is provided by appropriate use of series and shunt resistances of copper and manganin.
- The simple compensation circuit uses a resistance in series with the movable coil, as shown in the Fig.(a).
- The resistor is called a swamping resistor. It is made up of manganin having practically zero temperature coefficient, combined with copper in the ratio of 20/1 or 30/1.



Fig(a) Simple temperature compensation

- The resultant resistance of coil and the swamping resistor increases slightly as temperature increases, just enough to compensate the change in springs and magnet due to temperature.
- Thus the effect of temperature is compensated.
- More complicated but complete cancellation of temperature effects can be obtained by using the swamping resistors in series and parallel combination as shown in the Fig.(b)
- In this circuit, by correct proportioning of copper and manganin parts, complete cancellation of the temperature effects can be achieved.



Fig(b) Improved temperature compensation

MOVING-IRON INSTRUMENTS

➢ Moving-iron Instruments:

✓ The brief description of different components of a moving-iron instrument is given below

Moving element: a small piece of soft iron in the form of a vane or rod.

Coil: to produce the magnetic field due to current flowing through it and also to magnetize the iron pieces.

Control torque is provided by spring or weight (gravity)

Damping torque is normally pneumatic, the damping device consisting of an air chamber and a moving vane attached to the instrument spindle.

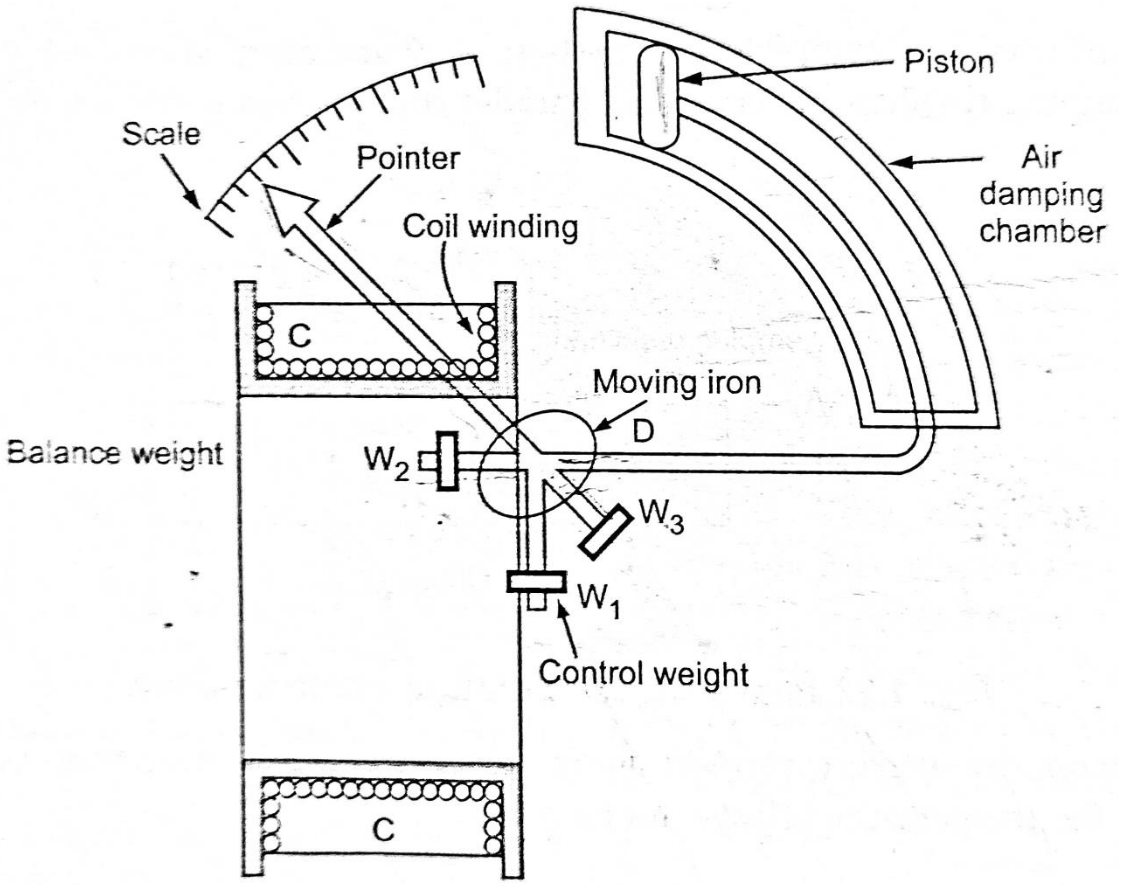
Deflecting torque produces a movement on an aluminum pointer over a graduated scale.

✓ There are two types of moving iron instruments

1. **Attraction type moving iron instrument**
2. **Repulsion type moving iron instrument**

Moving iron attraction type instruments

- The basic working principle of these instruments is very simple that a soft iron piece if brought near the magnet gets attracted by the magnet. The construction of the attraction type instrument is shown in the Fig.(a)
- It consists of a fixed coil C and moving iron piece D. The coil is flat and has a narrow slot like opening.
- The moving iron is a flat disc which is eccentrically mounted on the spindle.
- The spindle is supported between the jewel bearings. The spindle carries a pointer which moves over a graduated scale. The number of turns of the fixed coil are dependent on the range of the instrument. For passing large current through the coil only few turns are required.
- The controlling torque is provided by the springs but gravity control may also be used for vertically mounted panel type instruments.



Fig(a).Attraction type moving iron instrument

- The damping torque is provided by the air friction. A light aluminium piston is attached to the moving system.
- It moves in a fixed chamber. The chamber is closed at one end. It can also be provided with the help of vane attached to the moving system
- The operating magnetic field in moving iron instruments is very weak. Hence eddy current damping is not used since it requires a permanent magnet which would affect on distort the operating field.

Moving Iron Repulsion Type Instrument

- These instruments have two vanes inside the coil, the one is fixed and other is movable.
- When the current flows in the coil, both the vanes are magnetized with like polarities induced on the same side. Hence due to the repulsion of like polarities, there is a force of repulsion between the two vanes causing the movement of the moving vane.
- The repulsion type instruments are the most commonly used instruments.

The two different designs of repulsion type instruments are :

I. Radial vane type
II. Co-axial vane type

- The Figure(a) shows the radial vane repulsion type instrument.
- Out of the other moving iron mechanisms, this is the most sensitive and has most linear scale.
- The two vanes are radial strips of Iron. The fixed vane is attached to the coil. The movable vane is attached to the spindle and suspended in the Induction field of the coil.
- The needle(pointer) of the instrument is attached to this vane.
- Even though the current through the coil is alternating, there is always repulsion between the like poles of the fixed and the movable vane.
- Hence the deflection of the pointer is always in the same direction.
- The deflection is effectively proportional to the actual current and hence the scale is calibrated directly to read amperes or volts.
- The calibration is accurate only for the frequency for which it is designed because the impedance is different for different frequencies.

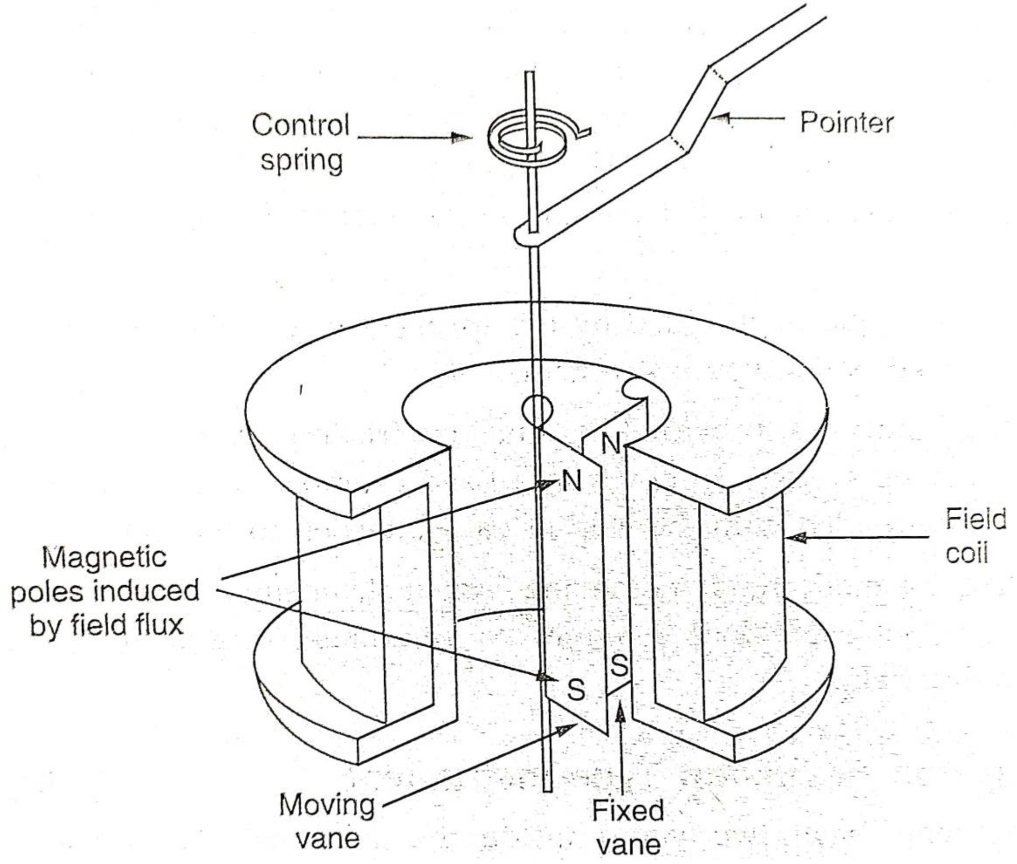


Fig.(a)Radial vane repulsion type instrument

Concentric Vane Repulsion Type Instrument

- As shown in the figure The instrument has two concentric vanes.
- One is attached to the coil frame rigidly while the other can rotate coaxially inside the stationary vane.
- Both the vanes are magnetized to the same polarity due to the current in the coil. Thus the movable vane rotates under the repulsive force.
- As the movable vane is attached to the pivoted shaft, the repulsion results in a rotation of the shaft.
- The pointer deflection is proportional to the current in the coil.
- The concentric vane type instrument is moderately sensitive and the deflection is proportional to the square of the current through coil.
- Thus the instrument is said to have square law response. Thus the scale of the instrument is non-uniform in nature.
- Thus whatever may be the direction of the current in the coil, the deflection in the moving iron instruments is in the same direction.
- Hence moving iron instruments can be used for both a.c. and d.c. (voltage & current)measurements. Due to square law response, the scale of the moving iron instrument is non-uniform.

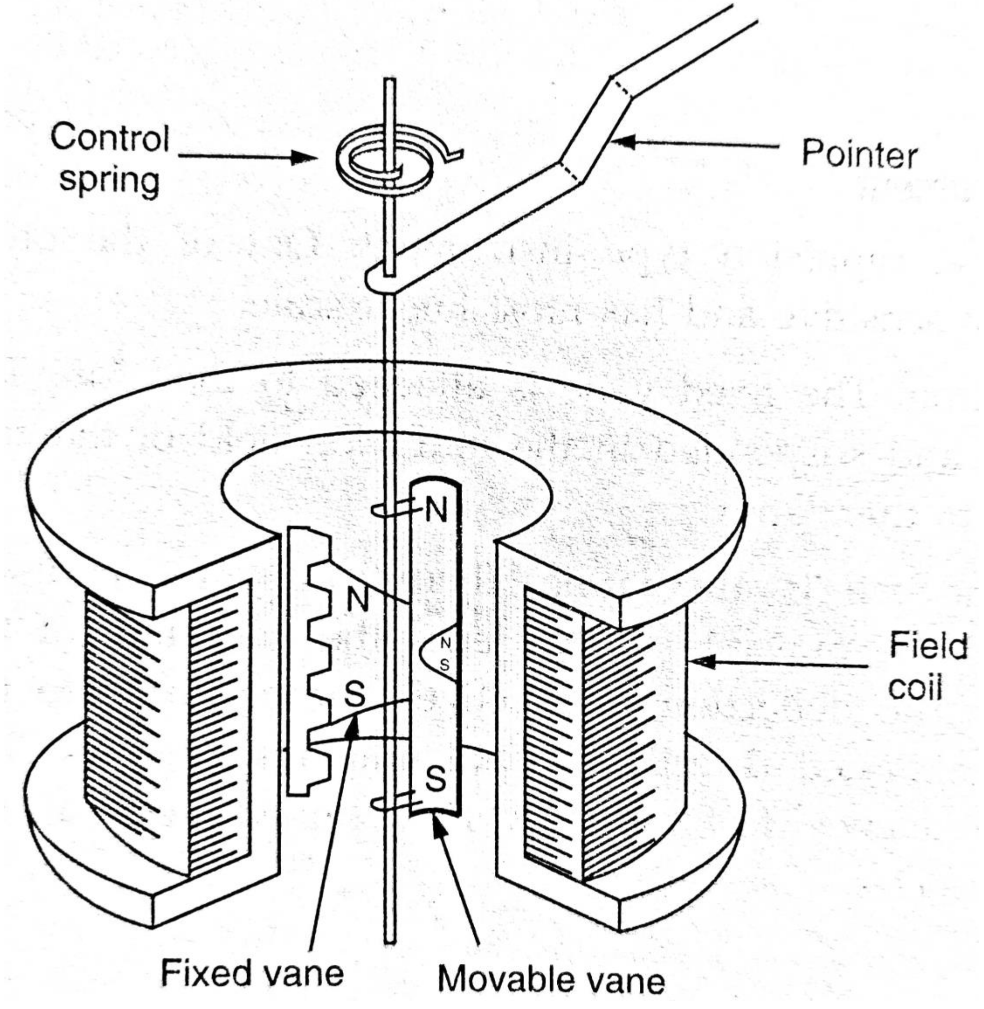


Fig.Concentric vane repulsion type instrument

Torque Equation of Moving Iron Instruments

Consider a small increment in current supplied to the coil of the instrument. Due to this current let dϴ be the deflection under the deflecting torque T_d. Due to such deflection, some mechanical work will be done.

Mechanical work = T_d dϴ

There will be a change in the energy stored in the magnetic field due to the change in inductance. This is because the vane tries to occupy the position of minimum reluctance hence the force is always in such a direction so as to increase the inductance of coil.

The inductance is inversely proportional to the reluctance of the magnetic circuit of coil.

Let I = Initial current
L = Instrument inductance
ϴ = Deflection
dI = Increase in current
dϴ = change in deflection
dL = change in inductance

In order to effect an increment dI in the current, there must be an increase in the applied voltage given by,

Torque Equation of Moving Iron Instruments

Suppose that, at any instant of time current flowing in the coil is I.

Thus the energy stored in the coil in the form of magnetic field = $(1/2)LI^2$.

As soon as the current changes to (I+dI), deflection in the pointer becomes dϴ

resulting into change in inductance of coil from L to (L+dL).

Let this deflection in pointer is due to deflection torque T_d .

Thus mechanical work done = T_d x dϴ ………………..(1)

Energy stored in Coil = $(1/2)(L+dL)(I+dI)^2$

Change in stored energy of coil = Final Stored Energy – Initial Stored Energy

$$= (1/2)(L+dL)(I+dI)^2 - (1/2)LI^2$$

$$= (1/2)[\ (L+dL)(I+dI)^2 - I^2L]$$

$$= (1/2)[\ (L+dL)(I^2+2IdI+(dI)^2 - I^2L]$$

$$= (1/2)[\ LI^2+2LIdI+L(dI)^2 + dL \text{ x } I^2+2IdI \text{ x } dL+dL \text{ x } (dI)^2 \quad - I^2L]$$

Neglecting second order and higher terms of differential quantities i.e. $L(dI)^2$, $2IdI$ x dL and dL x $(dI)^2$

$$= (1/2)[\ 2LIdI+dL \text{ x } I^2]$$

$$= LIdI + (1/2)dL \text{ x } I^2 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Again, just think, when there is a change of current from I to (I+dI), this change of current must be accompanied by change in emf of coil. Thus we can write as

$$e = d(LI) / dt$$

$$= IdL/dt + LdI/dt$$

But electrical energy supplied by the source = eIdt

$$= (IdL + LdI) \times I$$

$$= I^2dL + LIdI$$

According to law of conservation of energy, this electrical energy supplied by the source is converted into stored energy in the coil and mechanical work done for deflection of needle of Moving Iron Instruments.

Hence,

$$I^2dL + LIdI = \text{Change in stored energy} + \text{Work done}$$

$$\Rightarrow I^2dL + LIdI = LIdI + (1/2)dL \times I^2 + T_d \times d\Theta \quad \text{....[from Equ.(1) and (2)]}$$

$$\Rightarrow T_d \times d\Theta = (1/2)dL \times I^2$$

$$\Rightarrow \boldsymbol{T_d = (1/2)I^2(dL/d\Theta)}$$

Thus deflecting torque in Moving iron Instruments is given as

$$\boldsymbol{T_d = (1/2)I^2(dL/d\Theta)}$$

$$T_d = (1/2)I^2(dL/d\Theta)$$

From the above torque equation, we observe that the deflecting torque is dependent on the rate of change of inductance with the angular position of iron van and square of rms current flowing through the coil.

In moving iron instruments, the controlling torque is provided by spring.

Controlling torque due to spring is given as

$$T_c = K\Theta$$

Where K = Spring constant

Θ = Deflection in the needle(pointer)

In equilibrium state, deflecting and controlling torque shall be equal as below.

Deflecting Torque = Controlling Torque

$$\Rightarrow T_d = T_c$$

$$\Rightarrow (1/2)I^2(dL/d\Theta) = K\Theta$$

$$\Rightarrow \Theta = (1/2)(I^2/K)(dL/d\Theta)$$

Thus the deflection is proportional to the square of the current through the coil and instrument gives square law response.

Moving Iron Instruments

➢ Advantages

1) The instruments can be used for both a.c. and d.c. measurements.
2) As the torque to weight ratio is high, errors due to the friction are very less.
3) A single type of moving element can cover the wide range hence these instruments are cheaper than other types of instruments.
4) There are no current carrying parts in the moving system hence these meters are extremely rugged and reliable. These are capable of giving good accuracy. Modern moving iron instruments have a d.c. error of 2 % or less.
5) These can withstand large loads and are not damaged even under severe overload conditions.
6) The range of instruments can be extended.

➢ Disadvantages

1) The scale of the moving iron instruments is not uniform and is cramped at the lower end. Hence accurate readings are not possible at this end.
2) There are serious errors due to hysteresis, frequency changes and stray magnetic fields.
3) The increase in temperature increases the resistance of coil, decreases stiffness of the springs, decreases the permeability and hence affect the reading severely.
4) Due to the non linearity of B-H curve, the deflecting torque is not exactly proportional to the square of the current.
5) There is a difference between a.c. and d.c. calibrations on account of the effect of inductance of the meter. Hence these meters must always be calibrated at the frequency at which they are to be used. The usual commercial moving iron instrument may be used within its specified accuracy from 25 to 125 Hz frequency range.
6) Power consumption is on higher side.

Errors in Moving Iron Instruments

1) **Hysteresis errors :** Due to hysteresis effect, the flux density for the same current while ascending and descending values is different. While descending, the flux density is higher and while ascending it is lesser. So meter reads higher for descending values of current or voltage. So remedy for this is to use smaller iron parts which can demagnetize quickly or to work with lower flux densities.

2) **Temperature error :** The temperature error arises due to the effect of temperature on the temperature coefficient of the spring. This error is of the order of 0.02 % per °C change in temperature. Errors can cause due to self heating of the coil and due to which change in resistance of the coil. So coil and series resistance must have low temperature coefficient. Hence manganin is generally used for the series resistances.

3) **Stray magnetic field error :** The operating magnetic field in case of moving iron instruments is very low. Hence effect of external i.e. stray magnetic field can cause error. This effect depends on the direction of the stray magnetic field with respect to the operating field of the instrument.

4) **Frequency error :** These are related to a.c. operation of the instrument. The change in frequency affects the reactance of the working coil and also affects the magnitude of the eddy currents. This causes errors in the instrument.

5) **Eddy current error :** When instrument is used for a.c. measurements the eddy currents are produced in the iron parts of the instrument. The eddy current affects the instrument current causing the change in the deflecting torque. This produces the error in the meter reading. As eddy currents are frequency dependent, frequency changes cause eddy current error.

Example 4.33. *Find the deflection of a moving-iron ammeter having the following data :*

Control spring constant $= 8 \times 10^{-6}$ *Nm/rad.; Current* $= 6A$,

$$L = 6 + 3\theta - 0.5\,\theta^2\ \mu H.$$

Solution. *Given:* $C = 8 \times 10^{-6}$ Nm/rad.; $I = 6$A; $L = 6 + 3\theta - 0.5\,\theta^2\ \mu$H

Deflection, θ :

Now, $$\frac{dL}{d\theta} = (3 - \theta)\ \mu\text{H/rad. or } (3 - \theta) \times 10^{-6}\ \text{H/rad.}$$

We know that, $$\theta = \frac{1}{2}\frac{I^2}{C}\frac{dL}{d\theta} \qquad \text{...[Eqn. (4.78)]}$$

or, $$\theta = \frac{1}{2} \times \frac{6^2}{8 \times 10^{-6}} \times (3 - \theta) \times 10^{-6}$$

$$\theta = 2.25 \times 10^{6}(3 - \theta) \times 10^{-6} = 2.25(3 - \theta)$$

or, $$\theta = \frac{2.25 \times 3}{3.25} = 2.077\ \text{rad.} = \mathbf{119°}\ \textbf{(Ans.)}$$

Example 4.41. *A 15-volt moving-iron voltmeter has a resistance of 300 Ω and an inductance of 0.12 H. Assuming that this instrument reads correctly on D.C., what will be its readings on A.C. at 15 volts when frequency is (i) 25 Hz and (ii) 100 Hz ?*

Solution. On D.C., *only ohmic resistance* is involved and the voltmeter reads correctly. But on A.C., it is the *impedance* of the instrument which has to be taken into account.

(*i*) **When frequency is 25 Hz :**

Impedance at 25 Hz,

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi f L)^2}$$

$$= \sqrt{(300)^2 + (2\pi \times 25 \times 0.12)^2} = 300.6\,\Omega$$

$$\therefore \quad \textbf{Voltmeter reading} = 15 \times \frac{300}{300.6} = \mathbf{14.97\ V} \quad \textbf{(Ans.)}$$

(*ii*) **When frequency is 100 Hz :**

Impedance at 100 Hz,

$$Z = \sqrt{(300)^2 + (2\pi \times 100 \times 0.12)^2} = 309.3\,\Omega$$

$$\therefore \quad \textbf{Voltmeter reading} = 15 \times \frac{300}{309.3} = \mathbf{14.55\ V} \quad \textbf{(Ans.)}$$

Incidently, it may be noted that as the frequency is increased, the impedance of the voltmeter is also increased. Hence, the current is decreased and, therefore, the voltmeter readings are lower.

Example 4.40. *The coil of a 250 V moving-iron voltmeter has a resistance of 500 Ω and inductance of 1 H. The current taken by the instrument when placed on 250 V, D.C. supply is 0.05 A. Determine the percentage error when the instrument is placed on 250 V, A.C. supply at 100 Hz.*

Solution. Total ohmic resistance, $R = \dfrac{250}{0.05} = 5000\,\Omega$

(Original calibration of the instrument is with direct current)

Reactance of the coil, $X_L = 2\pi f L = 2\pi \times 100 \times 1 = 628\ \Omega$

Coil impedance, $Z = \sqrt{R^2 + X_L^2} = \sqrt{(5000)^2 + (628)^2} = 5039\,\Omega$

$\therefore$ Voltage reading on A.C. $= \dfrac{250 \times 5000}{5039} = 248\,\text{V}$

$\therefore$ Error $= 248 - 250 = -2\text{V}$

Percentage error $= \dfrac{2}{250} \times 100 = \mathbf{0.8\%}$ (*low*) (Ans.)

Range Extension of An Ammeter

Basic D.C. Ammeter

The basic d.c. ammeter is nothing but a D'Arsonval galvanometer. The coil winding of a basic movement is very small and light and hence it can carry very small currents. So as mentioned earlier, for large currents, the major part of current is required to be bypassed using a resistance called shunt. It is shown in the Figure.

The shunt resistance can be calculated as :

Let R_m = Internal resistance of coil

R_{sh}= Shunt resistance

I_m= Full scale deflection current

I_{sh}= Shunt current

I= Total current

Now I = I_{sh} + I_m

As the two resistances R_{sh} and R_m are in parallel, the voltage drop across them is same.

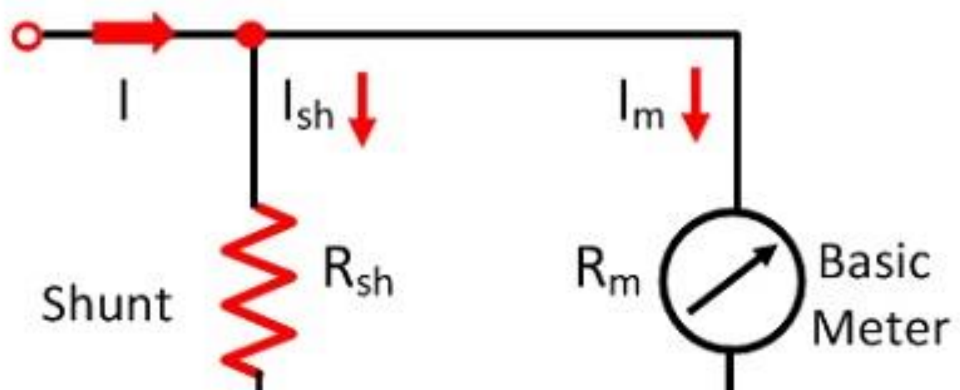
$$I_{sh}R_{sh} = I_m R_m$$

$$R_{sh} = \frac{I_m R_m}{I_{sh}}$$

But I_{sh}= I- I_m

$$R_{sh} = \frac{I_m R_m}{I - I_m}$$

$$R_{sh} = \frac{R_m}{m-1} \qquad where\ m = \frac{I}{I_m}$$



Basic Ammeter Circuit

The m is called multiplying power of the shunt and defined as the ratio of total current to the current through the coil. It can be expressed as,

$$m = \frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}}$$

$$R_{sh} = \frac{R_m}{m-1}$$

The shunt resistance may consist of a constant temperature resistance wire within the case of the meter or it may be external shunt having low resistance.

Thus to increase the range of ammeter 'm' times, the shunt resistance required is 1/(m-1) times the basic meter resistance. This is nothing but extension of ranges of an ammeter.

Magainin and Constantan use for making the shunt in the DC and AC instruments respectively.

- Why Shunt Connect in parallel with Ammeter?

- The ammeter designs for measuring the low current. For measuring the heavy current, the shunt is connected in parallel to the ammeter. The significant portion of the measure and current passes to the shunt because of the low resistance path and few amount of current passes through the ammeter.
- The shunt connects in parallel to the ammeter because of which the voltage drops across the meter and shunt remain the same. Thus, the movement of the pointer is not affected by the shunt.

➢ Precautions to be taken while using an Ammeter :

The following precautions must be taken while using an ammeter:

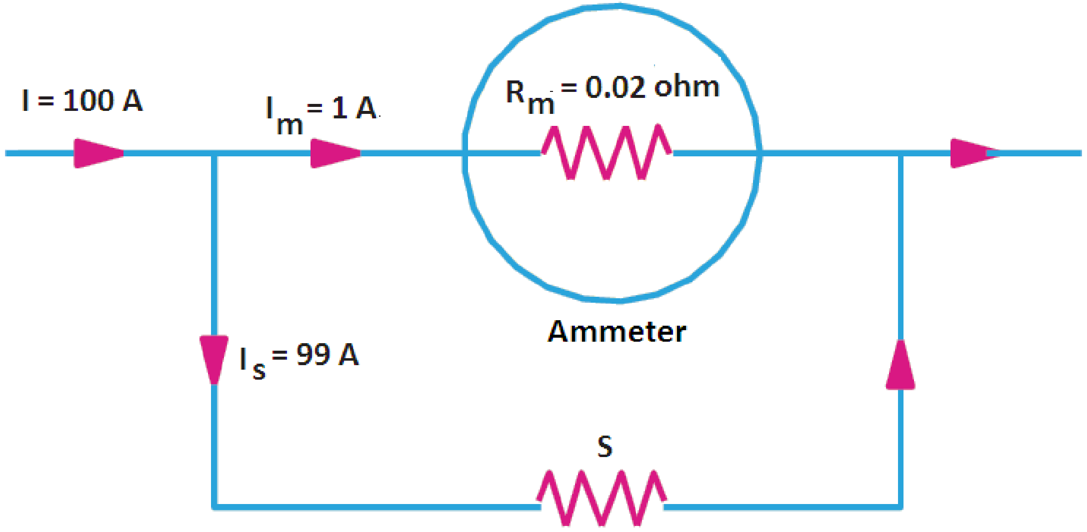
1) As the ammeter resistance is very low, it should never be connected across any source of e.m.f. Always connect an ammeter in series with the load.
2) The polarities must be observed correctly. The opposite polarities deflect the pointer in opposite direction against the mechanical stop and this may damage the pointer
3) While using multi range ammeter, first use the highest current range and then decrease the current range until sufficient deflection is obtained. So to increase the accuracy, finally select the range which will give the reading near full scale deflection

➢ Requirements of a Shunt:

1) The temperature coefficient of shunt and the meter should be low and should be as equal as possible.
2) The shunt resistances should be stable and constant with time.
3) The shunt resistances should not carry currents which will cause excessive temperature rise.
4) The type of material used to join the shunts should have low thermo dielectric voltage drop i.e. the soldering of joints should not cause a voltage drop.
5) Due to the soldering, the values of resistance should not be change.
6) The resistances should have low thermal electromotive force with copper.

(Q).A moving coil ammeter reading up to 1 ampere has a resistance of 0.02 ohm. How could this instrument be adopted to read current up to 100 amperes.

Solution: In this case,
Full-scale deflection current of the ammeter, $I_m = 1$ A
Line current to be measured, $I = 100$ A
Resistance of ammeter, $R_m = 0.02$ ohm
Let, the required shunt resistance = S



As seen from Figure, the voltage across the instrument coil and the shunt resistance is the same since both are joined in parallel.

$$\therefore\ I_m*R_m = S*I_s = S(I - I_m)$$

$$\text{or}\ \ S = I_m*R_m/(I - I_m)$$

$$= 1*0.02/(100 - 1) = 0.02/99 = 0.000202\ \ \text{Ans.}$$

Multirange Ammeters

- The range of the basic d.c. ammeter can be extended by using number of shunts and a selector switch. Such a meter is called multirange ammeter and is shown in the Fig. (a).
- R_1, R_2, R_3 and R_4, are four shunts. When connected in parallel with the meter, they can give four different ranges I_1, I_2, I_3 and I_4,.
- The selector switch S is multi position switch, having low contact resistance and high current carrying capacity. The make before break type switch is used for the range changing.
- If the ordinary switch is used, while range changing the switch remains open and full current passes through the meter.

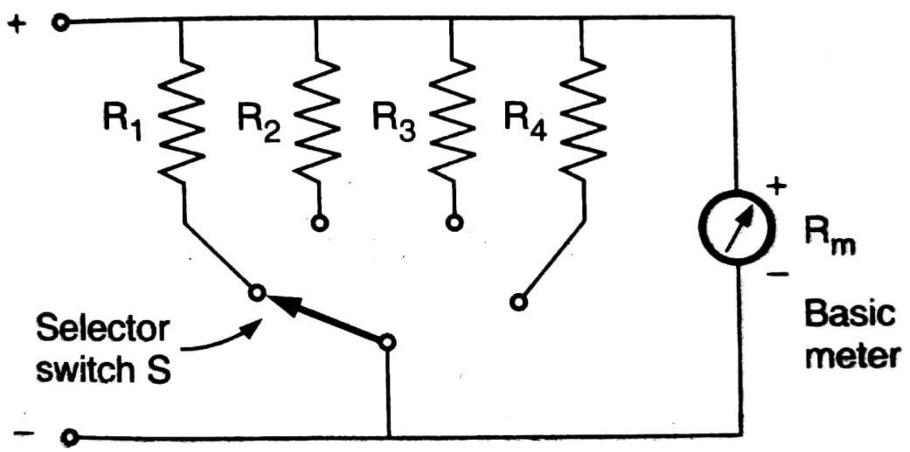


Fig.(a).Multirange Ammeter

- The meter may get damaged due to such high current. So make before break switch is used. The design of such switch is so that makes contact with next terminal before completely breaking the contact with the previous terminal.

- The multirange ammeters are used for the ranges upto 50 A. While using the multirange ammeter, highest range should be used first and thus the current range should be decreased till good upscale reading is obtained.
- All the shunts are very precise resistances and hence cost of such multirange ammeter is high.

- The mathematical analysis of basic d.c. ammeter is equally applicable to such multirange ammeter. Thus,

$$R_1 = \frac{R_m}{m_1 - 1}$$

$$R_2 = \frac{R_m}{m_2 - 1} \text{ and so on,}$$

Where m_1, m_2, m_3 …….. are the shunt multiplying powers for the currents I_1, I_2, I_3 ....

Example 1.7 : *Design a multirange d.c. milliammeter with a basic meter having a resistance 75 Ω and full scale deflection for the current of 2 mA. The required ranges are 0 - 10 mA, 0 - 50 mA and 0 - 100 mA.*

Solution : The first range is 0 - 10 mA,

$$I_1 = 10 \text{ mA}$$

While $I_m = 2 \text{ mA}$ and $R_m = 75\ \Omega$

$$R_1 = \frac{I_m R_m}{(I_1 - I_m)} = \frac{2 \times 75}{(10-2)}$$

$$= \mathbf{18.75\ \Omega}$$

The second range is 0 - 50 mA,

$$\therefore \quad I_2 = 50 \text{ mA}$$

$$R_2 = \frac{I_m R_m}{(I_2 - I_m)} = \frac{2 \times 75}{(50-2)}$$

$$= \mathbf{3.125\ \Omega}$$

The third range is 0 - 100 mA,

$$\therefore \quad I_3 = 100 \text{ mA}$$

$$R_3 = \frac{I_m R_m}{(I_3 - I_m)} = \frac{2 \times 75}{(100-2)} = \mathbf{1.53\ \Omega}$$

The designed multirange ammeter with a selector switch is shown in the Fig. 1.28.

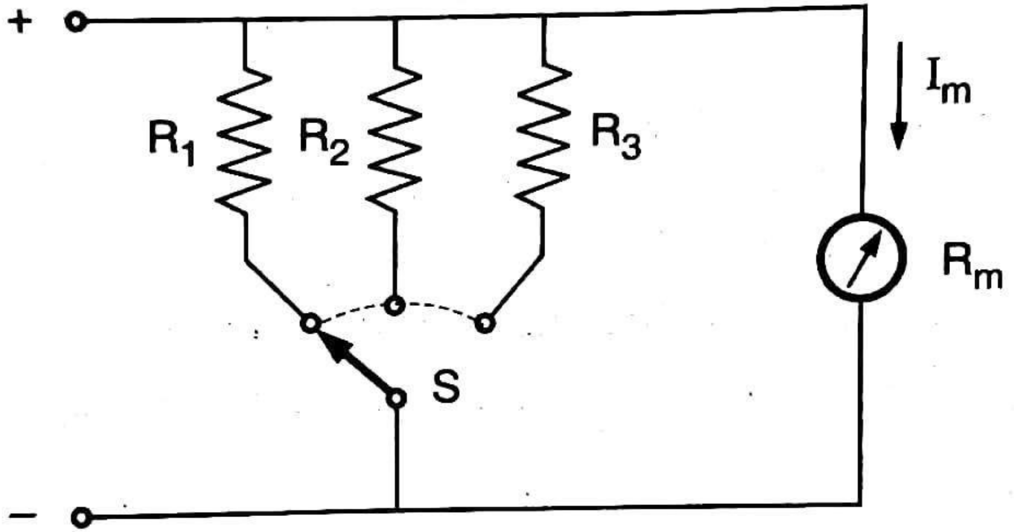
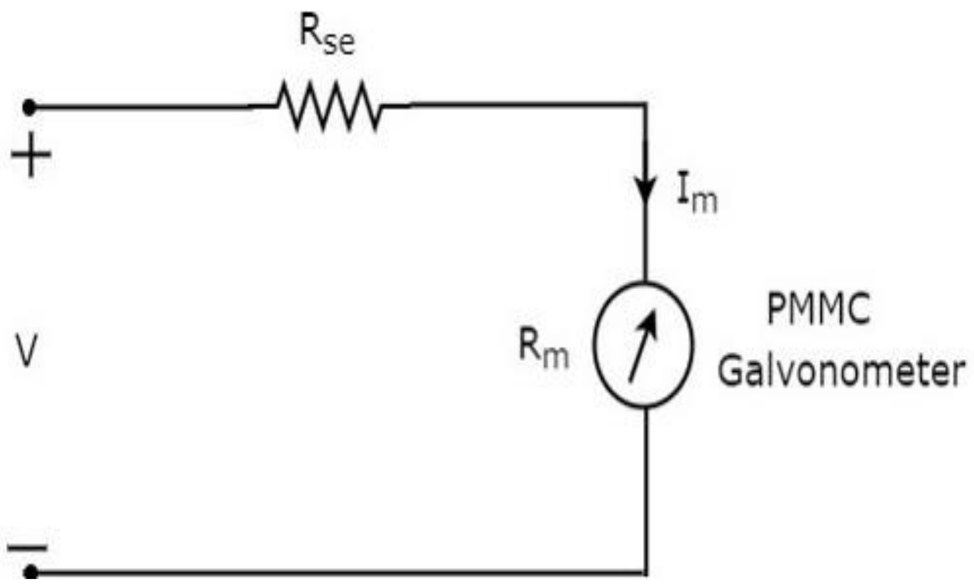


Fig. 1.28

Range Extension of Voltmeter

Basic D.C. Voltmeter-Voltmeter Multipliers

- The series resistance, which is used in DC voltmeter is also called series multiplier resistance or simply, multiplier. It basically limits the amount of current that flows through galvanometer in order to prevent the meter current from exceeding the full scale deflection value. The circuit diagram of DC voltmeter is shown in below figure.



We have to place this DC voltmeter across the two points of an electric circuit, where the DC voltage is to be measured.
Apply **KVL** around the loop of above circuit.

$$V - I_m R_{se} - I_m R_m = 0 \quad \dots\dots\dots\dots(1)$$

$$V - I_m R_m = I_m R_{se}$$

$$R_{se} = \frac{V - I_m R_m}{I_m}$$

$$R_{se} = \frac{V}{I_m} - R_m \quad \dots\dots(2)$$

Where,
Rse is the series multiplier resistance
V is the full range DC voltage that is to be measured
Im is the full scale deflection current
Rm is the internal resistance of galvanometer

The ratio of full range DC voltage that is to be measured, V and the DC voltage drop across the galvanometer, v_m is known as multiplying factor, m. Mathematically, it can be represented as

$$m = \frac{V}{v_m} \quad \text{..............................(3)}$$

From Equation (1), we will get the following equation for full range DC voltage that is to be measured, V.

$$V = I_m R_{se} + I_m R_m \quad \text{................(4)}$$

The DC voltage drop across the galvanometer, v_m is the product of full scale deflection current, I_m and internal resistance of galvanometer, R_m. Mathematically, it can be written as

$$v_m = I_m R_m \quad \ldots \ldots \ldots \ldots \ldots .(5)$$

substitute, Equation (4) and Equation (5) in Equation (3).

$$m = \frac{I_m R_{se} + I_m R_m}{I_m R_m}$$

$$m = \frac{R_{se}}{R_m} + 1$$

$$m - 1 = \frac{R_{se}}{R_m}$$

$$R_{se} = R_m(m - 1)$$

Precautions to be taken while using a Voltmeter

1) The voltmeter resistance is very high and it should always be connected across the circuit or component whose voltage is to be measured.
2) The polarities must be observed correctly. The wrong polarities deflect the pointer in the opposite direction against the mechanical stop and this may damage the pointer.
3) While using the multirange voltmeter, first use the highest range and then decrease the voltage range until the sufficient deflection is obtained.
4) Take care of the loading effect. The effect can be minimised by using high sensitivity voltmeters.

Requirements of a Multiplier

1) Their resistance should not change with time.
2) The change in their resistance with temperature should be small.
3) They should be non-inductively wound for a.c. meters.

Commonly used resistive materials for construction of multiplier are manganin and constantan

(Q).A moving coil voltmeter reading up to 20 mV has a resistance of 2 ohms. How this instrument can be adopted to read voltage up to 300 volts.

Voltmeter resistance, $R_m = 2$ ohm

Full-scale voltage of the voltmeter, $v = R_m I_m = 20$ mV $= 0.02$ V

Full-scale deflection current, $I_m = v/R_m = 0.02/2 = 0.01$ A

Voltage to be measured, $V = 300$ V

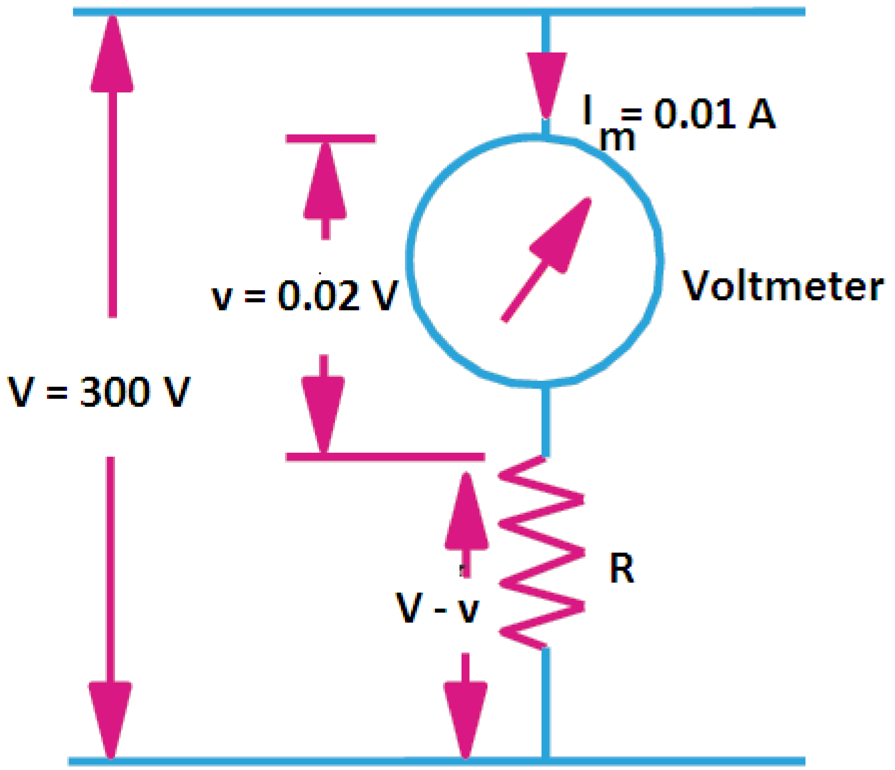
Let the series resistance required $= R$

Then as seen from figure, the voltage drop across R is $V - v$

$R * I_m = V - v$

or $R = (V - v)/I_m$

or $R = (300 - 0.02)/0.01 = 299.98/0.01 = 29998$ ohms Ans.



Example 4.49. *A milliammeter of 2.5 ohms resistance reads upto 100 milliamperes. Calculate the resistance which is necessary to enable it to be used as :*

(i) A voltmeter reading upto 10 V.

(ii) An ammeter reading upto 10 A.

Draw the connection diagram in each case.

Solution. Resistance of the milliammeter, $R_m = 2.5\ \Omega$

Maximum current of the milliammeter, $I_m = 100$ mA $= 0.1$ A.

(*i*) Voltage to be measured, $V = 10$ volts

Resistance to be connected in series.

$$\mathbf{R_s} = \frac{V}{I_m} - R_m = \frac{10}{0.1} - 2.5 = \mathbf{97.5\ \Omega} \quad \textbf{(Ans.)}$$

Connection diagram is shown in Fig. 4.37.

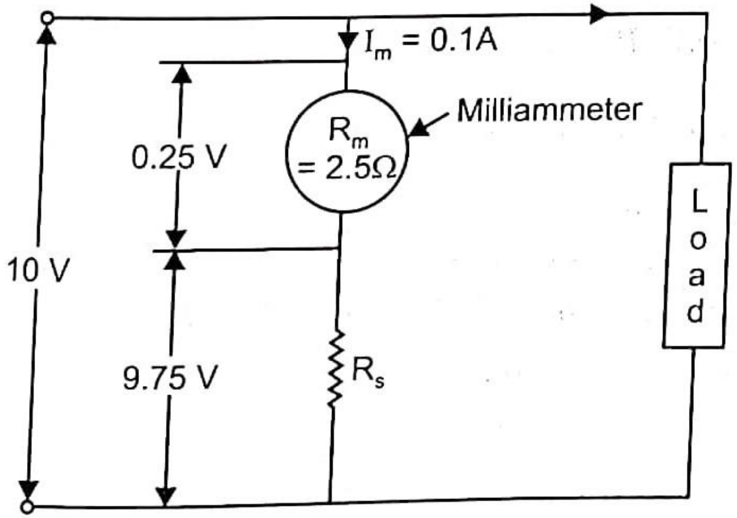


Fig. 4.37.

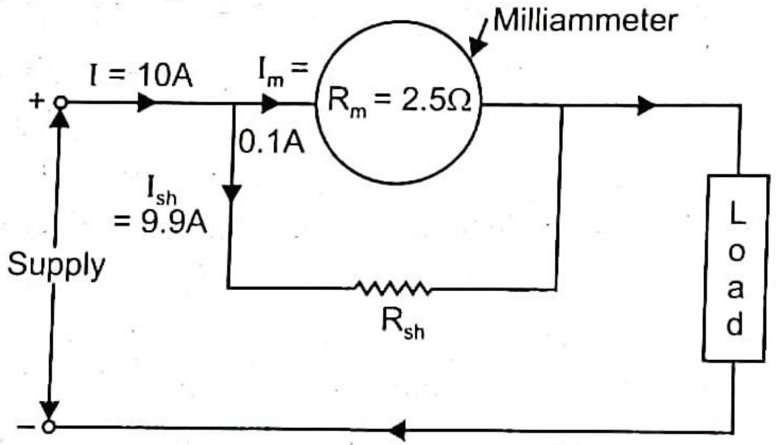


Fig. 4.38.

(*ii*) Current to be measured, $I = 10$ A

∴ Resistance to be connected in parallel,

$$R_{sh} = \frac{I_m R_m}{I - I_m} = \frac{0.1 \times 2.5}{10 - 0.1} = \mathbf{0.02525\ \Omega}$$

Connection diagram is shown in Fig. 4.38.

Multirange Voltmeters

- The range of the basic d.c. voltmeter can be extended by using number of multipliers and a selector switch. Such a meter is called multirange voltmeter and is shown in the Fig.(a).
- $The\ R_1, R_2, R_3$ and R_4 are the four series multipliers. When connected in series with the meter, they can give four different voltage ranges as V_1, V_2, V_3 and V_4 .
- The selector switch S is multiposition switch by which the required multiplier can be selected in the circuit.
- The mathematical analysis of basic d.c. voltmeter is equally applicable for such multirange voltmeter.

Thus,

$$R_1 = \frac{V_1}{I_m} - R_m$$

$$R_2 = \frac{V_2}{I_m} - R_m \quad \text{and so on}$$

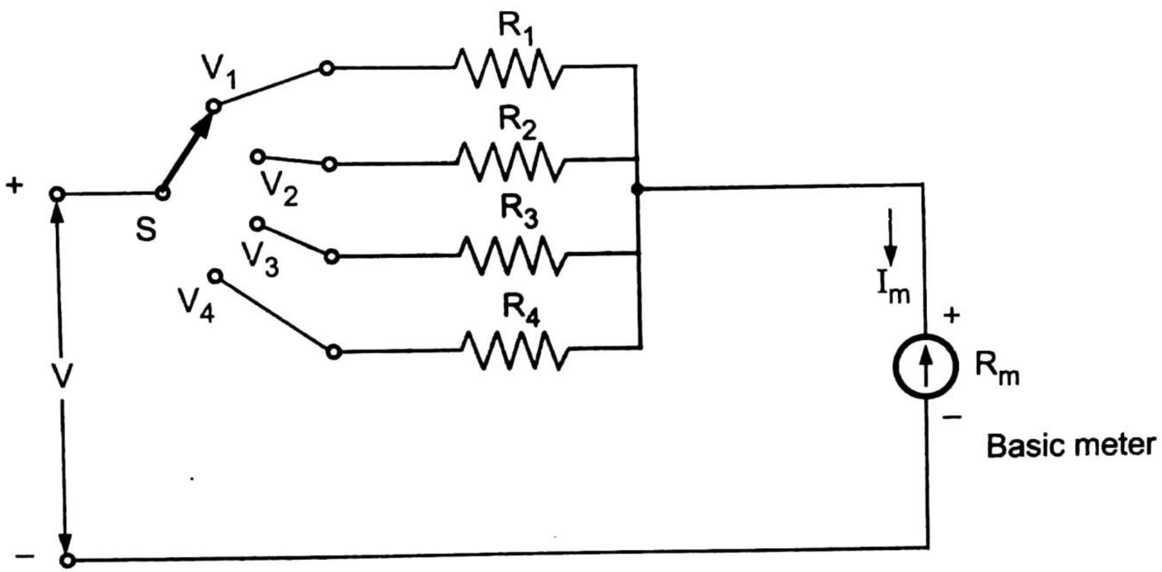


Fig.(a)Multirange voltmeter

ELECTROSTATIC INSTRUMENTS

ELECTROSTATIC INSTRUMENTS

➢ Working Principle of Electrostatic Type Instruments:

- The instrument whose working depends on the principle of attraction or repulsion of electrodes that carry electrical charges such type of instrument is known as the electrostatic instrument. In other words, the instrument which uses the static electric field for producing the deflecting torque is known as the electrostatic instrument.
- In electrostatic instruments, the deflecting torque is produced by action of electric field on charged conductors.
- The electrostatic instrument is used for measuring the high and low voltage and also the power of the given circuit.

These are Two types

(1). Linear Type Electrostatic Instrument

(2). Rotary type Electrostatic Instrument

❑ Linear Type Electrostatic Instrument

- The fig.(a) shows the linear electrostatic type instrument. The plates A become positively charged, and the plate B becomes negatively charged.
- The positive charge plates become fixed, and the negative plates become movable.
- The spring is connected to the negatively charged plates for controlling the movement.
- When the voltage is applied to the plate, then the force of attraction induces between them. The plate tries to moves towards A until the force becomes maximum. The C is the capacitance (in farad) between the plate.

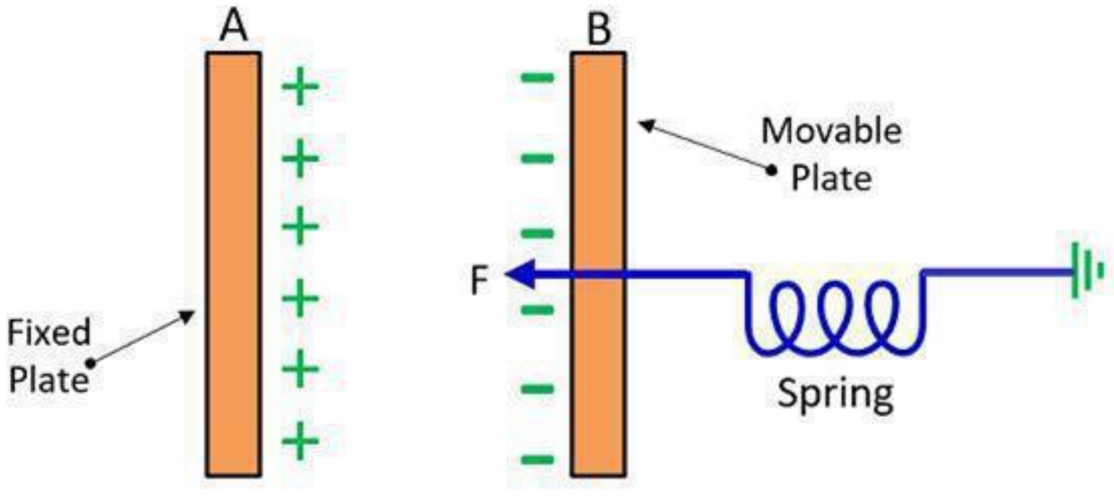


Fig.(a).Linear Type Electrostatic Instrument

❑ Rotary type Electrostatic Instrument

- In this type, there is force of attraction or repulsion between the electrodes which causes rotary motion of the moving electrode.
- In both the cases, the mechanism resembles a variable capacitor and the force or torque is due to the fact that the mechanism tends to move the moving electrode to such a position where the energy stored is maximum.

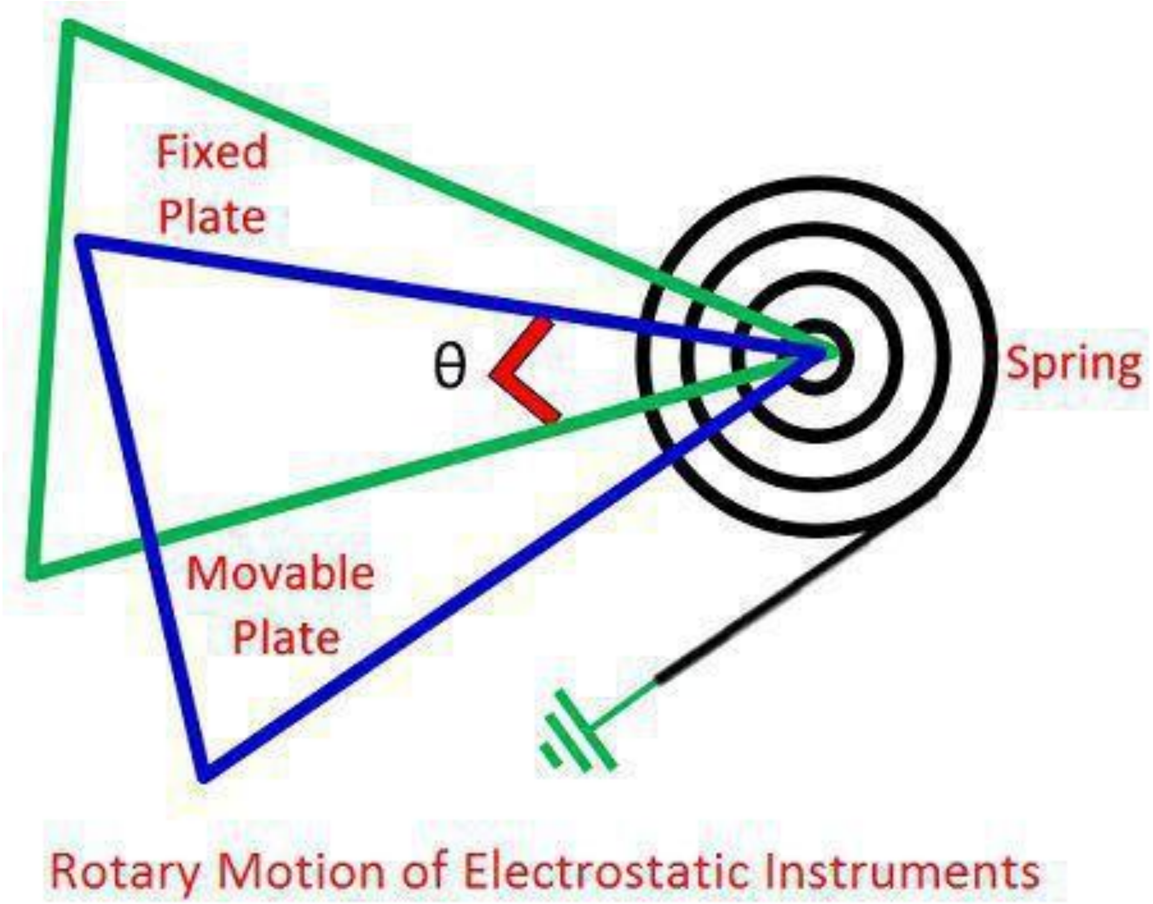


Fig.(b)

Force and Torque Equation of Electrostatic Type Instrument

1. Linear Motion: Referring to Figure (a)

- Now let us derive force equation for the linear electrostatic type instruments. Let us consider two plates as shown in the diagram .
- Plate A is positively charged and plate B is negatively charged. As mentioned above as per the possible condition (a) we have linear motion between the plates. The plate A is fixed and plate B is free to move.
- Let us assume there exists some force F between the two plates at equilibrium when electrostatic force becomes equal to spring force. At this point, the electrostatic energy stored in the plates is $\frac{1}{2}Cv^2$

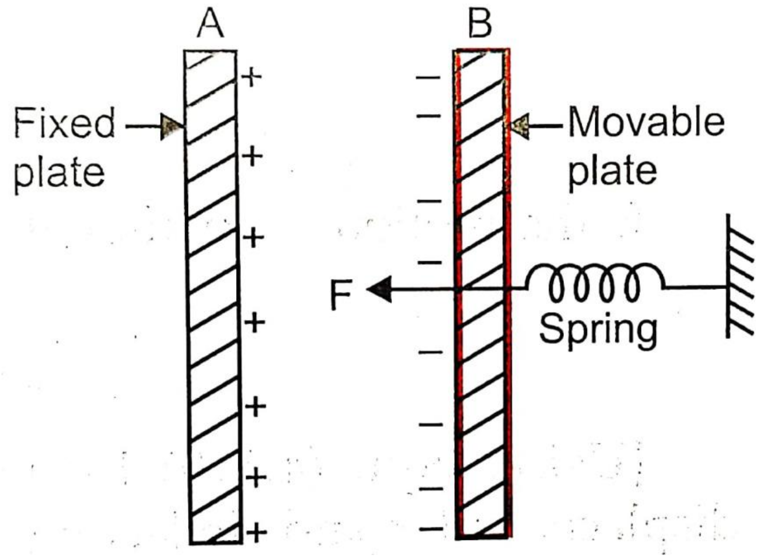


Fig. (a)

Now suppose we increase the applied voltage by an amount dV, due to this the plate B moves towards the plate A by a distance dx. The work done against the spring force due to displacement of the plate B be F.dx. The applied voltage is related to current as

$$i = c\frac{dv}{dt} + v\frac{dc}{dt}$$

From this value of electric current the input energy can be calculated as $vi\ dt = v^2 dc + cv\ dv$

From this we can calculate the change in the stored energy and that comes out to be

$$\frac{1}{2}v^2 dc + cv\,dv$$

By neglecting the higher order terms that appears in the expression.
Now applying the principle of energy conservation we have
input energy to the system = increase in the stored energy of the system + mechanical work done by the system. From this we can write,

$$v^2 dc + cv\,dv = \frac{1}{2}v^2 dc + cv\,dv + F dx$$

From the above equation the force can be calculated as

$$F = \frac{1}{2}v^2 \frac{dc}{dx} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

2. Rotational Motion-Referring to Figure (b)

- In order to find out the expression for deflecting torque in case of rotary type **electrostatic instruments**, just replace the in the equation (1) F by T_d and dx by dΘ.
- Now rewriting the modified equation we have deflecting torque is equals to

$$T_d = \frac{1}{2}v^2 \frac{dc}{d\Theta}$$

Now at steady state (Equilibrium position) $T_c = T_d$
If the instrument is spring controlled or has a suspension then we have controlling torque is given by the expression $T_c = K \times \Theta$
where k = spring constant θ = deflection .

$$T_c = T_d$$

$$K \times \Theta = \frac{1}{2} v^2 \frac{dc}{d\Theta}$$

The deflection Θ can be written as

$$\Theta = \frac{1}{2} v^2 \frac{dc}{K.d\Theta}$$

- Since the deflection is proportional to the square of the voltage to be measured, the instrument can be used on both ac and dc. The instrument exhibits a square law response and hence the scale is non-uniform.

Advantages of Electrostatic Instruments

1. These instruments draws negligible amount of power from the mains.
2. Suitable for both A.C and D.C. & measure r.m.s. values
3. They have no frequency and waveform errors as the deflection is proportional to square of voltage and there is no hysteresis.
4. There are no errors caused by the stray magnetic field as the instrument works on the electrostatic principle.
5. They are particularly suited for high voltage.

Disadvantages of Electrostatic Instruments

1. The use of electrostatic instruments is limited to certain special applications, particularly in ac circuits of relatively high voltage, where the current drawn by other instruments would result in erroneous indication. A protective resistor is generally used in series with the instrument in order to limit the current in case of a short circuit between plates.
2. Low deflecting torque especially when the voltage required to be measured is less than 500V. However, Kelvin multicellular voltmeter can be designed for use for measurements of voltages as low as 30 V.
3. These instruments are expensive, large in size and are not robust in construction.
4. Their scale is not uniform.
5. The operating force is small.

Classification of electrostatic instruments

There are two general types of such instruments

1. The quadrant type Used upto 20 kV
2. The attracted disc type. Used upto 500 kV.

Range extension of electrostatic voltmeters

- The range of electrostatic voltmeters can be extended by the use of multipliers which are in form of:
  1. Resistance potential divider ------ Can be used both for direct and alternating voltages.
  2. Capacitance potential divider ------ Useful only for alternating voltages.

1. Resistance potential divider :

- This divider consists of a high non-inductive resistance across a small portion which is attached to the electrostatic voltmeter as shown in Fig.(c)

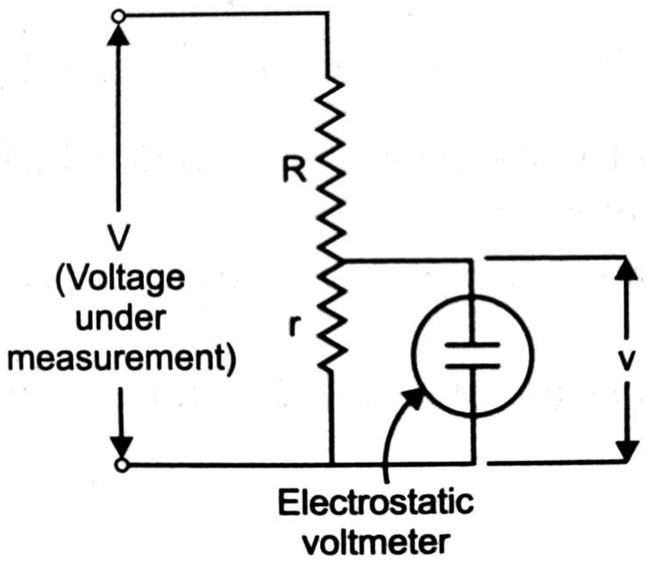


Fig.(c)Resistance potential divider

Let

R = The resistance of the whole of the potential divider across which the voltage V under measurement is applied

r = The resistance of the portion of the divider across which voltmeter is connected

Then the multiplying factor is given by

$$\frac{V}{v} = \frac{R}{r} \qquad \text{.................(1)}$$

- The above eqn. (1) is true for D.C. circuits but for A.C. circuits, the capacitance of the voltmeter (which is in parallel with r). has to be taken into account.
- Since this capacitance is variable, it is advisable to calibrate the voltmeter along with its multiplier.

2. Capacitance potential divider:

In this method, the voltmeter may be connected in series with a single capacitor C and put across the voltage V which is to be measured [Fig.(a)] or a number of capacitor may be joined in series to form the potential divider and the voltmeter may be connected across one of the capacitors as shown in Fig. (b).

Referring to Fig.(a), the multiplying factor is given by

$$\frac{V}{v} = \frac{\text{Reactance of total circuit}}{\text{Reactance of voltmeter}}$$

Now, capacitance of the total circuit = $\frac{CC_v}{C+C_v}$

and, its reactance = $\frac{1}{\omega \times capacitance} = \frac{C+C_v}{\omega CC_v}$

Reactance of the voltmeter = $\frac{1}{\omega C_v}$

$$\therefore \quad \frac{V}{v} = \frac{(C+C_v)/\omega CC_v}{1/\omega C_v} = \frac{C+C_v}{C}$$

$\therefore$ Multiplying factor, $\frac{V}{v} = \frac{C+C_v}{C} = 1 + \frac{C_v}{C}$

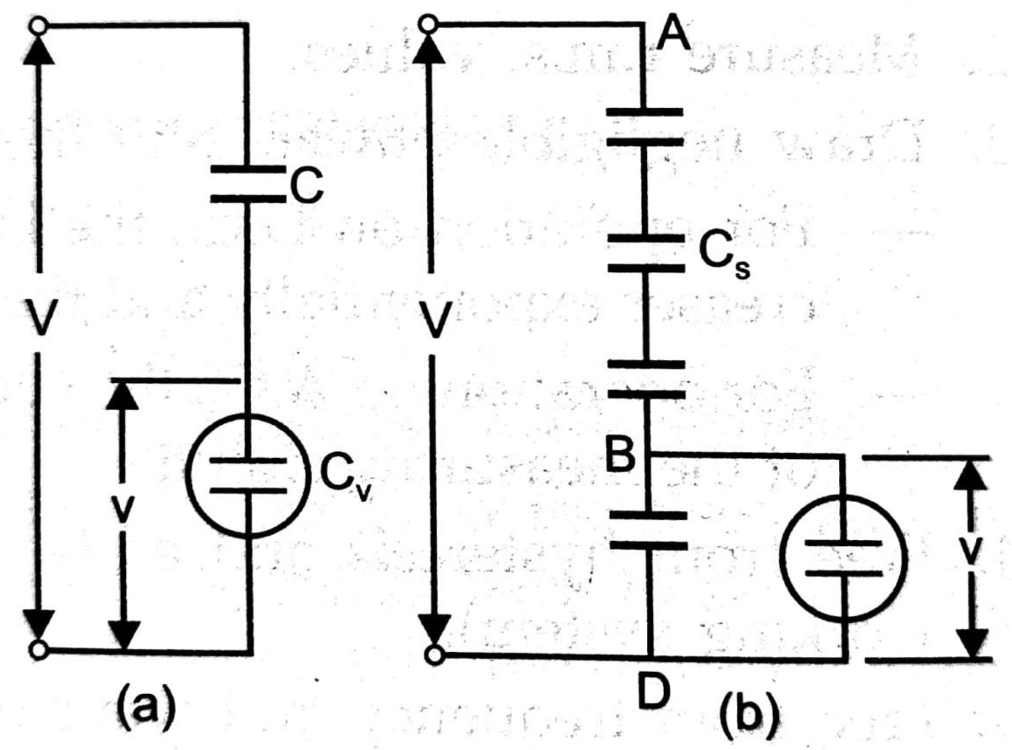


Fig.Capacitance potential divider

Example 4.67. *An absolute electrometer uses a movable circular plate 80 mm in diameter. During a voltage measurement the distance between the plates is 3.2 mm and the force of attraction is 0.003 N. The medium is air.* $\in_0 = 8.85 \times 10^{-12}$ *F/m. Determine V.*

Solution. *Given* : $A = \frac{\pi}{4} \times (0.08)^2 = 5.0265 \times 10^{-3}$ m^2; d = 3.2 mm or 0.0032 m; F = 0.03 N;
$\in = \in_0 = 8.85 \times 10^{-12}$ F/N

We know that,

$$V = \sqrt{\frac{2Fd^2}{\in A}} \qquad \text{[Eqn. (4.116)]}$$

or,

$$V = \sqrt{\frac{2 \times 0.03 \times (0.0032)^2}{8.85 \times 10^{-12} \times 5.0265 \times 10^{-3}}} = \mathbf{1175\ V} \quad \textbf{(Ans.)}$$

Example 4.68. *In an electrometer, the movable plate is 110 mm in diameter. When 12 kV is applied between the movable plate and the fixed plate, the force is 0.006 N. Find the change in capacitance for 1.2 mm movement of the movable plate.*

Solution. *Given* : Diameter of movable plate = 110 mm or 0.11 m; V = 12 kV; F = 0.006 N; x = 1.2 mm.

We know that,
$$F = \frac{1}{2} V^2 \frac{dC}{dx} \qquad \text{[Eqn. (4.110)]}$$

$$0.006 = \frac{1}{2} \times (12 \times 10^3)^2 \times \frac{dC}{dx}$$

or,
$$\frac{dC}{dx} = \frac{2 \times 0.006}{(12 \times 10^3)^2} = 83.33 \times 10^{-12} \text{ F/m}$$

$\therefore$ Change in capacitance,
$$dC = 83.33 \times 10^{-12} \times (1.2 \times 10^{-3})$$
$$\simeq 0.1 \times 10^{-12} \text{ F} \quad \text{or} \quad \mathbf{0.1\ pF} \quad \textbf{(Ans.)}$$

INSTRUMENT TRANSFORMERS

Introduction

- In heavy currents and high voltage a.c. circuits, the measurement can not be done by using the method of extension of ranges of low range meters by providing suitable shunts.
- In such conditions, specially constructed accurate ratio transformers called instrument transformers.
- These can be used, irrespective of the voltage and current ratings of the a.c. circuits.
- These transformers not only extend the range of the low range instruments but also isolate them from high current and high voltage a.c. circuits.
- This makes their handling very safe.

Advantages and Disadvantages of Instrument Transformers

- Advantages and Disadvantages of Instrument Transformers as compared with ammeter shunts and voltmeter multipliers are as follows:

Advantages:

1) The normal range voltmeter and ammeter can be used along with these transformers to measure high voltage and currents.
2) The rating of low range meter can be fixed irrespective of the value of high voltage or current to be measured.
3) These transformers isolate the measurement from high voltage and current circuits. This ensures safety of the operator and makes the handling of the equipments very easy and safe.
4) These can be used for operating many types of protecting devices such as relays or pilot lights
5) Several instruments can be fed economically by single transformer.

Disadvantage :

1) The only disadvantage of these instrument transformers is that they can be used only for a.c. circuits and not for d.c. circuits.

Ratios of Instrument Transformer

- The various ratios defined for the instrument transformers are,

1. Actual ratio [R]

The actual transformation ratio is defined as the ratio of the magnitude of actual primary phasor to the corresponding magnitude of actual secondary phasor.

$$R = \frac{\text{Magnitude of actual primary current}}{\text{Magnitude of actual secondary current}} \quad \text{... For C.T.}$$

$$R = \frac{\text{Magnitude of actual primary voltage}}{\text{Magnitude of actual secondary voltage}} \quad \text{... For P.T.}$$

The actual ratio is also called transformation ratio.

2. Nominal ratio [K_n]

The nominal ratio is defined as the ratio of rated primary quantity to the rated secondary quantity, either current or voltage.

$$K_n = \frac{\text{Rated primary current}}{\text{Rated secondary current}} \quad \text{... For C.T.}$$

$$K_n = \frac{\text{Rated primary voltage}}{\text{Rated secondary voltage}} \quad \text{... For P.T.}$$

3. Turns ratio [n]

$$n = \frac{\text{Number of turns of secondary winding}}{\text{Number of turns of primary winding}} \quad \text{... For C.T.}$$

$$n = \frac{\text{Number of turns of primary winding}}{\text{Number of turns of secondary winding}} \quad \text{... For P.T.}$$

Ratio Correction Factor (RCF) : It is the ratio of transformation i.e. actual ratio to the nominal ratio.

Burden of an Instrument Transformer

- The nominal ratio of an instrument transformer, does not remain constant in practice as the load on the secondary changes.
- It changes because of effect of secondary current, power factor and magnetising as well as core loss components of current and this causes errors in the measurements.
- For the particular class of transformers the specific loading at rated secondary winding voltage is specified such that the errors do not exceed the limits. Such a permissible load is called burden of an instrument transformer.
- Thus the permissible load across the secondary winding expressed in volt-amperes at the rated secondary winding voltage or current, such that errors do not exceed the limits is called burden of an instrument transformer.

$$\text{Total secondary winding burden} = \frac{(\text{Secondary winding induced voltage})^2}{\text{Total impedance of secondary circuit including load and winding}}$$

➢ **Note:**

- In using instrument transformers for current (or voltage) measurements, we must know the ratio of primary current (or voltage) to the secondary current (or voltage).
- These ratios give the multiplying factor for finding the primary values from the instrument readings on the secondary side.
- However, for energy or power measurements, it is essential to know not only the transformation ratio but also the phase angle between primary and secondary currents (or voltages) because it necessitates further correction to the meter reading.

Instrument Transformers are generally classified as

i) Current transformers and

ii) Potential transformers

Current Transformer

Current Transformer

➢ Current Transformers (C.T.)

- A **current transformer** (**CT**) is a type of transformer that is used to reduce or multiply an alternating current (AC). It produces a current in its secondary which is proportional to the current in its primary.
- The large alternating currents which cannot be sensed or passed through normal ammeters and current coils of watt meters, energy meters can easily be measured by use of current transformers along with normal low range instruments.
- A transformer is a device which consists of two windings called primary and secondary.
- It transfers energy from one side to another with suitable change in the level of current or voltage.
- A current transformer basically has a primary coil of one or more turns of heavy cross-sectional area. In some, the bar carrying high current may act as a primary. This is connected in series with the line carrying high current.
- The secondary of the transformer is made up of a large number of turns of fine wire having small cross-sectional area.
- This is usually rated for 5 A. This is connected to the coil of normal range ammeter.
- Symbolic representation of a current transformer is as shown in the Fig.(a).

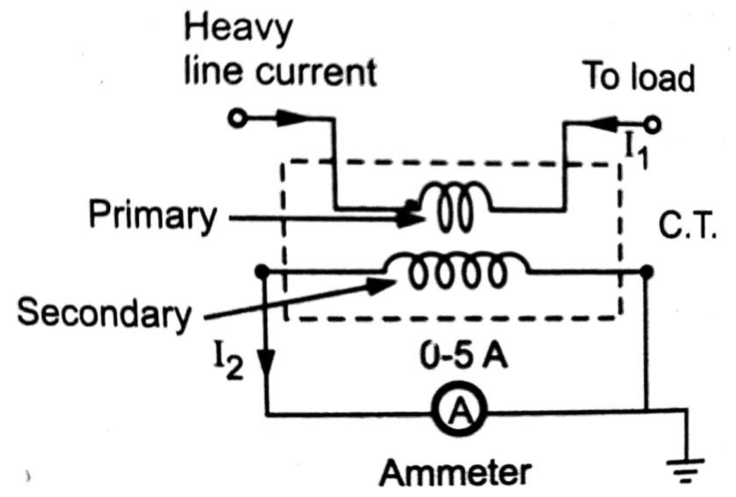
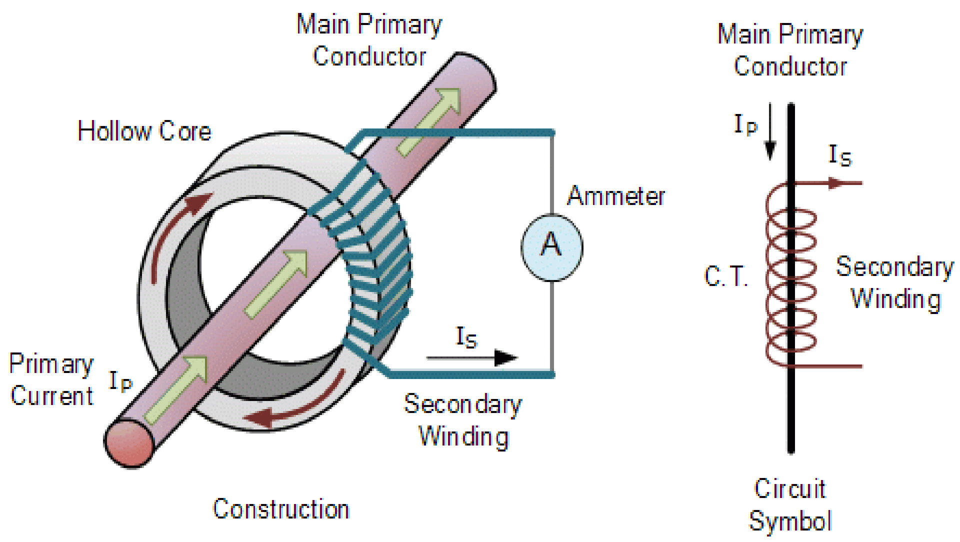


Fig (a).Current Transformer



Current Transformer

➢ Working Principle:

- The principle of the instrument transformer is fundamentally the same as that of the power transformer.
- The primary ampere turns to produce a magnetic flux in the iron core which in turn induces an EMF in the secondary winding; this causes a current to flow through the burden connected to the secondary terminals.
- These transformers are basically step up transformers i.e. stepping up a voltage from primary to secondary.
- Thus the current reduces from primary to secondary.
- So from current point of view, these are step down transformers, stepping down the current value considerably from primary to secondary.

Let

N_1 = Number of turns of primary

N_2 = Number of turns of secondary

I_1 = Primary current

I_2 = Secondary current

For a transformer,

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

- As N_2 is very high compared to N_1, the ratio I_1 to I_2 is also very high for current transformers.
- Such a current ratio is indicated for representing the range of current transformer.
- For example, consider a 500:5 range then it indicates that C.T. Steps down the current from primary to secondary by a ratio 500 to 5.

$$\therefore \quad \frac{I_1}{I_2} = \frac{500}{5}$$

- Knowing this current ratio and the meter reading on the secondary, the actual high line current flowing through the primary can be obtained.

(Q) A 250 : 5, current transformer is used along with an ammeter. If ammeter reading is 2.7 A, estimate the line current?

Solution : $$\frac{I_1}{I_2} = \frac{250}{5}$$

But as ammeter is in secondary, $I_2 = 2.7$ A

$$\therefore \quad \frac{I_1}{2.7} = \frac{250}{5}$$

$$\therefore \quad I_1 = 135 \text{ A}$$

So line current is 135 A.

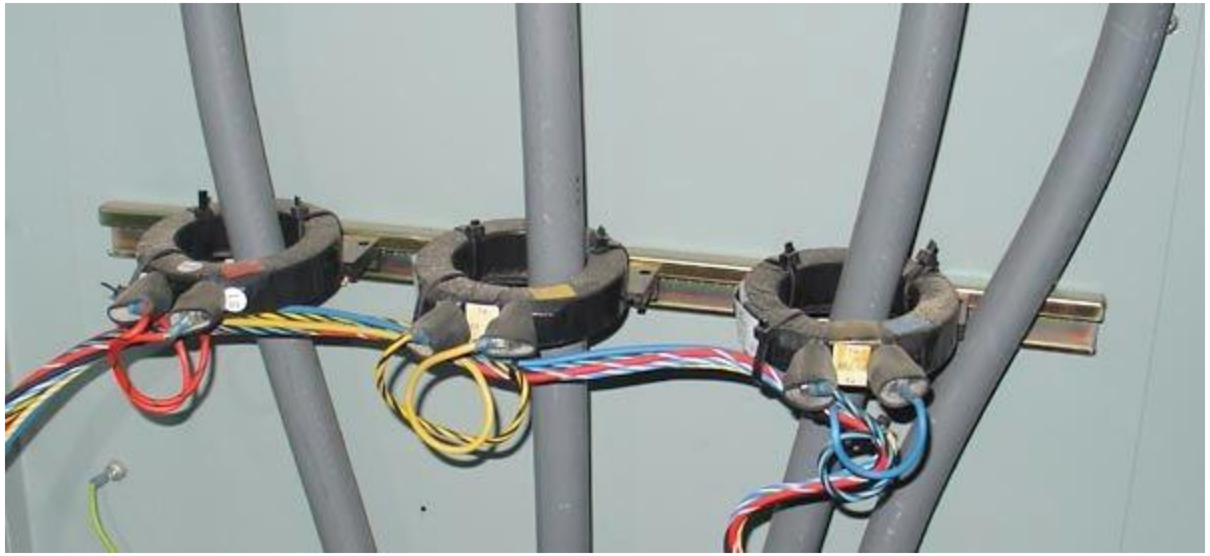


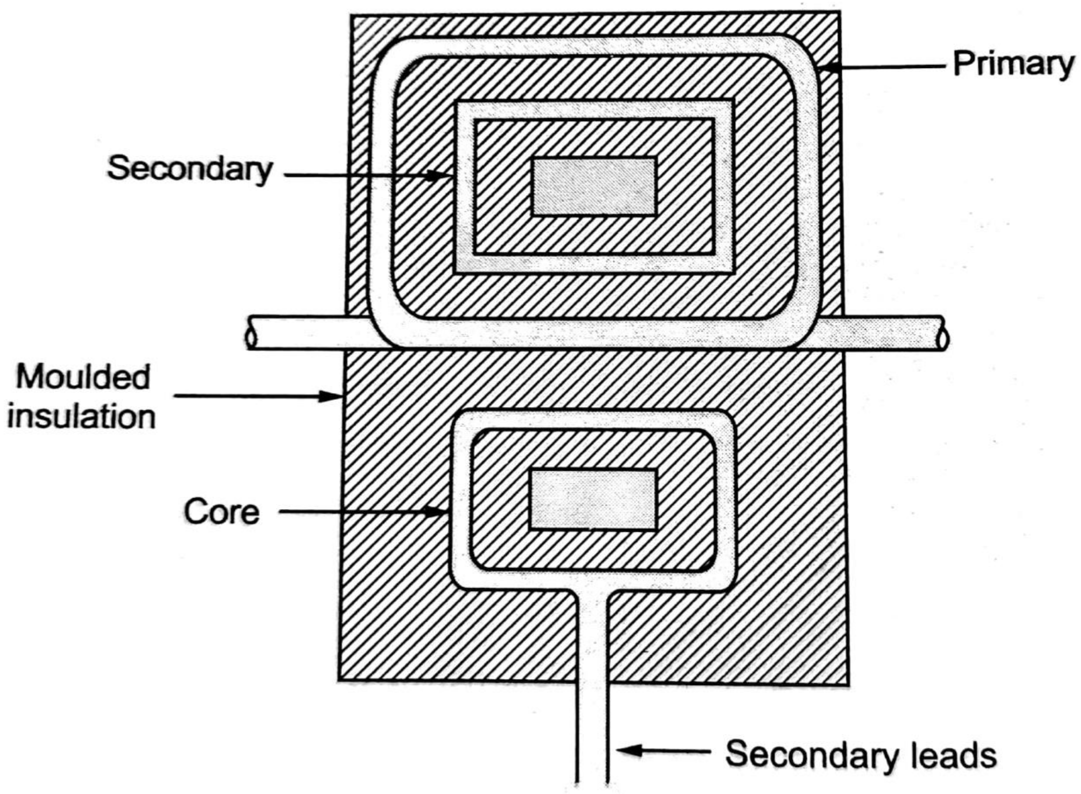
Fig.Current transformers used in metering equipment for three-phase 400-ampere electricity supply

Current Transformer

➢ Construction of Current Transformers

- There are two types of constructions used for the current transformers which are,
  1. Wound type
  2. Bar type

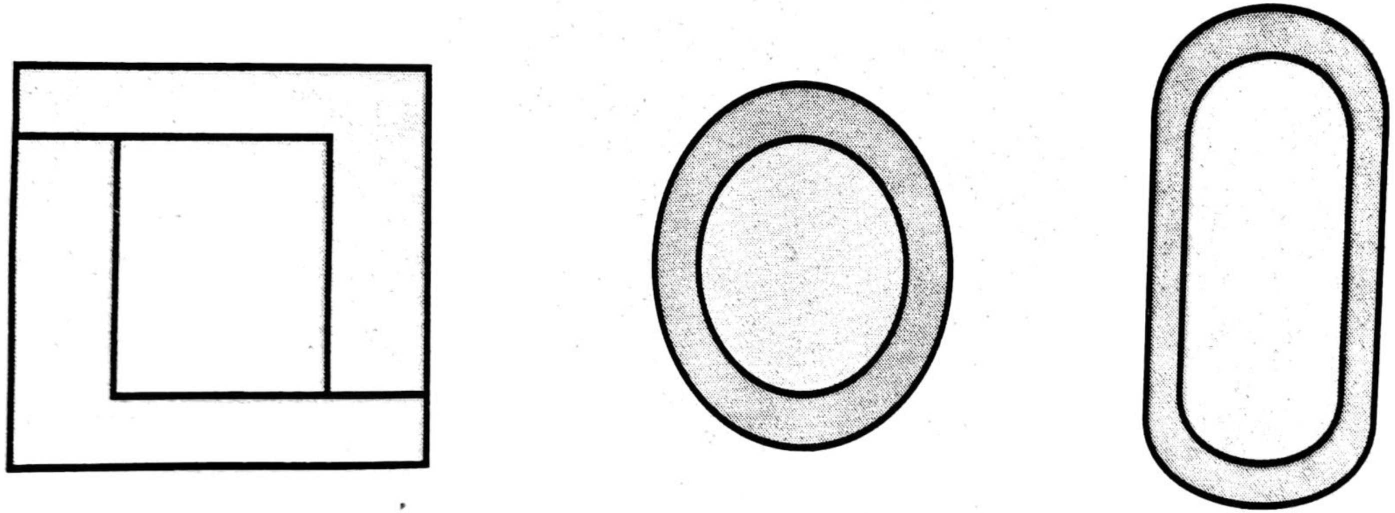
  1. Wound Type Current Transformer
- In wound type construction, the primary is wound for more than one full turn, on the core. The construction is shown in the Fig(d).
- In a low voltage wound type current transformer, the secondary winding is wound on a bakelite former.
- The heavy primary winding is directly wound on the top of the secondary winding with a suitable insulation in between the two.
- Otherwise the primary is wound completely separately and then taped with suitable insulating material and assembled with the secondary on the core.



Fig(d).Wound Type Current Transformer

Current Transformer

- The current transformers can be ring type or window type. Some commonly used shapes for the stampings of window type current transformers are shown in the figure.



(a) Rectangular (b) Ring (c) Stadium

Fig. CT for operation on a 110 kV grid

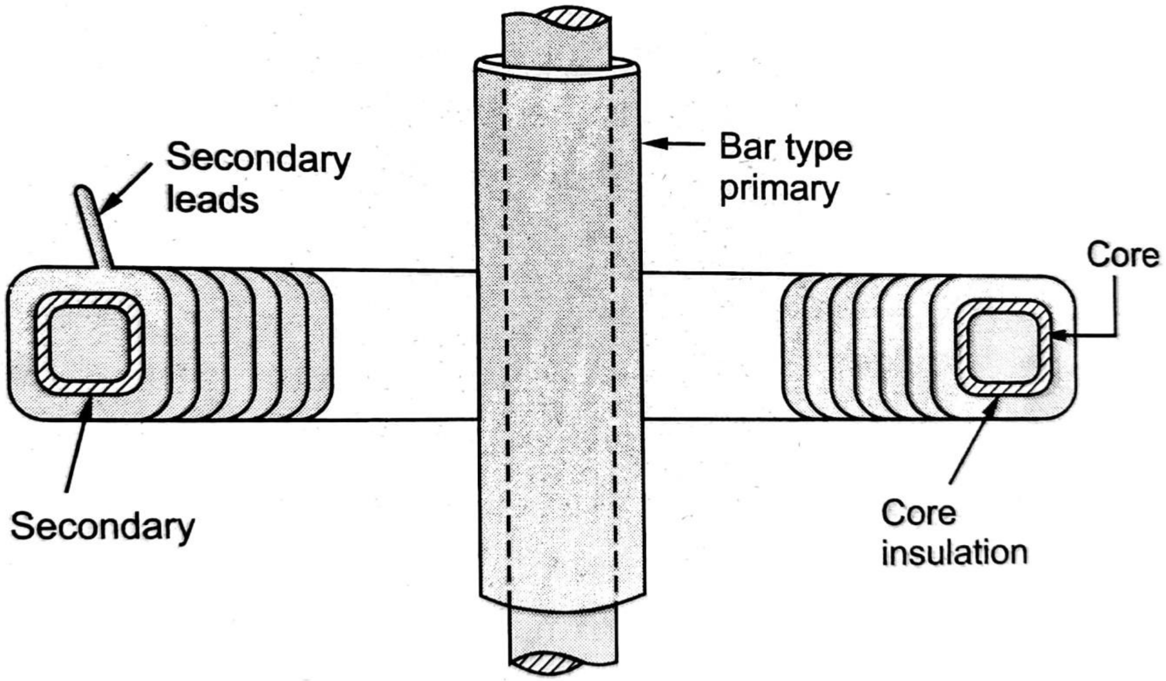
- The core material for wound type is nickel-iron alloy or an oriented electrical steel.
  Before installing the secondary winding on core it is insulated with the help of end collars and circumferential wraps of pressboards.
- Such pressboards provide additional insulation and protection to the winding from damage due to the sharp corners.

Current Transformer

2.Bar Type Current Transformer:

- In this type of current transformer, the primary winding is nothing but a bar of suitable size.
- The construction is shown in the Fig.(e).
  The insulation on the bar type primary is bakelized paper tube or a resin directly moulded on the bar.
- Such bar type primary is the integral part of the current transformer.
- The core and the secondary windings are same in bar type transformer.

- The stampings used for the laminations in current transformers must have high cross-sectional area than the ordinary transformers.
- Due to this, the reluctance of the interleaved corners remains as low as possible.
- Hence the corresponding magnetizing current is also small. The windings are placed very close to each other so as to reduce the leakage reactance.
- To avoid the corona effect, in bar type transformer, the external diameter of the tube is kept large.



Fig(e).Bar Type Current Transformer

Current Transformer

- The windings are so designed that without damage, they can withstand short circuit forces which may be caused due to short circuit in the circuit in which the current transformer is inserted.
- For small line voltages, the tape and varnish are used for insulation. For line voltages above 7 kV the oil immersed or compound filled current transformers are used.

Why Secondary of C.T. Should not be Open?

- It is very important that the secondary of C.T. should not be kept open. Either it should be shorted or must be connected in series with a low resistance coil such as current coils of wattmeter, coil of ammeter etc.
- If it is left open, then current through secondary becomes zero hence the ampere turns produced by secondary which generally oppose primary ampere turns becomes zero.
- As there is no counter m.m.f., unopposed primary m.m.f. (ampere turns) produce high flux in the core.
- This produce excessive core losses, heating the core beyond limits.
- Similarly heavy e.m.f.s will be induced on the primary and secondary side. This may damage the insulation of the winding.
- This is danger from the operator point of view as well.
- It is usual to ground the C.T. on the secondary side to avoid a danger of shock to the operator.

  Hence never open the secondary winding circuit of a current transformer while its primary winding is energized.

Derivation of Actual Ratio of Current Transformer

- Consider the equivalent circuit of a current transformer as shown in Fig.(a) along with the load

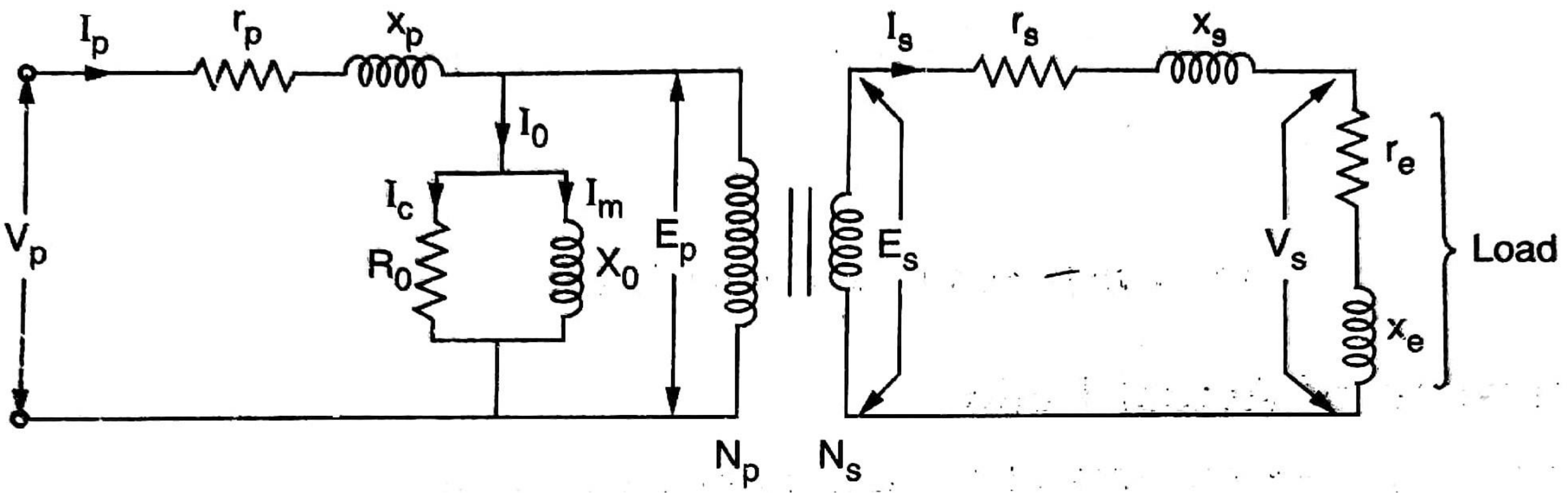


Fig.(a).Equivalent circuit of a current transformer

$$n = \text{Turns ratio} = \frac{\text{secondary turns}}{\text{primary turns}} = \frac{N_S}{N_P}$$

Where,

r_p – Resistance of primary winding

x_p – Reactance of primary winding

r_s – Resistance of secondary winding

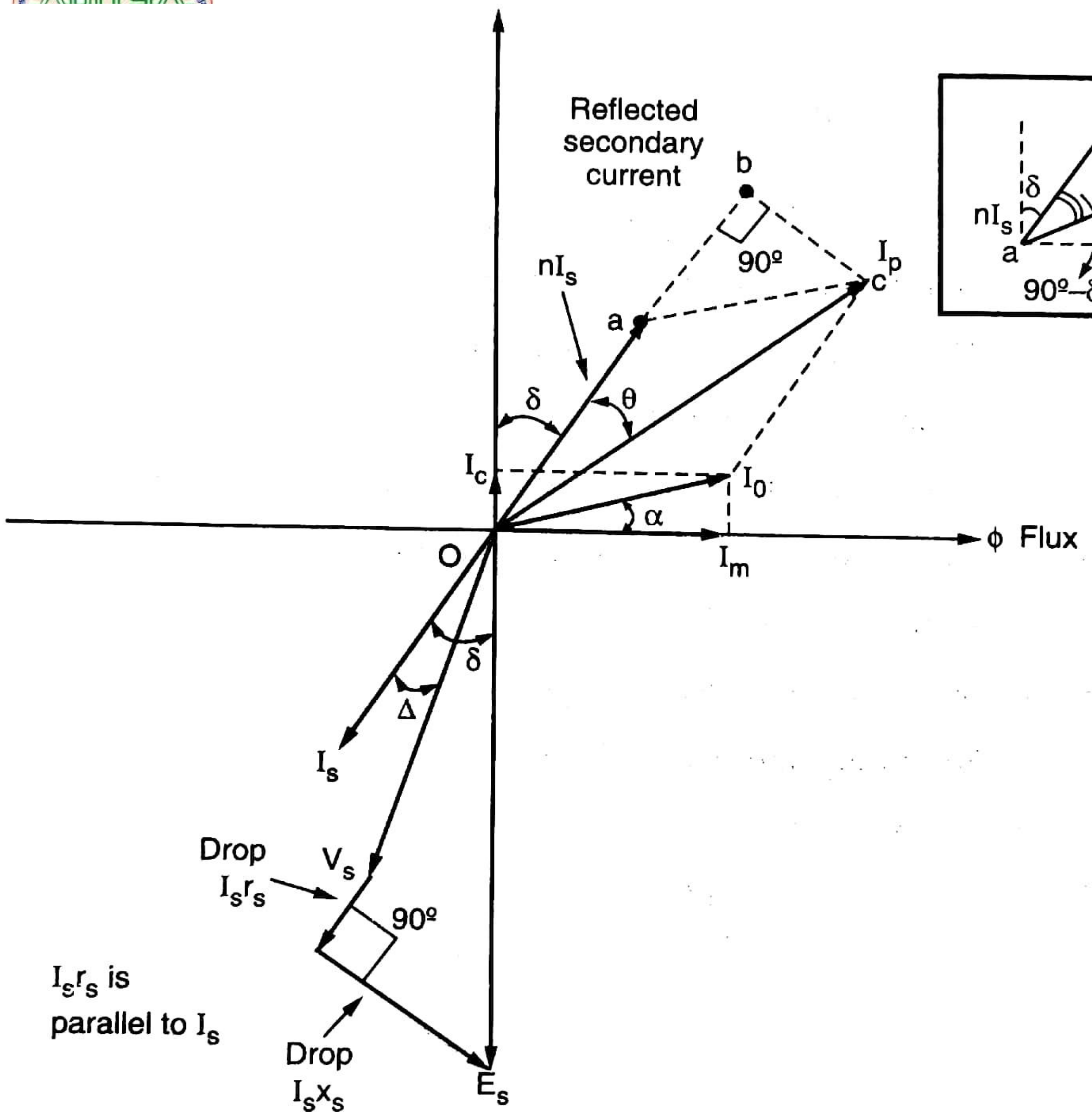
x_s – Reactance of secondary winding

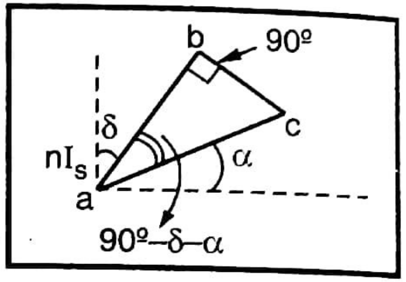
r_e – Resistance of external burden i.e. load on secondary

x_e – Reactance of external burden i.e. load on secondary

Derivation of Actual Ratio of Current Transformers

E_p – Primary induced voltage
E_S – Secondary induced voltage
V_S – Secondary terminal voltage
I_P – Primary current
I_S – Secondary current
I_0 – No load current or exciting current
I_C – Core loss component of I_0 , i.e. I_0 COS $\emptyset_0$
I_m – Magnetising component of I_0 , i.e. I_0 sin $\emptyset_0$
$\emptyset$ – Working flux of transformer
δ – Angle between E_S and I_S
– Phase angle of total impedance of secondary including burden = $\tan^{-1}\left(\frac{x_s+x_e}{r_s+r_e}\right)$
θ – Phase angle of transformer
Δ – Phase angle of load or burden i.e. $r_e + j\,x_e = \tan^{-1}\frac{x_e}{r_e}$
α – Angle between I_0 and working flux $\emptyset$





- Consider the phasor diagram of the transformer with a lagging Power factor load ,as shown in Fig (a).

Fig.(a) Phasor diagram of current transformer

Consider $\angle$ bac as shown in the small section which is,

$$\angle bac = 90° - \delta - \alpha, \quad ac = I_0, \quad Oa = n\, I_s, \quad Oc = I_p$$

$$\therefore \quad bc = ac \sin(90° - \delta - \alpha) = I_0 \sin\left[90° - (\delta + \alpha)\right] = I_0 \cos(\delta + \alpha)$$

$$\therefore \quad ab = ac \cos(90° - \delta - \alpha) = I_0 \cos\left[90° - (\alpha + \delta)\right] = I_0 \sin(\delta + \alpha)$$

From right angle triangle Obc,

$$(Oc)^2 = (Ob)^2 + (bc)^2 = (Oa + ab)^2 + (bc)^2$$

$$= \left[n\, I_s + I_0 \sin(\delta + \alpha)\right]^2 + \left[I_0 \cos(\delta + \alpha)\right]^2$$

$$= n^2 I_s^2 + 2n\, I_s\, I_0 \sin(\delta + \alpha) + I_0^2 \sin^2(\delta + \alpha) + I_0^2 \cos^2(\delta + \alpha)$$

i.e.

$$I_p = \sqrt{n^2 I_s^2 + 2n\, I_s\, I_0 \sin(\delta + \alpha) + I_0^2} \qquad \text{... (1)}$$

$$\therefore \quad \text{Actual ratio} = R = \frac{I_p}{I_s} \qquad \text{... As per definition}$$

$$\therefore \quad R = \frac{\sqrt{n^2 I_s^2 + 2n\, I_s\, I_0 \sin(\delta + \alpha) + I_0^2}}{I_s} \qquad \text{... (2)}$$

Practically for properly designed transformer $I_0 << n\, I_s$,

$$\therefore \qquad R = \frac{\sqrt{n^2\, I_s^2 + 2n\, I_s\, I_0 \sin(\delta+\alpha) + I_0^2 \sin^2(\delta+\alpha)}}{I_s}$$

... Adjusting I_0^2 as $I_0^2 \sin^2(\delta+\alpha)$

$$\therefore \qquad R = \frac{n\, I_s + I_0 \sin(\delta+\alpha)}{I_s} = n + \frac{I_0}{I_s} \sin(\delta+\alpha) \qquad \text{... (3)}$$

This is approximate value of actual ratio but practically very close to actual result.

The equation (3) can be further expanded as,

$$R = n + \frac{I_0}{I_s}[\sin\delta\cos\alpha + \cos\delta\sin\alpha]$$

But $I_0 \cos\alpha = I_m$ and $I_0 \sin\alpha = I_c$

$$\therefore \qquad R = n + \frac{I_m}{I_s} \sin\delta + \frac{I_c}{I_s} \cos\delta$$

$$\therefore \qquad \boxed{R = n + \frac{I_m \sin\delta + I_c \cos\delta}{I_s}} \qquad \text{... (4)}$$

Note that δ is **positive** for **lagging** p.f. load while **negative** for **leading** p.f. load.

Derivation of Phase angle (θ) of Current Transformers

- The phase angle o is defined as the angle between reversed secondary current phasor i.e. reflected secondary current phasor and the primary current.
- Sign convention : θ is positive if reflected secondary current leads primary current. θ is negative if secondary current lags primary current.
- As shown in phasor diagram in Fig(a).

$$\therefore \quad \theta = n I_s \wedge I_p$$

From the phasor diagram,

$$\tan\theta = \frac{bc}{Ob} = \frac{bc}{Oa+ab} = \frac{I_0\cos(\delta+\alpha)}{nI_s + I_0\sin(\delta+\alpha)}$$

Now $\tan\theta \approx \theta$ as θ is very small.

$$\therefore \quad \theta = \frac{I_0\cos(\delta+\alpha)}{nI_s + I_0\sin(\delta+\alpha)} \text{ radians} \qquad \ldots (5)$$

But $I_0 << nI_s$ hence neglecting from denominator,

$$\theta = \frac{I_0\cos(\delta+\alpha)}{nI_s} = \frac{I_0[\cos\delta\cos\alpha - \sin\delta\sin\alpha]}{nI_s}$$

$$\therefore \quad \theta = \frac{I_m\cos\delta - I_c\sin\delta}{nI_s} \text{ radians} \qquad \ldots (6)$$

Converting to degrees,

$$\boxed{\theta = \frac{180^\circ}{\pi}\left[\frac{I_m\cos\delta - I_c\sin\delta}{nI_s}\right] \text{ degrees}} \qquad \ldots (7)$$

Current Transformer

➢ Errors in Current Transformer

- For an instrument transformers, it is necessary that the transformation ratio must be exactly equal to turns ratio and phase of the secondary terms (voltage and current) must be displaced by exactly 180° from that of the primary terms (voltage and current).

✓ Two types of errors affect these characteristics of an instrument transformer which are,

1. Ratio error
2. Phase angle error

1.Ratio error:

- In practice it is said that current transformation ratio I_2/I_1 is equal to the turns ratio N_1/N_2 . But actually it is not so. The current ratio is not equal to turns ratio because of magnetizing and core loss components of the exciting current. It also gets affected due to the secondary current and its power factor. Due to this fact, large error is introduced in the measurements done by the instrument transformers. Such an error is called ratio error.

$$\% \text{ Ratio error} = \frac{\text{Nominal ratio} - \text{Actual ratio}}{\text{Actual ratio}} \times 100 = \frac{K_n - R}{R} \times 100$$

2. Phase angle error:

- The phase of secondary voltage is to be displaced by exactly 180° from that of primary voltage, for PT. but actually it is not so. The error introduced due to this fact is called phase angle error. It denoted by angle θ by which the phase difference between primary and secondary is different from 180°.

The phase angle error is given by, $$\theta = \frac{180°}{\pi}\left[\frac{I_m \cos\delta - I_c \sin\delta}{n\, I_s}\right] \text{ degrees}$$

Current Transformer

❖ A current transformer has turns ratio 1:399 and is rated as 2000/5 A. The core loss component is 3 A and magnetizing component is 8 A, under full load conditions.
Find the phase angle and ratio errors under full load condition if secondary circuit power factor is 0.8 leading

Solution: $I_c = 3\ A,\quad I_m = 8\ A,\ \cos\delta = 0.8$ **leading**

$\delta = -36.86989^{o}$, negative as leading

$$n = \frac{N_s}{N_p} = \frac{399}{1}$$

$$K_n = \frac{I_p}{I_s} = \frac{2000}{5} = 400$$

$$R = n + \frac{I_m \sin\delta + I_c \cos\delta}{I_s} \quad \text{where} \quad I_s = 5\ A \ \text{(full load)}$$

$$= 399 + \frac{8\sin(-36.869^{\circ}) + 3\times 0.8}{5} = 398.52$$

$$\%\ \text{ratio error} = \frac{K_n - R}{R}\times 100 = \mathbf{0.3713}\ \%$$

$$\theta = \frac{180}{\pi}\left[\frac{I_m \cos\delta - I_c \sin\delta}{nI_s}\right] \text{degrees}$$

$$= \frac{180}{\pi}\left[\frac{8\times 0.8 - 3\times\sin(-36.8698^{\circ})}{399\times 5}\right]$$

$$= 0.2355^{0} = 14.13'$$

Current Transformer

➢ Applications of Current Transformers

- The current transformers are used in a wide variety of applications ranging from power system control to the precise current measurement in industrial, medical, automotive and telecommunication systems. Some of the applications include

1) Extending the range of measuring instruments such as ammeter, energy meter, KVA meters, wattmeter, etc.
2) Differential circulating current protection systems.
3) Distance protection in power transmission systems.
4) Over current fault protection.

Potential Transformer

Oil-immersed 66kv potential transformer

Potential Transformers (P.T.)

- Potential transformer is a voltage step-down transformer which reduces the voltage of a high voltage circuit to a lower level for the purpose of measurement. These are connected across or parallel to the line which is to be monitored.
- The primary winding consists of a large number of turns which is connected across the high voltage side or the line in which measurements have to be taken or to be protected.
- The secondary winding has lesser number of turns which is connected to the voltmeters, or potential coils of wattmeter and energy meters, relays and other control devices. These can be single phase or three phase potential transformers.
- Irrespective of the primary voltage rating, these are designed to have the secondary output voltage of 110 V.

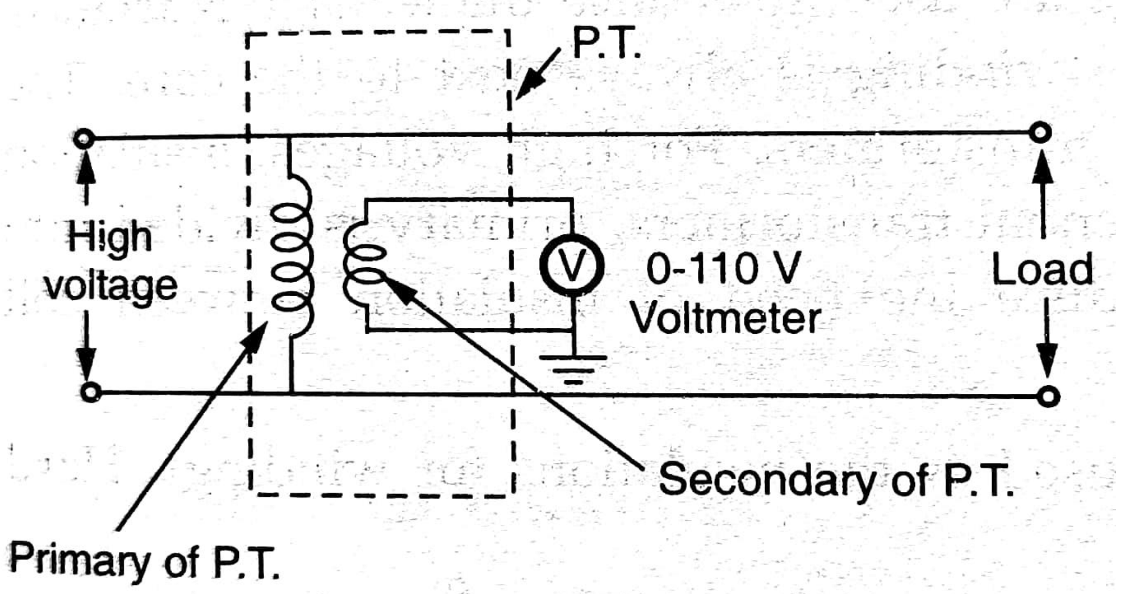


Fig.(a) Potential transformer

- One end of the secondary is always grounded for safety purpose. The connections are shown in the Fig. (a). As a normal transformer, its ratio can be specified as,

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

From the above equation, if the voltmeter reading and transformation ratio are known, then high voltage side voltage can be determined.

Potential Transformer

(Q).A 11000:110, potential transformer is used along with a volt meter reading 87.5V.Estimate the value of line voltage?

Solution: For a P.T.

$$\frac{V_1}{V_2} = \frac{11000}{110}$$

and V_2= 87.5 V

$$\therefore \quad \frac{V_1}{87.5} = \frac{11000}{110}$$

$\therefore$ V_1=8750 V

This is the value of high voltage to be measured

Potential Transformer

➢ Construction of Potential Transformer

- The potential transformer use larger core and conductor sizes compared to conventional power transformer.
- In potential transformer, economy of material is not an important consideration at the time of design.
- The accuracy is an important consideration.
  The shell type or core type construction is preferred for potential transformer.
- The shell type is used for low voltage while core type for high voltage transformers.
- At the time of assembly special core is required to reduce the effect of air gap at the joints.

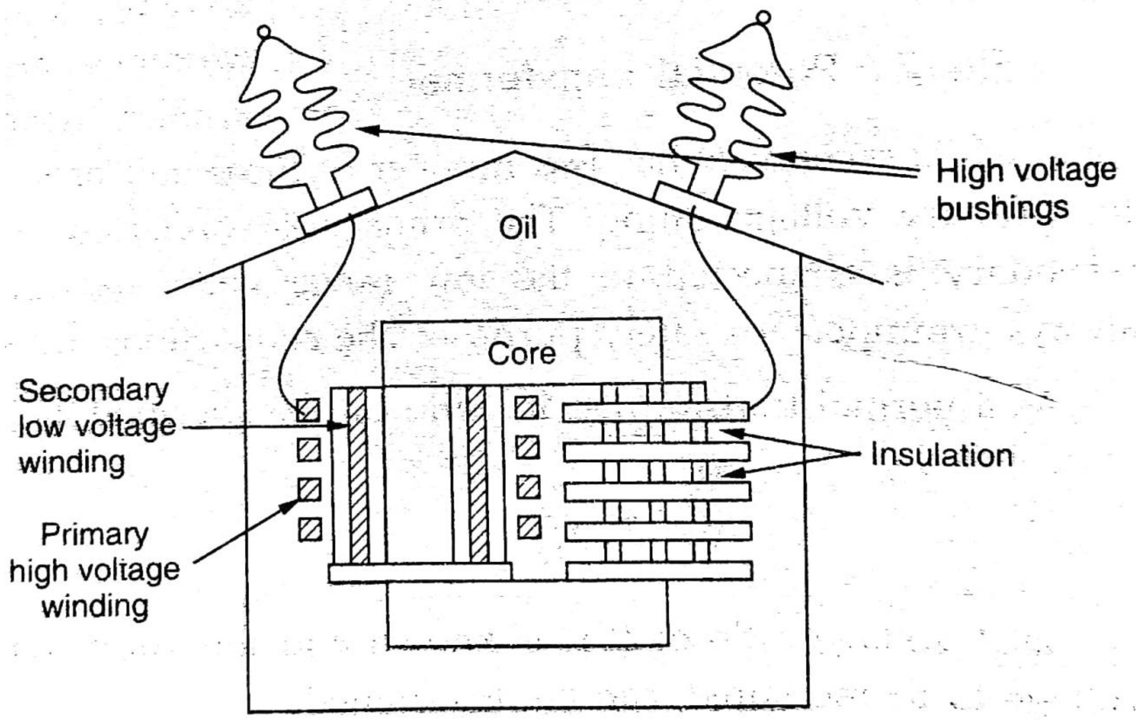


Fig.(b) Single phase potential transformer

- The coaxial primary and secondary windings are used, to reduce the leakage reactance.The secondary winding which is a low voltage winding is always next to the core. The primary winding is a single coil in low voltage transformers.
- For high voltages, insulation is the main problem. Hence in high voltage potential transformers, primary is divided into number of small sections of short coils to reduce the need of insulation between coil layers.
- The cotton tape and varnished cambric are used as the insulations for windings. Hard fiber separators are used in between the coils. The oil immersed potential transformers are used for the voltage levels above 7 kV.
  For oil filled potential transformers, oil filled bushings Core are used. Two bushings are required when no side of the line is at earth potential.The overall construction of single phase, two winding potential transformer is shown in the Fig.(b).

Derivation of Actual Ratio of Potential Transformer

- The loading of potential transformer is very small in practice hence exciting current I_0 is of the order of I_s i.e. secondary winding current. While in a normal power transformer I_0 is very small compared to I_s .
- The equivalent circuit of potential transformer is shown in the Fig.(a).

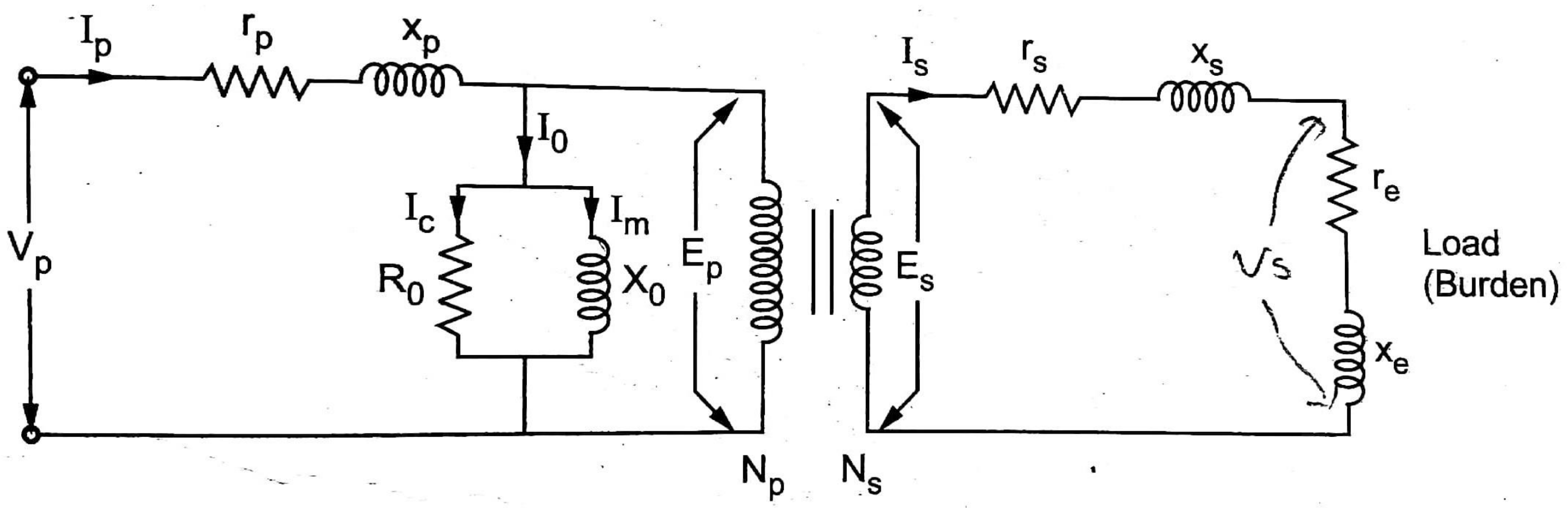


Fig. (a). Equivalent circuit of potential transformer

Potential Transformer

Φ – working flux.
N_p – Primary turns
N_s – Secondary turns
I_p – Primary current
I_s – Secondary current
I_m – Magnetizing component of I_0
I_c – Core loss component of I_0
I_0 – No load current or Excitation current
r_s – Secondary winding resistance
x_s – Secondary winding reactance
r_p – Primary winding resistance
x_p – Primary winding reactance
r_e – Resistance of external burden i.e. load on secondary
x_e – Reactance of external burden i.e. load on secondary
E_p – Primary induced emf.
E_s – Secondary induced emf.
Δ – Phase angle of load or burden i.e. $r_e + j\, x_e \;= \tan^{-1}\frac{x_e}{r_e}$
V_s – Secondary terminal voltage.
V_p – Primary applied voltage.
For P.T. – Turns ratio (n) = $\frac{N_p}{N_s} = \frac{E_p}{E_s}$

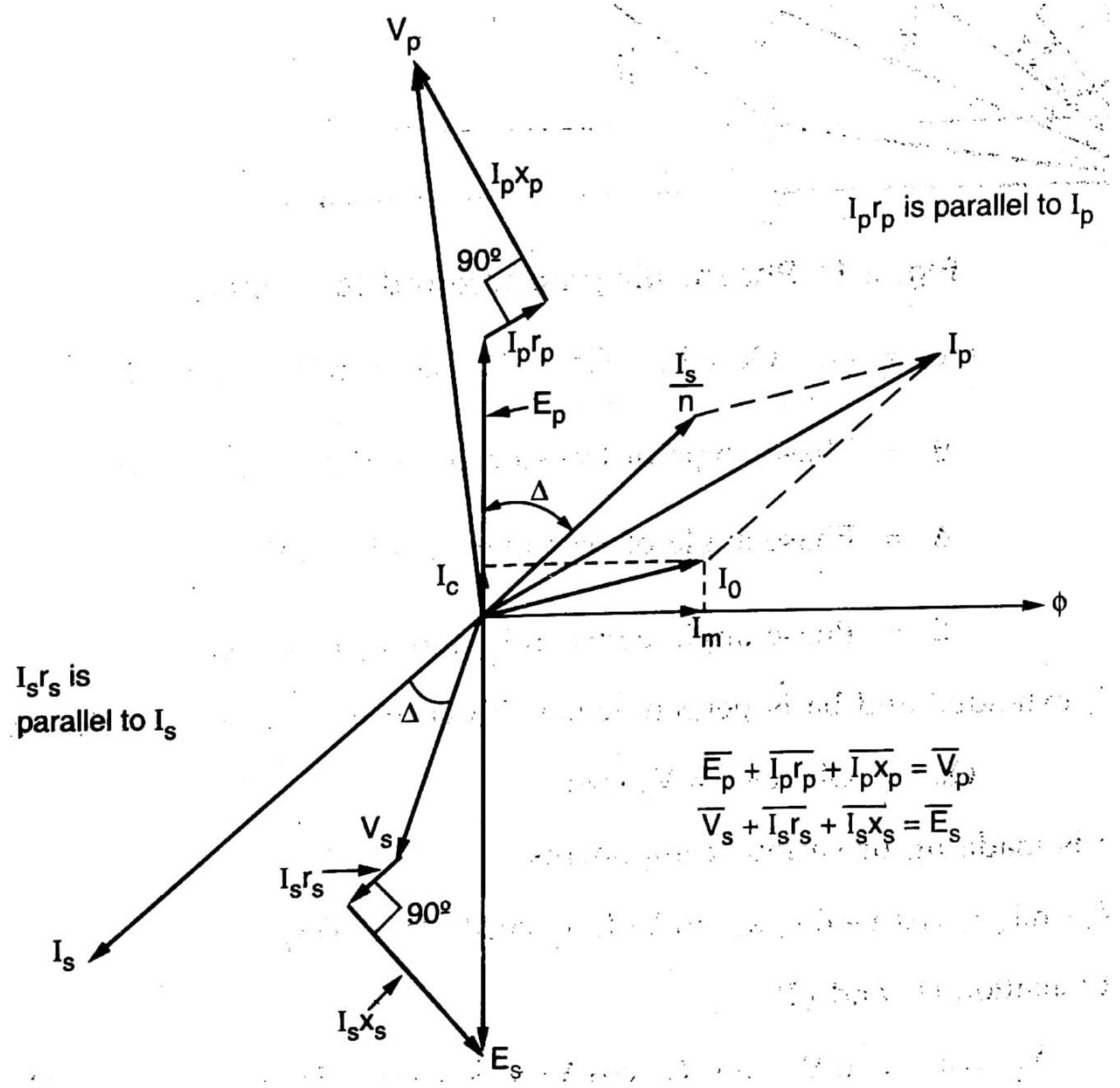


Fig.(b). Phasor diagram of a Potential transformer

Potential Transformer

- Consider the phasor diagram with all the quantities referred to the primary side. In the phasor diagram Fig.(c),

$$Oe = I_0,\quad Oc = I_p,\quad Of = \frac{I_s}{n},\quad Og = n\,V_s,\quad Ob = V_p$$

θ = Phase angle of transformer = V_p^ V_s , reversed

Δ = Phase angle of secondary load = n V_s ^ $\frac{I_s}{n}$

β = Phase angle between I_p and V_s reversed

Oa is n V_s , extended and ba is perpendicular drawn from b on n V_s, extended.

$\therefore$ Oa = Ob cos θ = V_p cosθ ....(1)

Now Oa is made up of various components.

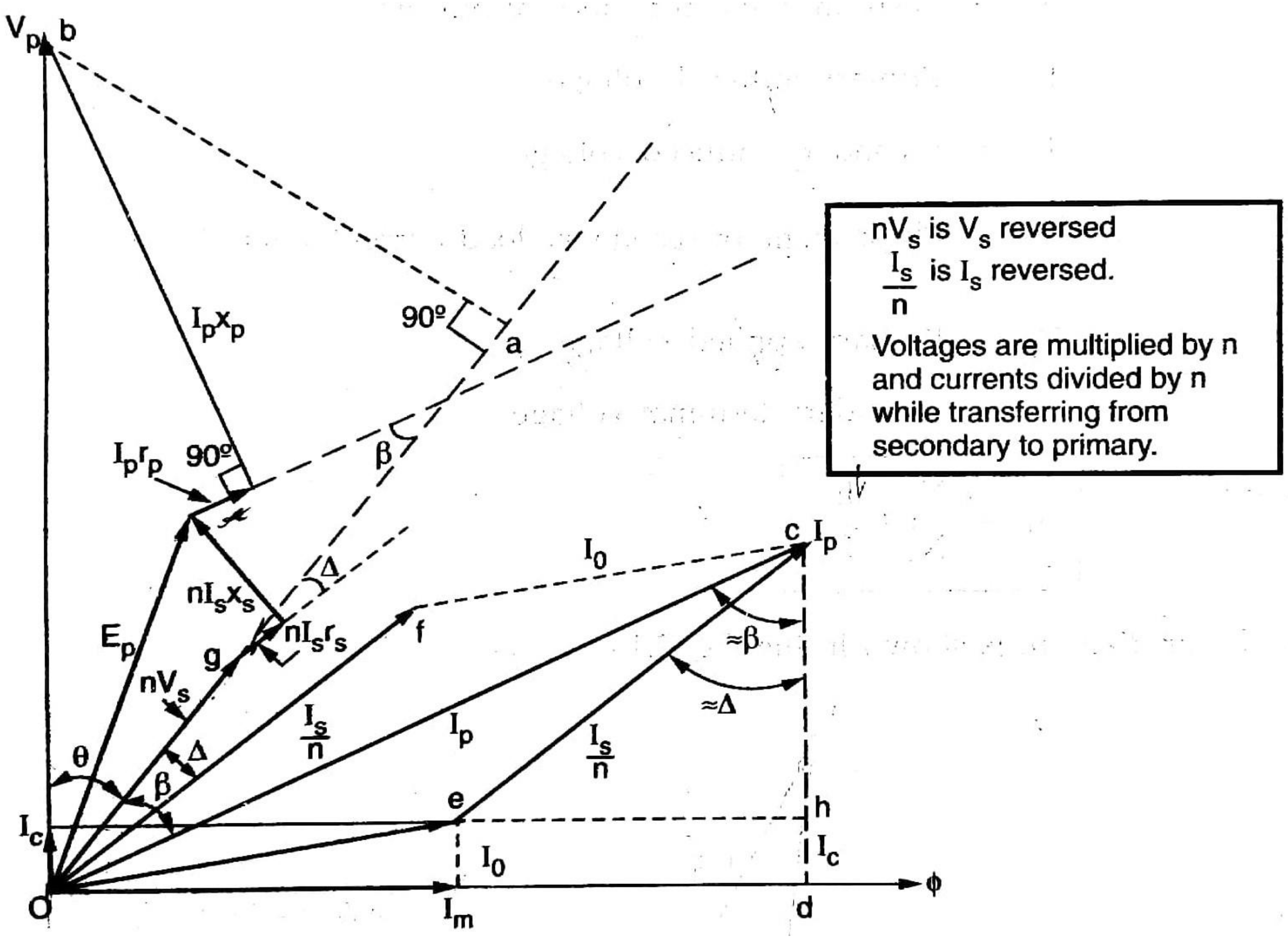


Fig.(c). Phasor diagram referred to primary

Potential Transformer

$$Oa = n\,V_s + n\,I_s\,r_s\cos\Delta + n\,I_s\,x_s\sin\Delta + I_p\,r_p\cos\beta + I_p\,x_p\sin\beta \qquad \dots (2)$$

Equating equation (1) and (2)

$$\therefore \quad V_p\cos\theta = n\,V_s + n\,I_s(r_s\cos\Delta + x_s\sin\Delta) + I_p(r_p\cos\beta + x_p\sin\beta) \qquad \dots (3)$$

For potential transformer the load is nothing but a voltmeter hence is very small hence both V_P as well as n V_S , can be assumed to be perpendicular to $\emptyset$.

Thus approximately, $\angle Ocd \approx \beta$ and $\angle ecd \approx \Delta$

$$\therefore \quad cd = I_p\cos\beta = ch + hd = \frac{I_s}{n}\cos\Delta + I_c \qquad \dots (4)$$

and

$$I_p\sin\beta = I_m + \frac{I_s}{n}\sin\Delta \qquad \dots (5)$$

As θ is very very small, cos θ = 1

$$\therefore \quad V_P\cos\theta = V_P \qquad \dots (6)$$

Using in equation (3), all the results of equations (4), (5) and (6)

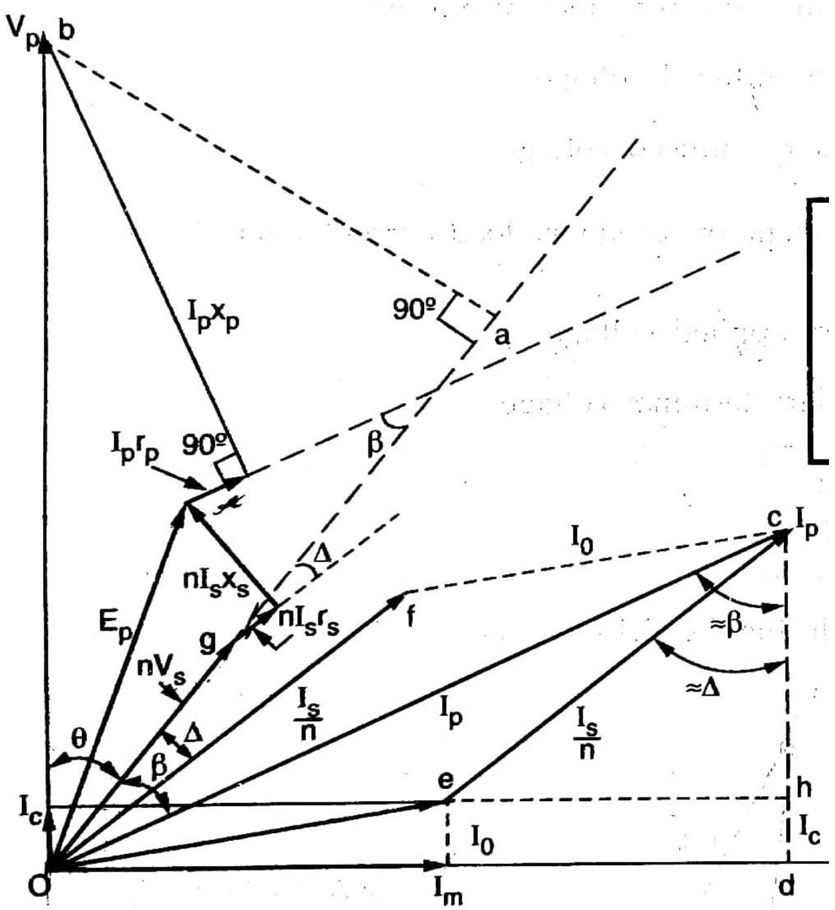


Fig.(c). Phasor diagram referred to primary

Potential Transformer

$$V_p = n V_s + n I_s (r_s \cos\Delta + x_s \sin\Delta) + r_p\left[\frac{I_s}{n}\cos\Delta + I_c\right] + x_p\left[I_m + \frac{I_s}{n}\sin\Delta\right]$$

$$= n V_s + I_s \cos\Delta\left(n r_s + \frac{r_p}{n}\right) + I_s \sin\Delta\left(n x_s + \frac{x_p}{n}\right) + I_c r_p + I_m x_p$$

$$\therefore \quad V_p = n V_s + \frac{I_s}{n}\cos\Delta\left(n^2 r_s + r_p\right) + \frac{I_s}{n}\sin\Delta\left(n^2 x_s + x_p\right) + I_c r_p + I_m x_p \qquad \ldots (7)$$

Now $n^2 r_s + r_p = R_{1e}$ = Equivalent resistance referred to primary

$n^2 x_s + x_p = X_{1e}$ = Equivalent reactance referred to primary

$$\therefore \quad V_p = n V_s + \frac{I_s}{n}\left[R_{1e}\cos\Delta + X_{1e}\sin\Delta\right] + I_c r_p + I_m x_p \qquad \ldots (8)$$

Thus the actual ratio is,

$$R = \frac{V_p}{V_s} = n + \frac{\frac{I_s}{n}\left[R_{1e}\cos\Delta + X_{1e}\sin\Delta\right] + I_c r_p + I_m x_p}{V_s} \qquad \ldots (9)$$

Potential Transformer

- **The result also can be derived in terms of parameters referred to secondary. The equation of V_P, can be written as,**

$$V_p = n V_s + \frac{n^2}{n} I_s \cos\Delta\left(r_s + \frac{r_p}{n^2}\right) + \frac{n^2}{n} I_s \sin\Delta\left(x_s + \frac{x_p}{n^2}\right) + I_c r_p + I_m x_p$$

$$\text{But} \quad r_s + \frac{r_p}{n^2} = R_{2e} = \text{Equivalent resistance referred to secondary}$$

$$x_s + \frac{x_p}{n^2} = X_{2e} = \text{Equivalent reactance referred to secondary}$$

$$\therefore \quad V_p = n V_s + n I_s [R_{2e} \cos\Delta + X_{2e} \sin\Delta] + I_c r_p + I_m x_p \quad \ldots (10)$$

$$R = \frac{V_p}{V_s} = n + \frac{n I_s [R_{2e} \cos\Delta + X_{2e} \sin\Delta] + I_c r_p + I_m x_p}{V_s} \quad \ldots (11)$$

Potential Transformer

➢ Derivation of Phase Angle θ

From the phasor diagram shown in the Fig. (c).

$$\tan\theta = \frac{ab}{Oa} = \frac{I_p\, x_p \cos\beta - I_p\, r_p \sin\beta + n\, I_s\, x_s \cos\Delta - n\, I_s\, r_s \sin\Delta}{n\, V_s + n\, I_s\, r_s \cos\Delta + n\, I_s\, x_s \sin\Delta + I_p\, r_p \cos\beta + I_p\, x_p \sin\beta}$$

In the expression of Oa, the terms other than n V_s , are very small and can be neglected. Similarly as θ is very small, tan θ = θ

$$\therefore \quad \theta = \frac{I_p\, x_p \cos\beta - I_p\, r_p \sin\beta + n\, I_s\, x_s \cos\Delta - n\, I_s\, r_s \sin\Delta}{n\, V_s}$$

$$= \frac{x_p\left[\frac{I_s}{n}\cos\Delta + I_c\right] - r_p\left[I_m + \frac{I_s}{n}\sin\Delta\right] + n\, I_s\, x_s \cos\Delta - n\, I_s\, r_s \sin\Delta}{n\, V_s}$$

$$= \frac{I_s \cos\Delta\left(\frac{x_p}{n} + n\, x_s\right) - I_s \sin\Delta\left(\frac{r_p}{n} + n\, r_s\right) + I_c\, x_p - I_m\, r_p}{n\, V_s}$$

$$= \frac{\frac{I_s \cos\Delta}{n}\left(x_p + n^2 x_s\right) - \frac{I_s \sin\Delta}{n}\left(r_p + n^2 r_s\right) + I_c\, x_p - I_m\, r_p}{n\, V_s}$$

Now $r_p + n^2 r_s = R_{1e}$ and $x_p + n^2 x_s = X_{1e}$

$$\therefore \qquad \theta = \frac{\frac{I_s}{n}\cos\Delta\, X_{1e} - \frac{I_s}{n}\sin\Delta\, R_{1e} + I_c x_p - I_m r_p}{n V_s}$$

$$\therefore \qquad \theta = \frac{\frac{I_s}{n}(X_{1e}\cos\Delta - R_{1e}\sin\Delta) + I_c x_p - I_m r_p}{n V_s} \text{ radians} \qquad \ldots (12)$$

Interms of quantities referred to secondary,

$$R_{2e} = \frac{R_{1e}}{n^2} \quad \text{and} \quad X_{2e} = \frac{X_{1e}}{n^2}$$

$$\therefore \qquad \theta = \frac{n I_s (X_{2e}\cos\Delta - R_{2e}\sin\Delta) + I_c x_p - I_m r_p}{n V_s}$$

$$\theta = \frac{I_s}{V_s}(X_{2e}\cos\Delta - R_{2e}\sin\Delta) + \frac{I_c x_p - I_m r_p}{n V_s} \text{ radians} \qquad \ldots (13)$$

- It can be noted that the phase angle θ is treated positive when V_S , revered i.e n V_S , leads the primary winding voltage V_P . The θ is treated negative when n V_S , lags the primary winding voltage V_P.
- Once R and are obtained then the errors in potential transformers are,

  % ratio error = $\frac{K_n - R}{R} \times 100$ and Phase angle error = θ radians

(Q).A single phase potential transformer has a turns ratio of 3810/63. The nominal secondary voltage is 63 V and the total equivalent resistance and leakage reactance referred to the secondary side are 2 Ω and 1 Ω respectively. Calculate the ratio and phase angle errors when the transformer is supplying a burden of 100 + j 200 Ω

Solution: $R_{2e} = 2\Omega$

$X_{2e} = 1\Omega$

$V_S = 63\ V$

$$n = \frac{3810}{63} = 60.4761$$

Neglecting no load component of current

$$R = n + \frac{nI_s[R_{2e}\cos\Delta + X_{2e}\sin\Delta]}{V_s}$$

$$\text{Burden} = r_e + jx_e = 100 + j\,200\ \Omega$$

$$\therefore \quad \Delta = \tan^{-1}\frac{x_e}{r_e} = 63.434948^\circ$$

$$\therefore \quad R = 60.4761 + \frac{60.4761\,[2\times\cos\Delta + \sin\Delta]}{\frac{V_s}{I_s}}$$

$$\text{Now} \quad \frac{V_s}{I_s} = Z_s = \sqrt{100^2 + 200^2} = 223.6067\ \Omega$$

$$\therefore \quad R = 60.9599 \text{ and } K_n = \text{Nominal ratio} = n = 60.4761$$

$$\therefore \quad \%\text{ ratio error} = \frac{K_n - R}{R}\times 100 = -0.799\ \% \approx -\mathbf{0.8}\ \%$$

$$\theta = \frac{I_s}{V_s}[X_{2e}\cos\Delta - R_{2e}\sin\Delta] \text{ in radians}$$

$$= \frac{1}{223.6067}[\cos 63.43 - 2\sin 63.43]$$

$$= -5.9\times10^{-3}\text{ rad} = \frac{-5.9\times10^{-3}\times180^\circ}{\pi}\text{ degrees}$$

$$= \mathbf{-0.338^\circ} \quad \text{... Phase angle error}$$

Potential Transformer

Applications of Potential(Voltage) Transformers

1) Electrical Metering systems
2) Electrical protection systems
3) Distance protection of feeders
4) Synchronizing generators with grid
5) Impedance protection of generators

- The class of potential transformers used for metering is called as measurement voltage or potential transformers.
- On other hand PTs used for protection called as protection voltage transformers. In some cases PTs are used for both metering and protection purposes, in such cases, one secondary winding is connected to metering and other secondary winding is used for protection.

Thank You

ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

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ELECTRICAL MEASUREMENTS AND INSTRUMENTATION
Syllabus

UNIT–I	Analog Ammeter and Voltmeters
UNIT–II	Analog Watt meters and Power Factor Meters
UNIT–III	Measurements of Electrical parameters
UNIT–IV	Transducers
UNIT–V	Digital meters

Text Books:

- Electrical & Electronic Measurement & Instruments by A.K.Sawhney Dhanpat Rai & Co.Publications
- Electrical and Electronic Measurements and instrumentation by R.K.Rajput, S.Chand
- Electrical Measurements and measuring Instruments by E.W. Golding and F.C.Widdis, fifth Edition, Wheeler Publishing
- Electrical Measurements by Buckingham and Price, Prentice – Hall

ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

UNIT-II

Analog Watt meters and Power Factor Meters

UNIT–II

Analog Watt meters and Power Factor Meters

Topics:

- Electrodynamometer type wattmeter (LPF and UPF),
- Power factor meters: Dynamometer and M.I type (Single phase and Three phase)
- construction, theory, torque equation,
- advantages and disadvantages
- Numerical problems

Analog Watt meters

➢ Introduction

- The **wattmeter** is an instrument for measuring the electric power in watts of any given circuit.

$$P = VI\ Cos\emptyset$$

- These wattmeters are used in various laboratories, industries for specific purposes.
- These wattmeters plays an important role in measurement of power in distribution and transmission of power.

➢ Types of Wattmeters:

1) Electrodynamometer Type Wattmeter
2) Induction Type Wattmeter
3) Electrostatic Type Wattmeter

➢ Electrodynamometer Type Wattmeter

- A dynamometer type wattmeter is basically a moving coil instrument. But in this case, the operating field is produced by the fixed coil instead of a permanent magnet.
- A wattmeter in which fixed coils produce the operating field is known as a dynamometer type wattmeter.
- The traditional analog wattmeter is an electrodynamic instrument
- A dynamometer type wattmeter is most commonly employed for measurement of power in a.c as well as d.c circuits.

➢ Principle of Electrodynamometer Type Wattmeter:

- ❑ Dynamometer type wattmeter works on very simple principle and this principle can be stated as when any current carrying conductor is placed inside a magnetic field, it experiences a mechanical force and due to this mechanical force deflection of conductor takes place.
- ❑ It is based on the principle that mechanical force exists between two current carrying conductors.

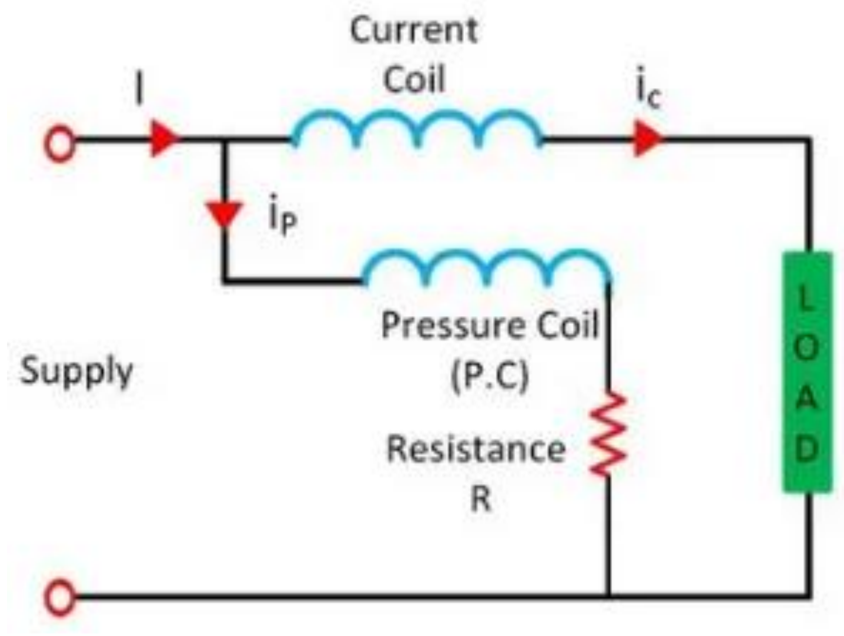
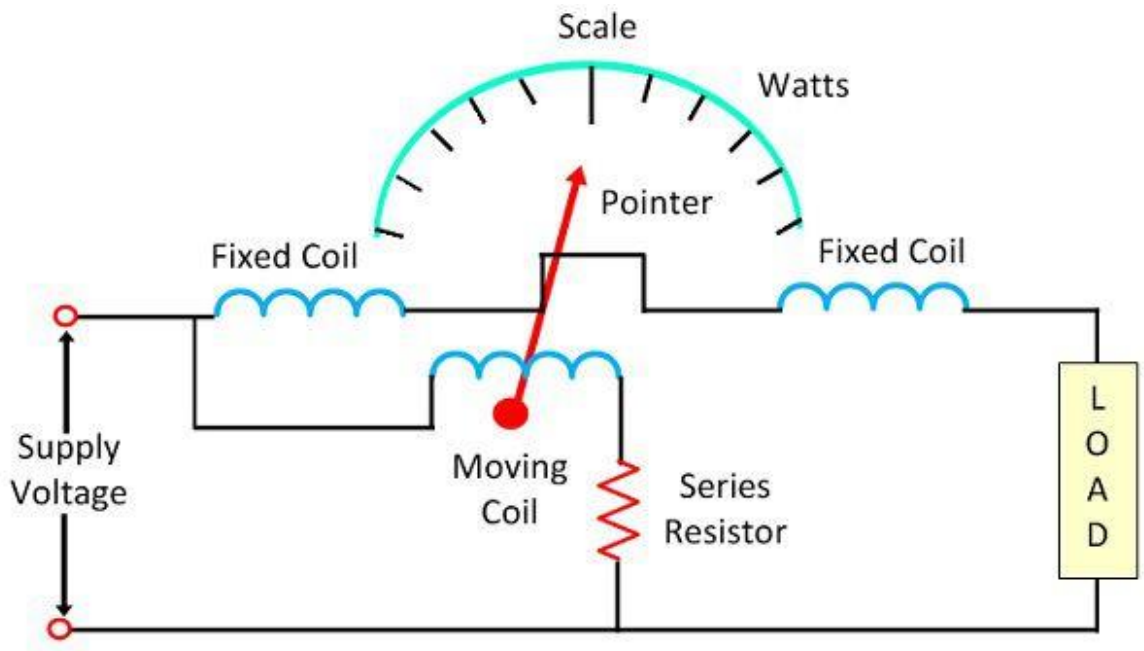


Fig.circuit diagram of the electrodynamometer wattmeter

Construction of Dynamometer type wattmeter:

- It essentially consists of two coils, namely fixed coil and moving coil.
- The fixed coil is split into two equal parts which are placed close together and parallel to each other.
- The fixed coils are connected in series with the load and carry the circuit current. It is, therefore called current coil.
- The moving coil is pivoted between the two fixed coils and is placed on the spindle to which the pointer is attached.
- The moving coil is connected across the load and carries current proportional to the voltage. It is therefore called potential coil.
- The current flows through the coil increases their temperature. So Generally, a high resistance is connected in series with potential coil to limit the current through it.



Electrodynameter Wattmeter

- Only spring controlled systems are used in these types of wattmeter. Gravity controlled system cannot be employed because there will be appreciable amount of errors.
- Air friction damping is used, as eddy current damping will distort the weak operating magnetic field and thus it may leads to error.
- The instruments use a linear scale because their moving coil moves linearly. The apparatus uses the knife edge pointer for removing the parallax error which causes because of oversights.

➢ Working of Dynamometer type wattmeter:

- When power is to be measured in a circuit, the instrument is suitably connected in the circuit. The current coil is connected in series with load so that it carries the circuit current. The potential coil is connected across the load so that it carries current proportional to the voltage.
- Due to the current in the coils, mechanical force exists between them. The result is that the moving coil, moves the pointer over the scale. The pointer comes to rest at a position where deflecting torque is equal to the controlling torque.
- Reversing the current, reverses the field due to fixed coil as well as the current in the moving coil so that the direction of the deflection torque remains unchanged. Therefore, such instruments can be used for the measurement of a.c as well as d.c power.

➢ Advantages of Dynamometer Type Wattmeter

1) It can be used both on AC and DC circuits.
2) It has a uniform scale.
3) We can obtain a high degree of accuracy through careful design.

➢ Disadvantages of Dynamometer Type Wattmeter

1) At low power factors, the inductance of the potential coil causes serious errors.
2) The reading of the instrument may be affected by stray fields acting on the moving coil. To prevent it, magnetic shielding is provided by enclosing the instrument in an iron case.

Single Phase Dynamometer Wattmeter

- An electrodynamometer type wattmeter is used to measure power. It has two coils, fixed coil which is current coil and moving coil which is pressure coil or voltage coil.
- The current coil carries the current of the circuit while pressure coil carries current proportional to the voltage in the circuit. This is achieved by connecting a series resistance in voltage circuit.

The connections of an electrodynamometer wattmeter in the circuit are shown in the Fig. (a).

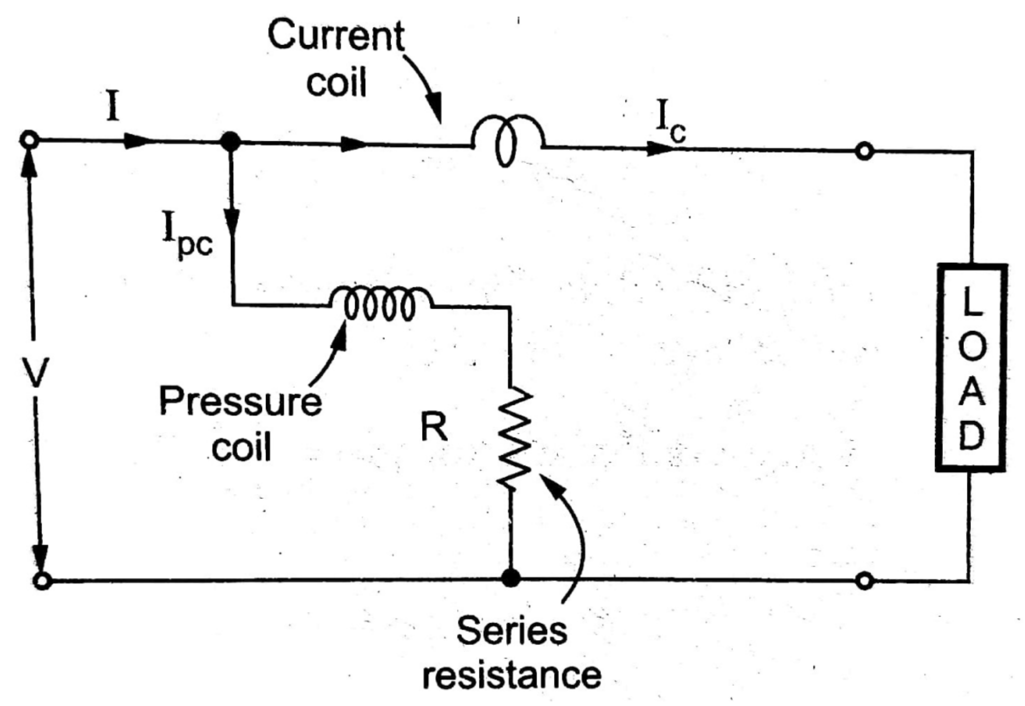


Fig. (a) Electrodynamometer wattmeter

I_c = Current through current coil
I_{pc} = Current through pressure coil
R = Series resistance
V = R.M.S. value of supply voltage
I = R.M.S. value of current
$\emptyset$ = phase angle between V and I

Effect of mutual inductance on the deflecting torque :

- The mutual inductance between the fixed and moving coils varies as the moving coil moves and the deflecting torque is affected due to change in mutual inductance.
- Hence it is necessary to study the effect of change of mutual inductance on the deflecting torque.

As shown in Fig(a).

Let

i_1, i_2 = Instantaneous values of currents in two coils, and

V, I = R.M.S. values of voltage and current being measured and

M = Mutual inductance between the current coil and potential or pressure coil.

Then, the instantaneous torque of a dynamometer type of instrument,

$$T_i = i_1 i_2 \frac{dM}{d\theta}$$

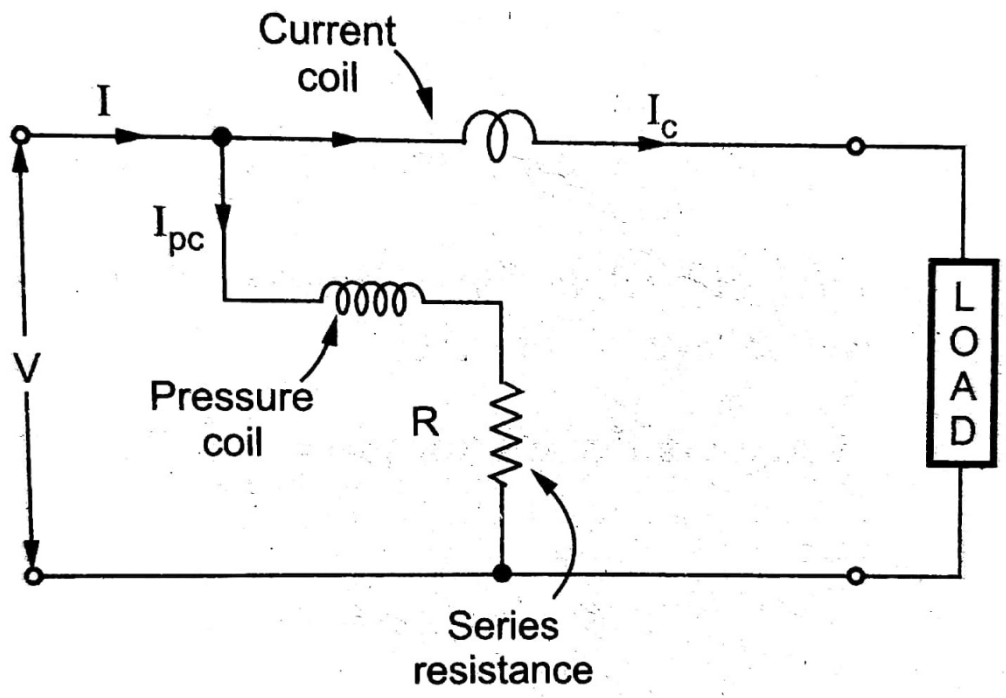


Fig.(a)circuit of dynamometer wattmeter

Torque Equation

According to theory of electrodynamic instruments,

$$T_i = i_1 i_2 \frac{dM}{d\theta} \qquad \text{........(1)}$$

Let v = Instantaneous voltage across the pressure coil circuit = $V_m \sin \omega t = \sqrt{2}\, V \sin \omega t$ ....(2)

Due to high series resistance, pressure coil is treated to be purely resistive.so current I_{pc} is in phase with the voltage V

i_{pc} = Instantaneous value = $\dfrac{v}{R_p}$ where $R_p = r_{pc} + R$

$$i_{pc} = \frac{\sqrt{2}\, V}{R_p} \sin \omega t = \sqrt{2}\, I_{pc} \sin \omega t \qquad \text{.......(3)}$$

If the current in the current coil lags the voltage by angle Ø then instantaneous value of the current through coil is

$$\therefore \quad i_c = \sqrt{2} I_c \sin(\omega t - \phi) \qquad \ldots (4)$$

Now $i_1 = i_c$ and $i_2 = i_{pc}$ hence,

$$T_i = [\sqrt{2} I_{pc} \sin \omega t][\sqrt{2} I_c \sin(\omega t - \phi)] \frac{dM}{d\theta}$$

$$= 2 I_c I_{pc} \sin(\omega t) \sin(\omega t - \phi) \frac{dM}{d\theta}$$

$$\therefore \quad T_i = I_c I_{pc} [\cos \phi - \cos(2\omega t - \phi)] \frac{dM}{d\theta} \qquad \ldots (5)$$

- Thus instantaneous torque has a component of power which varies as twice the frequency of current and voltage(refer to term containing $2\omega t$).

$$T_d = \text{Average deflecting torque} = \frac{1}{T}\int_0^T T_i \, d(\omega t)$$

$$= \frac{1}{T}\int_0^T I_c I_{pc} \left[\cos\phi - \cos(2\omega t - \phi)\right] \frac{dM}{d\theta} \, d(\omega t)$$

$$\therefore \quad T_d = I_c I_{pc} \cos\phi \frac{dM}{d\theta} \qquad \ldots\ldots (6)$$

Where $\quad I_{pc} = \frac{V}{R_p}$

For spring controlled wattmeter,

$$T_c = K\,\theta \qquad \ldots\ldots\ldots(7)$$

At balance position $T_d = T_c$

$$I_c I_{pc} \cos\phi \frac{dM}{d\theta} = K\,\theta$$

$$\therefore \quad \boxed{\theta = \frac{1}{K} I_c I_{pc} \cos\phi \frac{dM}{d\theta} = K_1 I_c I_{pc} \cos\phi}$$

where $\quad K_1 = \frac{1}{K}\frac{dM}{d\theta}$

$$\therefore \quad \boxed{\theta = K_1 I_c \frac{V}{R_p} \cos\phi = K_2 P}$$

where $\quad K_2 = \frac{K_1}{R_p}$ and $P = V I_c \cos\phi = \text{Power}$

$$\therefore \quad \boxed{\theta \propto P}$$

Thus the wattmeter deflection when calibrated gives the Power consumption of the circuit

Errors in Electrodynamometer Wattmeter

The following are the errors in the Electrodynamometer Wattmeter

1.Pressure Coil Inductance – The pressure coil of the Electrodynamometer has some inductance. Because of the inductance, the current of the pressure coils lags behind the voltage. Thus, the power factor of the wattmeter becomes lagging, and the meter reads high reading.

2.Pressure Coil Capacitance – The pressure coil has capacitances along with the inductance. This capacitance increases the power factor of the instrument. Hence causes the error in the reading.

3.Error due to Mutual Inductance Effect – The mutual inductance between the pressure and current coil produces an error.

4.Eddy Current Error – The eddy current induces in the coil creates its own magnetic field. This field affects the main current flows through the coil. Thus, the error occurs in the reading.

5.Stray Magnetic Field – The stray magnetic field disturbs the main magnetic field of the Electrodynamic Wattmeter. Thus, affect their reading.

6.Temperature Error – The variation in temperature will change the resistance of the pressure coil. The movement of the spring, which provides the controlling torque also affected because of the temperature change. Thereby, the error occurs in the reading.

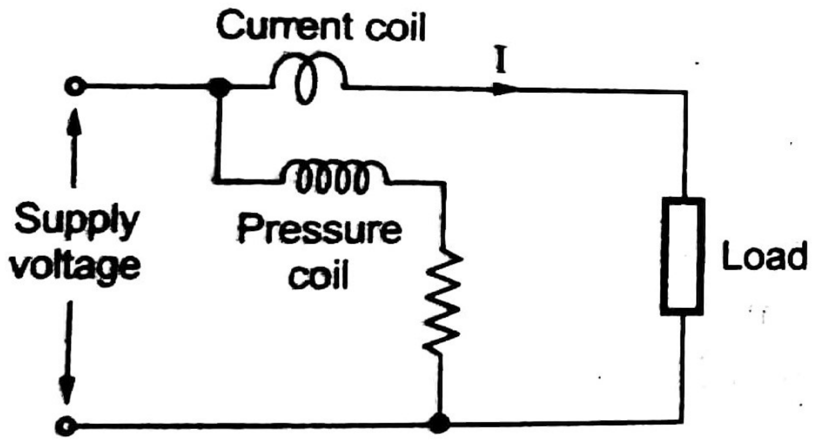
The calibration of the electrodynamometer wattmeter is same both for the AC and DC measurement.

7.Errors may be due connections.(i.e. pressure coil is connected after current coil)

8.Errors caused by vibration of moving system

Error due to method of connection

- In connection shown in the Fig. (a), pressure coil is connected on the supply side and therefore the voltage applied to the pressure coil is the voltage across the load plus the voltage drop across current coil.
- Thus wattmeter measures power loss in its current coil in addition to power consumed by load.



(a)

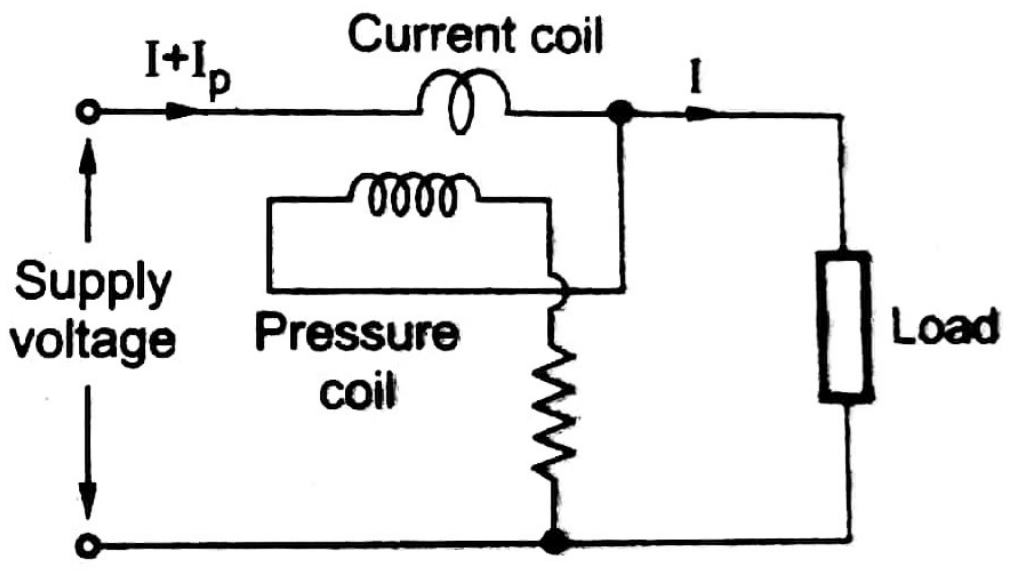
- Power indicated by wattmeter = Power consumed by load + Power loss in current coil

  ∴ Power indicated by wattmeter = Power consumed by load + I^2R_c

- With small load current, the voltage drop in current coil is small so connections in Fig. (a) introduces small error.
- Alternatively if load current is large, the pressure coil current is very small as compared with load current.
- Hence power loss in pressure coil circuit is small as compared with power consumed by load. Thus connection shown in Fig.(a) is preferable for small currents while for large currents the connections shown in Fig. (b) are preferable.

➢ Error due to method of connection

- If wattmeter connections are as shown in the Fig.(b) the current coil is on supply side and hence it carries pressure coil current plus the load current.
- Thus wattmeter reads in addition to power consumed in load, the power loss in pressure coil.



(b)

$$\text{Power indicated by wattmeter} = \text{Power consumed by the load} + \text{Power loss pressure coil circuit}$$
$$= \text{Power consumed by the load} + V^2/\,R_p$$

- For large currents the connections shown in Fig. (b) are preferable.
- But if load current is high and the power factor is small, connection shown in Fig. (b) results in large error as the total power measured is small.
- In this case, a compensating coil may be used for compensation of error

ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

LPF Watt Meter

Types of Electrodynamometer Type Wattmeter

Based on application of electrical power measurement

1) UPF (Unity Power Factor) wattmeter :These meters are used to measure power in resistive circuits. i.e. Cos Ø =1(UPF loads)
2) LPF (Low Power Factor) wattmeter is used for highly inductive circuits here p.f is low.

➢ Low power factor(LPF) Electro-dynamometer type wattmeters:

- If any circuit is operating at low power factor then power in that circuit is difficult to measure with ordinary electrodynamometer wattmeters. The reading of the wattmeter is inaccurate on account of following reasons,

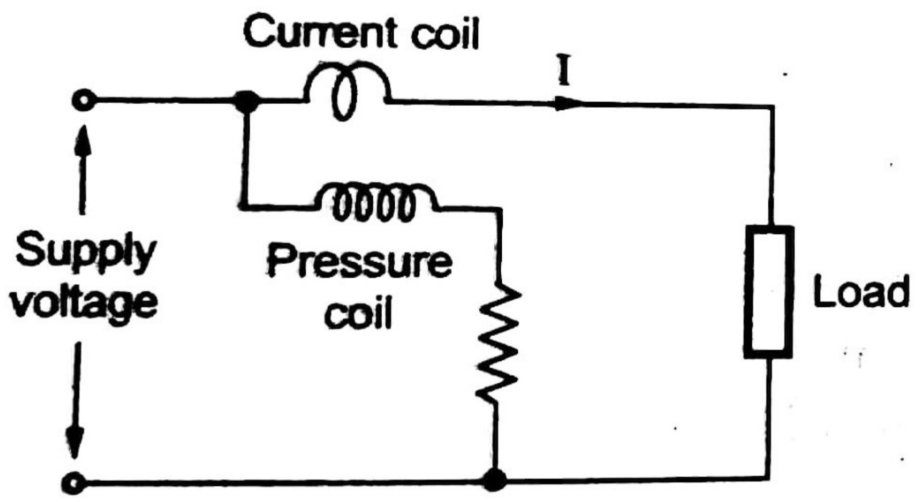
1. The deflecting torque on the moving system is small as the power factor is low even though the current and pressure coils are fully excited.
2. The inductance of pressure coil introduces considerable error at low power factors.

- ❑ In order to get accurate reading from the wattmeter when it is measuring low power, extra adjustments are required to be made so that there will be compensation of the errors.
- ❑ There are three major modifications
  1) Reduction of pressure coil resistance
  2) Compensation of error due to pressure coil connections
  3) Compensation of error due to pressure coil inductance

Modifications in ordinary wattmeter

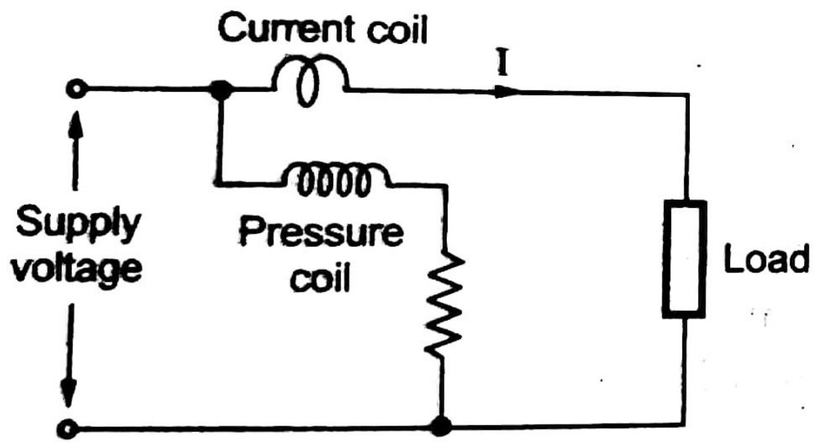
1) Reduction of pressure coil resistance

- In case of ordinary wattmeter a high resister is connected in series with the pressure coil in order to limit the current through coil as shown in Fig.(a)
- In case of low power factor wattmeter, pressure coil is designed to having a low value of resistance so that a high value of current passes through the coil.
- This current produces high deflecting torque (T_d).
- In low power factor wattmeter, the value of pressure coil current is 10 times the current in case of high power factor wattmeters.

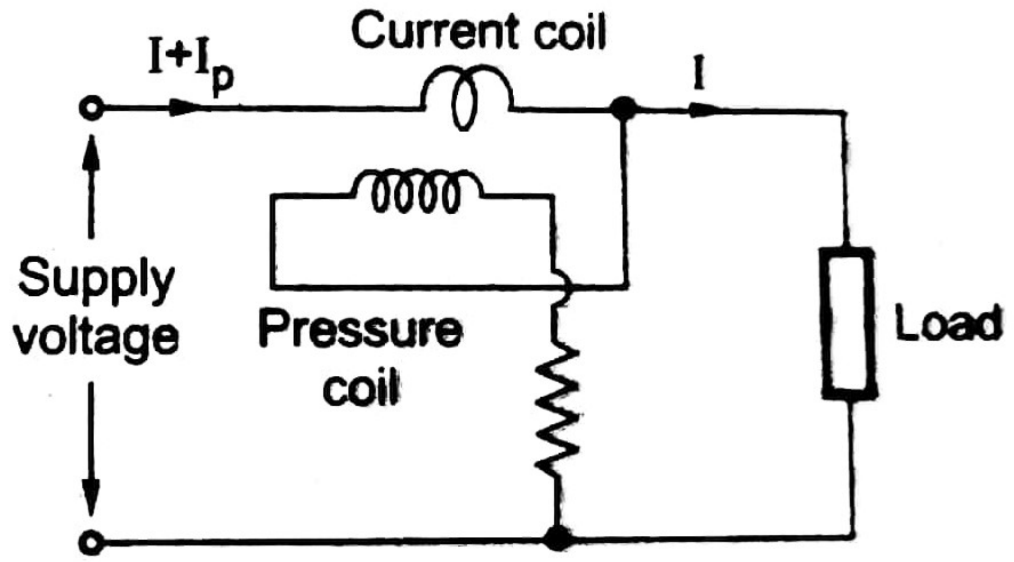


(a)

2.Compensation of error due to pressure coil connections



(a)



(b)

- In connection shown in the Fig. (a), pressure coil is connected on the supply side and therefore the voltage applied to the pressure coil is the voltage across the load plus the voltage drop across current coil.
- Thus wattmeter measures power loss in its current coil in addition to power consumed by load.

$$P_W = P_L + {I_L}^2 R_{CC}$$

- If wattmeter connections are as shown in the Fig.(b) the current coil is on supply side and hence it carries pressure coil current plus the load current.
- Thus wattmeter reads in addition to power consumed in load, the power loss in pressure coil.

$$P_W = P_L + \frac{V^2}{R_{PC}}$$

- When power to be measured is low then the current in the circuit is high as the power factor is low.
- Thus in this case pressure coil cannot be connected to supply side as otherwise large error will be produced because of large current flowing in current coil and corresponding power loss in current coil circuit is measured by wattmeter.
- If pressure coil is connected to load side, power consumed by pressure coil is measured by wattmeter which is appreciable in comparison with power to be measured which is small.
- Hence it is necessary to compensate for pressure coil current in low power factor wattmeter.
- The compensated wattmeter is shown in Fig. (a).

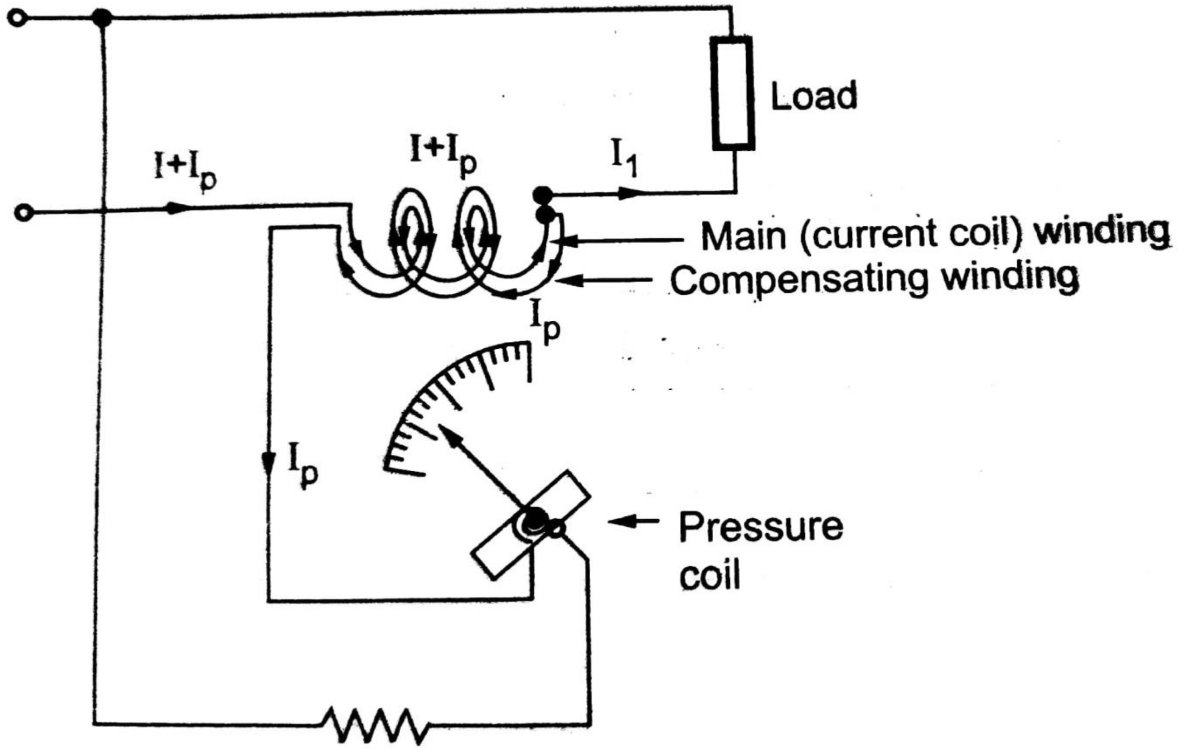


Fig. (a)

- As shown in the Fig. (a), the compensating coil is connected in series with the potential coil and is made as identical and coincident with current coil as possible. The current coil carries current I+Ip and produces its own field proportional to this current. The compensating coil carries current Ip and produces field proportional to this current. This field acts in opposite direction to the field produced by current coil.
- Thus the resultant field is due to current I only. Hence error due to pressure coil current is neutralized. Thus at no load condition, the wattmeter should not deflect as the resultant current coil field is zero.

- In order to compensate the error due to pressure coil connections, a compensating coil is connected in series with the pressure coil as shown in Fig(c).
- When the pressure coil current I_P flows through the compensating coil, a magnetic field is induced in it.
- This a magnetic field of compensating coil opposes the field of current coil thus the error caused by the pressure coil current flowing through the current coil is neutralised.

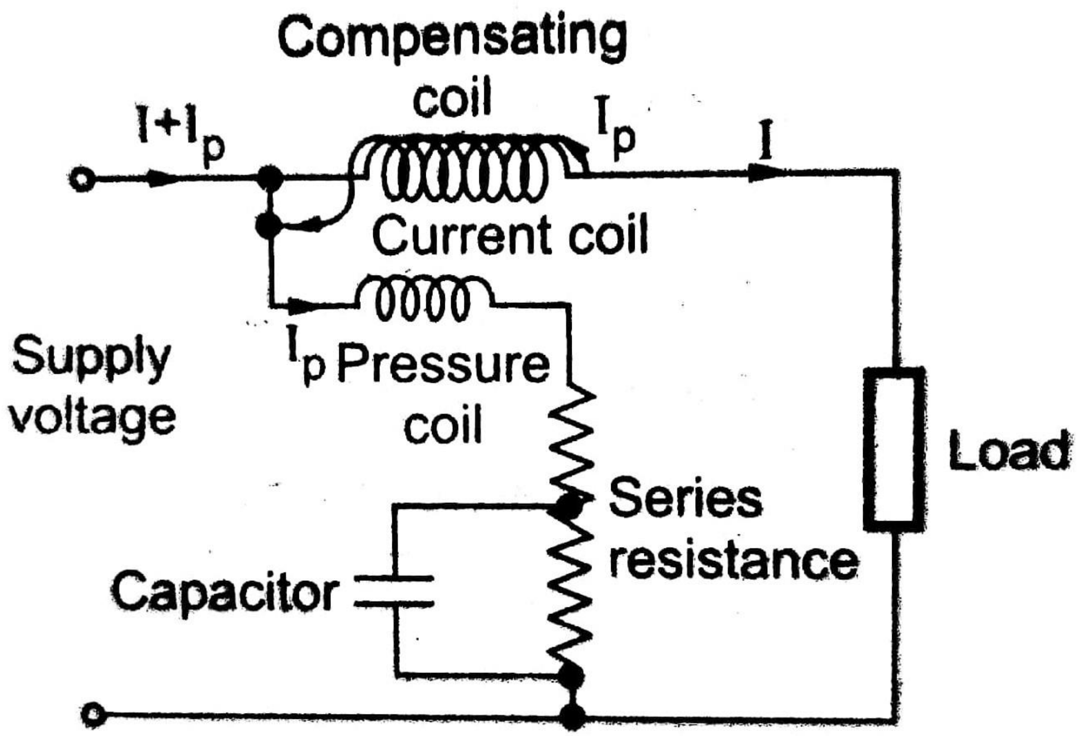


Fig.(c)Low power factor wattmeter

3.Compensation of error due to pressure coil inductance:

- A small amount of inductance is present in the pressure coil of the wattmeter, this inductance causes error in the reading.
- This type of error can be reduced by connecting the capacitor in parallel with the pressure coil series resistance as shown in Fig.(c).so that the capacitive reactance cancel out the inductive reactance of the pressure coil. Then pressure coil becomes purly resistive.

Three phase dynamometer type wattmeter

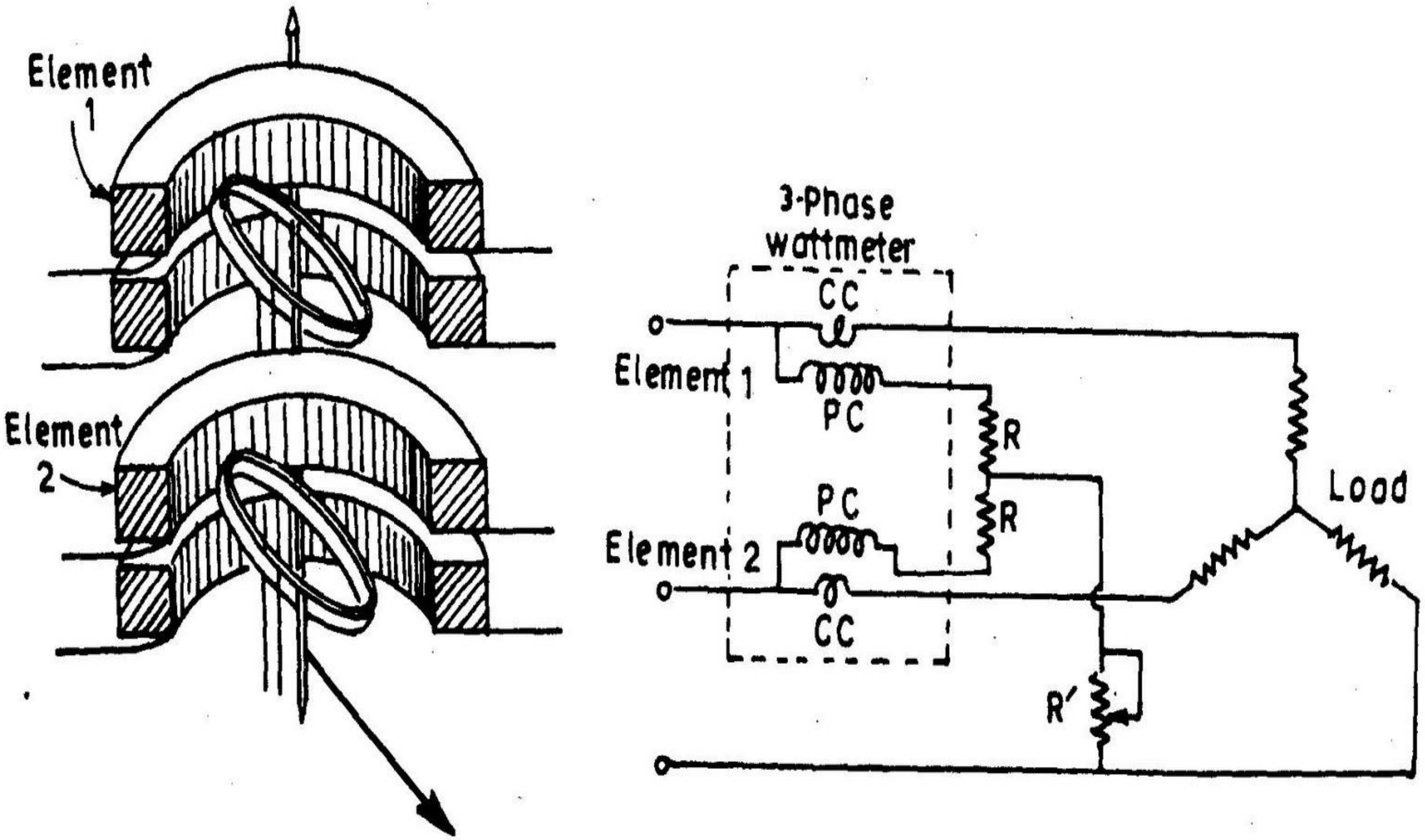


Fig.(a).Three phase dynamometer type wattmeter

Three phase dynamometer type wattmeter

- A dynamometer type three-phase wattmeter consists of two separate wattmeter movements mounted together in one case with the two moving coils mounted on the same spindle.
- The arrangement is shown in Fig.(a)
- There are two current coils and two pressure coils.
- A current coil together with its pressure coil is known as an element. Therefore, a three phase wattmeter has two elements.
- The connections of two elements of a 3 phase wattmeter are the same as that for two wattmeter method using two single phase wattmeter.
- The torque on each element is proportional to the power being measured by it.
- The total torque deflecting the moving system is the sum of the deflecting torque of the two elements.
- Hence the total deflecting torque on the moving system is proportional to the total Power.

- In order that a 3 phase wattmeter read correctly, there should not be any mutual interference between the two elements.
- A laminated iron shield may be placed between the two elements to eliminate the mutual effects.

ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

Measurement of Power

Measurement of Power

➢ Measurement of Power in 3- Phase A.C. Circuits

❑ The power in 3-phase load can be measured by using the following methods

1) Three wattmeter method
2) Two wattmeter method
3) One wattmeter method

➢ Measurement of Power in Single Phase A.C. Circuits

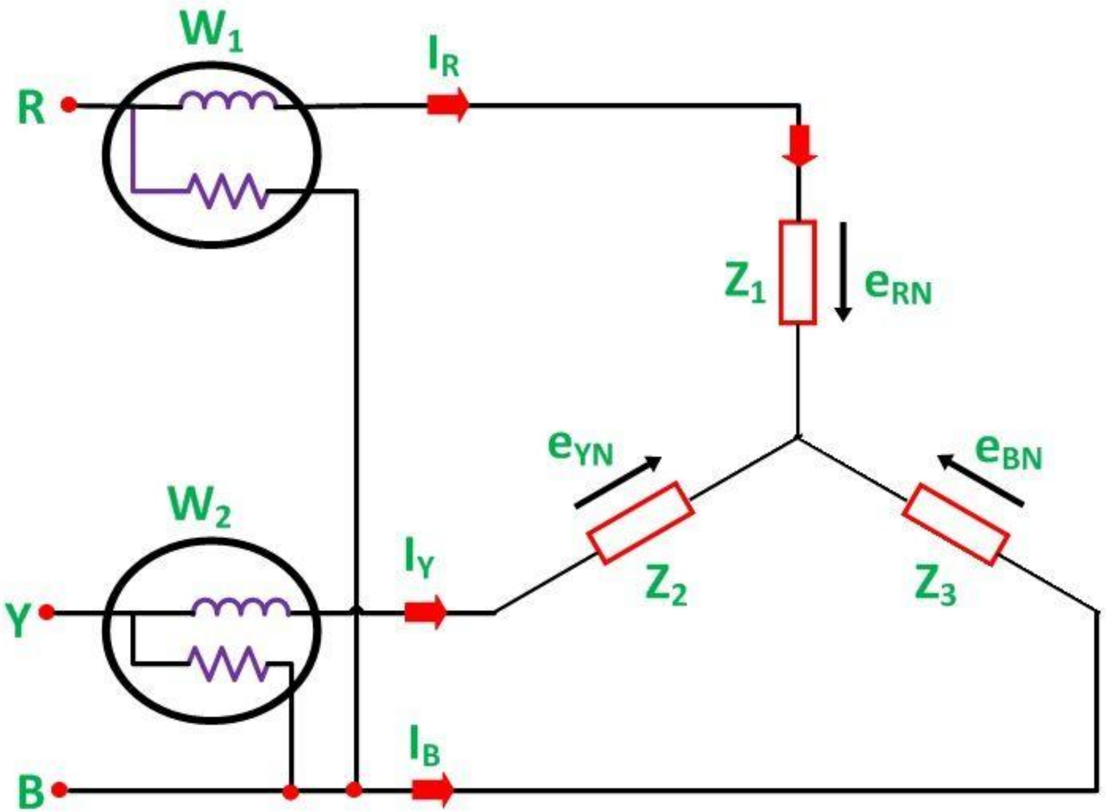
❑ The various methods used for measurement of power in single phase A.C.circuits are

1) Measurement of power without wattmeters
   i. Three-voltmeter method
   ii. Three-ammeter method
2) Measurement of power with wattmeters
3) Measurement of power in conjunction with instrument transformers

Measurement of Three Phase Power: Two Wattmeter Method

➢ Measurement of Power by Two Wattmeter Method in Star Connection

- **Two Wattmeter** Method can be employed to measure the power in a 3 phase, three wire star or delta connected the balanced or unbalanced load.
- In Two wattmeter method the current coils of the wattmeter are connected with any two lines, say R and Y and the potential coil of each wattmeter is joined on the same line, the third line i.e. B as shown in Fig(a).



Fig(a).Two Wattmeter Method for star connected load

Measurement of Power by Two Wattmeter Method in Star Connection

Considering the above figure (A) in which Two Wattmeter W_1 and W_2 are connected, the instantaneous current through the current coil of Wattmeter, W_1 is given by the equation shown below.

$$W_1 = i_R$$

Instantaneous potential difference across the potential coil of Wattmeter, W_1 is given as

$$W_1 = e_{RN} - e_{BN}$$

Instantaneous power measured by the Wattmeter, W_1 is

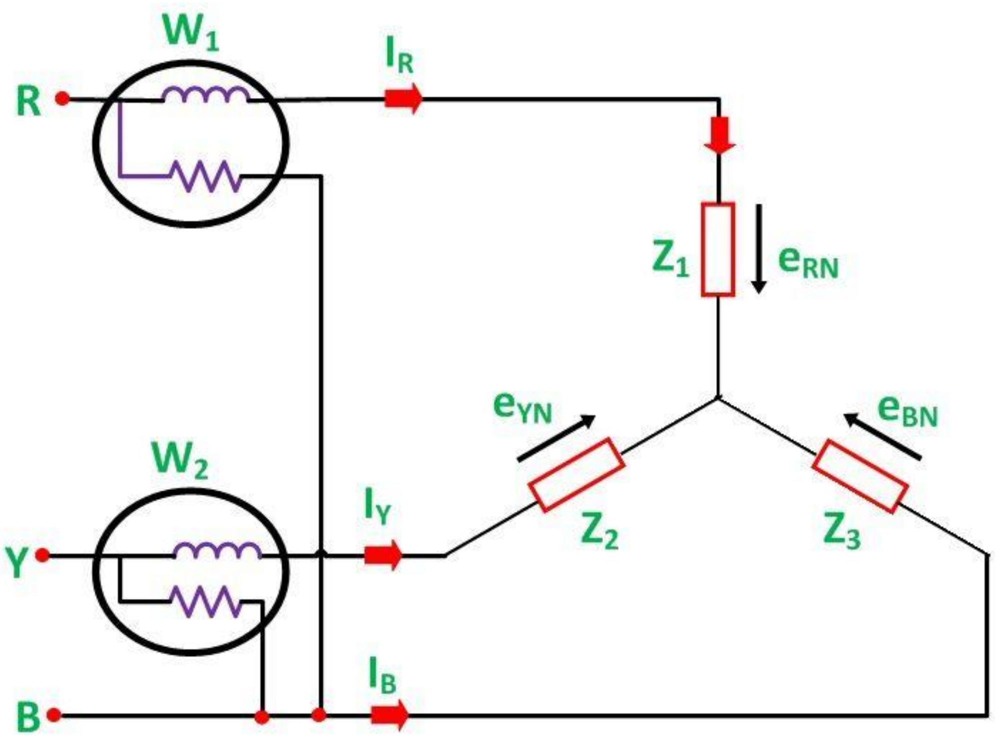
$$W_1 = i_R (e_{RN} - e_{BN}) \;\ldots\ldots\ldots (1)$$

The instantaneous current through the current coil of Wattmeter, W_2 is given by the equation

$$W_2 = i_Y$$

Instantaneous potential difference across the potential coil of Wattmeter, W_2 is given as

$$W_2 = e_{YN} - e_{BN}$$



Fig(a).Two Wattmeter Method for star connected load instantaneous power measured by the Wattmeter, W_2 is

$$W_2 = i_Y\,(e_{YN} - e_{BN}) \quad \ldots\ldots\ldots..(2)$$

Therefore, the Total Power Measured by the Two Wattmeters W_1 and W_2 will be obtained by adding the equation (1) and (2).

$$W_1 + W_2 = i_R\,(e_{RN} - e_{BN}) + i_Y\,(e_{YN} - e_{BN})$$

$$W_1 + W_2 = i_R e_{RN} + i_Y e_{YN} - e_{BN}\,(i_R + i_Y) \quad \text{or}$$

$$W_1 + W_2 = i_R e_{RN} + i_Y e_{YN} + i_B e_{BN} \qquad (\text{i.e. } i_R + i_Y + i_B = 0)$$

$$W_1 + W_2 = P$$

Where P – the total power absorbed in the three loads at any instant.

Measurement of Power by Two Wattmeter Method in Delta Connection

The instantaneous current through the coil of the Wattmeter, W_1 is given by the equation

$$W_1 = i_R = i_1 - i_3$$

Instantaneous potential difference across the potential coil of the Wattmeter, W_1 will be

$$W_1 = e_{RB}$$

Therefore, the instantaneous power measured by the Wattmeter, W_1 will be given as

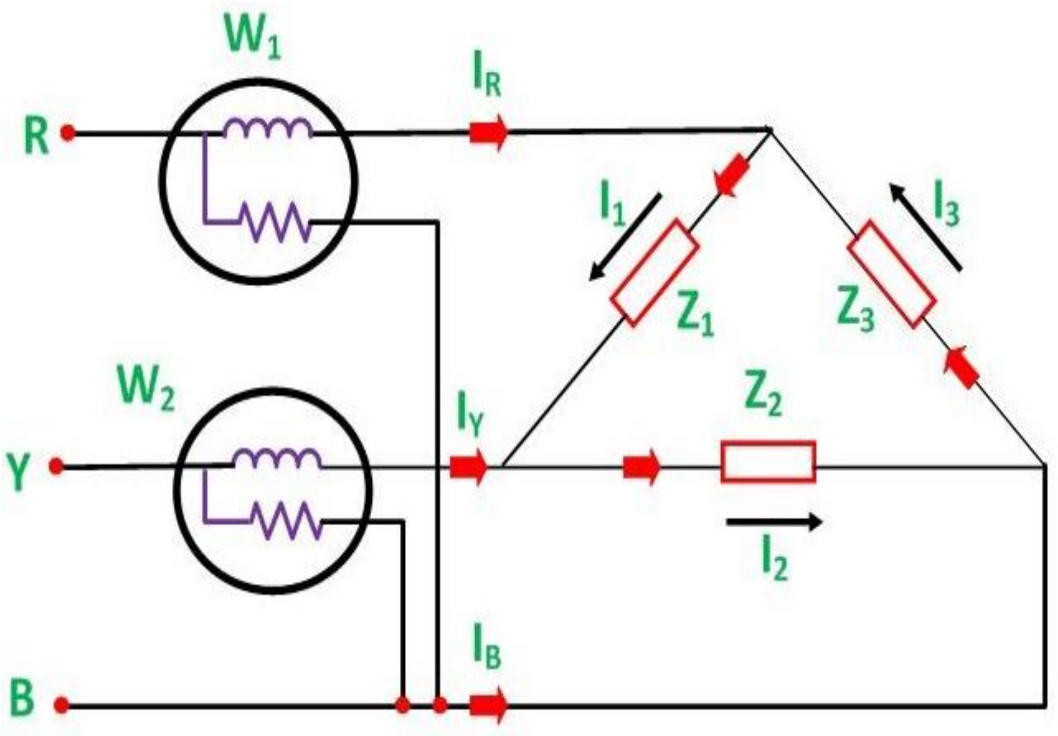
$$W_1 = e_{RB}\,(i_1 - i_3) \;\ldots\ldots\ldots (3)$$

The instantaneous current through the current coil of the Wattmeter, W_2 is given as

$$W_2 = i_Y = i_2 - i_1$$

The instantaneous potential difference across the potential coil of Wattmeter, W_2 is

$$W_2 = e_{YB}$$



Fig(b).Two Wattmeter Method for delta connected load

Measurement of Power by Two Wattmeter Method in Delta Connection

Therefore, the instantaneous power measured by Wattmeter, W_2 will be

$$W_2 = e_{YB}\,(i_2 - i_1) \;\ldots\ldots\ldots (4)$$

Hence, to obtain the total power measured by the Two Wattmeter the two equations, i.e. equation (3) and (4) has to be added.

$$W_1 + W_2 = e_{RB}\,(i_1 - i_3) + e_{YB}\,(i_2 - i_1)$$

$$W_1 + W_2 = i_1 e_{RB} + i_1 e_{YB} - i_3 e_{RB} - i_1 e_{YB}$$

$$W_1 + W_2 = i_2 e_{YB} + i_3 e_{BR} - i_1 (e_{YB} + e_{BR}) \qquad (\text{i.e.} -e_{RB} = e_{RB})$$

$$W_1 + W_2 = i_1 e_{RY} + i_2 e_{YB} + i_3 e_{BR} \quad (\text{i.e } e_{RY} + e_{YB} + e_{BR} = 0)$$

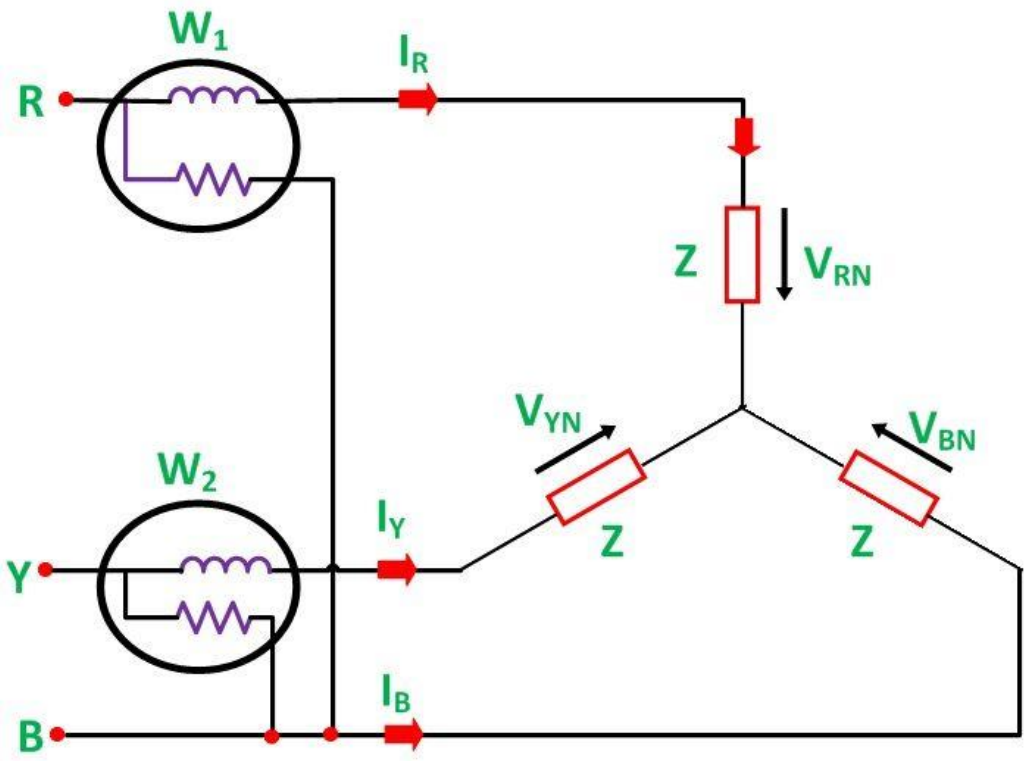
$$W_1 + W_2 = P$$

- The power measured by the Two Wattmeter at any instant is the instantaneous power absorbed by the three loads connected in three phases.
- In fact, this power is the average power drawn by the load since the Wattmeter reads the average power because of the inertia of their moving system.

Measurement of Three Phase Power

Two Wattmeter Method – Balanced Load Condition

- The **Two Wattmeter Method** is explained, taking an example of a balanced load. In this, we have to prove that the power measured by the Two Wattmeter i.e. the sum of the two wattmeter readings is equal to $\surd 3 V_L I_L \cos\emptyset$ which is the actual power consumed in a 3 phase balanced load.



Fig,Star- connected load

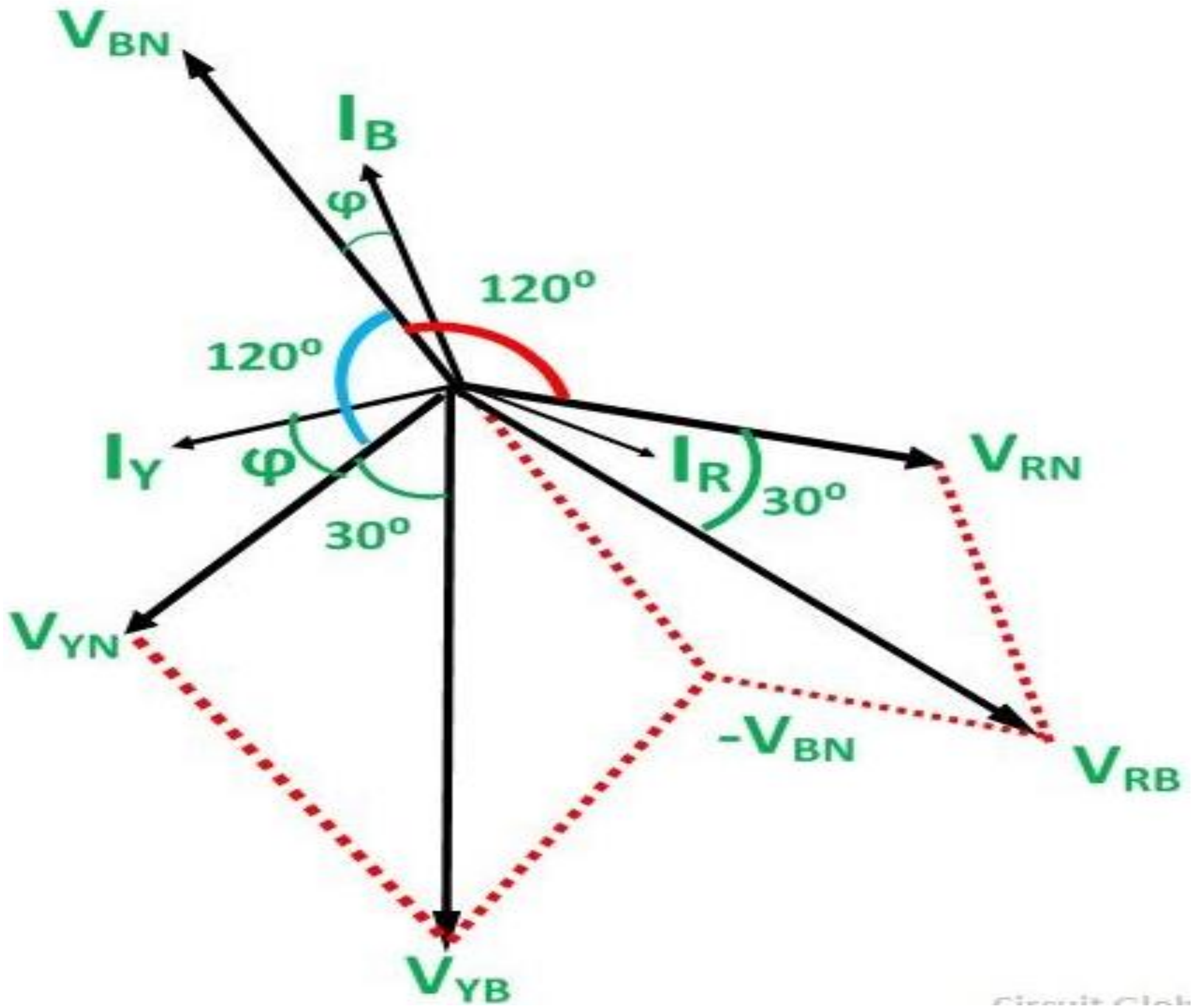


Fig.(a).Phasor diagram

- The load is considered as an inductive load, and thus, the phasor diagram of the inductive load is shown in Fig.(a)

The three voltages V_{RN}, V_{YN} and V_{BN}, are displaced by an angle of 120 degrees electrical as shown in the phasor diagram. The phase current lag behind their respective phase voltages by an angle φ.
Now, the current flowing through the current coil of the Wattmeter, W_1 will be given as

$$W_1 = I_R$$

Potential difference across the pressure or potential coil of the Wattmeter, W_1 will be

$$W_1 = \overline{V_{RB}} = \overline{V_{RN}} - \overline{V_{BN}}$$

To obtain the value of V_{RB}, reverse the phasor V_{BN} and add it to the phasor V_{RN} as shown in the phasor diagram.
The phase difference between V_{RB} and I_R is $(30^o - \varphi)$

Therefore, the power measured by the Wattmeter, W_1 is

$$W_1 = V_{RB} I_R \cos\,(30^\circ - \varphi)$$

Current through the current coil of the Wattmeter, W_2 is given as

$$W_2 = I_Y$$

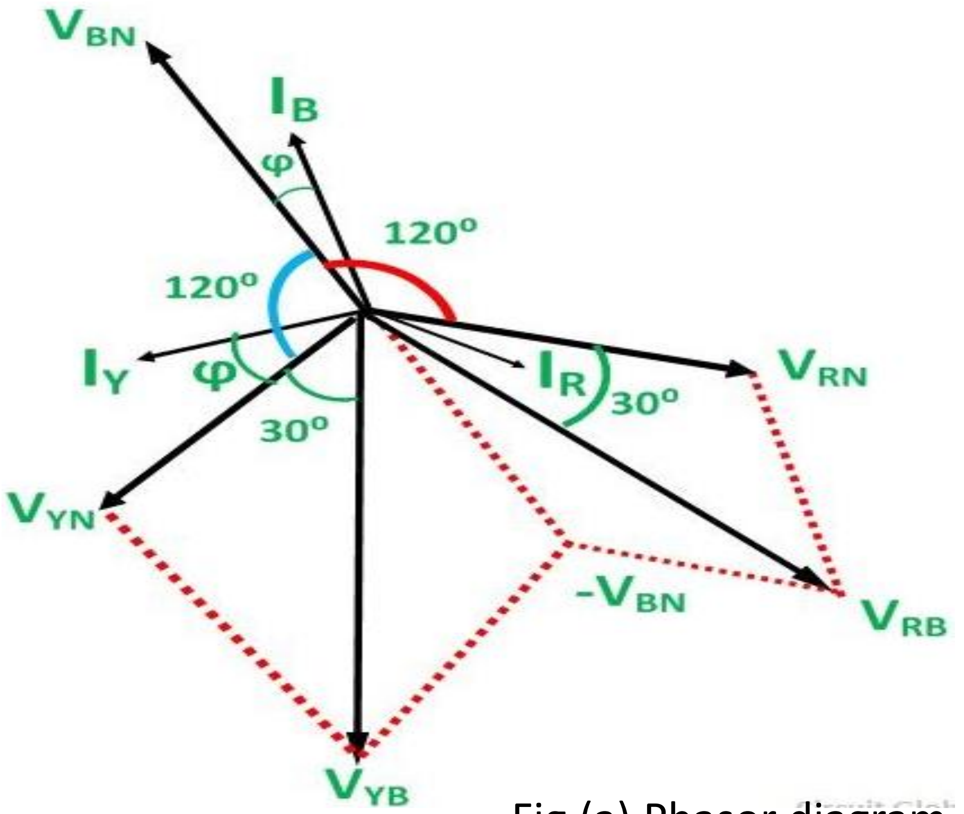


Fig.(a).Phasor diagram

Potential difference across the Wattmeter, W_2 is

$$W_2 = \overline{V_{YB}} = \overline{V_{RN}} - \overline{V_{BN}}$$

The phase difference V_{YB} and I_Y is $(30^o + \phi)$.

Therefore, the power measured by the Wattmeter, W_2 is given by the equation shown below.

$$W_2 = V_{YB} I_Y \cos(30^\circ + \varphi)$$

Since, the load is in balanced condition, hence,

$$I_R = I_Y = I_B = I_L \quad \text{and}$$

$$V_{RY} = V_{YB} = V_{BR} = V_L$$

Therefore, the wattmeter readings will be

$$W_1 = V_L I_L \cos(30^\circ - \varphi) \quad \text{and}$$

$$W_2 = V_L I_L \cos(30^\circ + \varphi)$$

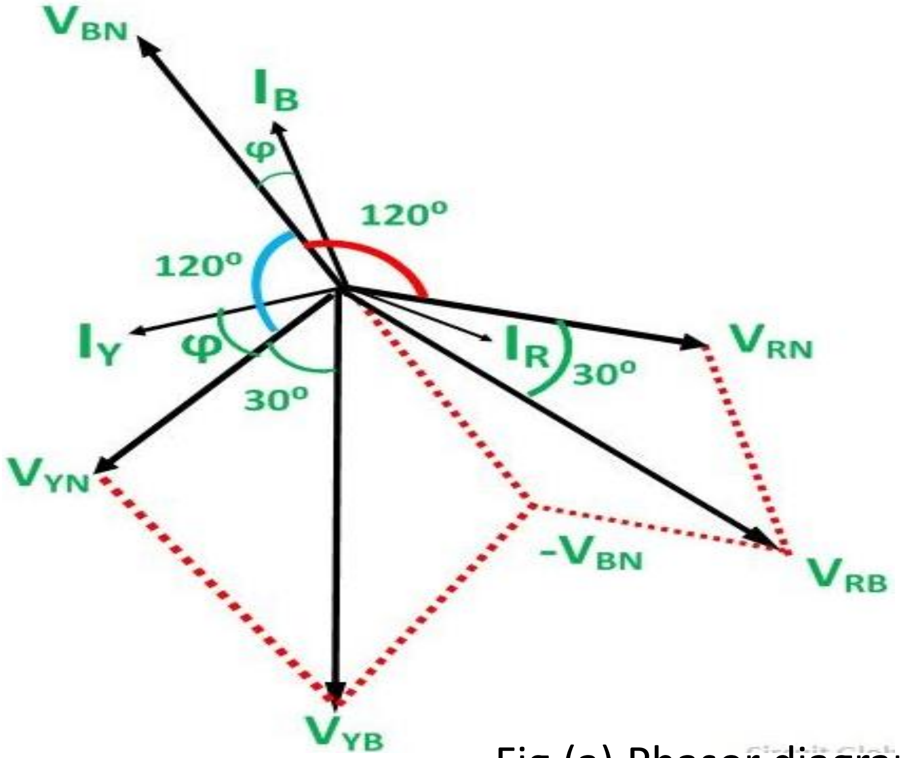


Fig.(a).Phasor diagram

Now, the sum of two Wattmeter readings will be given as

$$W_1 + W_2 = V_L I_L \cos(30^\circ - \varphi) + V_L I_L \cos(30^\circ + \varphi)$$

$$W_1 + W_2 = V_L I_L [\cos(30^\circ - \varphi) + \cos(30^\circ + \varphi)] \quad \text{or}$$

$$W_1 + W_2 = V_L I_L [\cos 30^\circ \cos\varphi + \sin 30^\circ \sin\varphi + \cos 30^\circ \cos\varphi - \sin 30^\circ \sin\varphi] \text{ or}$$

$$W_1 + W_2 = V_L I_L (2 \cos 30^\circ \cos\varphi) \quad \text{or}$$

$$W_1 + W_2 = V_L I_L \left(2 \frac{\sqrt{3}}{2} \cos\varphi\right)$$

$$W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\varphi$$

$$W_1 + W_2 = P \; \ldots \ldots (1)$$

The above equation (1) gives the total power absorbed by a 3 phase balanced load.

Thus, the sum of the readings of the two Wattmeters is equal to the power absorbed in a 3 phase balanced load.

ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

Determination of Power Factor from Wattmeter Readings

Determination of Power Factor from Wattmeter Readings

As we know that,

$$W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\varphi \;\ldots\ldots\ldots (2)$$

Now,

$$W_1 - W_2 = V_L I_L \left[\cos(30^\circ - \varphi) - \cos(30^\circ + \varphi)\right] \quad \text{or}$$

$$W_1 - W_2 = V_L I_L \left[\cos 30^\circ \cos\varphi + \sin 30^\circ \sin\varphi - \cos 30^\circ \cos\varphi + \sin 30^\circ \sin\varphi\right] \text{ or}$$

$$W_1 - W_2 = 2\, V_L I_L \sin 30^\circ \sin\varphi$$

$$W_1 - W_2 = V_L I_L \sin\varphi \;\ldots\ldots\ldots (3)$$

Dividing equation (3) by equation (2) we get,

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\varphi}{\sqrt{3}\, V_L I_L \cos\varphi} \quad \text{or}$$

$$\tan\varphi = \sqrt{3}\, \frac{W_1 - W_2}{W_1 + W_2}$$

Power factor of the load is given as

$$\cos\varphi = \cos\tan^{-1}\sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

- **Determination of Reactive Power by Two Wattmeter Method**

To get the reactive power, multiply equation (3) by √3.

$$\sqrt{3}\,(W_1 - W_2) = \sqrt{3}\,V_L I_L \sin\varphi = P_r$$

Therefore, the Reactive Power is given by the equation shown below.

$$P_r = \sqrt{3}\,(W_1 - W_2)$$

➢ Effect of P.F. on Wattmeter Readings

For a lagging p.f.

$$W_1 = V_L I_L \cos(30 - \emptyset)$$

$$W_2 = V_L I_L \cos(30 + \emptyset)$$

Consider different cases,

Case(i)

$$\text{Cos } \emptyset = 0 \qquad \emptyset = 90^0$$

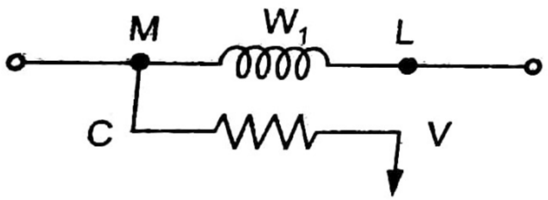
$$W_1 = V_L I_L \cos(30 - 90) = +\frac{1}{2} V_L I_L$$

$$W_2 = V_L I_L \cos(30 + 90) = -\frac{1}{2} V_L I_L$$

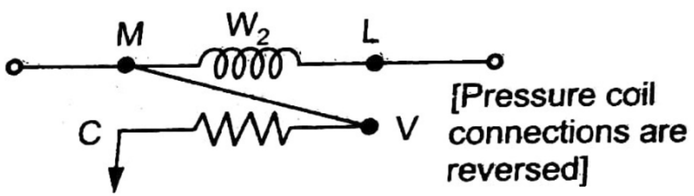
i.e., $W_1 + W_2 = 0$

$$|W_1| = |W_2| \qquad but \qquad W_2 = -W_1$$

- Wattmeter cannot show negative reading as it has only positive scale.
- Indication of negative reading is that pointer tries to deflect in negative direction i.e. to the left of zero. In such case, reading can be converted to positive by interchanging either
  pressure coil connections i.e. (C↔V) or by interchanging current coil connections (M↔L)
- Remember that interchanging connections of both the coils will have no effect on the wattmeter reading
- Such a reading obtained by interchanging connections of either of the two coils will be positive on wattmeter but must be taken as negative for calculations.



For +ve reading



For –ve reading

So on wattmeter $W_1 = W_2$ but W_2 must be taken a negative as this reading will be obtained by reversing connections of any one coil.

Case(ii)

$\text{Cos } \emptyset = 0.5, \qquad \emptyset = 60^0$

$W_1 = V_L I_L \cos(30 - 60) = V_L I_L \cos 30$

= Positive

$W_2 = V_L I_L \cos(30 + 60) = 0$

$\therefore \qquad W_1 + W_2 = W_1$ = Total power

One water shows zero reading for $\text{Cos } \emptyset = 0.5$

For all power factors between 0 to 0.5 W_2 shows negative and W_1 shows positive, for lagging p.f.

Case(iii)

$\text{Cos } \emptyset = 1, \qquad \emptyset = 0^0$

$W_1 = V_L I_L \cos(30 + 0) = V_L I_L \cos 30 = +ve$

$W_2 = V_L I_L \cos(30 - 0) = V_L I_L \cos 30 = +ve$

$\therefore$ $both$ W_1 and W_2 are equal and positive.

For all power factors between 0.5 to 1 both wattmeters gives positive reading.

Range of p.f.	Range of ' ϕ '	W_1 sign	W_2 sign	Remark
$\cos \phi = 0$	$\phi = 90^o$	positive	negative	$\lvert W_1 \rvert = \lvert W_2 \rvert$
$0 \angle \cos \phi \angle 0.5$	$90^o \angle \phi \angle 60^o$	positive	negative	
$\cos \phi = 0.5$	$\phi = 60^o$	positive	0	
$0.5 \angle \cos \phi \angle 1$	$60^o \angle \phi \angle 0^o$	positive	positive	
$\cos \phi = 1$	$\phi = 0^o$	positive	positive	$W_1 = W_2$

Advantages of Two Wattmeter Method

The various advantages of two wattmeter method are,

1) The method is applicable for balanced as well as unbalanced loads.
2) Neutral point for star connected load is not necessary to connect the wattmeters.
3) The delta connected load, need not be opened for connecting the wattmeters.
4) Only two wattmeters are sufficient to measure total 3 phase power.
5) If the load is balanced not only the power but power factor also can be determined.
6) Total reactive volt amperes can be obtained using two wattmeter readings for balanced loads.

Disadvantages of Two Wattmeter Method

The few disadvantages of this method are,

1) Not applicable for three phase, 4 wire system
2) The signs of W_1 and W_2 must be identified and noted down correctly otherwise it may lead to the wrong results.

$$S = \text{Apparent power} = \sqrt{3}\, V_L\, I_L \qquad \text{VA or kVA}$$

$$P = \text{Active power} = \sqrt{3}\, V_L\, I_L \cos\phi = W_1 + W_2 \qquad \text{W or kW}$$

$$Q = \text{Reactive power} = \sqrt{3}\, V_L\, I_L \sin\phi = \sqrt{3}\,(W_1 - W_2) \quad \text{VAR or kVAR}$$

Numerical Problems

1) Two wattmeters are being used to measure power of a balanced load of 30 A at power factor 0.8 lagging being supplied by a 3-phase, 3-wire, 440 V supply. Calculate :
   (i) Power consumed
   (ii) Reading of wattmeter No. 1
   (iii) Reading of wattmeter No. 2.

Solution. Line voltage, $E_L = 440$ V

Line current $I_L = 30$ A

Power factor, $\cos\phi = 0.8$

Phase angle, $\phi = \cos^{-1} 0.8 = 36.87°$

(i) Power consumed

$$P = \sqrt{3}\, E_L I_L \cos\phi = \sqrt{3} \times 440 \times 30 \times 0.8$$

= 18290 W = 18.29 kW

(ii) Reading of wattmeter No. 1

$$= E_L I_L \cos(30° - \phi) = 440 \times 30 \cos(30° - 36.87°)$$

= 13105 W or **13.105 kW** **(Ans.)**

(iii) Reading of wattmeter No. 2.

$$= E_L I_L \cos(30° + \phi) = 440 \times 30 \times \cos(30° + 36.87°)$$

= 5185 W or **5.185 kW (app.)** **(Ans.)**

(2).The power to a three phase induction motor was measured by two wattmeter method with the readings 3400 and 1200 watts respectively. Calculate the total power and power factor?

Solution

$$W_1 = 3400 \text{ w} \quad \text{and} \quad W_2 = 1200 \text{ w}$$

$$P = W_1 + W_2 = 3400 + 1200 = 4600 \text{ w}$$

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}\right]\right\} = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\,(3400 - 1200)}{(3400 + 1200)}\right]\right\}$$

$$= \cos\{\tan^{-1}(0.8283)\} = \cos(39.6374°)$$

$$= 0.77$$

(3).In a balanced three phase system, the power is measured by two wattmeter method. The ratio of the two wattmeter reading is 2 : 1. Determine the power factor of the system

Solution

$$W_1 : W_2 = 2 : 1$$

i.e., $W_1 = 2W_2$

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}\right]\right\} = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\,(2W_2 - W_2)}{(2W_2 + W_2)}\right]\right\}$$

$$= \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\times 1}{3}\right]\right\} = \cos\{30^\circ\} = \mathbf{0.866}$$

(4).A certain circuit takes 10 A at 200 V and the power absorbed is 1000 W. If the current coil of the wattmeter has a resistance of 0.15 Ω and its pressure coil has a resistance of 5000 Ω and inductance of 0.3 H, find

i) the error due to resistance for each of the two possible methods of connection.

ii) the error due the inductance if the frequency of 50 Hz.

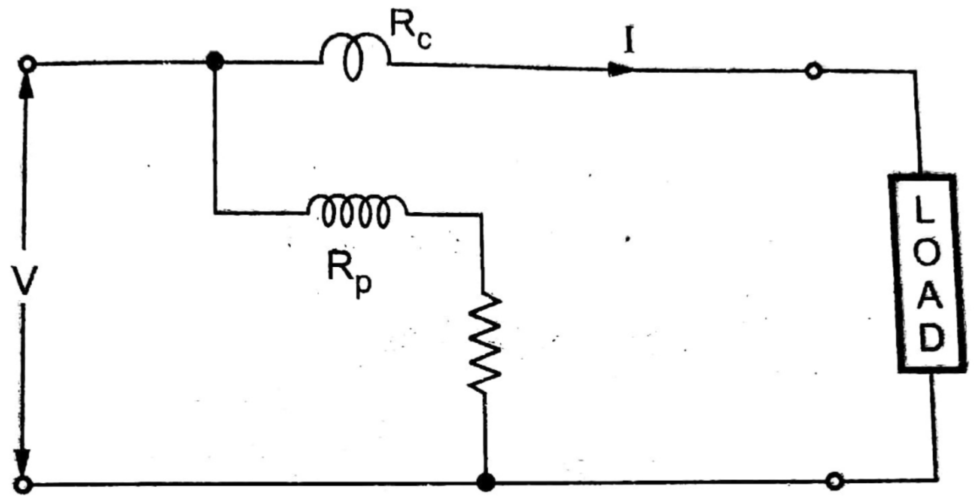
iii) the total error in each case.

Solution

$R_C = 0.15\ \boldsymbol{\Omega}$ $\quad R_P = 5000\ \boldsymbol{\Omega}$

$L_P = 0.3\ H$ $\quad$ I = 10 A $\quad$ V= 200V

(i)Error due to method of connection



Power indicated by wattmeter = Power consumed by load + $I^2 R_c$

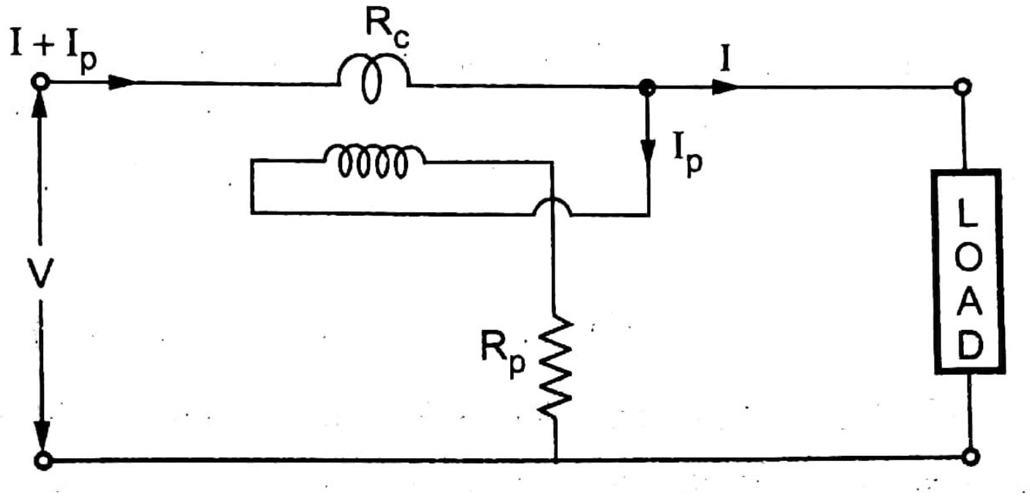
= 1000 + $(10)^2 \times 0.15 = 1015\ w$

∴ Error = 15 w

$$\therefore \%\ \text{Error} = \frac{\text{Measured} - \text{True}}{\text{True}} \times 100$$

$$= \frac{15}{1000} \times 100$$

$$= 1.5\%$$



Power indicated by wattmeter $=$ Power consumed by the load $+$ Power loss pressure coil circuit

$=$ Power consumed by the load $+ V^2/ R_p$

$$= 1000 + \frac{(200)^2}{5000} = 1008\, w$$

$\therefore$ Error = 8 w

$$\therefore \%\ \text{Error} = \frac{\text{Measured} - \text{True}}{\text{True}} \times 100$$

$$= \frac{8}{1000} \times 100 = 0.8\%$$

ii) $X_{Lp} = 2\pi f L_p = 2\pi \times 50 \times 0.3 = 94.2477\ \Omega$

$\therefore \quad \beta = \tan^{-1}\dfrac{X_{Lp}}{R_p} = \tan^{-1}\left[\dfrac{94.2477}{5000}\right] = 1.0798°$

$\therefore$ [Actual wattmeter reading] $= [1 + \tan\phi \tan\beta] \times$ True power

True power $= VI \cos\phi$

$\therefore \quad \cos\phi = \dfrac{1000}{10 \times 200} = 0.5 \quad$ i.e. $\phi = 60°$

$\therefore$ [Actual wattmeter reading] $= [1 + \tan 60° \tan 1.0798°] \times 1000 = 1032.6462$ W

$\therefore \quad$ Error $= 1032.6462 - 1000 =$ **32.6462 W**

$\therefore \quad$ % error $= \dfrac{32.6462}{1000} \times 100 =$ **3.2646 %**

iii) Total error

[Total error when pressure coil on supply side] $= 15 + 32.6462 =$ **47.6462 W**

$=$ **4.7646%**

[Total error when pressure coil on load side] $= 8 + 32.6462 =$ **40.6462 W**

$=$ **4.06462 %**

Measurement of Three Phase Power: Three Wattmeter Method

- **Power measurement** in an AC circuit is measured with the help of a Wattmeter. A Wattmeter is an instrument which consists of two coils called **Current coil** and **Potential coil.**
- The current coil having low resistance is connected in series with the load so that it carries the load current. The potential coil having the resistance is connected across the load and carries the current proportional to the potential difference.
- For measuring the power in a 3 phase or Poly Phase system, more than one wattmeter is required, or more than one readings are made by one wattmeter. If more than one wattmeter is connected for the measurement, the process becomes convenient and easy to work with instead of taking various readings with one wattmeter.
- The number of wattmeters required to measure power in a given polyphase system is determined by Blondel's Theorem.

- **According to Blondel's theorem** – When power is supplied by the K wire AC system, the number of wattmeters required to measure power is one less than the number of wire i.e. (K-I), regardless the load is balanced or unbalanced.
- Hence, Three wattmeters are required to measure power in three phase, four wire system, whereas, only two wattmeters are required to measure the power in 3 phase, 3 wire system.

- Three Wattmeter method is employed to measure power in a 3 phase, 4 wire system.However, this method can also be employed in a 3 phase, 3 wire delta connected load, where power consumed by each load is required to be determined separately.

- The connections **for star connected loads** for measuring power by Three wattmeter method is shown below.

- The pressure coil of all the Three wattmeters namely W_1, W_2 and W_3 are connected to a common terminal known as the neutral point. The product of the phase current and line voltage represents as phase power and is recorded by individual wattmeter.
- The total power in a Three wattmeter method of power measurement is given by the algebraic sum of the readings of Three wattmeters. i.e.

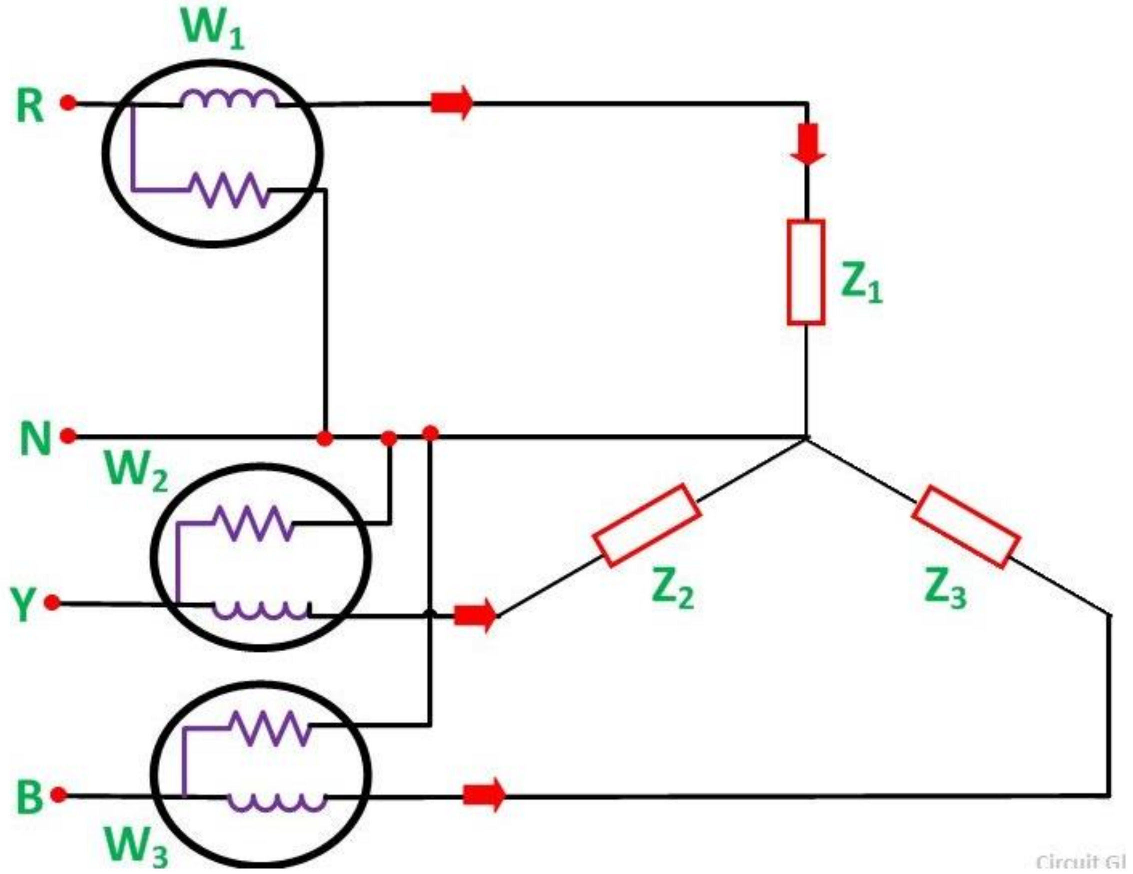
$$W_1 = V_1 I_1$$
$$W_2 = V_2 I_2$$
$$W_3 = V_3 I_3$$

If load is balanced, $W_1 = W_2 = W_3$

If load is unbalanced, $W_1 \neq W_2 \neq W_3$

Total power = $W_1 + W_2 + W_3$ ….for unbalanced loads

= $3W_1$ *or* $3W_2$ *or* $3W_3$ ….for balanced loads



Fig, 3-Ph Star- connected load

- Except for 3 phase, 4 wire unbalanced load, 3 phase power can be measured by using only Two Wattmeter Method.

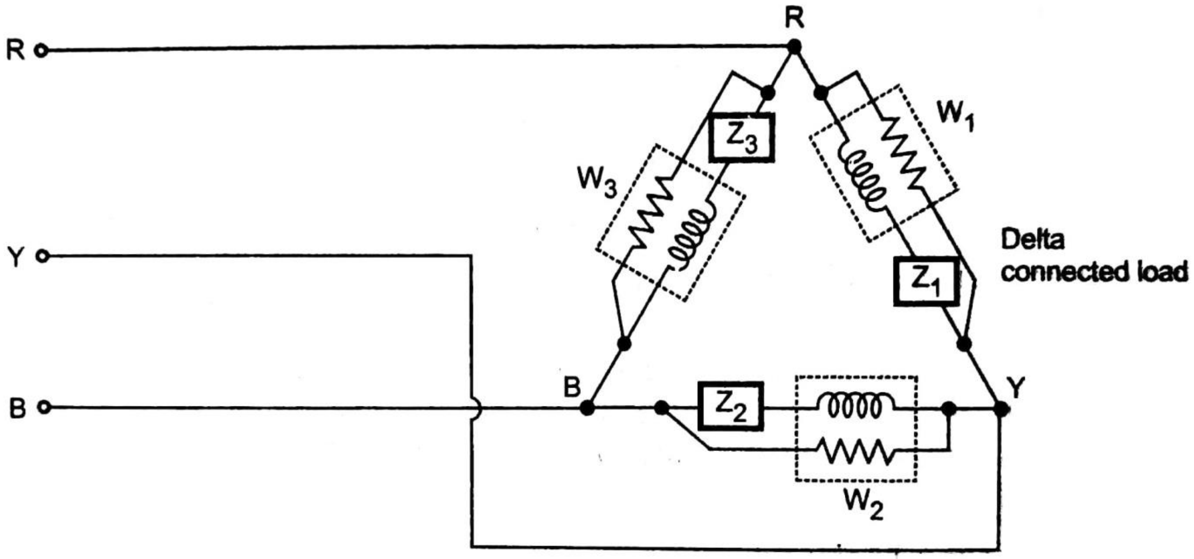
➢ The connections **for Delta connected loads** for measuring power by Three wattmeter method is shown in Fig(a).

- In this method, the power can be measured for unbalanced load but the disadvantage of requirement of neutral point for star load and arrangement for Insertion of current on in closed delta, still continues.
- Thus practically single wattmeter as well as three wattmeter methods are rarely used for the industrial loads, due to their limitations,

If load is balanced, $W_1 = W_2 = W_3$

Total power P= $W_1 + W_2 + W_3$ ….for unbalanced loads

$= 3W_1 \; or \; 3W_2 \; or \; 3W_3$



Fig(a) 3-Ph Delta- connected load

Single or One Wattmeter Method

- This can be only used for balanced three phase load. When the load is balanced, total power can be calculated as,

$$P = 3\, V_{ph}\, I_{ph} \cos\emptyset = 3 \times (\text{watt meter reading})$$

- Wattmeter must be connected in such a way that its current coil must carry I_{ph} and its voltage coil must be across V_{ph}

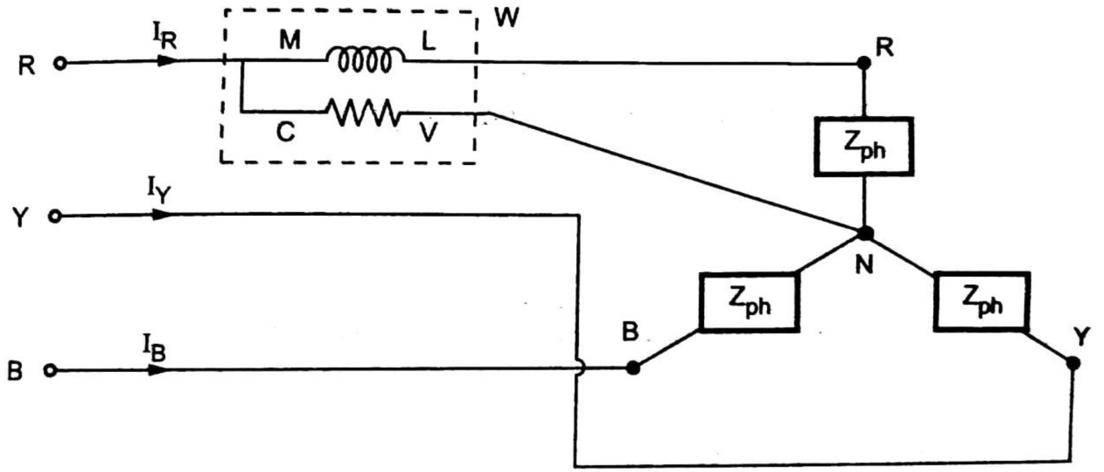


Fig.(a). Wattmeter measuring single phase power

Star Connected Load

Now $I_c = I_R = I_L = I_{ph}$ as load is star connected

$$V_{pc} = V_{RN} = V_{ph}$$

$$W = V_{pc}\, I_c \cos(V_{pc} \wedge I_c)$$

$$W = V_{RN}\, I_R \cos(V_{RN} \wedge I_R) = V_{ph}\, I_{ph} \cos\emptyset$$

$\therefore$ Total power, $P = 3\, V_{ph}\, I_{ph} \cos\emptyset$

$= 3\,W$ watts

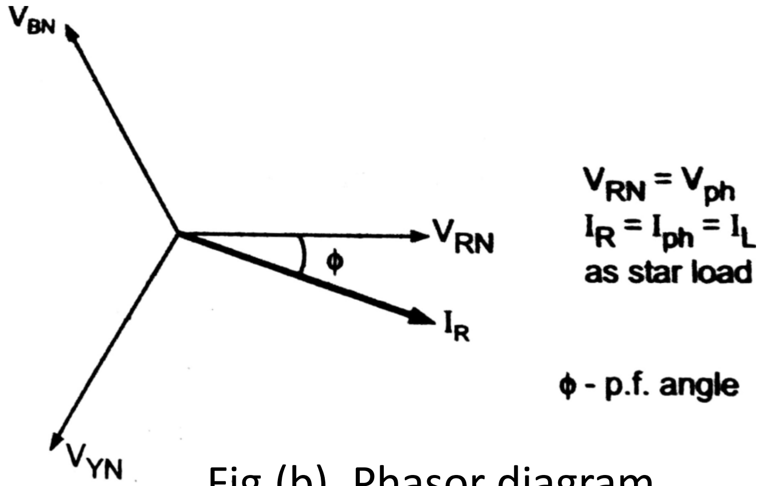


Fig.(b). Phasor diagram

➢ Delta Connected Load

In this connection

$$I_c = I_{YB} = I_{ph}$$

$$V_{pc} = V_{YB} = V_{ph}$$

$$W = V_{pc}\, I_c \cos\left(V_{pc} \wedge I_c\right)$$

$$\therefore \quad W = V_{YB}\, I_{YB} \cos(V_{YB} \wedge I_{YB}) = V_{ph}\, I_{ph} \cos\emptyset$$

Total power = 3 W watts

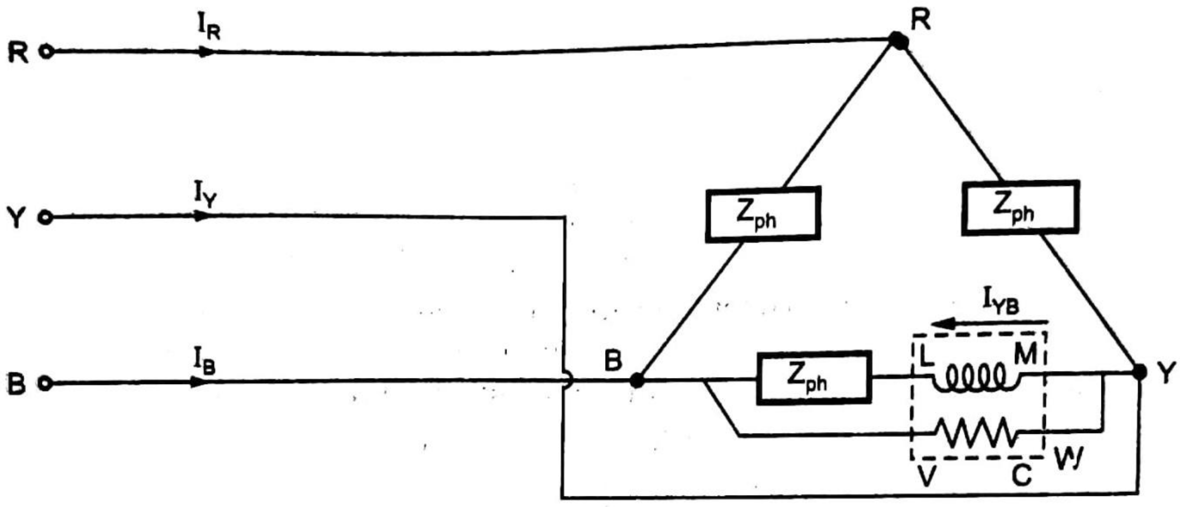


Fig.(a). Wattmeter measuring single phase power

If load is having $\cos\emptyset$ lagging power factor,phasor diagram can be shown in Fig(b).

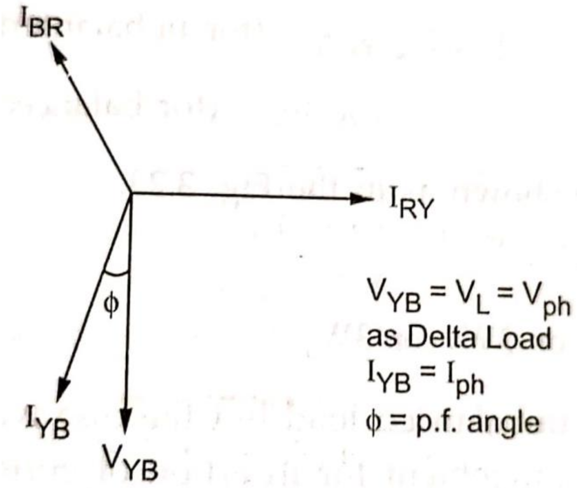


Fig.(b). Phasor diagram

❑ Disadvantages

1) Applicable only for balanced loads.
2) For using this method, for star connected loads, neutral point must be available otherwise voltage coil can not be connected so as to measure phase voltage.

   Similarly for delta connected load, it must be possible to open the closed delta so as to insert current coil to measure phase current. This may not be possible in practice.

Extension of Range of Wattmeters using Instrument Transformers

Ratios of Instrument Transformer

- The various ratios defined for the instrument transformers are,

1. Actual ratio [R]

The actual transformation ratio is defined as the ratio of the magnitude of actual primary phasor to the corresponding magnitude of actual secondary phasor.

$$R = \frac{\text{Magnitude of actual primary current}}{\text{Magnitude of actual secondary current}} \quad \text{... For C.T.} \qquad R = \frac{\text{Magnitude of actual primary voltage}}{\text{Magnitude of actual secondary voltage}} \quad \text{... For P.T.}$$

The actual ratio is also called transformation ratio.

2. Nominal ratio [K_n]

The nominal ratio is defined as the ratio of rated primary quantity to the rated secondary quantity, either current or voltage.

$$K_n = \frac{\text{Rated primary current}}{\text{Rated secondary current}} \quad \text{... For C.T.} \qquad K_n = \frac{\text{Rated primary voltage}}{\text{Rated secondary voltage}} \quad \text{... For P.T.}$$

3. Turns ratio [n]

$$n = \frac{\text{Number of turns of secondary winding}}{\text{Number of turns of primary winding}} \quad \text{... For C.T.} \qquad n = \frac{\text{Number of turns of primary winding}}{\text{Number of turns of secondary winding}} \quad \text{... For P.T.}$$

Ratio Correction Factor (RCF) : It is the ratio of transformation i.e. actual ratio to the nominal ratio.

Extension of Range of Wattmeters using Instrument Transformers

- For very high voltage circuits, the high rating wattmeters are not available to measure the power.
- The range of wattmeter can be extended using instrument transformers, in such high voltage circuits.
- The connections are shown in Fig.(a)
- The primary winding of C.T. is connected in series with the load and secondary is connected in series with an ammeter and the current coil of a wattmeter.
- The primary winding of P.T. is connected across the supply and secondary is connected across voltmeter and the pressure coil of the wattmeter. One secondary terminal of each transformer and the casings are grounded.

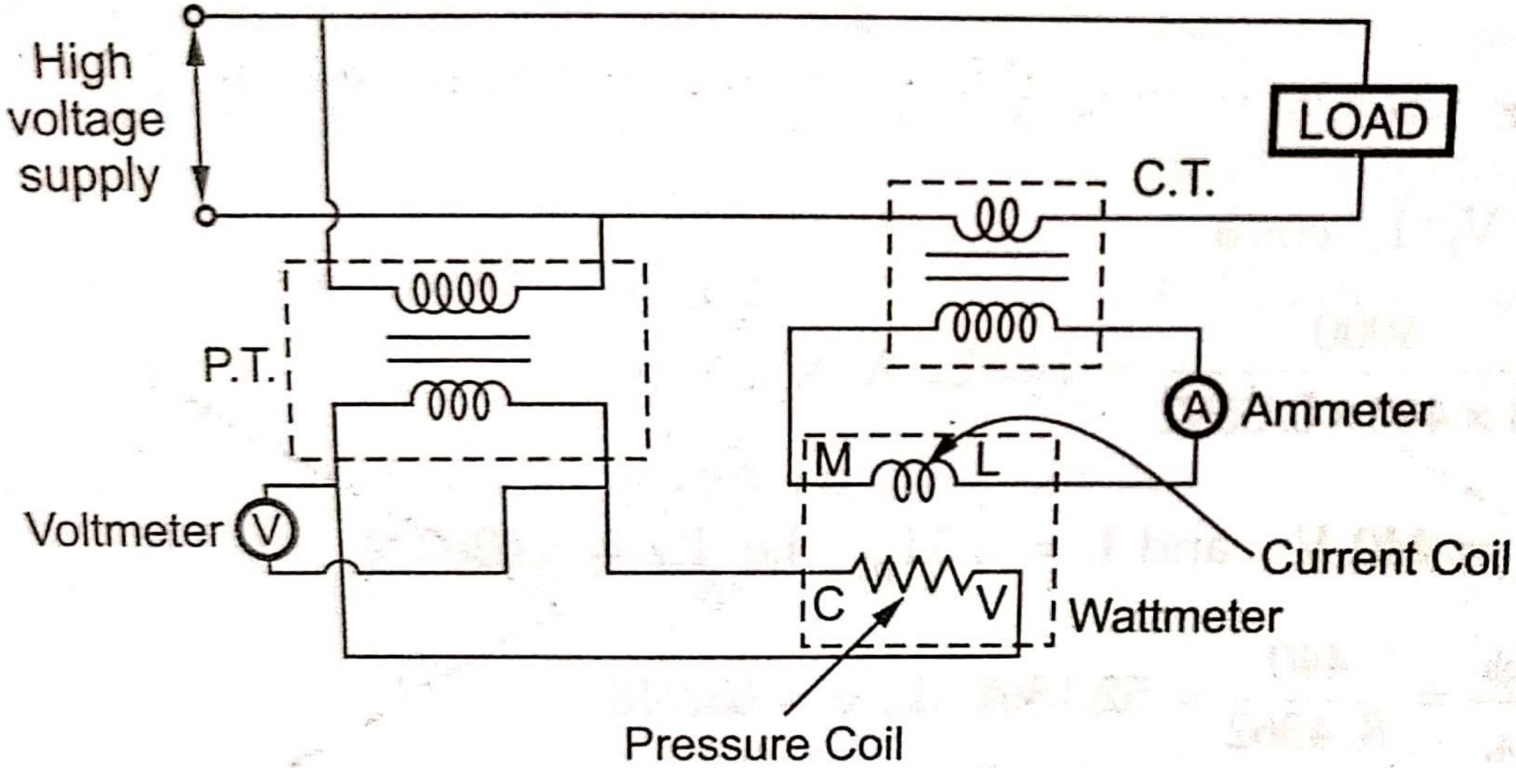


Fig. (a). Power measurement using C.T. and P.T.

- Now both C.T. and P.T. have errors like ratio error and phase angle error. For precise measurements, these errors must be considered. If not considered, these errors may cause inaccurate measurements.
- The correction must be applied to such errors to get the accurate results.

➢ Phasor Diagrams and Correction Factors

Consider the various parameters as,

V = Voltage across the load

I = Load current

∅ = Phase angle between V and I

V_S = Voltage across secondary of P.T.

= Wattmeter pressure coil voltage

I_S = Current in secondary of CT.

= Wattmeter current coil current

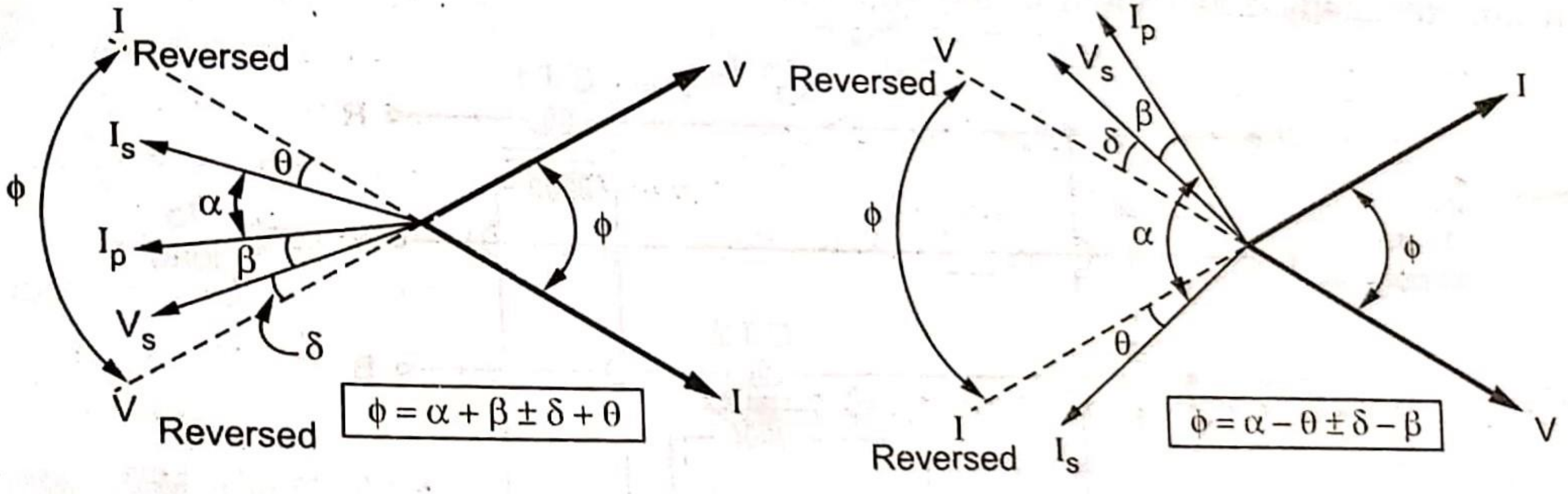
I_P = Current in pressure coil of wattmeter

α = Phase angle between currents in current coil and pressure coil of wattmeter

δ= Phase angle of P.T.

θ = Phase angle of CT.

β = Angle by which I_P lags V_S, due to inductance of pressure coil



(a) Lagging p.f. (b) Leading p.f.

The phasor diagrams for lagging and leading P.F.Joads are shown in the Fig, (a) and (b) respectively

Lagging power factor : For lagging p.f., θ is positive i.e., I_S, leads reversed I while phase of P.T. may be positive or negative i.e. δ can be positive or negative.
For phasor diagram shown in Fig.(a) δ is negative i.e. V_S, lags reversed V hence,

$$\left.\begin{array}{lll} \phi = \alpha + \beta + \delta + \theta & \ldots & \delta \text{ negative} \\ \phi = \alpha + \beta - \delta + \theta & \ldots & \delta \text{ positive} \end{array}\right\} \frac{\text{Lagging}}{\text{p.f.}}$$

Leading power factor : For leading p.f. the Ø is given by

$$\left.\begin{aligned}\phi &= \alpha - \beta - \delta - \theta \qquad \ldots \qquad \delta \text{ negative}\\ \phi &= \alpha - \beta + \delta - \theta \qquad \ldots \qquad \delta \text{ positive}\end{aligned}\right\} \frac{\text{Leading}}{\text{p.f.}}$$

Correction Factor : The correction factor, neglecting transformation ratio errors is,

$$K = \frac{\cos \emptyset}{\cos \beta \cos \alpha}$$

where $\alpha = \phi - \beta - \delta - \theta$ ... lagging p.f. (with δ negative)

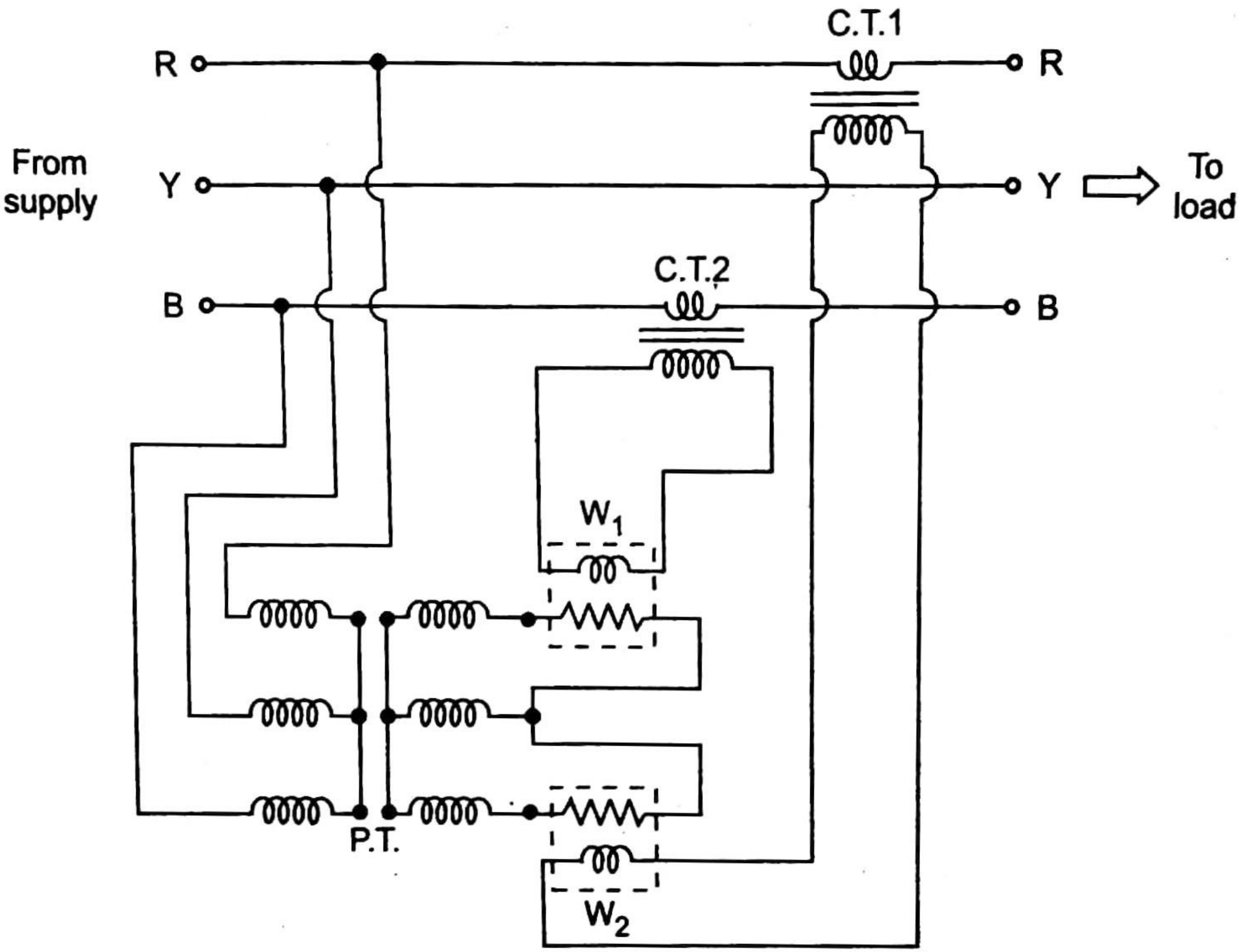
and $\alpha = \phi + \beta + \delta + \theta$ ... leading p.f. (with δ negative)

Key Point: *Change the sign of δ in the expression of α if δ is positive.*

$$\therefore \quad \text{True Power} = K \times \begin{matrix}\text{actual ratio}\\ \text{of C.T.}\end{matrix} \times \begin{matrix}\text{actual ratio}\\ \text{of P.T.}\end{matrix} \times \begin{matrix}\text{wattmeter}\\ \text{reading}\end{matrix}$$

$$\boxed{\text{True Power} = K \times \begin{bmatrix}\text{R.C.F}\\ \text{of C.T.}\end{bmatrix} \times \begin{bmatrix}\text{R.C.F.}\\ \text{of P.T.}\end{bmatrix} \times \begin{bmatrix}\text{nominal ratio}\\ \text{of C.T.}\end{bmatrix} \times \begin{bmatrix}\text{nominal ratio}\\ \text{of P.T.}\end{bmatrix} \times \begin{bmatrix}\text{wattmeter}\\ \text{reading}\end{bmatrix}}$$

- Three phase power measurement using C.T. and P.T.



(1).A reading of 400 W is indicated on a 100 V / 5 A wattmeter used in connection with voltage and current transformers of nominal ratio 100 / 1 and 20 / 1 respectively. If the wattmeter pressure coil has a resistance of 400 Ω and an inductance of 20 mH and the ratio errors and the phase differences of the voltage and current transformers are + 1% and 50 min and - 0.5 % and 100 min respectively. Compute the true value of the power measured. The load phase angle is 60° lagging and the frequency is 50 Hz.

Solution: $\delta = +50' = +0.833°, \theta = 100' = +1.667°, K_n(P.T.) = 100, K_n(C.T.) = 20$

$$X_{Lp} = 2\pi f L_p = 2\pi \times 50 \times 20 \times 10^{-3} = 6.2831\,\Omega$$

$$\beta = \tan^{-1}\left[\frac{X_{Lp}}{R_P}\right] = \tan^{-1}\left[\frac{6.2831}{400}\right] = 0.9°$$

$$\phi = 60° \text{ lagging, Wattmeter reading} = 400 \text{ W.}$$

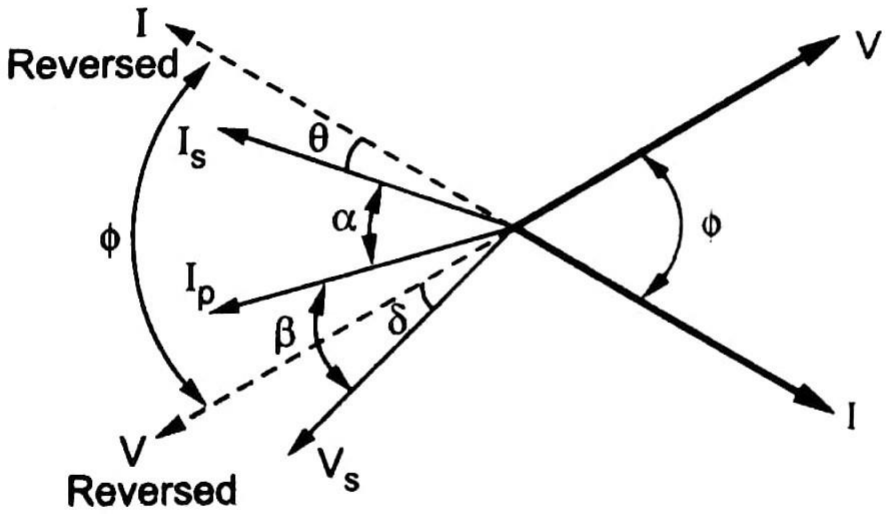
Note : δ is positive hence V_s leads V reversed by δ.

$$\phi = \theta + \alpha + \beta - \delta$$

$$60° = 1.667° + \alpha + 0.9° - 0.833°$$

$$\therefore \quad \alpha = 58.266°$$

$$K = \frac{\cos\phi}{\cos\beta \cos\alpha} = \frac{\cos 60°}{\cos(0.9°)\cos(58.266°)} = 0.9507$$



Phasor diagram

$$\% \text{ ratio error} = \frac{K_n - R}{R} \times 100$$

$$\therefore \quad 1 = \frac{100 - R}{R} \times 100 \quad \text{i.e. } R = 99.0099$$

$$\therefore \quad \text{R.C.F. of P.T.} = \frac{R}{K_n} = \frac{99.0099}{100} = \mathbf{0.990099}$$

$$\% \text{ ratio error} = \frac{K_n - R}{R} \times 100$$

$$\therefore \quad -0.5 = \frac{20 - R}{R} \times 100 \quad \text{i.e. } R = 20.1005$$

$$\therefore \quad \text{R.C.F. of C.T.} = \frac{R}{K_n} = \frac{20.1005}{20} = \mathbf{1.005}$$

$$\therefore \quad \begin{bmatrix} \text{True} \\ \text{power} \end{bmatrix} = K \times \begin{bmatrix} \text{R.C.F} \\ \text{of P.T.} \end{bmatrix} \times \begin{bmatrix} \text{R.C.F.} \\ \text{of C.T.} \end{bmatrix} \times \begin{bmatrix} K_n \text{ of} \\ \text{C.T.} \end{bmatrix} \times \begin{bmatrix} K_n \text{ of} \\ \text{P.T.} \end{bmatrix} \times \begin{bmatrix} \text{Wattmeter} \\ \text{reading} \end{bmatrix}$$

$$= 0.9507 \times 0.990099 \times 1.005 \times 100 \times 20 \times 400$$

$$= \mathbf{756794.8439 \ W}$$

ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

Power Factor Meters

Power Factor Meter

- The power factor meter measures the power factor of a transmission system. The power factor is the cosine of the angle between the voltage and current. The power factor meter determines the types of load using on the line, and it also calculates the losses occur on it.

- The power factor of the transmission line is measured by dividing the product of voltage and current with the power. And the value of voltage, current and power is easily determined by the voltmeter, ammeter and wattmeter respectively. This method gives high accuracy, but it takes time.

- The power factor of the transmission line is continuously changed with time. Hence it is essential to take the quick reading. The power factor meter takes a direct reading, but it is less accurate. The reading obtained from the power factor meter is sufficient for many purposes to expect precision testing.

- The power factor meter has the moving system called pointer which is in equilibrium with the two opposing forces. Thus, the pointer of the power factor meter remains at the same position which is occupied by it at the time of disconnection.

Types of Power Factor Meters

The power factor meter is of two types. They are

1.Electrodynamometer

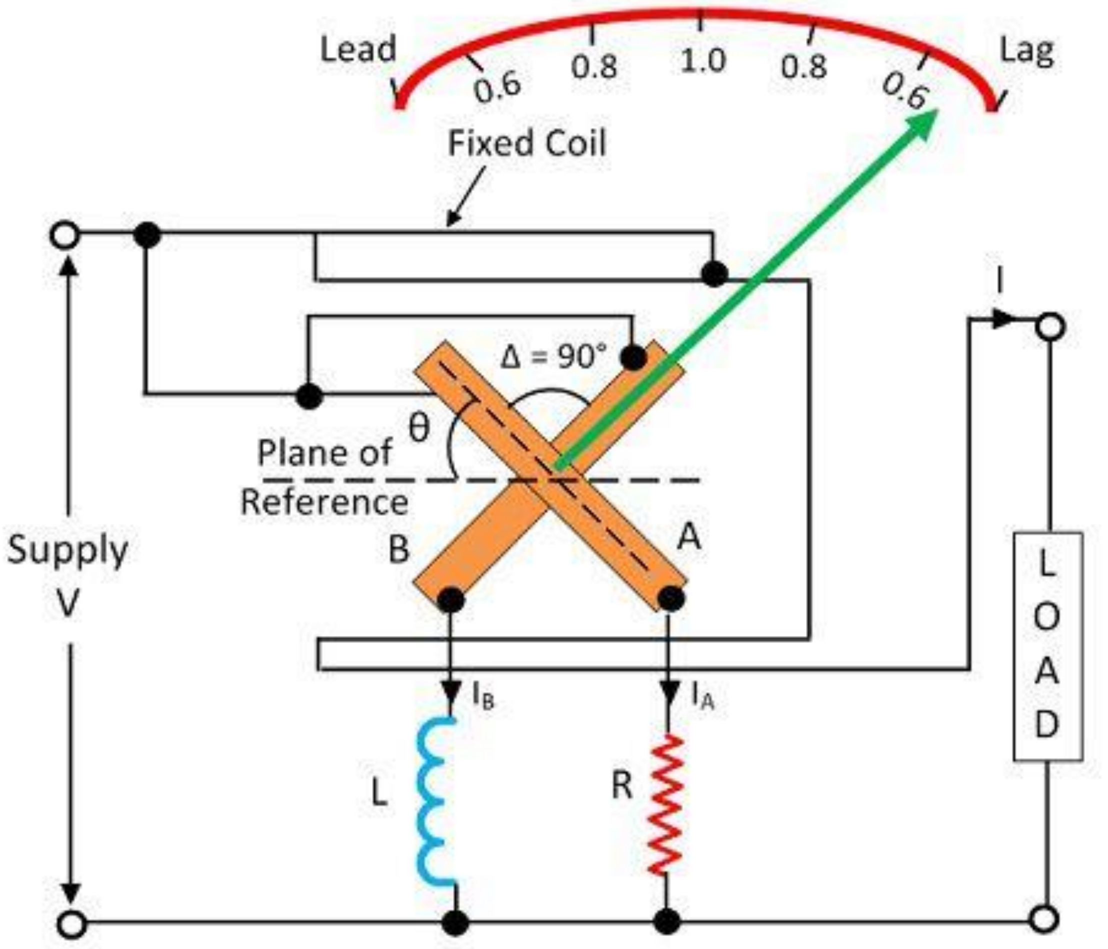
1. Single Phase Electrodynamometer
2. Three Phases Electrodynamometer

2.Moving Iron Type Meter

1. Rotating Field type
2. Alternating Field type

Single Phase Electrodynamometer Power Factor Meter

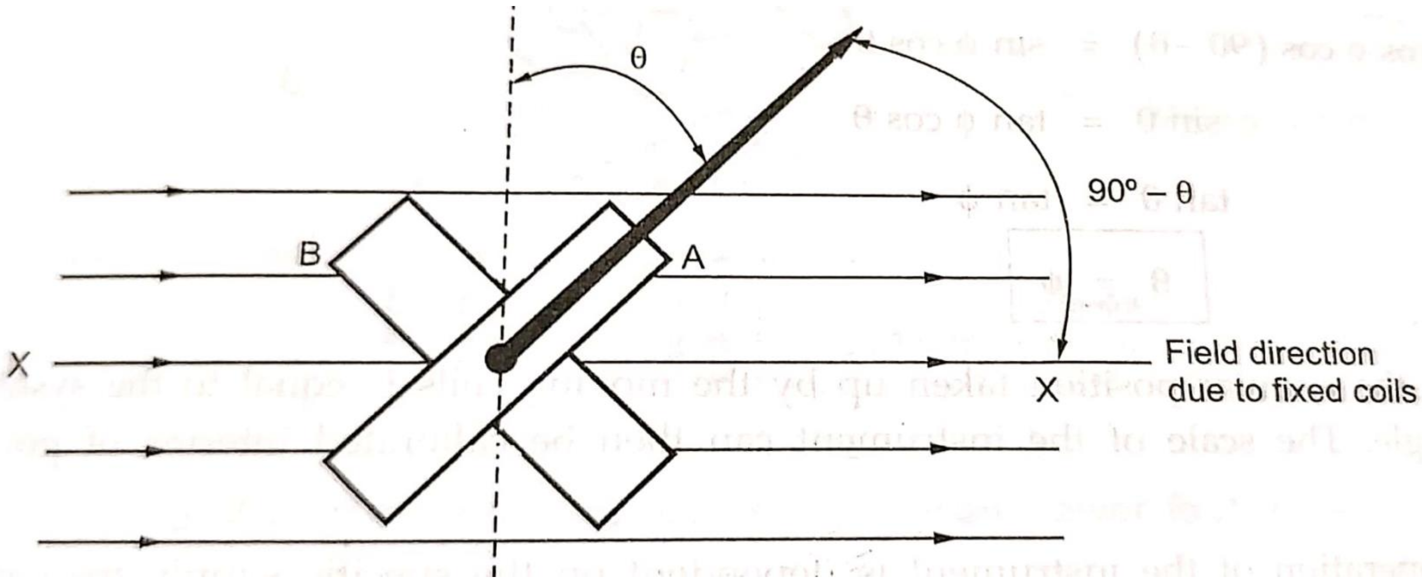
- **The construction** of the single phase electrodynamometer is shown in the figure.
- The meter has fixed coil which acts as a current coil.
- This coil is split into two parts and carry the current under test. The magnetic field of the coil is directly proportional to the current flow through the coil.
- The meter has two identical pressure coils A and B. Both the coils are pivoted on the spindle.
- The pressure coil A has no inductive resistance connected in series with the circuit, and the coil B has highly inductive coil connected in series with the circuit.
- The current in the coil A is in phase with the circuit while the current in the coil B lag by the voltage nearly equal to 90º. The connection of the moving coil is made through silver or gold ligaments which minimize the controlling torque of the moving system.
- The meter has two deflecting torque one acting on the coil A, and the other is on coil B. The windings are so arranged that they are opposite in directions. The pointer is in equilibrium when the torques are equal.



Single Phase Electrodynamometer Type Power Factor Meter

Working of P.F Meter

- Consider the position of the moving system as shown in Fig(a)
- Assume that the current through the coil B lags the voltage exactly by 90^0.
- Also assume that the field produced by the fixed coils is uniform and in the direction X-X as shown in the Fig.(a).
- Due to interaction of the fields produced by the currents through various coils, both the coils A and B experience a torque.



Fig(a)

- The windings are arranged in such a manner that the torques experienced by coil A and B are opposite to each other. Hence the pointer attains an equilibrium position when these two torques are equal.

Similar to a dynamometer type wattmeter, torque on coil A is given by,

$$T_A = \text{K VI} \cos \emptyset \cos (90°-\theta) \quad ....(1)$$

where

K = Constant

$\emptyset$ = Power factor angle

θ = Angular deflection from the plane of reference

The equation is similar to the torque equation of a dynamometer type instrument.
The current through coil A is in phase with system voltage V and it moves in a magnetic field which is proportional to system current I and dM/dθ which is generally constant for radial field is not constant for parallel field and is proportional to cos (90°- θ).

Similarly current in coil B lags the supply voltage by 90° and it moves in same field.

Hence the torque on B is proportional to cos (90° - Ø) i.e. sin Ø and cos θ.

$$T_B = K\ VI\ \sin\emptyset \cos\theta \quad ....(2)$$

In equilibrium position, $T_A = T_B$

$$\therefore \quad K\ VI \cos\emptyset \cos(90°-\theta) = K\ VI\ \sin\emptyset \cos\theta$$

$$\cos\emptyset \cos(90°-\theta) = \sin\emptyset \cos\theta$$

$$\sin\theta = \tan\emptyset \cos\theta$$

$$\tan\theta = \tan\emptyset$$

$$\therefore \quad \theta = \emptyset$$

Thus the angular position taken up by the moving coils is equal to the system power factor angle.

Three Phase Electrodynamometer Power Factor Meter

- A dynamometer type three-phase power factor meter gives correct readings only when the load is balanced. The basic principle of this instrument is the same as that of the single-phase dynamometer type power factor meter. The only difference is in its construction.

➢ Construction & working of a 3 Phase Power Factor Meter

- It consists of two fixed coils *FF* connected in series in one of the phases and carries the line current as shown in the Fig(a)
- The two identical moving coils *A* and *B* are fixed with their planes 120° apart and are connected across the two remaining phases respectively through high resistances as shown in the Fig(a)
- In this case, there is no necessity for phase splitting by artificial means, since the required phase displacement between the currents in the moving coils can be obtained from the supply itself.

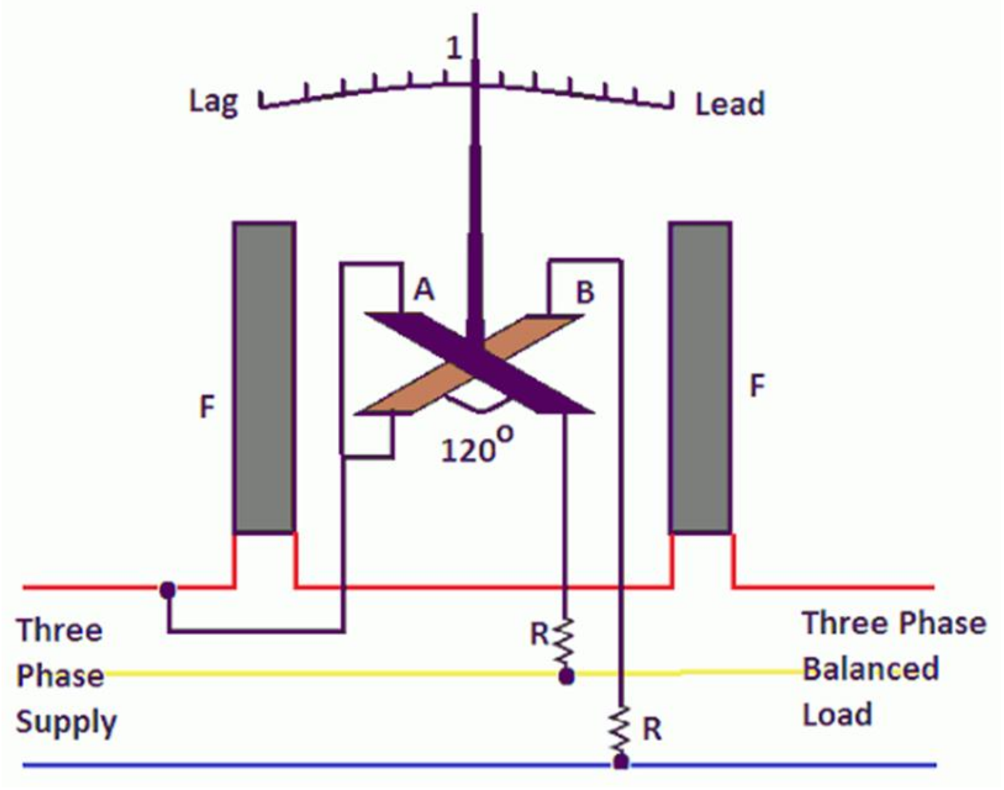


Fig.(a)3 ph Electrodynamometer Power Factor Meter

- The moving coil A is connected across R and Y phases, the moving coil B is connected across R and B phases of the supply through series resistance ,then as the phase difference between the currents through the two moving coils is automatically created due to the phase difference between the supply.
- The moving coil A is connected through series resistance to Y phase and moving coil B is connected through series resistance to B phase of the supply.
- The resistance used in series with the moving coils are adjusted such that the current of I_Y it should be equal to I_B.

➢ Let the load is balanced 3 phase with lagging power factor

Hence each phase current lags the phase voltage by angle Ø

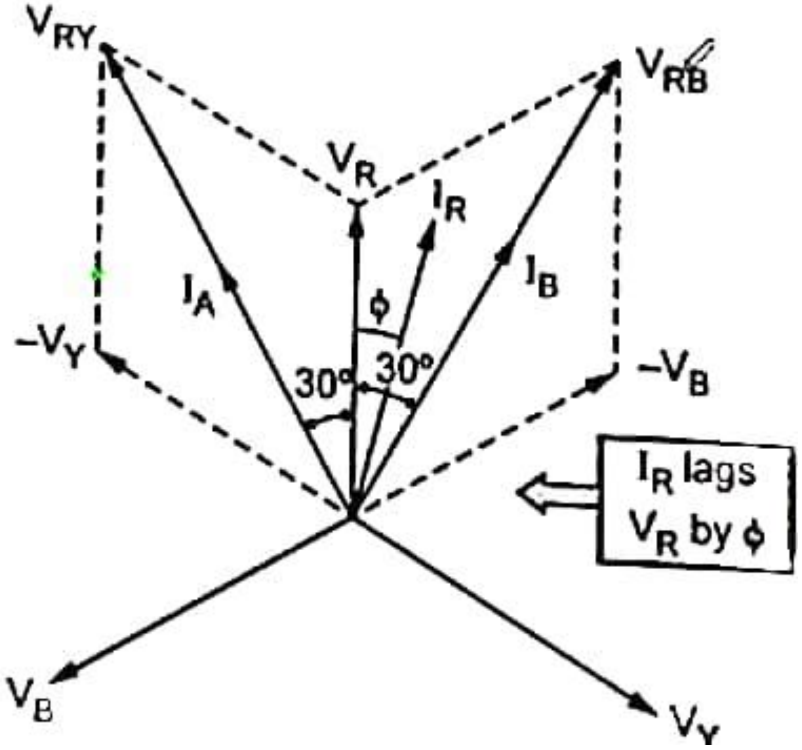
Let V_{RY} = the voltage across the moving coil A,

V_{RB} = the voltage across the moving coil B,

$\emptyset = Phase\ angle\ of\ load$

$\theta = Deflection$ of pointer from plane of reference

As the supply is balanced, $V_{RY} = V_{YB} = V_{BR}$



Fig(b).Phasor diagram

Let the load is star connected,

$\therefore\ \overline{V_{RY}} = \overline{V_R} - \overline{V_Y}$ and $\overline{V_{BR}} = \overline{V_B} - \overline{V_R}$ hence $\overline{V_{RB}} = \overline{V_R} - \overline{V_B}$

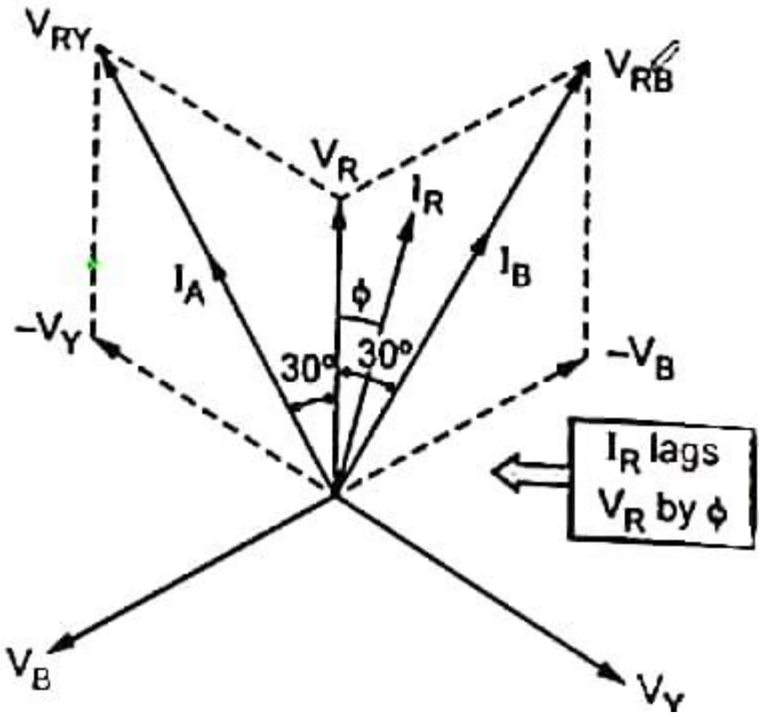
and $V_R = V_Y = V_B$ are the phase voltages.

$I_L = I_{ph} = I_R = I_Y = I_B$

The phasor diagram is shown in Fig(b).

$I_A \propto V_{RY}$ and I_A & I_B are moving coil currents

$I_B \propto V_{RB}$ and I_R is the current through the fixed coils.



Fig(b).Phasor diagram

- The deflecting torque in dynamo meter type instrument is given by,

$$T = I_1\ I_2\ \cos\emptyset\ \frac{dM}{d\theta}$$

If the flux density of the magnetic field is uniform, then the mutual inductance M is proportional to the $\cos\theta$.

$$M = -\ M_{max}\cos\theta$$

$$\frac{dM}{d\theta} = M_{max} \sin\theta$$

$$T = I_1 \, I_2 \, \cos\emptyset \; M_{max} \sin\theta$$

Hence the torque acting on coil A,

$$T_A = I_R \, I_A \, M_{max} \cos(I_R \, {}^\wedge I_A) \sin(60^0 + \theta)$$

$I_A = KV_{RY}$ and $I_R \, {}^\wedge I_A = 30^0 + \emptyset$

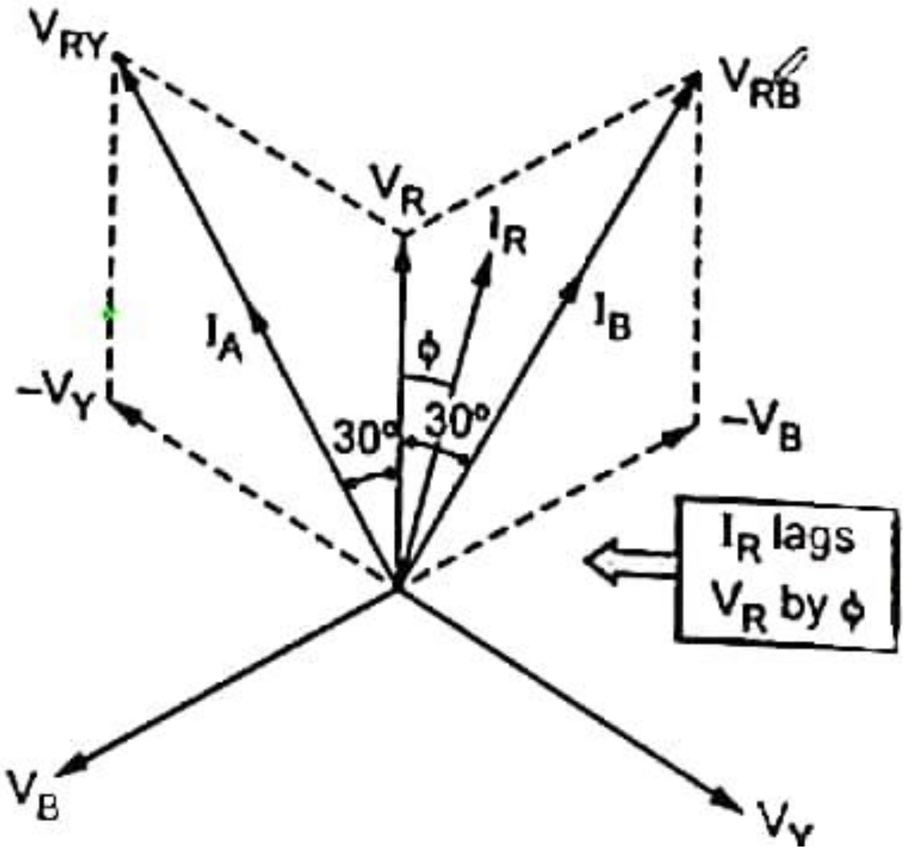
$$T_A = KV_{RY} \, I_R \, M_{max} \cos(30^0 + \emptyset) \sin(60^0 + \theta)$$

But $V_{RY} = \sqrt{3} \, V_R$

$$T_A = \sqrt{3} \, KV_R \, I_R \, M_{max} \cos(30^0 + \emptyset) \sin(60^0 + \theta)$$

Similarly torque acting on coil B,

$$T_B = I_R \, I_B \, M_{max} \cos(I_R \, {}^\wedge I_B) \sin(120^0 + \theta)$$



Fig(b).Phasor diagram

$I_B = KV_{RB}$ and $I_R \wedge I_B = 30^0 - \emptyset$

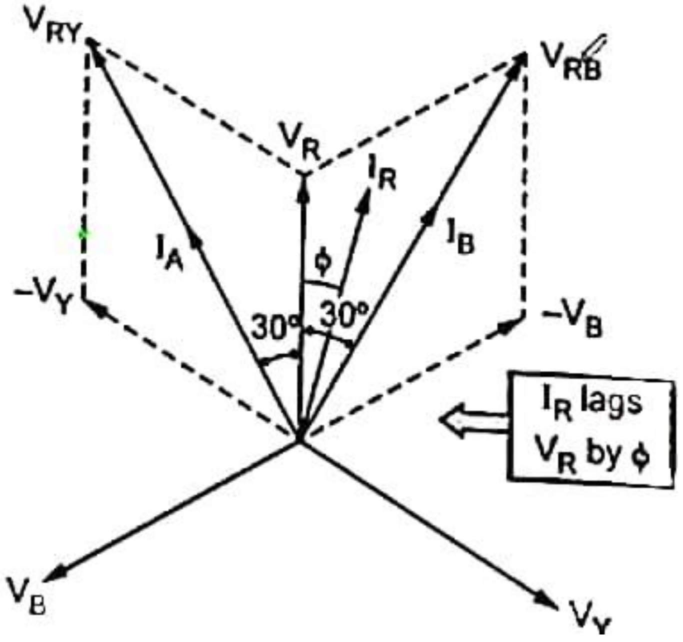
While $V_{RB} = \sqrt{3}\, V_B$

$T_B = \sqrt{3}\, KV_B\, I_R\, M_{max} \cos(30^0 - \emptyset) \sin(120^0 + \theta)$

The pointer attains the steady state θ when $T_A = T_B$

$V_B = V_R = V_{ph}$ for the balanced system hence $T_A = T_B$ gives,

$\sqrt{3}\, KV_R\, I_R\, M_{max} \cos(30^0 + \emptyset) \sin(60^0 + \theta) = \sqrt{3}\, KV_B\, I_R\, M_{max} \cos(30^0 - \emptyset) \sin(120^0 + \theta)$



Fig(b).Phasor diagram

$\cos(30^0 + \emptyset) \sin(60^0 + \theta) = \cos(30^0 - \emptyset) \sin(120^0 + \theta)$

Finally, after solving the above equation, we get $\theta = \emptyset$

- When the three phase power factor meter is connected in the circuit, under balanced load conditions, the angle through which the pointer is deflected from the unity power factor position is equal to the phase angle of the circuit, because the two moving coils are fixed 120° apart.
- The deflections in three phase power factor meters are independent of frequency and waveform since the currents in the two moving coils are affected in the same way by any change of frequency.

➢ Advantages of Electrodynamic Type Power Factor Meters

1) Losses are less because of minimum use of iron parts and also give less error over a small range of frequency as compared to moving iron type instruments.
2) They high torque is to weight ratio.

➢ Disadvantages of Electrodynamic Type Power Factor Meters

1) Working forces are small as compared to moving iron type instruments.
2) The scale is not extended over 360^{o}.
3) Calibration of electrodynamometer type instruments are highly affected by the changing the supply voltage frequency.
4) They are quite costly as compared to other instruments.

Moving Iron Power Factor Meter

Moving Iron Power Factor Meter

- There are two types of moving iron power factor meters. They are
  - **Rotating field type**
  - **Alternating field type**

Rotating Field Type Moving Iron Power Factor Meter

- It consists of three fixed coils whose axes are displaced from each other by 120°.
- The coils are supplied from a three phase supply through current transformers (C.T.)
- The Fig. (a) shows the construction of rotating field type moving iron power factor meter.
- The coils F1, F2 and F3 are the fixed coils. The coil F1 is supplied from phase R, coil F2, from phase Y and coil F3 from phase B.

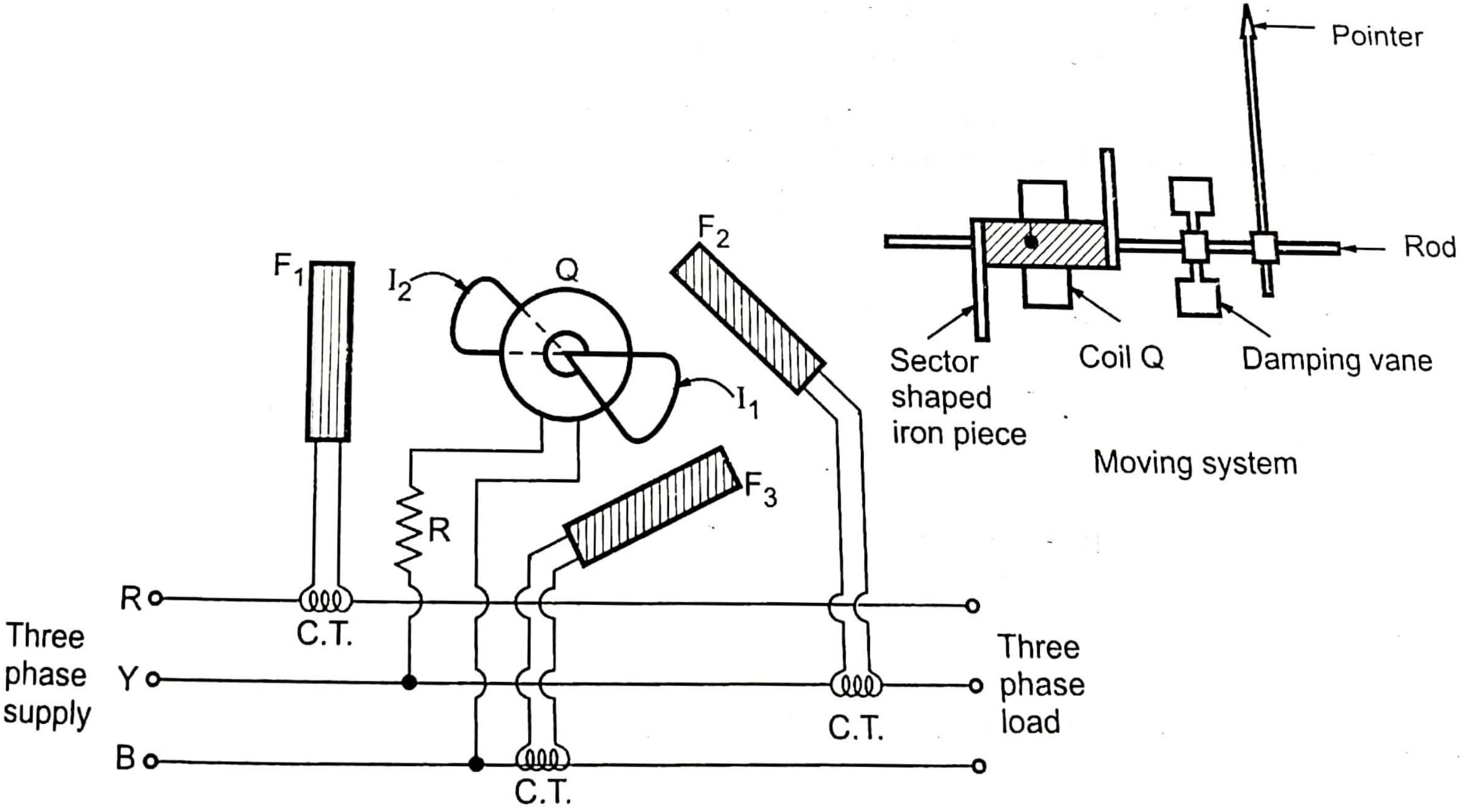


Fig.(a) Rotating field type moving iron power factor meter

Rotating Field Type Moving Iron Power Factor Meter

- The coil Q is placed at the centre of the three fixed coils and is connected across any two lines of the supply through a series resistance.

- Inside coil Q there is a short pivoted iron rod. The rod carries two sector shaped vanes I1, and I2, at its ends.
- The same rod carries damping vanes and a pointer.

- The coil Q and the iron system produce an alternating flux which interacts with the flux produced by the coils F1, F2 and F3.

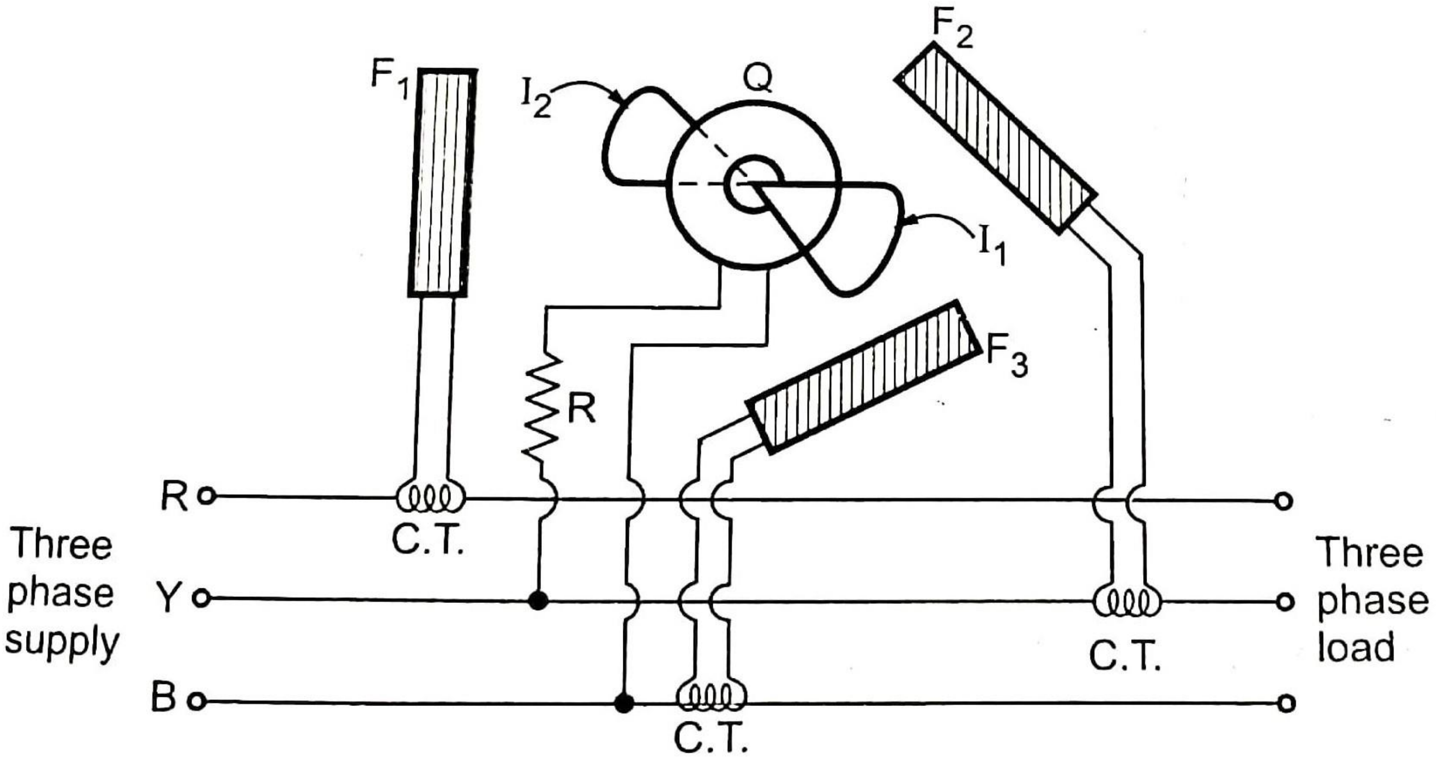


Fig.(a) Rotating field type moving iron power factor meter

Rotating Field Type Moving Iron Power Factor Meter

- Due to resistance R, the current in coil Q is in phase with the supply voltage. So the deflection of the moving system is approximately equal to the power factor angle of the three phase circuit.
- The flux produced by the coils F1, F2 and F3 is rotating magnetic flux which creates an induction motor action.

- It tries to keep moving system continuously rotating. But it sets moving system in a definite position due to use of high resistivity iron parts.
- Such high resistivity parts reduce the induced currents and stops the continuous rotation.
- The meter can be used for balanced loads.
- It is also called Westinghouse power factor meter. It is calibrated at the normal supply frequency and can cause serious errors if used at any other frequency.

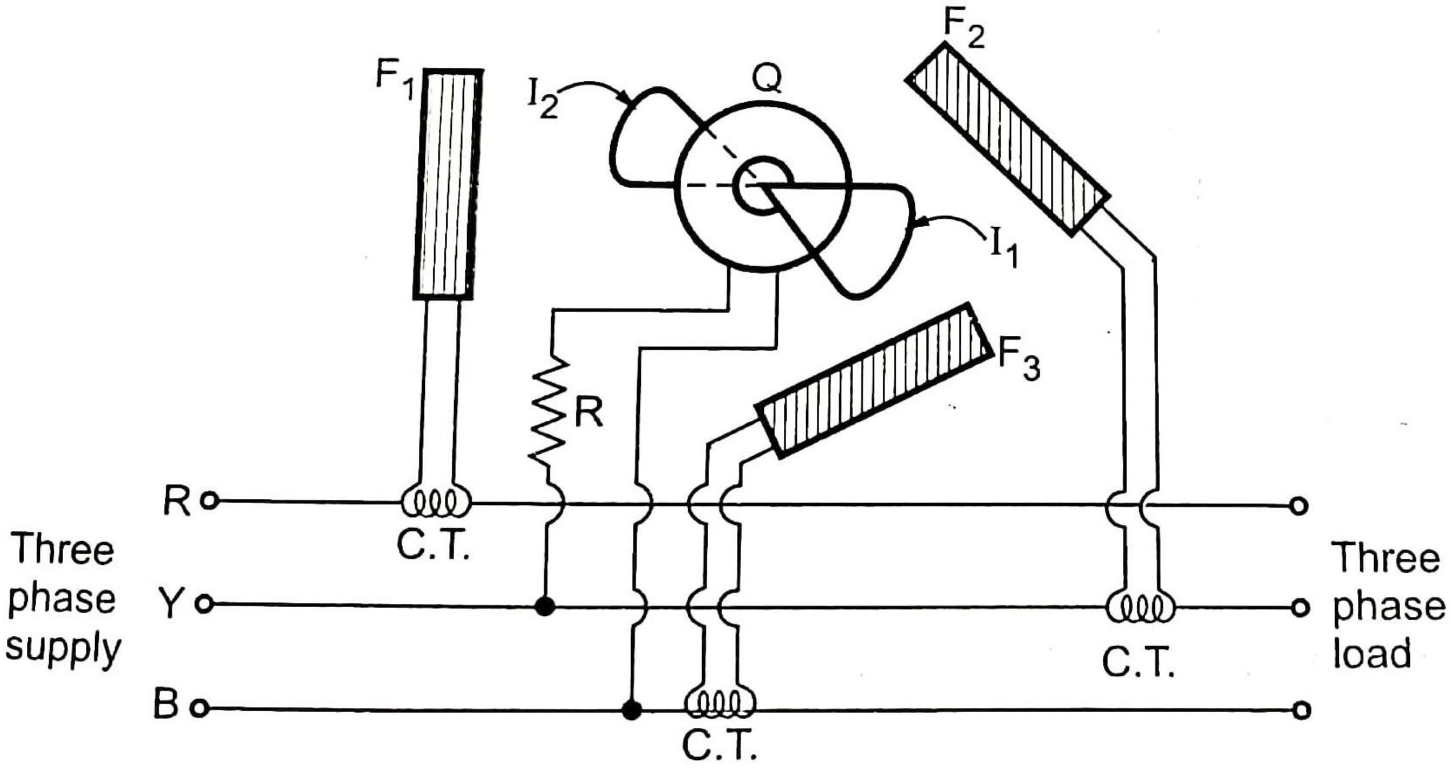
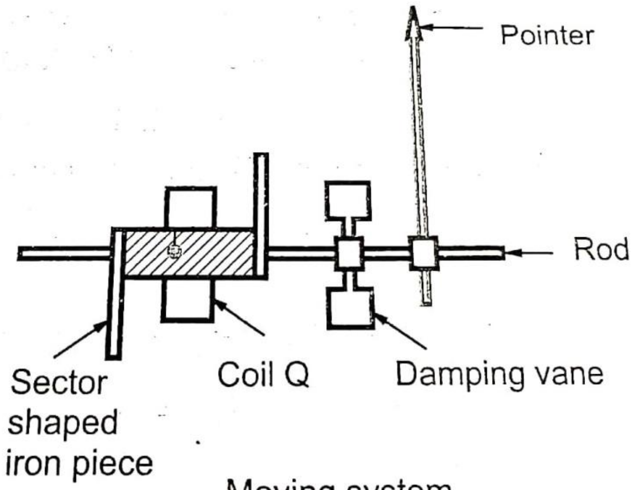


Fig.(a) Rotating field type moving iron power factor meter

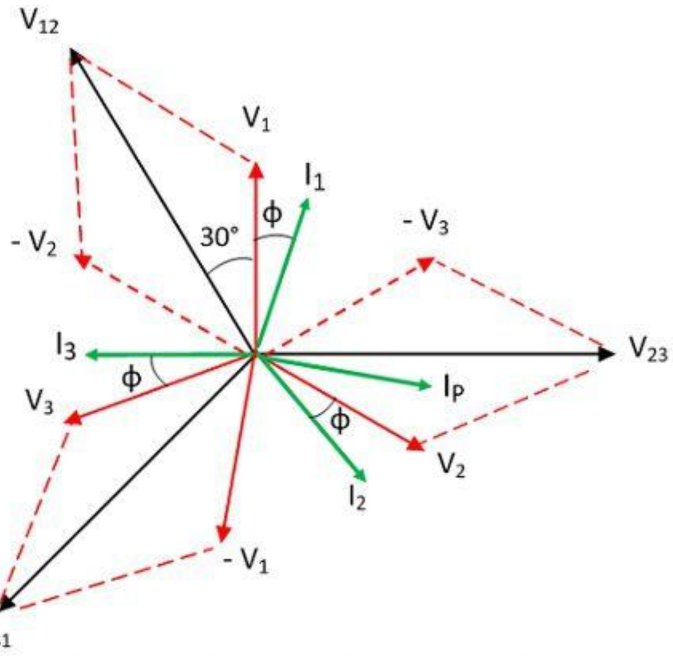
- The control springs are absent. The moving system is shown seperately in the Figure.



Moving system

- The total torque of the meter is zero for steady state deflection

$$[\mathrm{Cos}(90^\circ - \emptyset)Sin(90^\circ + \emptyset) + \mathrm{Cos}(330^\circ - \emptyset)Sin(210^\circ + \emptyset) + \mathrm{Cos}(210^\circ - \emptyset)Sin(330^\circ + \emptyset)] = 0$$



Phasor Diagram for Moving Iron Type Power Factor Meter

- The coil Q and the iron cylinders generate the alternating flux which interacts with the flux of the fixed coils.

- The interaction of the coil generates the moving system which determined the phase angle of the current.

- The vanes of the power factor meter are magnetized by the current of the moving coil which is in phase with the system line voltage.

Alternating Field Type Moving Iron Power Factor Meter

- This instrument consists of three moving irons and vanes, which are fixed to the common spindle The spindle carries the damping vanes and the pointer.

- The moving iron vanes are sector shaped.The arcs of these sectors have an angle of 120° with respect to each other. These iron sectors are separated from each other on the spindle by the non-magnetic pieces denoted as S.

- The Fig (a) shows the construction of this instrument where Q1, Q2 and Q3 are the iron sectors. These iron sectors are magnetized by the coils P1, P2 and P3. These are voltage coils.

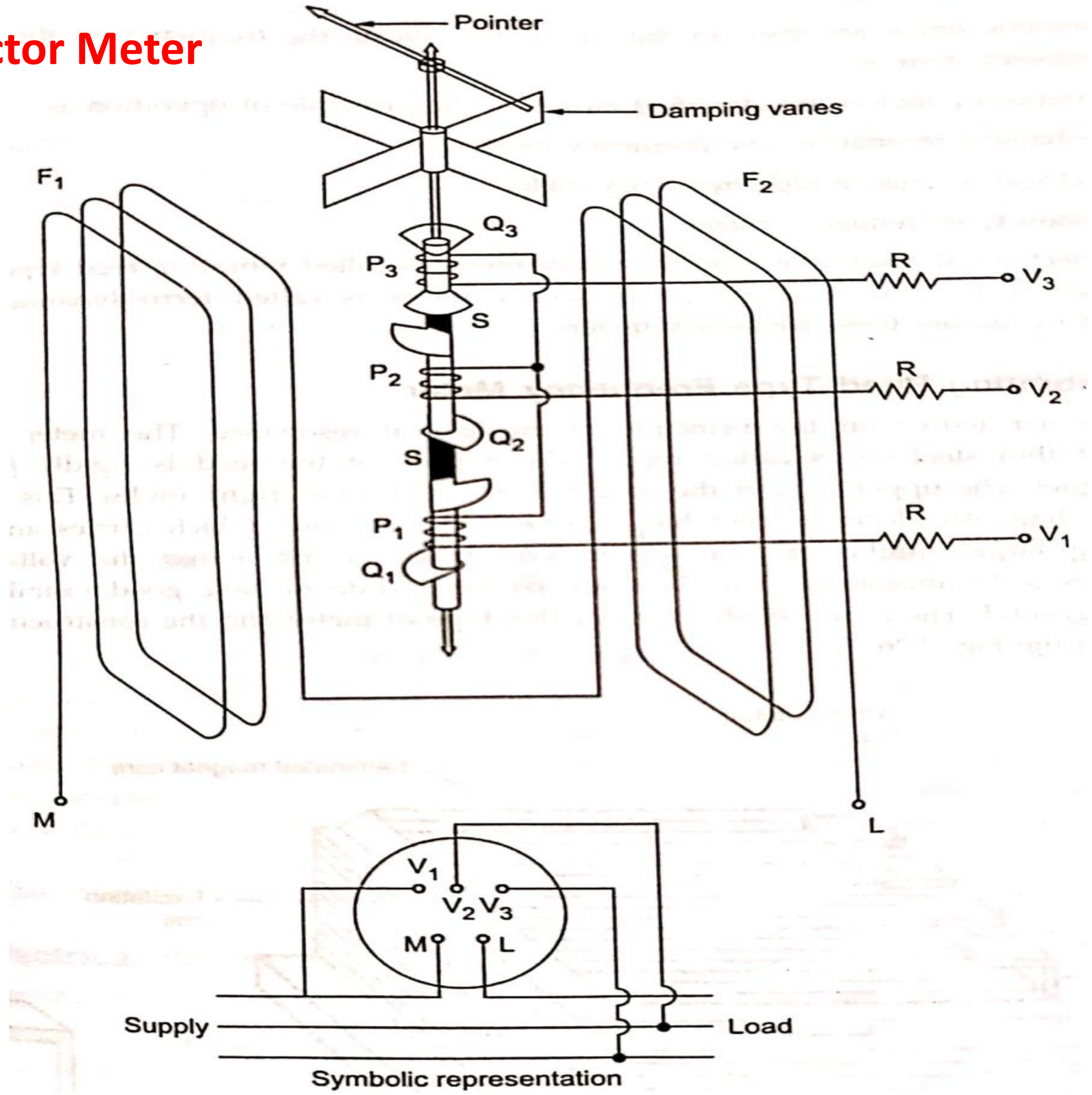


Fig.(a)Alternating Field Type M.I. Power Factor Meter

Alternating Field Type Moving Iron Power Factor Meter

- These coils are connected across the three phases. Thus the currents through them are proportional to the phase voltages of the three phase system.
- The current coil is divided into two equal parts F1 and F2, parallel to each other.
- The current coil carries one of the three line currents. One part F1 of the current coil is on one side of the moving system and other F2 on other side.

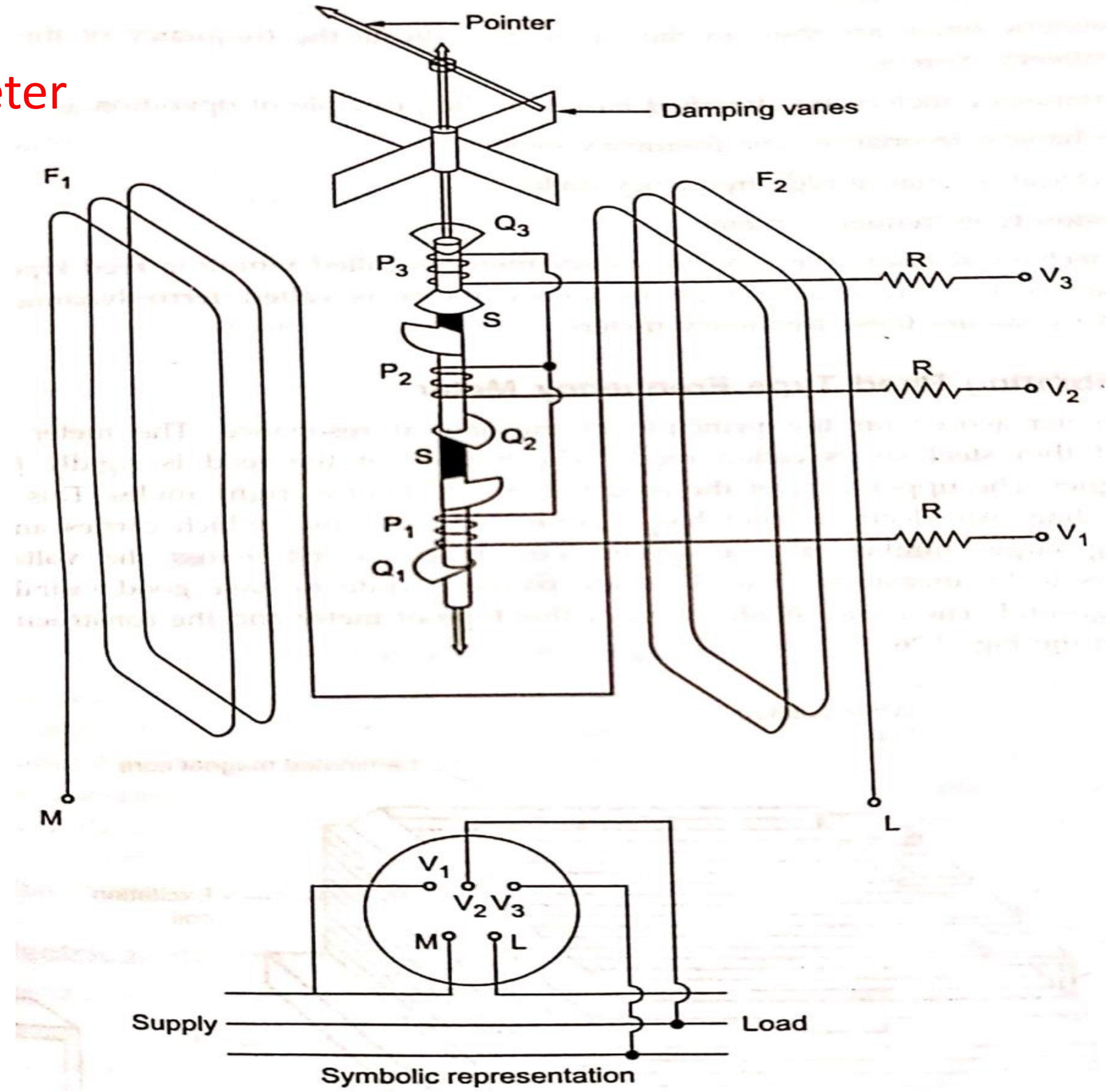


Fig.(a)Alternating Field Type M.I. Power Factor Meter

➢ Alternating Field Type Moving Iron Power Factor Meter

- When connected in the circuit, the moving system moves and attains such a position in which mean torque on one of the iron pieces gets neutralized by the torques produced by the other two iron pieces.
- In this position, the deflection of the pointer is equal to phase angle between the currents and voltages of the three phase system. The instrument is used for the balanced loads but can be modified for unbalanced loads.
- The voltage coils are at different levels hence the resultant flux is not rotating but alternating. This instrument is also called Nalder-Lipman power factor meter.

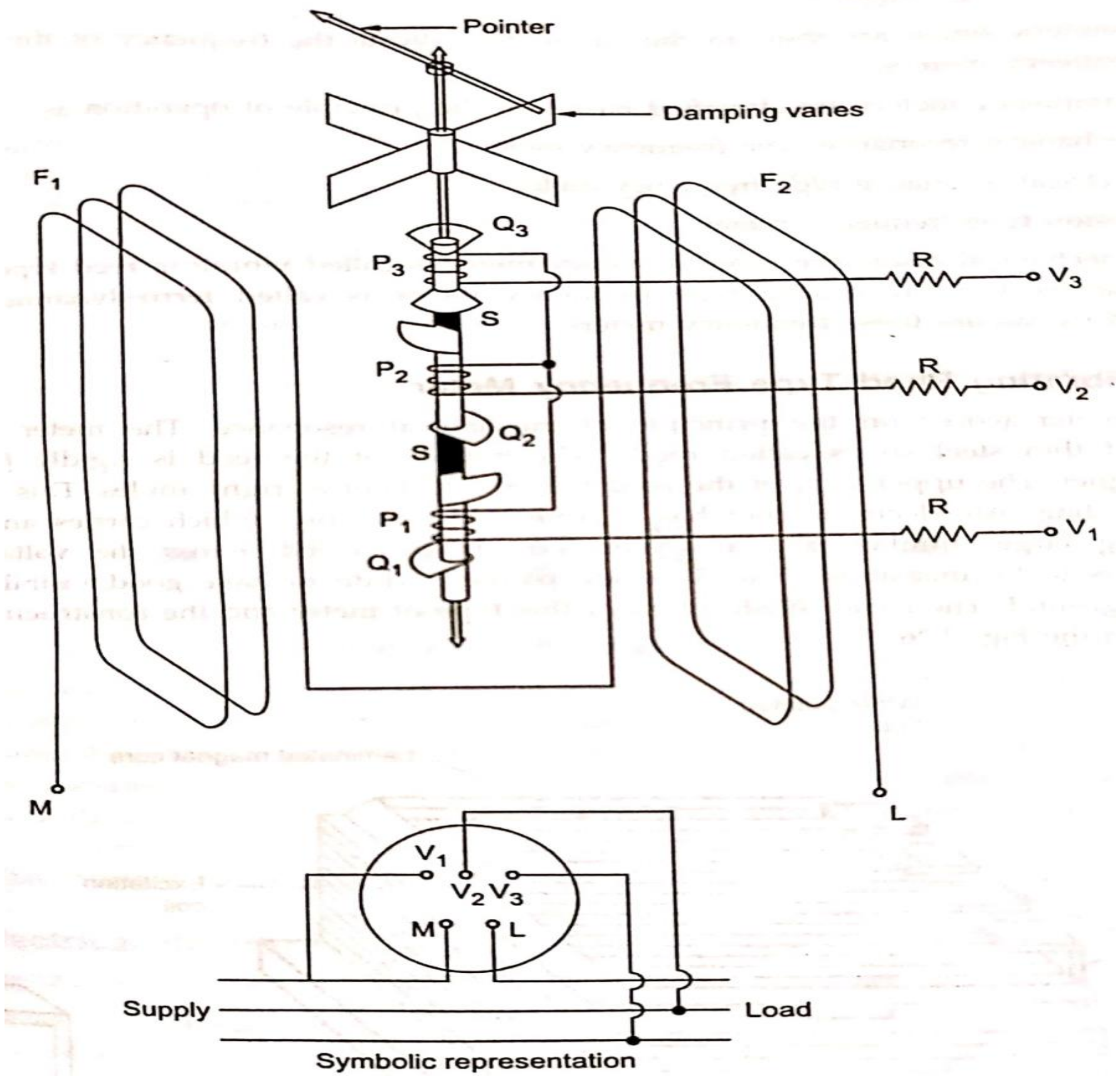


Fig.(a)Alternating Field Type M.I. Power Factor Meter

Advantages of Moving Iron Power Factor meter

1.The meter requires large working force as compared to the electrodynamometer type meter.
2.The coils of the moving iron instruments are fixed permanently.
3.The range of the scale extends up to 360º.
4.The construction of the meter is robust and simple.
5.The moving iron instrument is cheap as compared to electrodynamic meter.

Disadvantages of moving iron Power Factor meter

1) The loss occurs in the iron part of the meter. The losses depend on the load and the frequency of the meter.
2) The meter has low accuracy.
3) The calibration of the meter is affected because of the variation in supply frequencies, voltage and waveforms etc.

Thank you

ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

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ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

Syllabus

UNIT–I	Analog Ammeter and Voltmeters
UNIT–II	Analog Watt meters and Power Factor Meters
UNIT–III	Measurements of Electrical parameters
UNIT–IV	Transducers
UNIT–V	Digital meters

Text Books:

- Electrical & Electronic Measurement & Instruments by A.K.Sawhney Dhanpat Rai & Co.Publications
- Electrical and Electronic Measurements and instrumentation by R.K.Rajput, S.Chand
- Electrical Measurements and measuring Instruments by E.W. Golding and F.C.Widdis, fifth Edition, Wheeler Publishing
- Electrical Measurements by Buckingham and Price, Prentice – Hall

ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

UNIT-III

Measurements of Electrical parameters

UNIT–III

Measurements of Electrical parameters

Topics:

- **DC Bridges:**
  - Method of measuring low, medium and high resistance –
  - sensitivity of Wheat stone's bridge, Kelvin's double bridge for measuring low resistance,
  - Loss of charge method for measurement of high resistance,
  - Megger – measurement of earth resistance Numerical Problems.

- **AC Bridges:**
  - Measurement of inductance – quality factor, Maxwell's bridge, Hay's bridge, Anderson's bridge,
  - Measurement of capacitance and loss angle,Desauty's bridge, Schering Bridge, Wagner's earthing device, Wien's bridge-
  - Numerical Problems.

➢ INTRODUCTION

For several years the precision measurements of component values have been made by using various forms of bridges.

- Bridge circuits (DC or AC) are extensively used for measuring component values such as R (Resistance), L (Inductance) and C (Capacitance).
- A "bridge circuit" in its simplest form consists of a network of four resistance arms forming a closed circuit, with a D.C. source of current applied to two opposite junctions and a current detector connected to the other two junctions.

- Since the bridge circuit merely compares the value of an unknown component with that of an accurately known component (a standard), its measurement accuracy can be very high.
- This is because the readout of this comparison is based on the null indication at bridge balance, and is essentially independent of the characteristics of the null detector.
- The measurement accuracy is, therefore, directly related to the accuracy of the bridge component and not to that of the null detector used.

➢ Advantages of the bridge circuits are :

i. High measurement accuracy (as the measurement is done by comparing the unknown value with the standard value).

ii. The accuracy is independent of null detector's characteristics (it depends on the values of the components).

iii. The balance equation is independent of the magnitude of the input voltage or its source impedance, the sensitivity and impedance of the null detector or any impedance shunting the detector.

iv. The interchange of the source and detector does not affect the balance condition

v. The bridge circuit can be used in control circuits.

vi. One arm of the bridge contains a resistive element that is sensitive to the physical parameter(temperature, pressure, etc.) being controlled.

Types of Bridges

- Bridges are of the following two types :

1. D.C. bridges :
   - These bridges are used to measure the "resistances."
   - They use the D.C. voltage as the excitation voltage.

   Examples : (i) Wheatstone bridges.
   (ii) Kelvin bridges.

2. A.C. bridges :
   - These bridges are used to measure the "impedances" consisting of inductances and capacitances.
   - They use the alternating voltage as the excitation voltage.

   Examples :

   i. Maxwell's bridge;
   ii. Hay's bridge;
   iii. Schering Bridge;
   iv. Anderson bridge
   v. Desauty's bridge
   vi. Wien's bridge etc.

MEASUREMENT OF RESISTANCE

➢ Classification of Resistance

From the point of view of measurement, resistances can be classified as follows :

1. Low resistances: It is less than or equal to 1Ω

- Such resistances, in practice may be met with in the armatures and series winding of large machines, in ammeter shunts, cable lengths, contacts etc.

2. Medium resistances: It lies between 1Ω and 100 kΩ .

- In practice, the majority of the pieces of electrical apparatus used have resistances which lie between these limits.

3. High resistances: It is greater than 100 kΩ

- Measurements of high resistances are required for determination of
  (1) Insulation resistance of components and built-up electrical equipment of all types.
  (ii) Resistance of high resistance circuit elements.
  (iii) Volume resistivity and surface resistivity of a material.

➢ Measurement of Low Resistances

Following methods are used for the measurement of low resistances :

1. Ammeter-voltmeter method.
2. Potentiometer method.
3. Kelvin double bridge method.

Ammeter-Voltmeter method

- This method, which is the simplest of all, is in very common use for the measurement of low resistances when an accuracy of the order of 1% is sufficient.
- However, it must be realized that it is essentially a comparatively rough method, the accuracy being limited by those of the ammeter and voltmeter used, even if corrections are made for the "shunting" effect of the voltmeter.
- In this method, current through the resistor (X) under test and the potential drop across it are simultaneously measured. The readings are obtained by ammeter and voltmeter respectively. The attainable accuracy depends primarily on the accuracy and ranges of the instruments employed for measurement of current and voltage.

- From Fig(a), it may be observed that there are two ways in which the ammeter and voltmeter may be connected for measurement, and that, in either case the resistance of one of the instruments will affect the measurement to some extent.

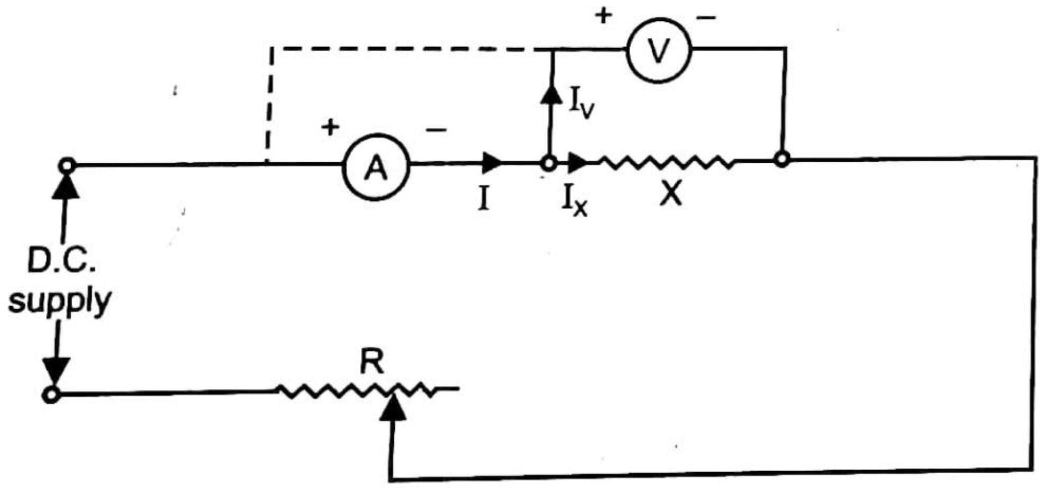


Fig.(a) Ammeter-voltmeter method of measuring low resistances

- **Case 1:** When the voltmeter is connected directly across the resistor, the ammeter measures current flowing through the unknown resistance X and the voltmeter.

In this case

Current through ammeter = Current through unknown resistance (X) + Current through voltmeter

$$I = I_x + I_v$$

$$I_x = I - I_v$$

.: True value of unknown resistance,

$$X_{true} = \frac{V}{I_X} = \frac{V}{I - I_V} = \frac{V}{I - \frac{V}{R_V}} = \frac{V}{I\left(1 - \frac{V}{IR_V}\right)}$$

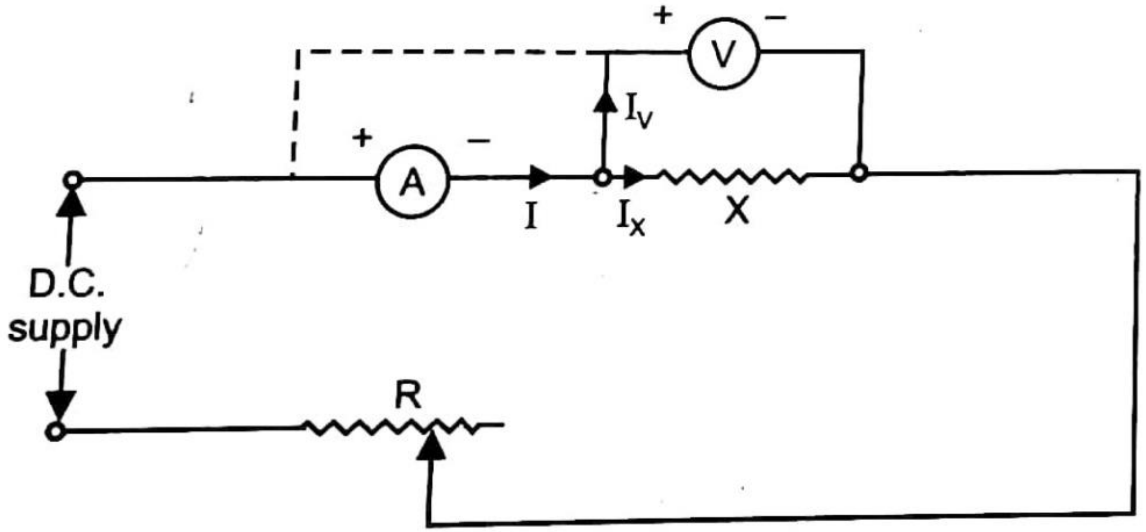


Fig.(a) Ammeter-voltmeter method of measuring low resistances

where,

V = The voltmeter reading,

R_v = The resistance of the voltmeter, and

I = The current indicated by the ammeter.

➢ **Case 2:** When the ammeter is connected so that it indicates only the current flowing through the the unknown resistance, the voltmeter measures voltage drop across the ammeter and unknown resistance X. In this case:

$$V = IR_A + IX = I(R_A + X)$$

$$X_{true} = \frac{V}{I} - R_A$$

R_A = The resistance of the ammeter

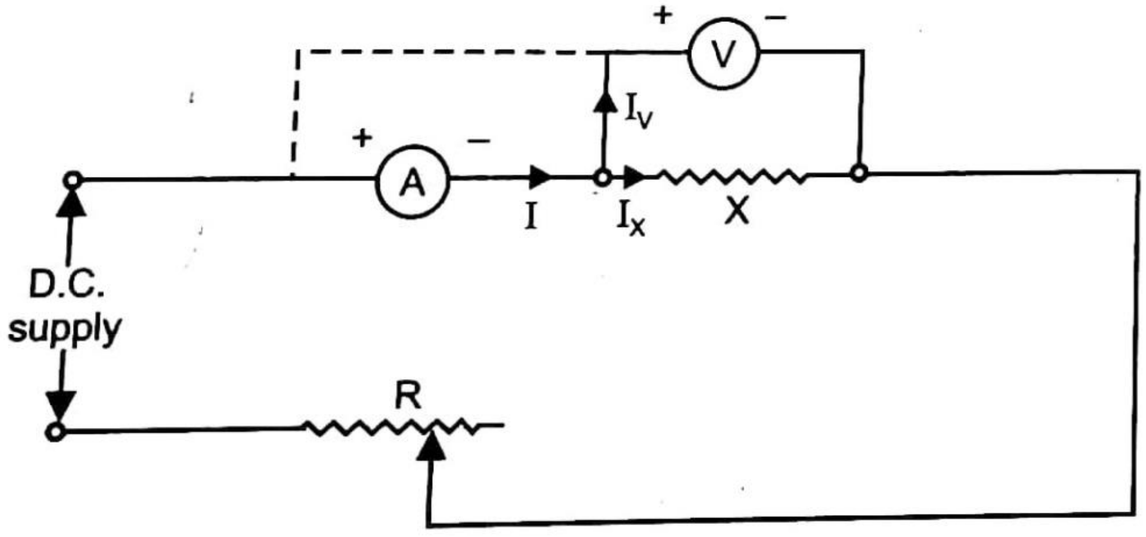


Fig.(a) Ammeter-voltmeter method of measuring low resistances

Q(1). The ammeter-voltmeter method is used to measure a resistance. With the voltmeter connected across the resistance the readings on the ammeter and voltmeter are 0.3 A and 2.4 V respectively. The resistance of the voltmeter is 450 $\boldsymbol{\Omega}$. Calculate :

(i) True value of resistance.

(ii) Percentage error in the value of resistance, if the voltmeter current is ignored.

Solution: Given : I = 0.3 A; V = 2.4 V; R_v = 450 $\boldsymbol{\Omega}$

(i) True value of resistance, X_{true} :

$$X_{true} = \frac{V}{I\left(1 - \frac{V}{IR_V}\right)}$$

$$= \frac{2.4}{0.3\left(1 - \frac{2.4}{0.3 \times 450}\right)} = \mathbf{8.14\ \Omega}$$

(ii) Percentage error in the value of resistance :

Measured value of unknown resistance,

$$X_m = \frac{V}{I} \quad \text{....ignoring } I_V$$

$$= \frac{2.4}{0.3} = 8\ \Omega$$

$$\%\ \text{age error} = \frac{X_m - X_{true}}{X_{true}} \times 100 = \frac{8 - 8.14}{8.14} \times 100$$

$$= -1.72\%\ \textit{i.e.}\ \mathbf{1.72\%\ (low)}$$

Potentiometer method

- In this method the unknown resistance is compared with a standard resistance of the same order of magnitude.
  Fig.(b) shows the circuit diagram for this method :
- The unknown resistance X, an ammeter A, a rheostat R to limit the current and a standard resistance S are connected, all in series with a low voltage, high current supply source.
- The value of S should be accurately known and of the same order of resistance and must be of the same or higher current rating than the one under test (X).
- The current flowing through the circuit is adjusted so that potential difference, if possible, across each of the resistor is about 1V.

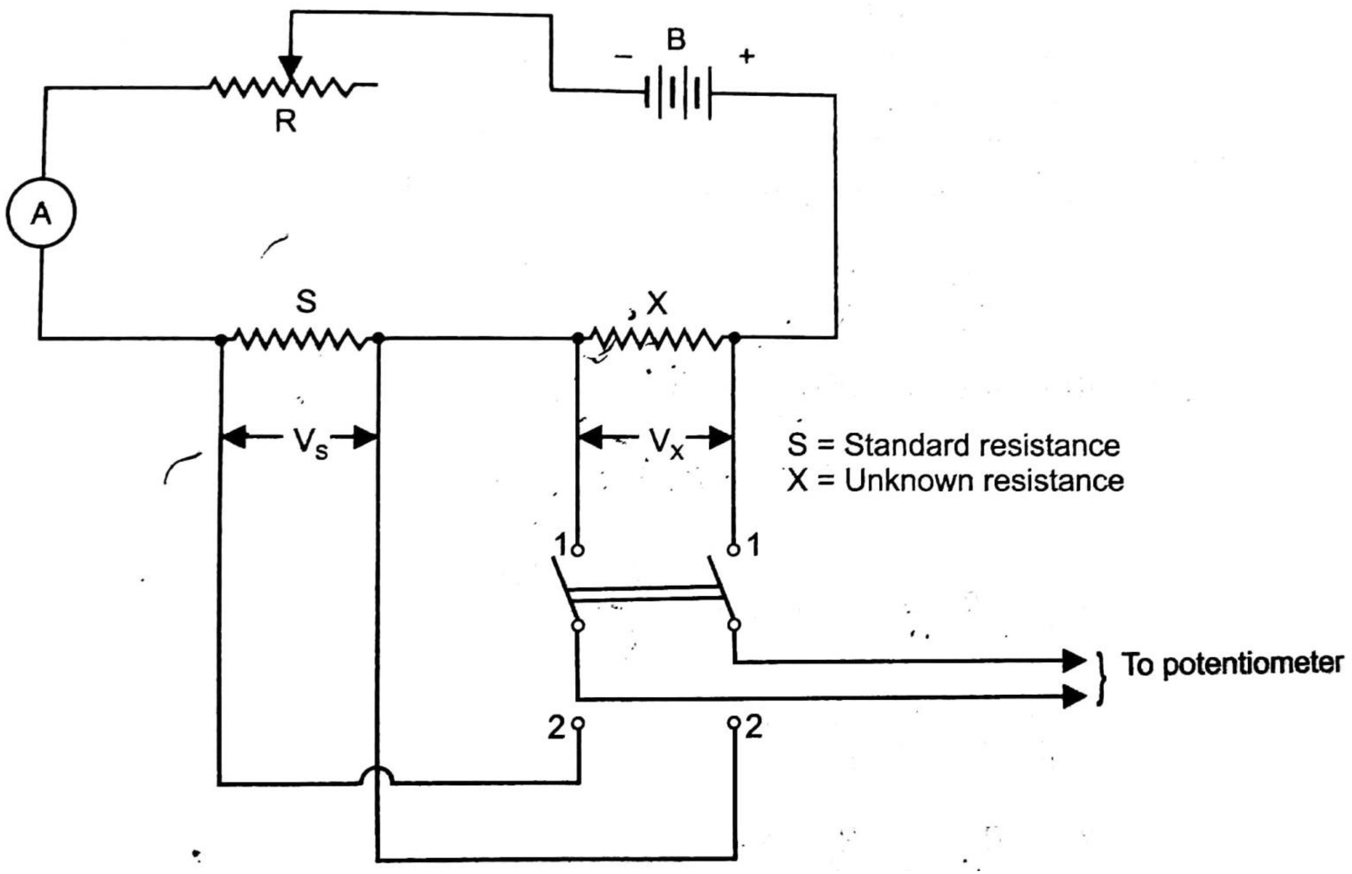


Fig.(b). Potentiometer method of measuring low resistances.

- The voltage drop across both the unknown resistor X and standard resistor S are measured by a potentiometer. The ratio of the two potentiometer readings gives the ratio of X to S.

Mathematically, $$\frac{X}{S} = \frac{\text{Potentiometer reading across } X}{\text{Potentiometer reading across } S} = \frac{V_X}{V_S}$$

- As the accuracy of this method depends on there being no change in current between the two readings, therefore, it is necessary that source of supply of current through the circuit be extremely stable. When the necessary precautions are taken, and a good potentiometer and sensitive galvanometer are used, very high accuracy can be obtained.

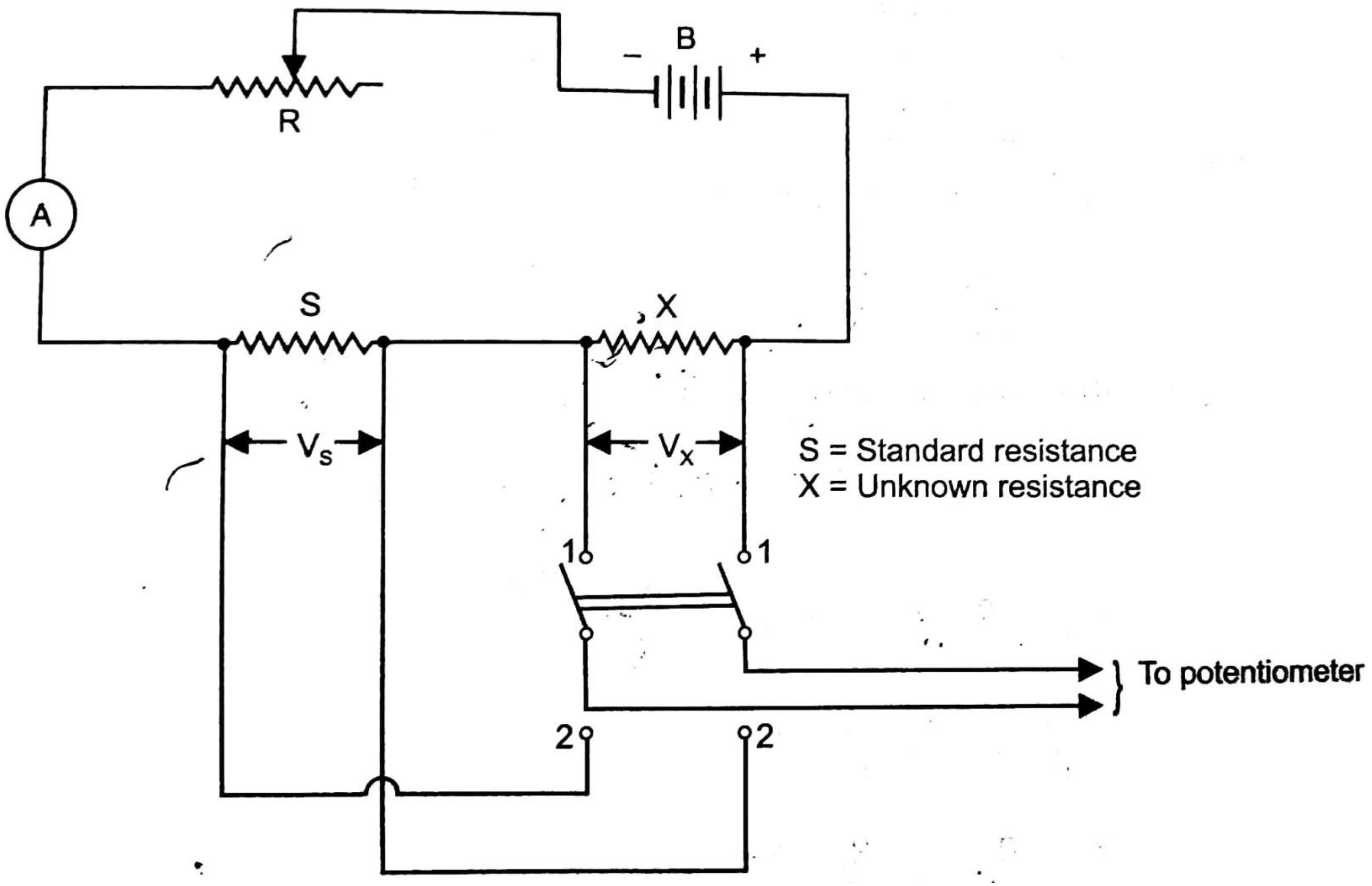


Fig.(b). Potentiometer method of measuring low resistances.

Q(2). In a measurement of resistance by potentiometer, the voltage drops across a resistor under test and across 0.024 Ω standard resistor were found to be 0.885 V and 11 V respectively. Determine the value of resistor under test.

Solution: Given : S = 0.024 Ω ; Vx = 0.885V; Vs = 1.1 V

Resistance of resistor under test, X:

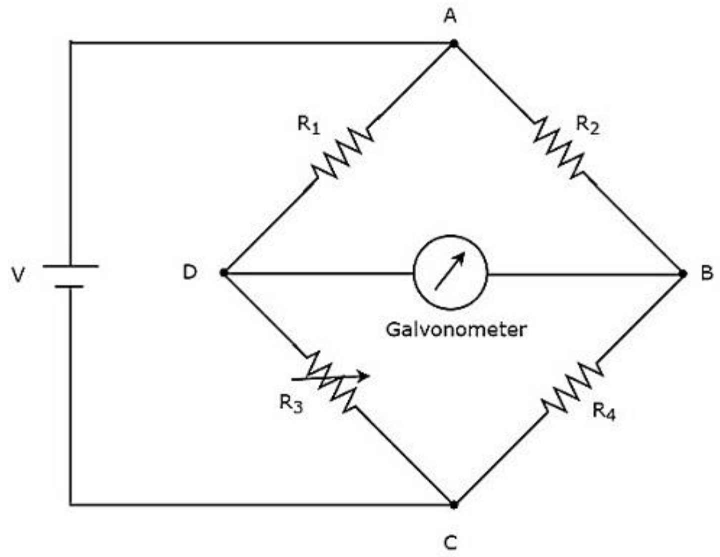
$$\frac{X}{S} = \frac{V_X}{V_S}$$

$$X = \frac{SV_X}{V_S} = \frac{0.024 \times 0.885}{1.1} = \mathbf{0.0193\ \Omega}$$

DIRECT-CURRENT (DC)BRIDGES

DC Bridges

- If the bridge circuit can be operated with only DC voltage signal, then it is a DC bridge circuit or simply **DC bridge**. DC bridges are used to measure the value of unknown resistance. The **circuit diagram** of DC bridge looks like as shown in figure.



- The above DC bridge has **four arms** and each arm consists of a resistor. Among which, two resistors have fixed resistance values, one resistor is a variable resistor and the other one has an unknown resistance value.
- The above DC bridge circuit can be excited with a **DC voltage source** by placing it in one diagonal. The galvanometer is placed in other diagonal of DC bridge. It shows some deflection as long as the bridge is unbalanced.
- Vary the resistance value of variable resistor until the galvanometer shows null (zero) deflection. Now, the above DC bridge is said to be a balanced one. So, we can find the value of **unknown resistance** by using nodal equations.

Measurement of Medium Resistances

The methods used for measurement of medium resistances are :

1. Ammeter-voltmeter method.
2. Substitution method.
3. Wheatstone bridge.
4. Carey-Foster slide-wire bridge method.

WHEATSTONE BRIDGE

WHEATSTONE BRIDGE

- This is an electrical DC bridge circuit used to measure resistance from approximately 1 ohm to the low megaohm range.
- It is the most accurate method available for measuring resistances and is popular for laboratory use.

➢ Basic operation:

- The bridge consists of four resistive arms together With a source of EMF and a null detector as shown in Fig(a).The galvanometer Is used as a null detector.

- The arms consisting the resistances R_1 and R_2 are called ratio arms. The arm consisting the standard known resistance R_3 is called standard arm. The resistance R_4 is the unknown resistance to be measured. The battery is connected between A and C while galvanometer is connected between B and D.

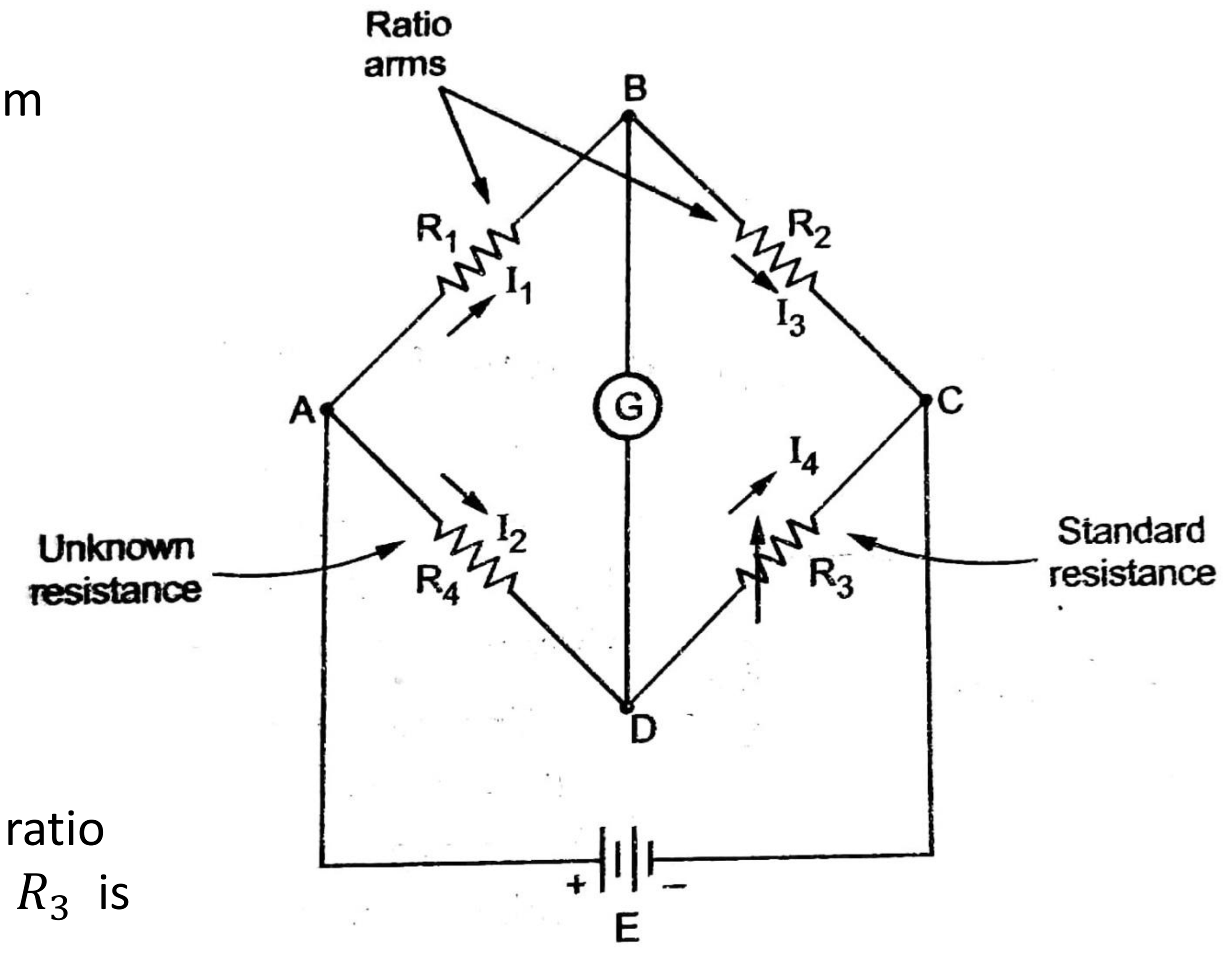


Fig.(a):Wheatstone Bridge

WHEATSTONE BRIDGE

- **Balance Condition:**
  - When the bridge is balanced, the galvanometer carries zero current and it does not show any deflection. Thus bridge works on the principle of null deflection or null indication.
  - To have zero current through galvanometer, the points B and D must be at the same potential. Thus potential across arm AB must be same as the potential across arm AD

Thus $I_1 R_1 = I_2 R_4$ …………………(1)

As galvanometer current is zero,

$I_1 = I_3 \quad and \quad I_2 = I_4$ …………(2)

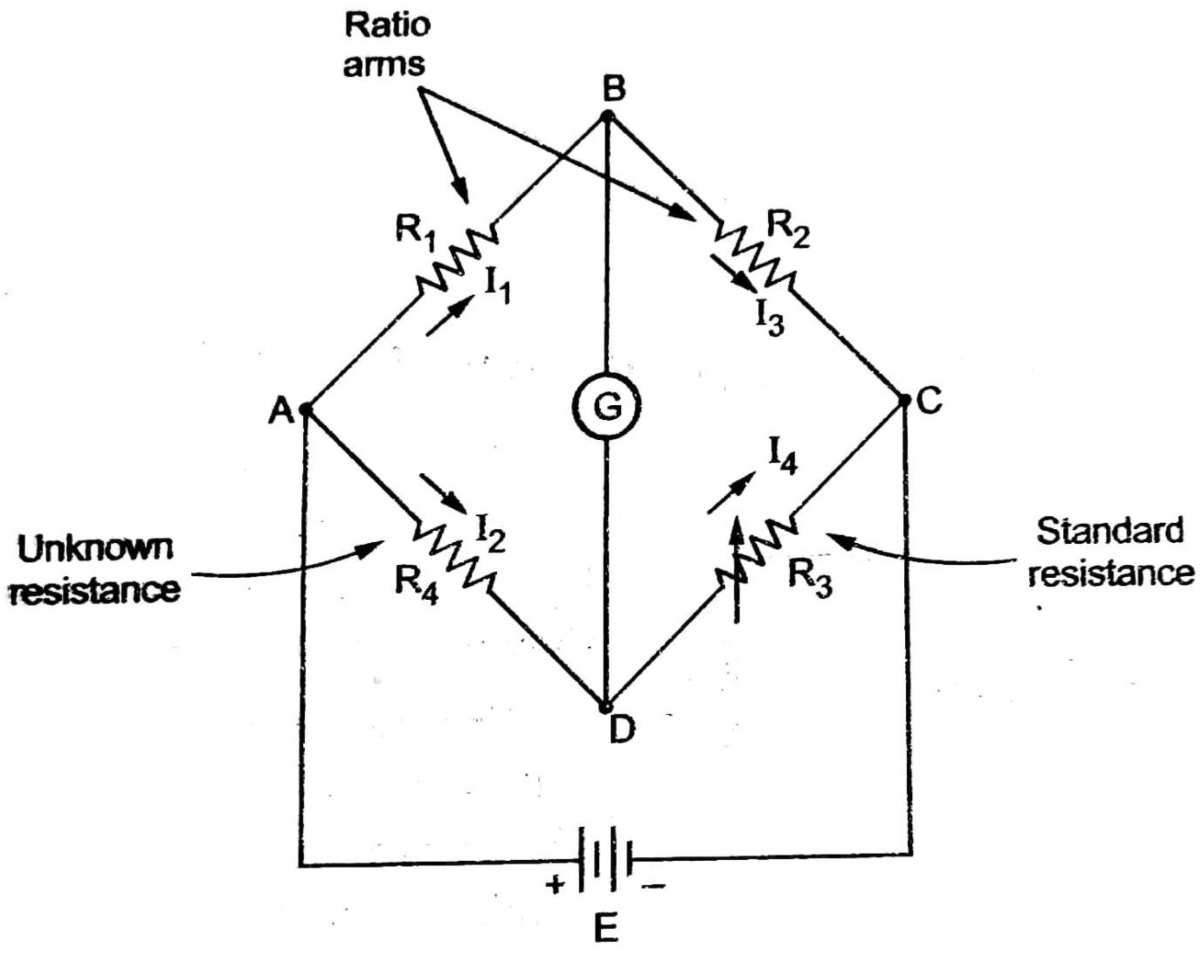


Fig.(a):Wheatstone Bridge

WHEATSTONE BRIDGE

Considering the battery path under balanced condition,

$$I_1 = I_3 = \frac{E}{R_1+R_2} \quad \text{...........(3)}$$

and $$I_2 = I_4 = \frac{E}{R_3+R_4} \quad \text{...............(4)}$$

Using equations (3) and (4) in equation (1),

$$\frac{E}{R_1+R_2} \times R_1 = \frac{E}{R_3+R_4} \times R_4$$

$$R_1(R_3 + R_4) = R_4(R_1 + R_2)$$

$$R_1R_3 + R_1R_4 = R_1R_4 + R_2R_4$$

$$\boxed{R_4 = \frac{R_1R_3}{R_2} = R_3\frac{R_1}{R_2}}$$

This is required balance condition of Wheatstone bridge.

- The following points can be observed,

1) It depends on the ratio of R_1, and R_2, hence these arms are called ratio arms.
2) As it works on null indication, the results are not dependent on the calibration and characteristics of galvanometer.
3) The standard resistance R_3 can be varied to obtain the required balance.

➢ Sensitivity of Wheatstone Bridge

- When the bridge is balanced, the current through galvanometer is zero. But when bridge is not balanced current flows through the galvanometer causing the deflection.
- The amount of deflection depends on the sensitivity of the galvanometer. This sensitivity can be expressed as amount of deflection per unit current.

$$\text{Sensitivity (S)} = \frac{\text{Deflection D}}{\text{Current I}}$$

- As the current is in microampere and deflection can be measured in mm, radians or degrees, the sensitivity is expressed as mm/μA, radians /μA or degrees /μA.
- More is the sensitivity of a galvanometer, more is its deflection for the same amount of current. Another way of representing the galvanometer sensitivity is the amount of deflection per unit voltage across the galvanometer. This is called voltage sensitivity of the galvanometer. Mathematically it is denoted as,

$$\boxed{S_v = \frac{\theta}{e}}$$

where
e = Voltage across galvanometer
θ = Deflection of galvanometer

- It is measured in degrees per volts or radians per volts.

- While the bridge sensitivity is defined as the deflection of the galvanometer per unit fractional change in the unknown resistance. It is denoted as S_B

$$S_B = \frac{\theta}{\Delta R/R}$$

where
$\Delta R/R$ = Unit fractional change in unknown resistance.

Wheatstone Bridge under Small Unbalance

- The bridge sensitivity can be calculated by solving the bridge for small unbalance.

At balance condition,

$$R_4 = \frac{R_1 R_3}{R_2} = R_3 \frac{R_1}{R_2}$$

$$\frac{R_4}{R_3} = \frac{R_1}{R_2}$$

- Let the resistance R_4 is changed by ΔR creating the unbalance. Due to this, the e.m.f. appears across the galvanometer.
- To obtain this e.m.f., let us use Thevenin's method.
- Remove the branch of galvanometer and obtain the voltage across the open circuit terminals.

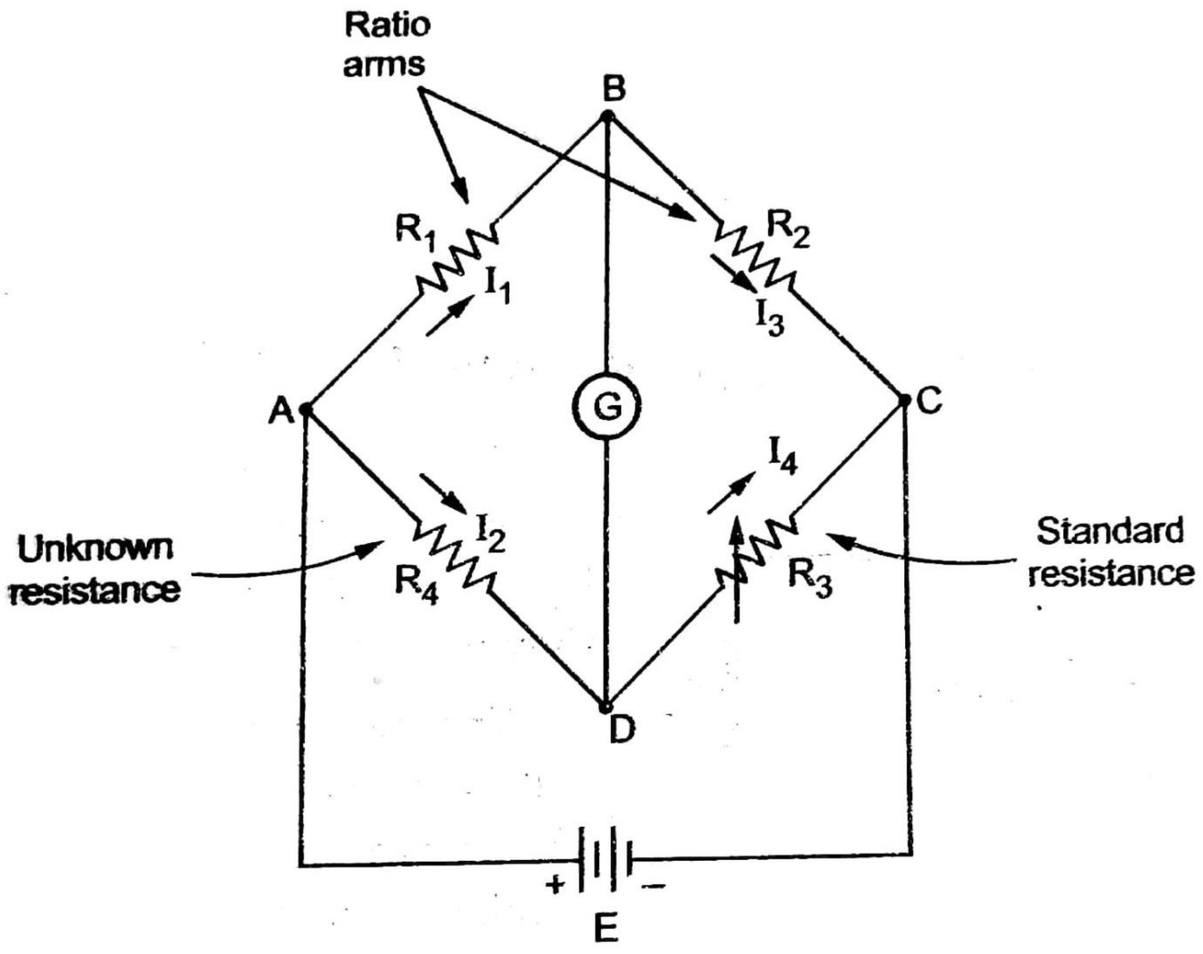


Fig.(a):Wheatstone Bridge

Wheatstone Bridge under Small Unbalance

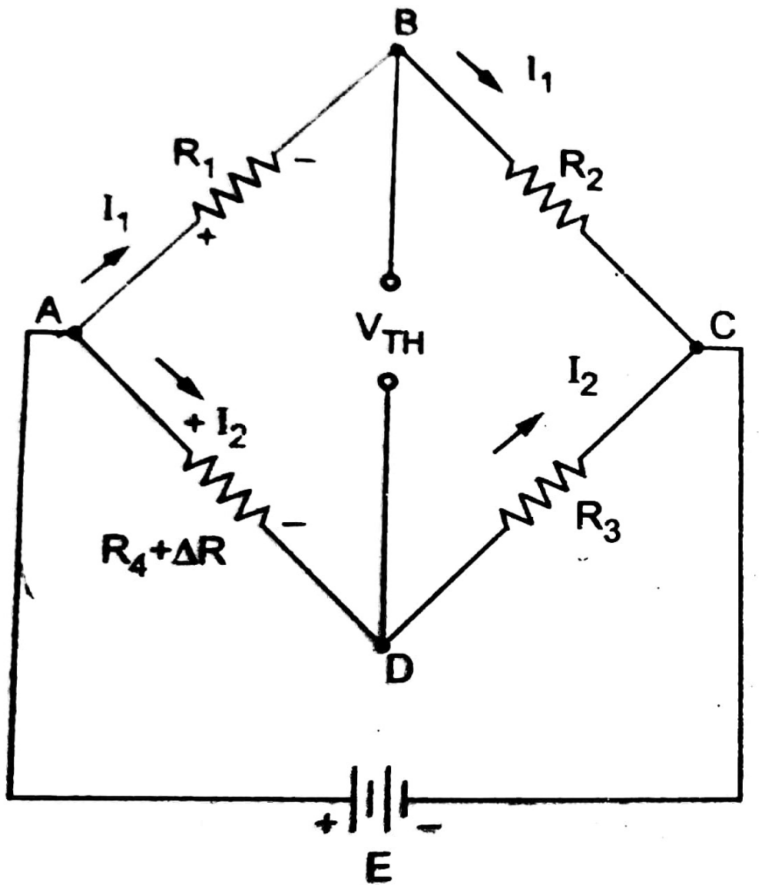


Fig.Bridge under Unbalance

$$E_{AB} = I_1 R_1 \quad \ldots(1)$$

$$I_1 = \frac{E}{R_1 + R_2} \quad \ldots(2)$$

$$E_{AD} = I_2 (R_4 + \Delta R) \quad \ldots(3)$$

$$I_2 = \frac{E}{R_3 + R_4 + \Delta R} \quad \ldots(4)$$

$$V_{BD} = V_{TH} = E_{AD} - E_{AB} \quad \ldots(5)$$

$$V_{TH} = \frac{E(R_4 + \Delta R)}{R_3 + R_4 + \Delta R} - \frac{E}{R_1 + R_2} R_1$$

$$V_{TH} = E\left\{\frac{R_4 + \Delta R}{R_3 + R_4 + \Delta R} - \frac{R_1}{R_1 + R_2}\right\} \quad \ldots(6)$$

as $\frac{R_4}{R_3} = \frac{R_1}{R_2}$ then $\frac{R_1}{R_1 + R_2} = \frac{R_4}{R_4 + R_3}$

Using above relation in equation(6)

$$V_{TH} = E\left\{\frac{R_4 + \Delta R}{R_3 + R_4 + \Delta R} - \frac{R_4}{R_3 + R_4}\right\}$$

$$= E\left\{\frac{R_3R_4 + R_3\Delta R + R_4^2 + R_4\Delta R - R_3R_4 - R_4^2 - R_4\Delta R}{(R_3 + R_4)(R_3 + R_4 + \Delta R)}\right\}$$

$$= \frac{ER_3\,\Delta R}{(R_3 + R_4)^2 + (R_3 + R_4)\Delta R}$$

But as ΔR is very small, $(R_3 + R_4)\Delta R << (R_3 + R_4)^2$

$$\therefore \quad \boxed{V_{TH} = V_g = \frac{ER_3\,\Delta R}{(R_3 + R_4)^2}} \qquad \text{...(7)}$$

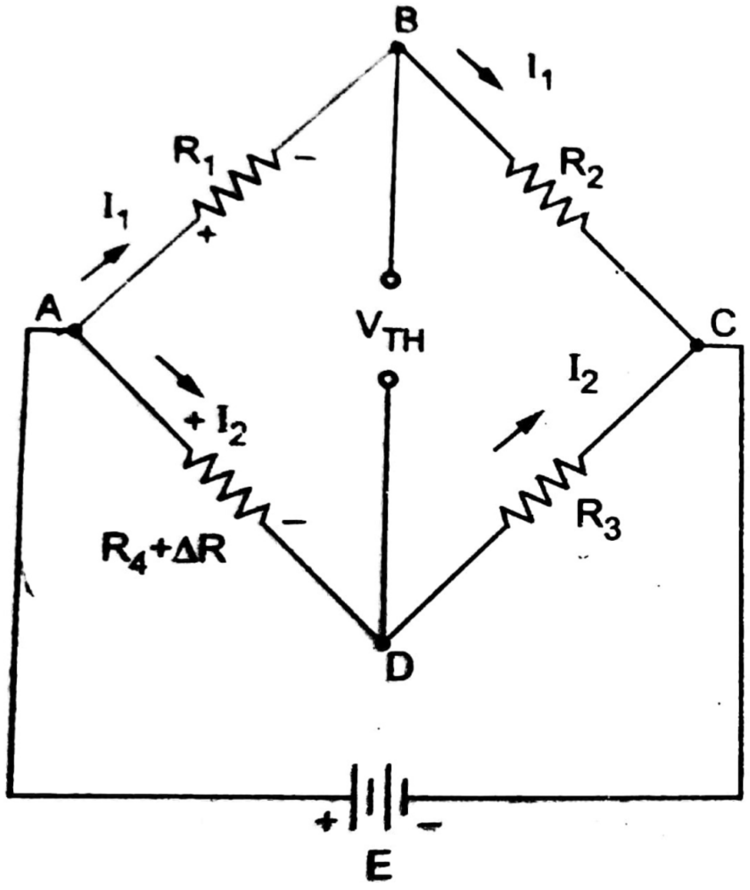


Fig.Bridge under Unbalance

Now $$S_B = \frac{\theta}{\Delta R / R} = \text{Bridge sensitivity}$$

and $\Delta R/R = \Delta R / R_4$ as there is change in R_4.

From the galvanometer sensitivity S_V,

$$\theta = S_V \times e \quad \text{where} \quad e = \text{Voltage across galvanometer} = V_g$$

Using θ in the expression of S_B,

$$\therefore \quad S_B = \frac{S_V V_g}{\Delta R / R_4} = \frac{S_V E R_3 \Delta R R_4}{(R_3 + R_4)^2} = \frac{S_V E R_3 R_4}{R_3^2 + 2 R_3 R_4 + R_4^2}$$

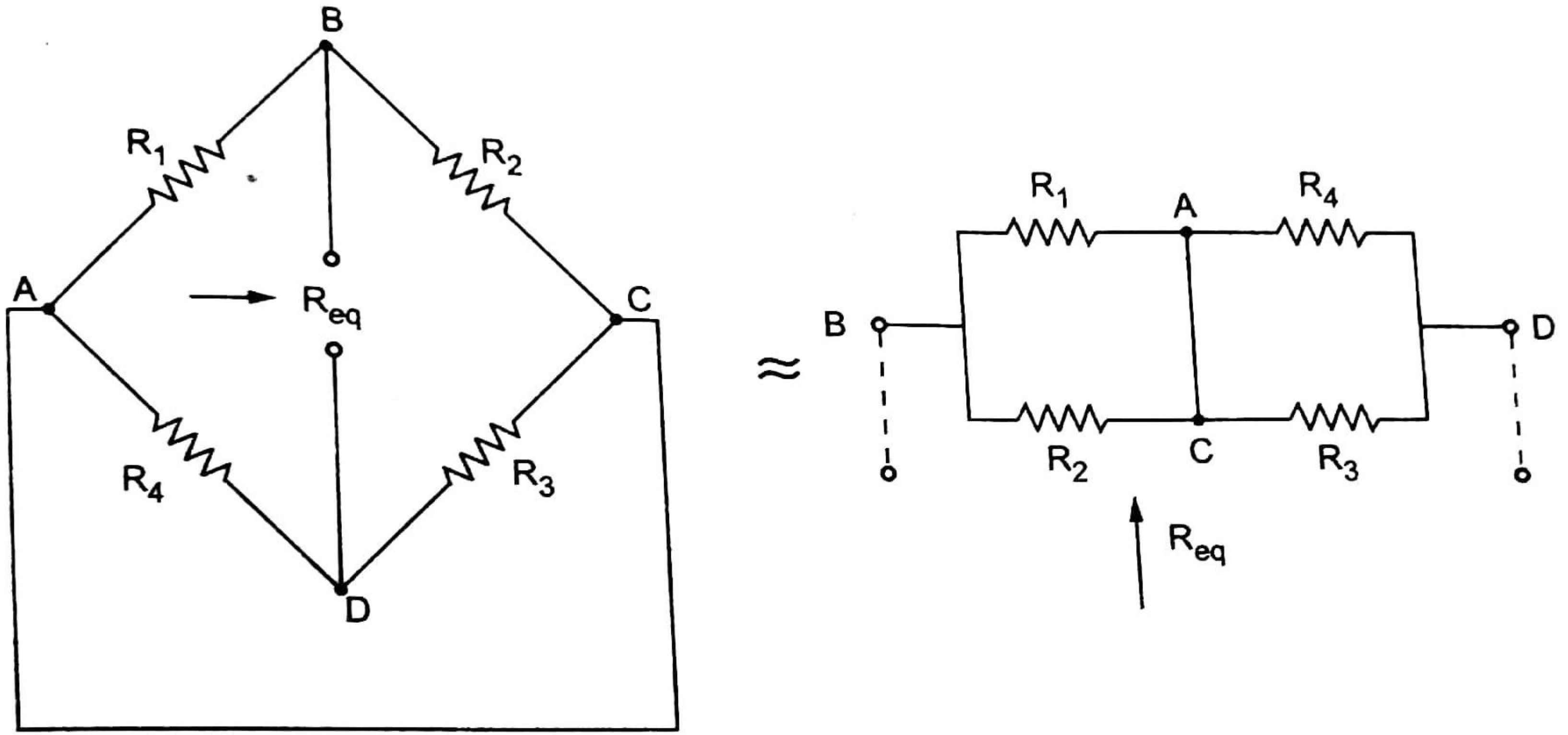
$$\therefore \quad \boxed{S_B = \frac{S_V E}{\frac{R_3}{R_4} + 2 + \frac{R_4}{R_3}}} \quad \text{...(8)}$$

- Thus the bridge sensitivity depends on the bridge parameters, the supply voltage and the voltage sensitivity of the galvanometer. Thus maximum sensitivity occurs when $\frac{R_3}{R_4} = 1$

  For higher or lower values of R3/R4, the sensitivity decreases considerably.

➢ Thevenin's Equivalent and Galvanometer Current -balanced Condition

- The Thevenin's voltage V_{TH} across the galvanometer is already obtained. Let us obtain equivalent resistance as viewed across the terminals BD, when battery E is replaced by short circuit. Thus circuit becomes,



$$R_{eq} = (R_1 || R_2) + (R_3 || R_4) \quad \ldots(9)$$

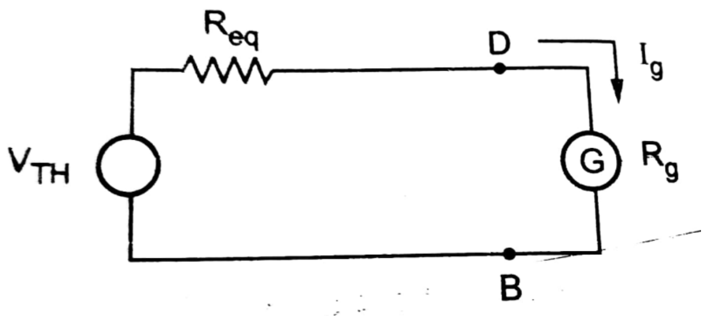
while

$$V_{TH} = E_{AD} - E_{AB} \quad \text{with } R_4 \text{ not changed by } \Delta R$$

$$= I_2 R_4 - I_1 R_1 = \frac{E}{R_3 + R_4} R_4 - \frac{E}{R_1 + R_2} R_1$$

$$\boxed{V_{TH} = E\left[\frac{R_4}{R_3 + R_4} - \frac{R_1}{R_1 + R_2}\right]} \quad \ldots(10)$$

Thus Thevenin's equivalent is as shown in the Fig(a)



Fig(a)

Let R_g = Galvanometer resistance

I_g = Galvanometer current

$$\therefore \quad \boxed{I_g = \frac{V_{TH}}{R_{eq} + R_g}} \quad \ldots(11)$$

where V_{TH} = Thevenin's voltage

➢ Galvanometer Current under Unbalanced Condition

Let the resistance R_4, is changed by ΔR which has caused the unbalance in the bridge.
As derived earlier,

$$V_{TH} = \frac{E R_3 \Delta R}{(R_3 + R_4)^2} \qquad ...(12)$$

and now

$$R_{eq} = (R_1 || R_2) + (R_3 || R_4 + \Delta R)$$

$$= \frac{R_3 (R_4 + \Delta R)}{R_3 + R_4 + \Delta R} + \frac{R_1 R_2}{R_1 + R_2}$$

Neglecting ΔR compared to R3 and R4,

$$R_{eq} = \frac{R_3 R_4}{R_3 + R_4} + \frac{R_1 R_2}{R_1 + R_2} \qquad ...(13)$$

$$I_g = \frac{V_{TH}}{R_{eq} + R_g} \qquad ...(14)$$

For bridge with equal arms R1 = R2 = R3 = R4 = R then

$$V_{TH} = \frac{E R \Delta R}{4 R^2} = \frac{E \Delta R}{4 R}$$

and

$$R_{eq} = \frac{R^2}{2 R} + \frac{R^2}{2R} = R$$

$$I_g = \frac{\frac{E \Delta R}{4R}}{R + R_g} = \frac{E(\Delta R / 4R)}{R + R_g} \qquad ...(15)$$

➢ S_B interms of Current Sensitivity of Galvanometer

The deflection of galvanometer for a small change in unknown resistance R_4 is,

$$\theta = S_V \, e = S_V \, V_g = \frac{S_V \, E R_3 \Delta R}{(R_3 + R_4)^2}$$

while $$S_V = \frac{S_i}{R_{eq} + R_g}$$

Where S_i = Current sensitivity of galvanometer

$$\theta = \frac{S_i \, E R_3 \Delta R}{(R_{eq} + R_g)(R_3 + R_4)^2}$$

and $$S_B = \frac{\theta}{\Delta R / R_4} = \frac{S_i E R_3 R_4}{(R_{eq} + R_g)(R_3 + R_4)^2} \qquad ...(16)$$

where $$R_{eq} = (R_1 || R_2) + (R_3 || R_4 + \Delta R)$$

This is the bridge sensitivity interms of current sensitivity of the galvanometer.

Practically R_{eq} can be assumed to be R_{TH} as ΔR is small compared to actual values of the resistances.

$$R_{eq} \approx R_{TH} = [R_1 || R_2] + [R_3 || R_4]$$

Measurement Errors

The Wheatstone bridge is used to measure the resistances in the range 1Ω to few megaohms. But certain errors occur during the measurement using the Wheatstone bridge.

These errors are as follows:

1) The main error is because of limiting errors of the three known resistances. Hence very precise resistances are required having tolerance of 1 % or even 0.1 %.
2) The insufficient sensitivity of the null detector may cause the error.
3) Heating effect : When the current passes through the resistances, due to the heating effect (I^2R) the temperature increases. Hence the values of the resistances of the bridge arms change due to the heating effect. The excessively high current may cause the permanent change in the resistance values. This may cause the serious error in the measurement. To avoid this, power dissipation in the arms must be calculated well in advance and currents must be limited to a safe value.

4).Thermal e.m.f. : In the galvanometer circuit the dissimilar metals come in contact and generate the thermal e.m.f.s. Such thermal e.m.f.s may cause the errors while measuring low value resistances. To prevent this, more sensitive galvanometers having copper coils and copper suspension systems are used.

5).The resistance of leads and contacts exterior to the actual bridge circuit adds the extra resistance and is the major cause of the errors, while measuring low resistance values. These errors may be reduced by using another bridge called Kelvin bridge.

- Advantages and Limitations of Wheatstone Bridge

  The various advantages of Wheatstone bridge are,

1) The results are not dependent on the calibration and characteristics of galvanometer as it works on null deflection.
2) The source e.m.f. and inaccuracies due to the source fluctuations do not affect the balance of the bridge. Hence the corresponding errors are completely avoided.
3) Due to null deflection method used, the accuracy and sensitivity is higher than direct deflection meters.

➢ Limitations of Wheatstone Bridge

1) The effect of lead resistance and contact resistance is very much significant while measuring low resistances.
2) The bridge cannot be used for high resistance measurement i.e. measurement in high megaohm range. This is because while such measurement the resistance presented by the bridge becomes so large that the galvanometer becomes insensitive to show any imbalance
3) Similarly heating effect due to large current also plays a major role. The excessive currents may generate heat which may cause the permanent change in the resistance.
4) The resistance used must be very precise having tolerance upto 1 % or 0.1 %, hence cost is high

➢ Applications of Wheatstone Bridge

1) The Wheatstone bridge is basically a d.c. bridge and used to measure the resistances in the range 1 ohm to low megaohm.
2) It is used to measure the d.c. resistance of various types of wires for the purpose of quality control of wire.
3) It is used to measure the resistance of motor winding, relay coils etc.
4) It is used by the telephone companies to locate the cable faults. The faults may be of the type line to line short or line to ground short.

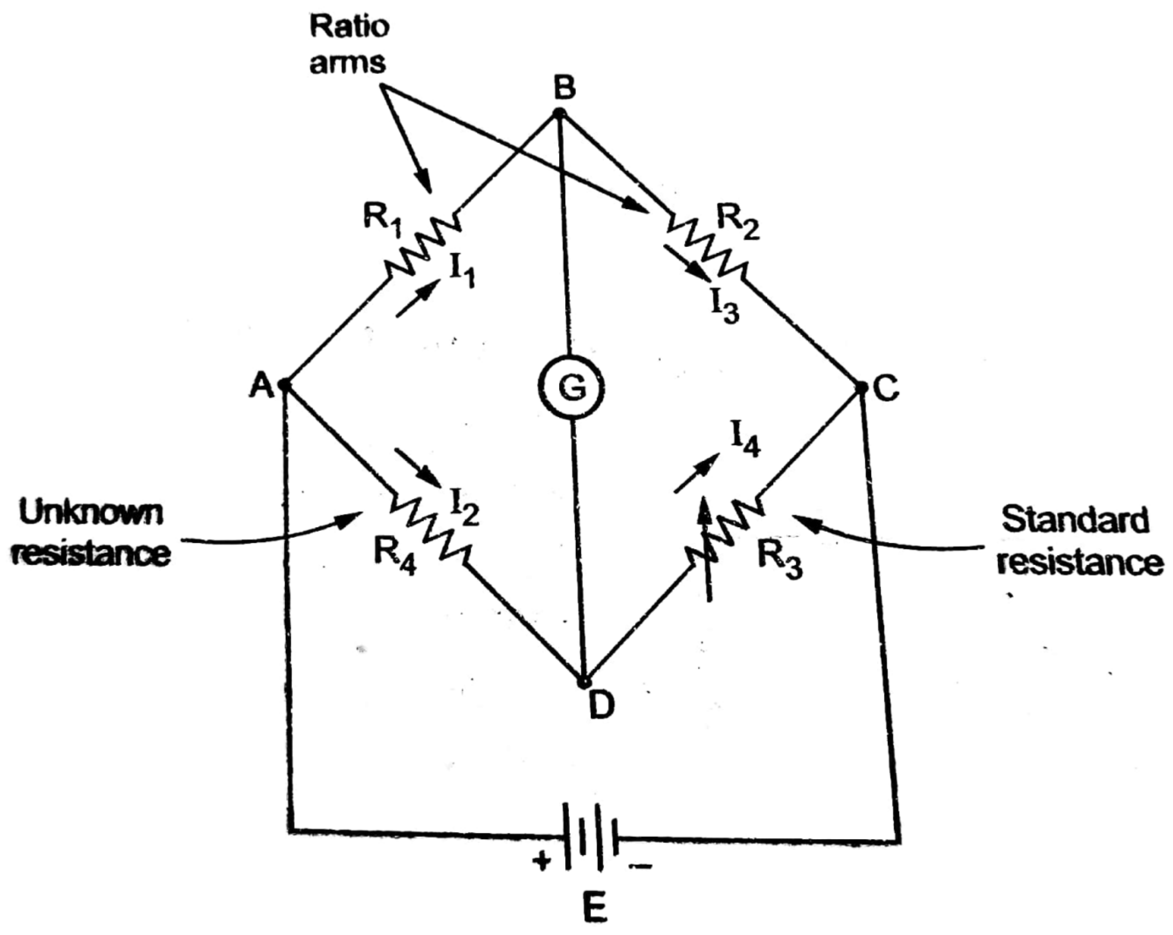
WHEATSTONE BRIDGE

(1Q). Fig(b) consists of the following parameters:
R1 = 10 k Ω; R2 = 2k Ω ; R3 = 5 k Ω and R4=Rx
Determine the unknown resistance Rx

Solution: Using the bridge balance equation, we have :

$$R_1 R_3 = R_2 R_4$$

$$R_4 = R_X = \frac{R_1 R_3}{R_2} = \frac{10 \times 5}{2} = 25K\Omega$$



Fig(b).Wheatstone Bridge

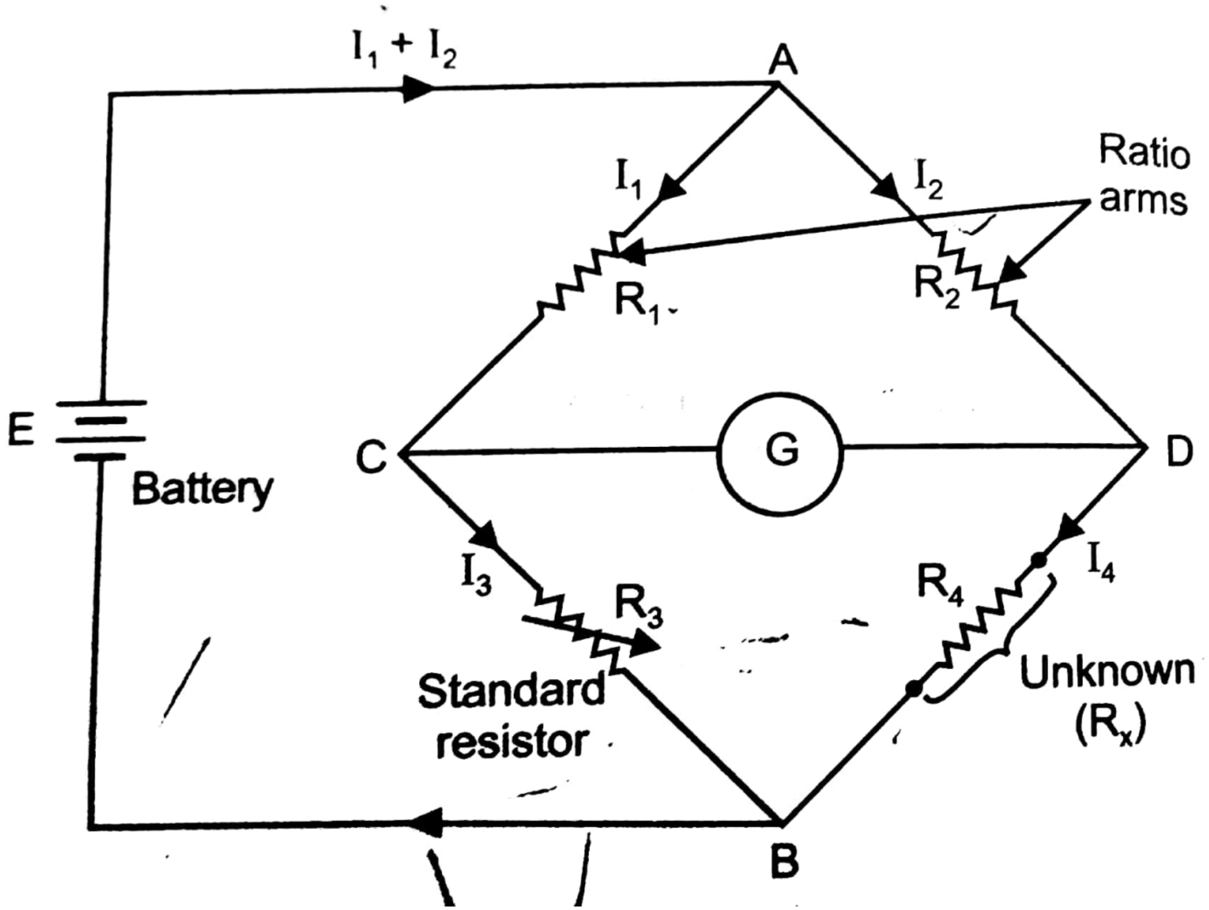
(2Q). Fig(a) consists of the following parameters:
R1 = 20 k ***Ω***; R2 = 30 k ***Ω*** ; R3 = 80 k ***Ω*** .
Determine the unknown resistance Rx

Solution: Using the bridge balance equation, we have :

$$R_1 R_X = R_2 R_3$$

$$R_X = \frac{R_2 R_3}{R_1} = \frac{30 \times 80}{20}$$

$$= 120 \text{ K } \boldsymbol{\Omega}$$



Fig(a).Wheatstone Bridge

(3Q).Calculate the current through the galvanometer for the bridge shown in the Fig.(a)

From the Fig.(a)

$R_1 = 7\ k\Omega$, $R_2 = 2\ k\Omega$,

$R_3 = 4\ k\Omega$, $R_4 = 20\ k\Omega$, $E = 8\ V$.

Use Thevenin's equivalent for I_g.

$$V_{TH} = V_{BD} = V_{AD} - V_{AB}$$
$$= I_2 R_4 - I_1 R_1$$
$$= \frac{E}{R_3 + R_4} R_4 - \frac{E}{R_1 + R_2} R_1$$
$$= 8\left\{\frac{20}{20+4} - \frac{7}{7+2}\right\}$$
$$= 0.444\ V$$

Thus B is positive w.r.t. D

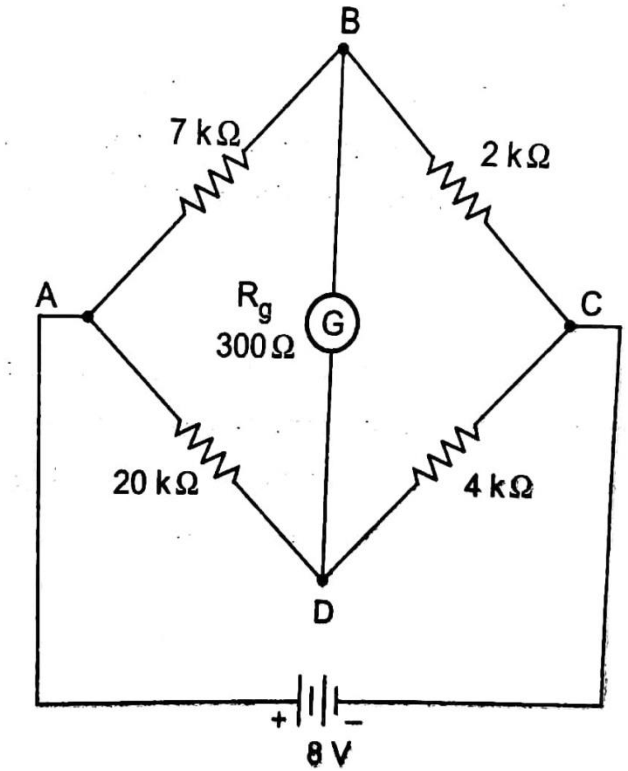


Fig.(a)

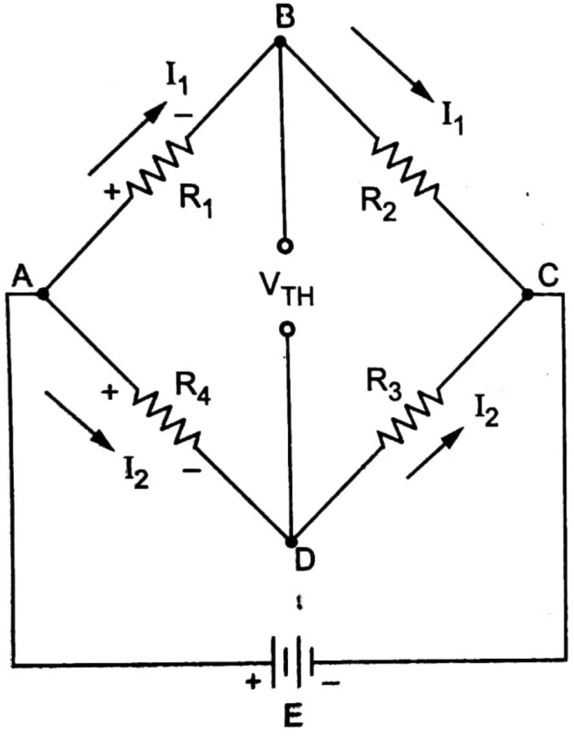


Fig.(b)

$$\text{Now } R_{eq} = [R_1 \| R_2] + [R_3 \| R_4] \quad \text{... with E shorted}$$

$$= \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} = 4.888 \text{ k}\Omega$$

$$I_g = \frac{V_{TH}}{R_{eq} + R_g} = \frac{0.444}{4.888 \times 10^3 + 300}$$

$$= 85.62 \ \mu\text{A}$$

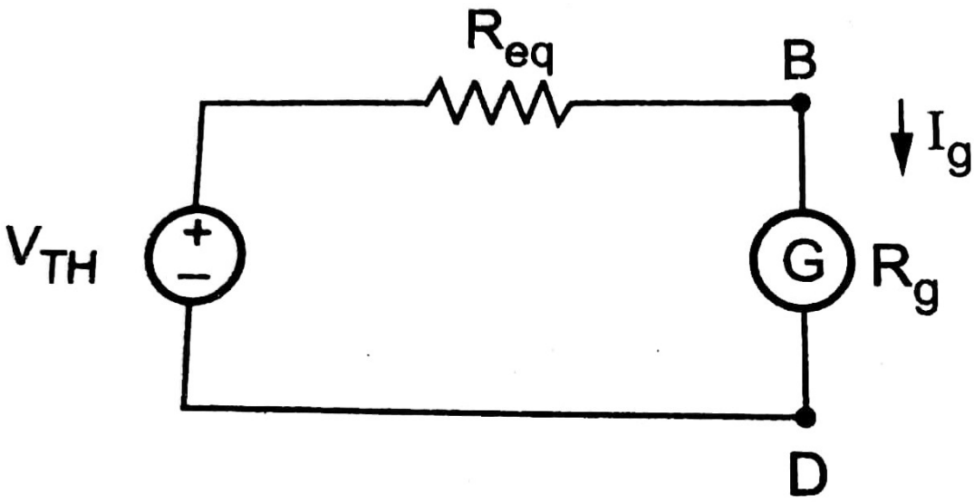
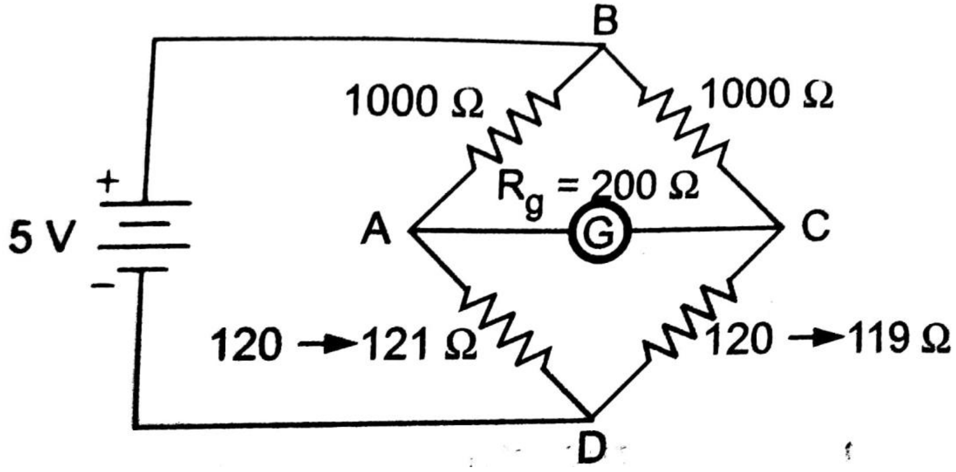


Fig.(c)

This is the current through the galvanometer

(4Q).The four arms of the Wheatstone bridge have the following resistances, AB=1000 **Ω**, BC =1000Ω, CD= 120 Ω, DA 120 **Ω** . The bridge is used for strain measurement and supplied from 5 V ideal battery. The galvanometer has sensitivity of 1 mm/μA with internal resistance of 200 **Ω** . Determine the deflection of the galvanometer if arm DA increases to 121 **Ω** and arm CD decreases to 119 **Ω**.



Solution:

Now $R_1 = 1000\ \Omega$ $R_2 = 1000\ \Omega$

$R_3 = 121\ \Omega$ $R_4 = 119\ \Omega$

Let us calculate Thevenin's equivalent due to change in R_3 and R_4.

$$V_{TH} = E\left[\frac{R_3}{R_1+R_3} - \frac{R_4}{R_2+R_4}\right]$$

$$= 5\left[\frac{121}{1000+121} - \frac{119}{1000+119}\right]$$

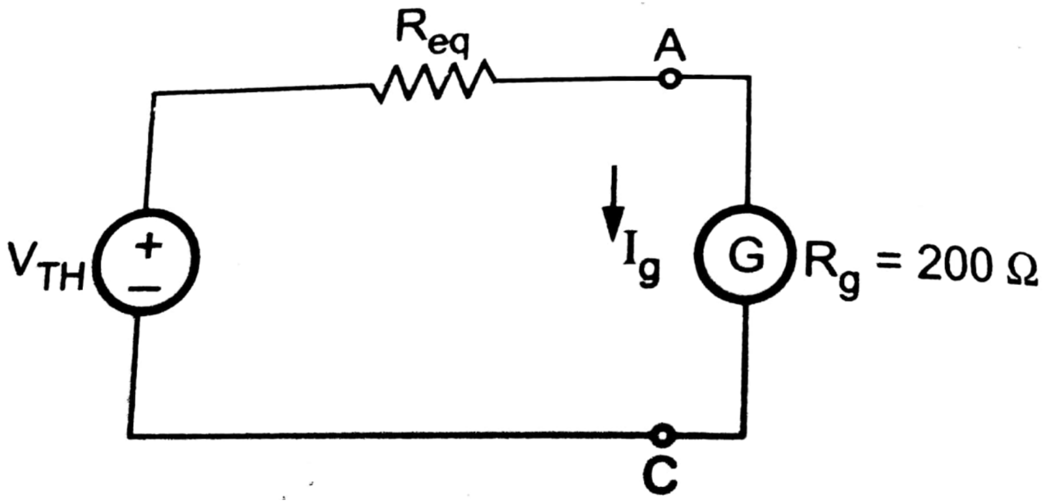
$$= 5\ [0.1079 - 0.1063] = \mathbf{7.975\ mV}$$

$$R_{eq} = \frac{R_1 R_3}{R_1+R_3} + \frac{R_2 R_4}{R_2+R_4}$$

$$= \frac{121\times1000}{121+1000} + \frac{119\times1000}{119+1000}$$

$$= 107.9393 + 106.3449 = \mathbf{214.2842\ \Omega}$$

Thevenin's equivalent circuit is



$$I_g = \frac{V_{TH}}{R_{eq} + R_g}$$

$$= \frac{7.975 \times 10^{-3}}{214.2842 + 200}$$

$$= \mathbf{19.24\ \mu A}$$

Now the deflection of the galvanometer is proportional to its sensitivity

$$S = \frac{D}{I}$$

$$D = S \times I = 1\ \text{mm}/\mu\text{A} \times 19.24\ \mu\text{A}$$

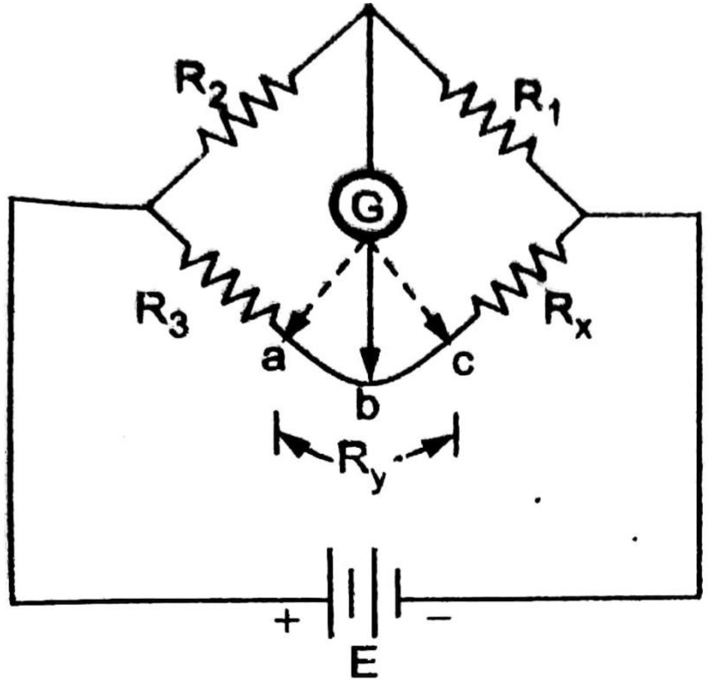
$$= \mathbf{19.24\ mm}$$

This is the deflection of the galvanometer

KELVIN BRIDGE

Kelvin Bridge

- It is used to measurement of Low Resistance(less than 1Ω)
- In the Wheatstone bridge, the bridge contact and lead resistance causes significant error, while measuring low resistances. Thus for measuring the values of resistance below 1Ω, the modified form of Wheatstone bridge is used, known as Kelvin bridge.
- The consideration of the effect of contact and lead resistances is the basic aim of the Kelvin bridge
- The Fig(a). is the basic circuit of the Kelvin bridge.
- The resistance R_y represents the resistance of the connecting leads from R3 to Rx. The resistance Rx is the unknown resistance to be measured
- The galvanometer can be connected to either terminal a, b or terminal c. When it is connected to a, the lead resistance Ry gets added to Rx hence the value measured by the bridge, indicates much higher value of Rx.
- If the galvanometer is connected to terminal c, then Ry, gets added to R3. This results in the measurement of Rx much lower than the actual value.



Fig(a) Kelvin bridge

Kelvin Bridge

The point b is in between the points a and c, in such a way that the ratio of the resistance from c to b and that from a to b is equal to the ratio of R1 and R2.

$$\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2} \qquad \text{...(1)}$$

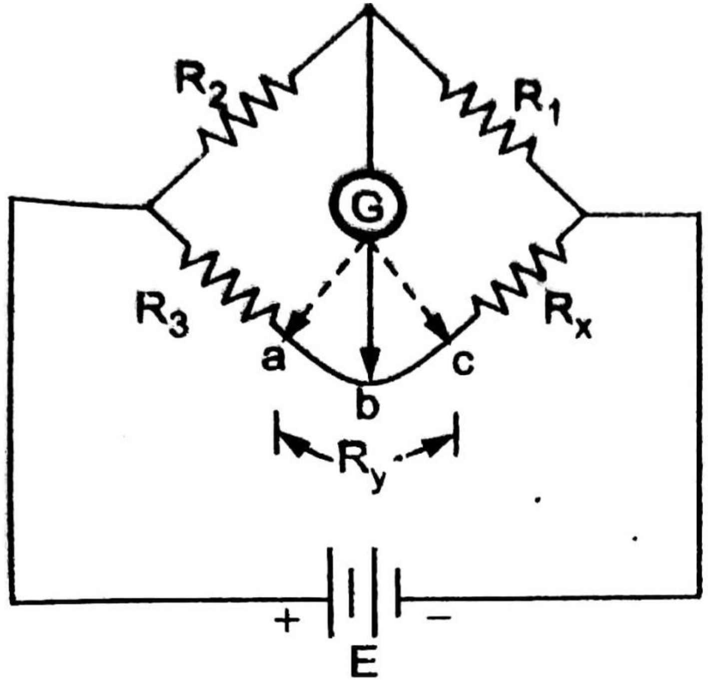
Now the bridge balance equation in its standard form is,

$$R_1 R_3 = R_2 R_x \qquad \text{...(2)}$$

But R_3 and R_x now are changed to $R_3 + R_{ab}$ and $R_x + R_{cb}$ respectively due to lead resistance.

$$\therefore \quad R_1(R_3 + R_{ab}) = R_2(R_x + R_{cb}) \qquad \text{...(3)}$$

$$\therefore \quad (R_x + R_{cb}) = \frac{R_1}{R_2}(R_3 + R_{ab}) \qquad \text{...(4)}$$



Fig(a) Kelvin bridge

Now we have, $\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$

$$\therefore \quad \frac{R_{cb}}{R_{ab}} + 1 = \frac{R_1}{R_2} + 1 \qquad \text{...adding 1 to both sides}$$

$$\frac{R_{cb} + R_{ab}}{R_{ab}} = \frac{R_1 + R_2}{R_2} \qquad \text{...(5)}$$

But $\quad R_{cb} + R_{ab} = R_y$

Substituting in equation (5) we get, ...(6)

$$\frac{R_y}{R_{ab}} = \frac{R_1 + R_2}{R_2}$$

Kelvin Bridge

$$R_{ab} = \frac{R_2 R_y}{R_1 + R_2} \quad \text{...(7)}$$

Now $R_{cb} + R_{ab} = R_y$

$$R_{cb} = R_y - R_{ab} \quad \text{...(8)}$$

Substituting eq(7) into eq(8)

$$R_{cb} = R_y - \frac{R_2 R_y}{R_1 + R_2} = R_y \left[1 - \frac{R_2}{R_1 + R_2}\right]$$

$$R_{cb} = \frac{R_1 R_y}{R_1 + R_2} \quad \text{...(9)}$$

Substituting these values of R_{cb} *and* R_{ab}
In the eq(4), we get

$$R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1}{R_2}\left(R_3 + \frac{R_2 R_y}{R_1 + R_2}\right)$$

$$R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1 R_3}{R_2} + \frac{R_1 R_y}{R_1 + R_2}$$

$$\boxed{R_x = \frac{R_1 R_3}{R_2}} \quad \text{...(10)}$$

- Thus equation (10) represents standard bridge balance equation for the Wheatstone bridge. Thus the effect of the connecting lead resistance is completely eliminated by connecting the galvanometer to an intermediate position 'b'.
- This principle forms the basis of the construction of Kelvin's Double Bridge which is popularly called Kelvin Bridge.

Kelvin's Double Bridge Method for Low Resistance Measurement

- This bridge consists of another set of ratio arms hence called double bridge. The Fig(b) shows the circuit diagram of Kelvin's double bridge. The second set of ratio arms is the resistances 'a' and 'b'. With the help of these resistances the galvanometer is connected to point 3.
- The galvanometer gives null indication when the potential of the terminal 3 is same as the potential of the terminal 4

Thus $\mathrm{E}_{45} = E_{513}$ ...(1)

Here E_{45} = Potential across R_2

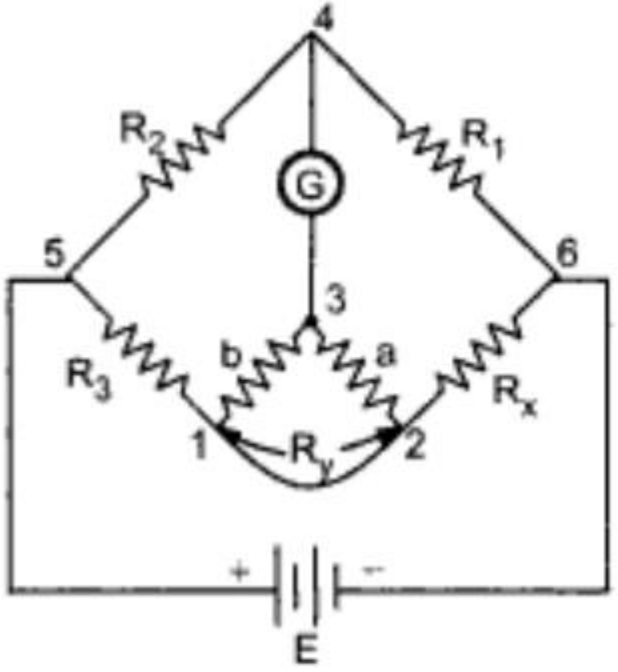
E_{513} = Potential across R_3 and b

The ratio of the resistances a and b is same as the ratio of R_1 and R_2.

$$\therefore \frac{a}{b} = \frac{R_1}{R_2} \qquad \text{...(2)}$$

Now $\mathrm{E}_{45} = R_2 \times \frac{E}{R_1+R_2}$ ...(3)

Consider the path from 5-1-2-6 back to 5 through the battery E. The resistance between the terminals 1-2 is the parallel combination of R_y and (a + b).



Fig(b) Kelvin's double bridge

Kelvin's Double Bridge

$$E = I \times [R_3 + R_y || (a+b) + R_x]$$

$$E = I\left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right] \quad ...(4)$$

Substituting eq (4) in eq(3),

$$E_{45} = \frac{R_2}{R_1+R_2} \times I\left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right] \quad ...(5)$$

For E_{513}, consider the path from the terminal 5 to 2 as shown in fig (c)

Galvanometer carries zero current

Now from the fig(c) we can write

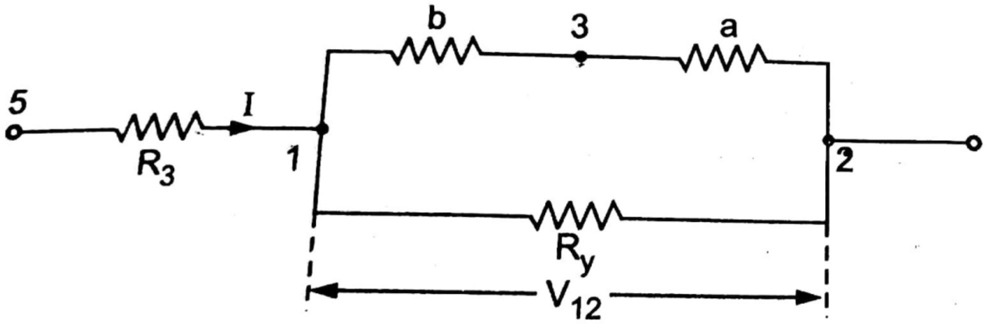


fig (c)

$$V_{12} = I \times \left[\frac{R_y(a+b)}{R_y+a+b}\right]$$

and $$V_{13} = \frac{b}{a+b} \cdot V_{12}$$

$$V_{13} = \frac{b}{a+b} \cdot I\left[\frac{R_y(a+b)}{R_y+a+b}\right] \quad ...(6)$$

$$E_{513} = I\,R_3 + V_{13}$$

Kelvin's Double Bridge

$$E_{513} = I R_3 + I \frac{b}{a+b}\left[\frac{R_y(a+b)}{R_y + a + b}\right]$$

$$E_{513} = I\left[R_3 + \frac{b}{a+b}\left[\frac{R_y(a+b)}{R_y + a + b}\right]\right] \quad \text{...(7)}$$

Now $\quad E_{45} = E_{513} \quad$ ...for balancing

$$\frac{I R_2}{R_1 + R_2}\left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right] = I\left[R_3 + \frac{b}{a+b}\left\{\frac{R_y(a+b)}{a+b+R_y}\right\}\right]$$

$$R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \frac{R_1 + R_2}{R_2}\left[R_3 + \frac{b}{\cancel{a+b}}\left\{\frac{R_y\cancel{(a+b)}}{a+b+R_y}\right\}\right]$$

$$R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \left[1 + \frac{R_1}{R_2}\right]\left[R_3 + \frac{b R_y}{R_x + a + b}\right]$$

$$R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = R_3 + \frac{R_1 R_3}{R_2} + \frac{b R_y}{R_y + a + b} + \frac{R_1 b R_y}{R_2(R_y + a + b)}$$

Kelvin's Double Bridge

$$R_x = \frac{R_1 R_3}{R_2} + \frac{bR_y}{R_y + a + b} + \frac{R_1 b R_y}{R_2 (R_y + a + b)} - \frac{(a+b) R_y}{(R_y + a + b)}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{bR_1 R_y}{R_2(R_y + a + b)} - \frac{aR_y}{(a + b + R_y)}$$

$$\boxed{R_x = \frac{R_1 R_3}{R_2} + \frac{bR_y}{(R_y + a + b)}\left[\frac{R_1}{R_2} - \frac{a}{b}\right]} \qquad \text{...(8)}$$

But $\frac{a}{b} = \frac{R_1}{R_2}$ thus $\frac{R_1}{R_2} - \frac{a}{b} = 0$

$$\boxed{R_x = \frac{R_1 R_3}{R_2}} \qquad \text{...(9)}$$

- This is the standard equation of the bridge balance. The resistances a, b and Ry are not present in this equation. Thus the effect of lead, and contact resistances is completely eliminated.

- The important condition for this bridge balance condition is that the ratio of the resistances of ratio arms must be same as the ratio of the resistances of the second ratio arms.
- In a typical Kelvin's double bridge, the range of a resistance covered is 1Ω to 10 $\mu\Omega$ with an accuracy of $\pm$ 0.05 % to $\pm$0.2%.

Kelvin's Double Bridge

1Q): In a Kelvin's double bridge, there is error due to mismatch between the ratios of outer and inner arm resistances. The bridge uses,

Standard resistance = 100.03 $\mu\Omega$

Inner ratio arms =100.31 Ω and 200 Ω

Outer ratio arms = 100.24 Ω and 200 Ω

The resistance of the connecting leads from standard to unknown resistance is 700 $\mu\Omega$

Calculate the unknown resistance under this condition.

Solution : From the given data,

$$R_3 = 100.03\ \mu\Omega,\ R_2 = 100.24\ \Omega,\ R_1 = 200\ \Omega$$

$$b = 100.31\ \Omega,\ a = 200\ \Omega,\ R_y = 700\ \mu\Omega$$

Thus unknown resistance is,

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_y}{[R_y + a + b]}\left\{\frac{R_1}{R_2} - \frac{a}{b}\right\}$$

$$= \frac{200 \times 100.03 \times 10^{-6}}{100.24} + \frac{100.31 \times 700 \times 10^{-6}}{[700 \times 10^{-6} + 200 + 100.31]}\left\{\frac{200}{100.24} - \frac{200}{100.31}\right\}$$

$$= 1.9958 \times 10^{-4} + (2.3381 \times 10^{-4})(1.3923 \times 10^{-3})$$

$$= 1.999 \times 10^{-4}\ \Omega = 199.905\ \mu\Omega$$

Measurement of High Resistances

➢ Methods for Measurement of High Resistances

- There are different methods that can be employed for the measurement of high resistances of the order of hundreds and thousands of megaohms is often required in equipments like insulation resistance of cables,machines,leakage resistance of capacitors etc.

Some of the important methods are as follows

1) Direct deflection method
2) Loss of charge method
3) Megaohm bridge
4) Megger

Loss of Charge Method

- This is very typical method of measuring the insulation resistance of very high value.The typical arrangement of resistance measurement is as shown in the Fig.(a)
- The resistance to be measured is shunted by a known value capacitor. The voltage across parallel combination is measured using electrostatic voltmeter.
- The circuit is driven by a d.c. voltage source of value V.
- This voltage is applied to the circuit through a switch. Initially switch is kept open. When the switch is closed at certain instant, the capacitor C starts charging.

The voltage across C is given by.

$$V_C = V\left(1 - e^{-\frac{t}{RC}}\right) \qquad \ldots(1)$$

Then at certain instant say t = t_1, switch is opened. Then capacitor C starts discharging through R. Then at instant voltage is given by,

$$V_C = V e^{-t/RC}$$

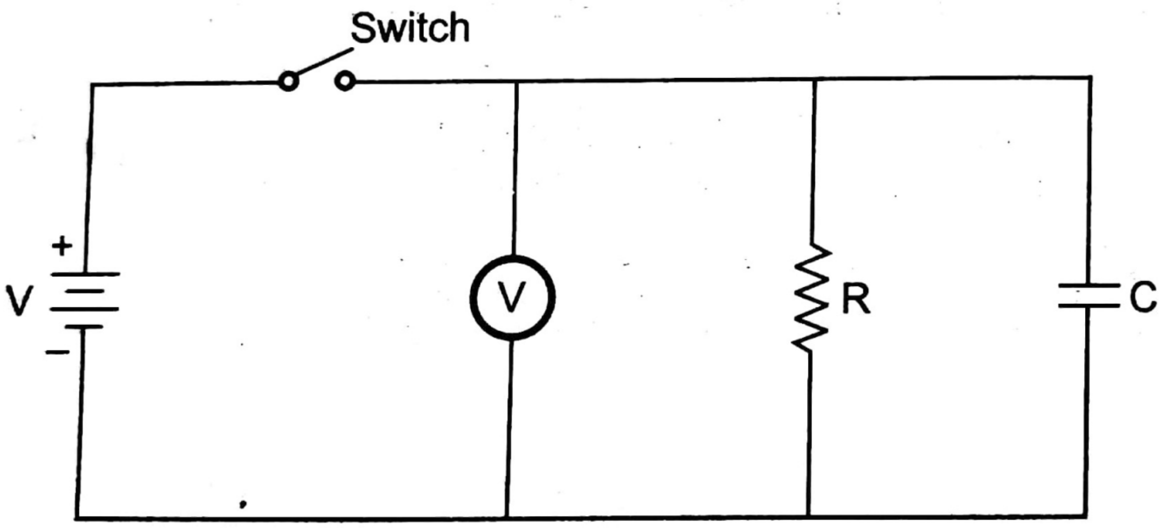


Fig.(a) Circuit arrangement of loss of charge method

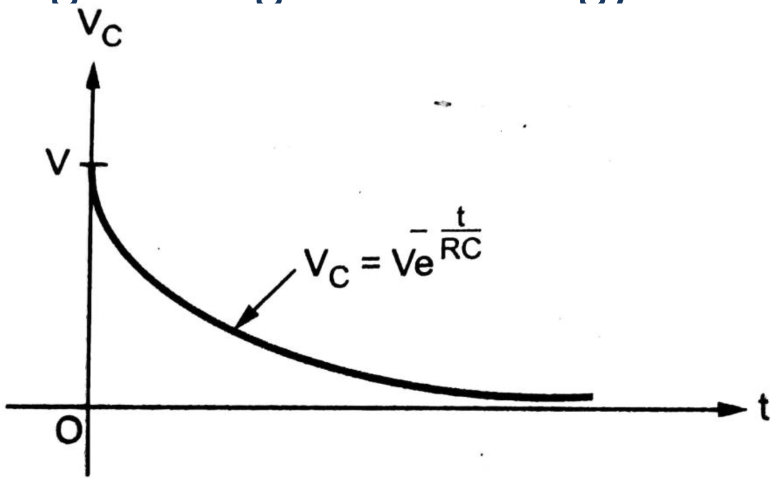
$$\frac{V_C}{V} = e^{-\frac{t}{RC}}$$

Simplifying

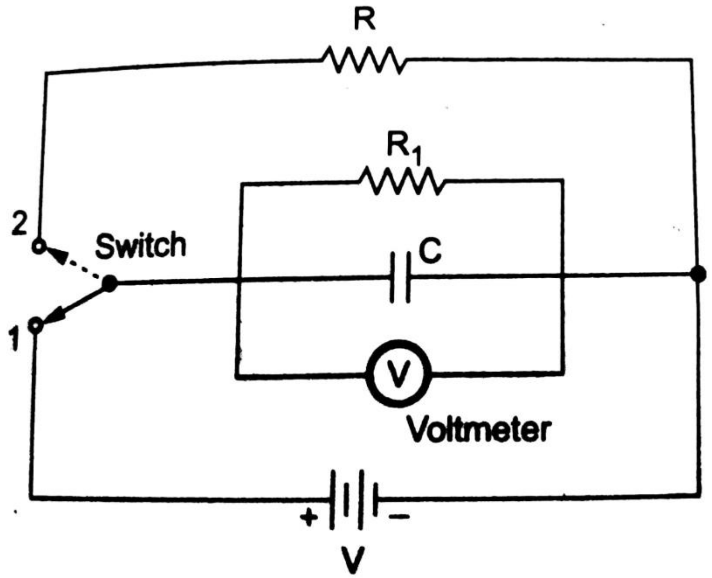
$$R = \frac{t}{C\, ln \frac{V}{V_C}} = \frac{0.4343\ t}{C\ \log_{10} \frac{V}{V_C}}$$

Loss of Charge Method

- If the value of resistance R is very large, then capacitor C requires more time for discharging as shown in fig(b). In such cases, the process becomes time consuming.
- This method can be used effectively for measurement of high resistances, but it needs a capacitor with a high leakage resistance.
- The typical circuit arrangement is as shown in Fig(c).
- The circuit consists high insulation resistance R to be measured along with capacitor of known value C shunted with electrostatic voltmeter and leakage resistance R_1.
- Initially, capacitor C is charged to suitable voltage say V_1 by moving switch to position 1. Then switch is moved to position 2. The capacitor starts discharging through parallel combination of R and R_1.
- At certain instant t voltage across capacitor C i.e. V_2 is measured using electrostatic voltmeter.
- Thus in time t, voltage across capacitor drops down from V_1 to V_2.
- Let the equivalent resistance through which C discharges be denoted by R' where R|| R_1



Fig(b) Variation of voltage across C with respect to time t



Fig(c) Measurement of high resistance using loss of charge method include leakage resistance of C

Loss of Charge Method

The expression for current any instant t is given by,

$$i = -\frac{dq}{dt} = -C\frac{dV}{dt} \qquad \text{...(2)}$$

but

$$i = \frac{\text{Potential drop across } R'}{R'} = \frac{V}{R'} \qquad \text{...(3)}$$

Comparing equations (2) and (3), we can write,

$$\frac{V}{R'} = -C\frac{dV}{dt} \quad \text{or} \quad \frac{dV}{V} = -\frac{dt}{RC}$$

Integrating both sides

$$[ln\ V]_{V_1}^{V_2} = \left[\frac{-t}{R'C}\right]_0^t$$

$$ln\frac{V_2}{V_1} = -\frac{t}{R'C}$$

$$V_2 = V_1\ e^{-\frac{t}{R'C}} \qquad \text{...(4)}$$

- Thus if time t is known, the resistance R' can be obtaining by measuring voltages V_1 and V_2. The same test is repeated with unknown resistance R removed.
- Then C discharges through only R_1.
- Then expression is given by,

$$V_2 = V_1\ e^{-\frac{t}{R_1 C}} \qquad \text{...(5)}$$

- Thus the value of the leakage resistance of the capacitor can also be found out using this method.
- Note that in this method, leakage resistance of the voltmeter can be neglected if its value of low. If it is high, then it must be considered along with R_1.

MEGGER

MEGGER

- Resistances of the order of 0.1MΩ and upwards are classified as high resistances. These high resistances are measured by portable, instrument known as Megger. It is also used for testing the insulation resistance of cables.
- The device enable us to measure electrical leakage in wire, results are very reliable as we shall be passing electric current through device while we are testing.
- The equipment basically uses for verifying the electrical insulation level of any device such as motors, cables, generators, windings, etc.

Types of Megger

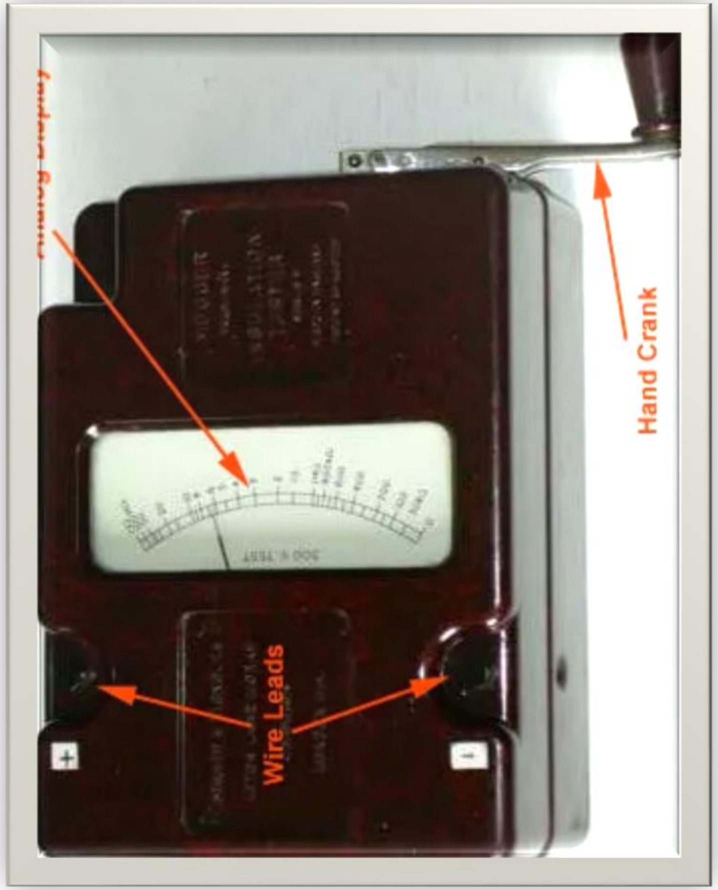
This can be separated into mainly two categories:-

1) Electronic Type (Battery Operated)
2) Manual Type (Hand Operated)

- ✓ But there is another types of megger which is motor operated type which does not use battery to produce voltage it requires external source to rotate a electrical motor which in turn rotates the generator of the megger.

MEGGER

Electronic Type Megger



Hand Operated Megger

Megger

Principle of Operation

- It is based on the principle of electromagnetic induction. The Fig(a) shows the construction of megger.
- When a current carrying conductor is placed in a uniform magnetic field it experiences a mechanical force whose magnitude depends upon the strength of current and magnetic field. While its direction depends on the direction of current and magnetic field.

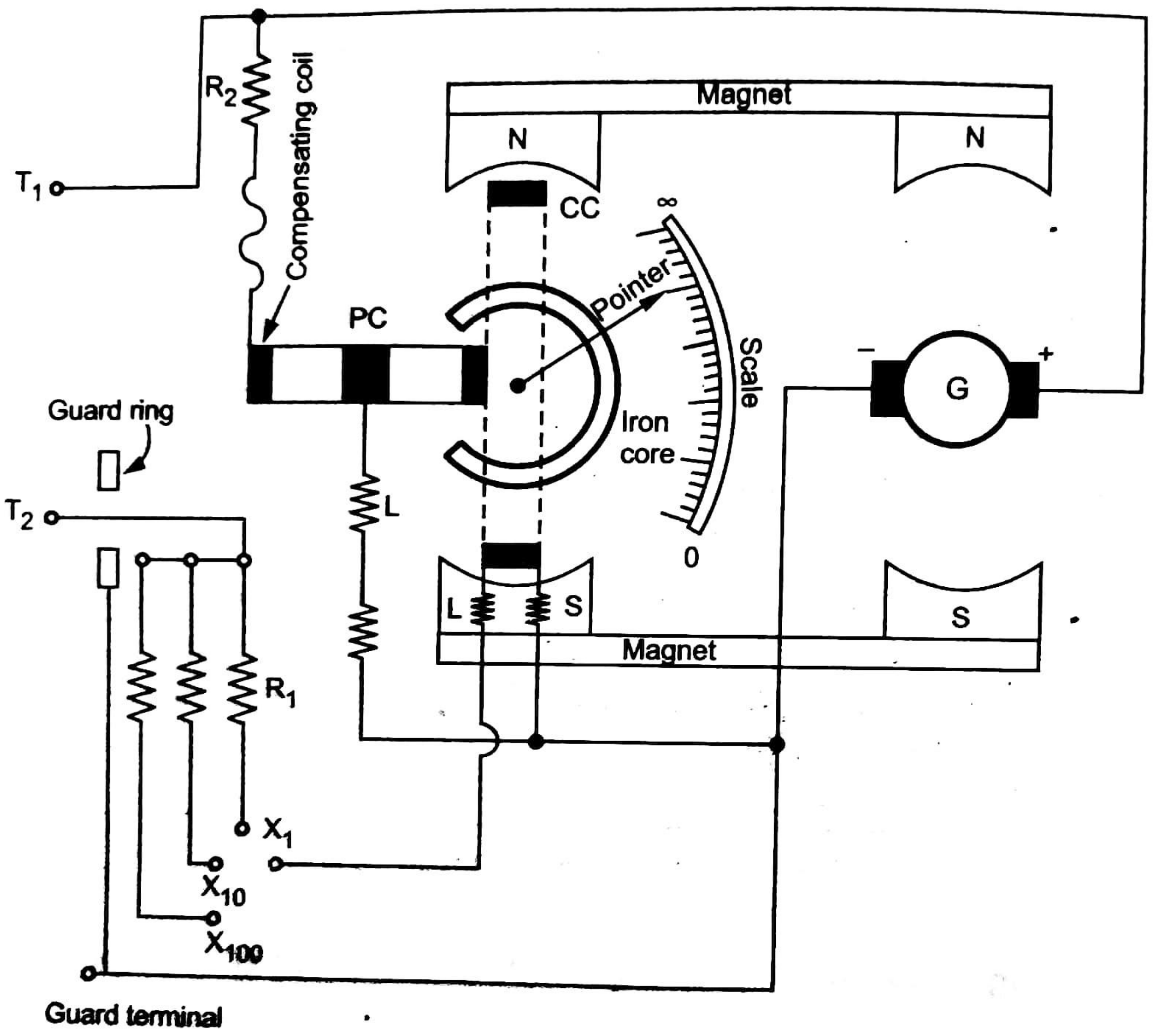
Construction

- It consists of a permanent magnet which provides the field for both the generator G and ohmmeter. The moving element of the ohmmeter consist of three coil viz. current or deflection coil, pressure or control coil and compensating coil. These coils are mounted on a central shaft which are free to rotate over a stationary C-shaped iron core.

- The coils are connected to the circuit through flexible leads called ligaments which do not produce a restoring torque on the moving element, consequently the moving element takes up any position over the scale when the generator handle is stationary.

Construction of Megger

- The current coil is connected in series with resistance R_1 between one generator terminal and the test terminal T_2.
- The series resistance R_1 protects the current coil in the event of the test terminals getting short circuited and also controls the range of the instrument.
- The pressure coil, in series with a compensating coil and protection resistance R_2 is connected across the generator terminals.
- The compensating coil is included in the circuit to ensure better scale proportions.
- The scale is calibrated reversely means the normal position of pointer indicates infinity while full scale deflection indicates zero resistance.

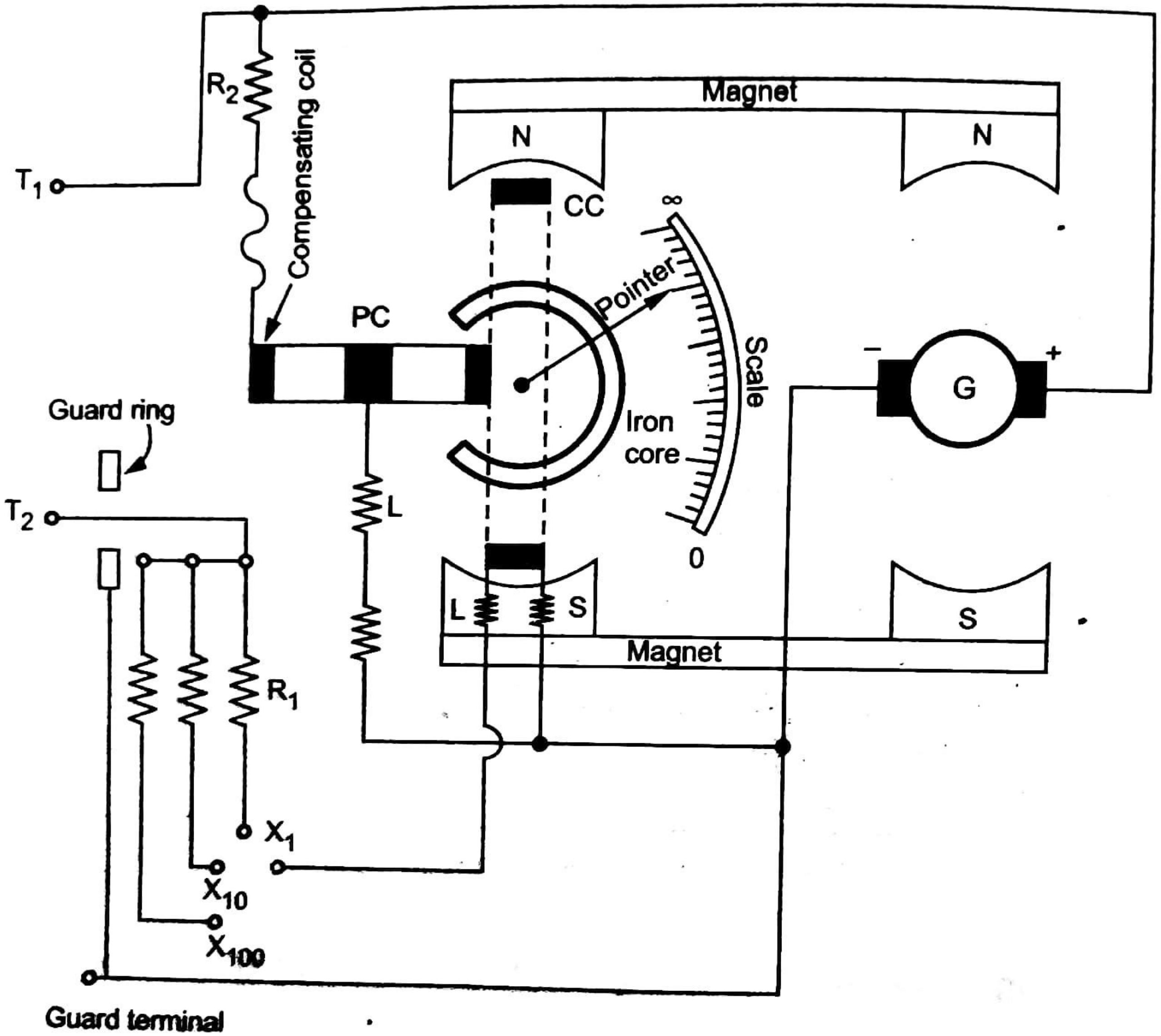


Fig(a) Megger

Megger

➢ Working of Megger

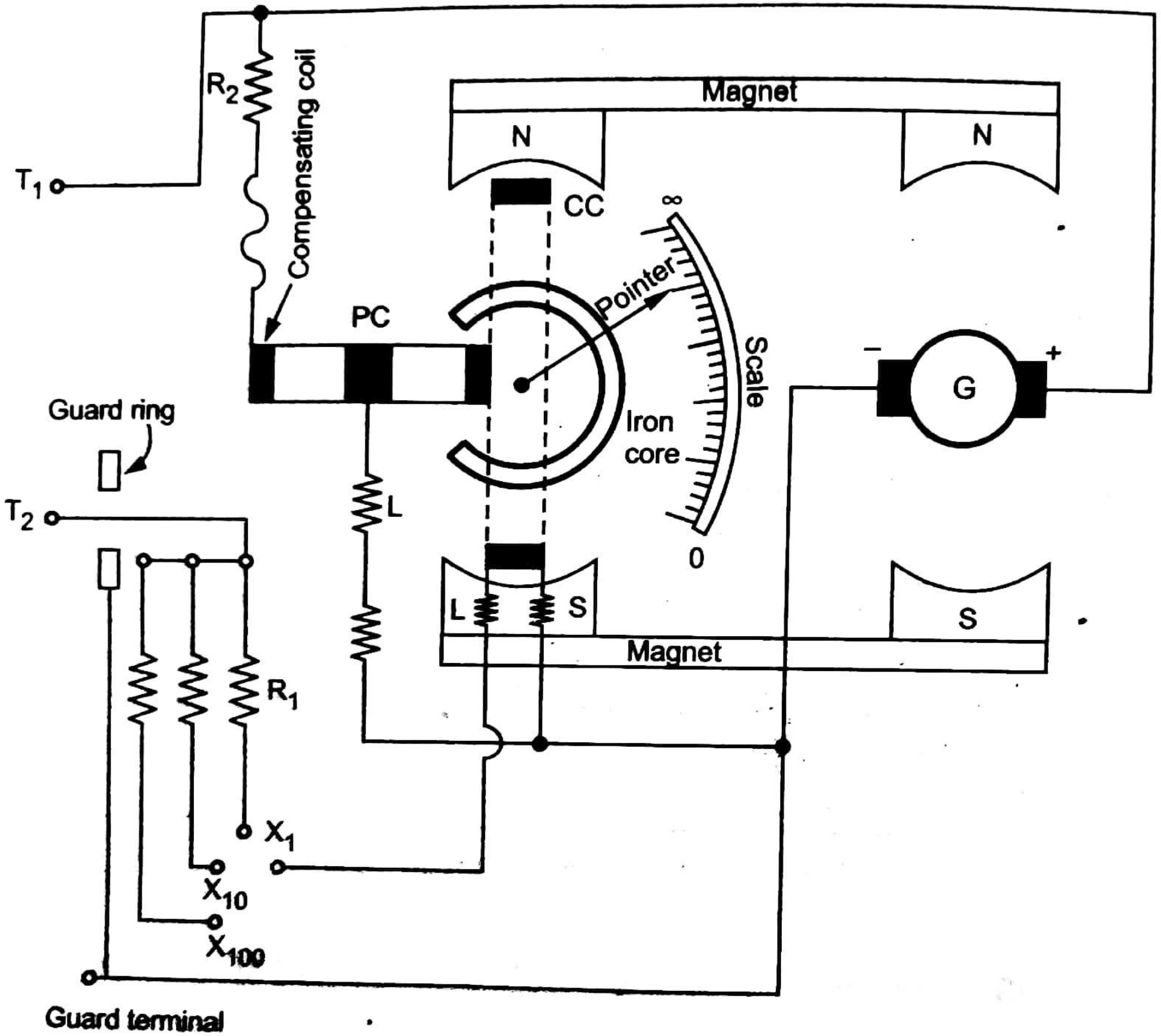
- When the current flows from the generator, through the pressure coil, the coil tends to set itself at right angles to the field of the permanent magnet.
- When the test terminals are open, corresponding to infinite resistance, no current flows through deflection coil.
  Thus the pressure coil governs the motion of the moving element making it move to its extreme anticlockwise position. The pointer comes to rest at the infinity end of the scale.
- When the test terminals are short circuited i.e. corresponding to zero resistance, the current from the generator flowing through the current coil is large enough to produce sufficient torque to overcome the counter-clockwise torque of the pressure coil. Due to this, pointer moves over a scale showing zero resistance.



Fig(a) Megger

Working of Megger

- When the high resistance to be tested is connected between terminals T_1 and T_2 the opposing torques of the coils balance each other so that pointer attains a stationary position at some intermediate point on scale. The scale is calibrated in megaohms so that the resistance is directly indicated by pointer.
- The guard ring is provided to eliminate the error due to leakage current. The supply to the meter is usually given by a hand-driven permanent magnet d.c. generator sometimes motor-driven generator may also be used.



Fig(a) Megger

Megger

- Applications
- The megger can be used to determine whether there is sufficiently high resistance between the conducting part of a circuit and the ground. This resistance is called insulation resistance.
- The megger can also be used to test continuity between any two points. When connected to the two points, if pointer shows full deflection then there is an electrical continuity between them.
- The equipment basically uses for verifying the electrical insulation level of any device such as motors, cables, generators, windings, etc.
- The device enable us to measure electrical leakage in wire

Megger

- **Advantages of Electronic Type Megger**

1) Level of accuracy is very high.
2) IR value is digital type, easy to read.
3) One person can operate very easily.
4) Works perfectly even at very congested space.
5) Very handy and safe to use.

- **Disadvantages of Electronic Type Megger**

1) Require an external source of energy to energies i.e. Dry cell.
2) Costlier in market.

- **Advantages of Hand Operated Megger**

1) Still keeps important in such high-tech world as it's an oldest method for IR value determination.
2) No external source required to operate.
3) Cheaper available in market.

- **Disadvantages of Hand Operated Megger**

1) At least 2 person required to operate i.e. one for rotation of crank other to connect megger with electrical system to be tested.
2) Accuracy is not up to the level as it's varies with rotation of crank.
3) Require very stable placement for operation which is a little hard to find at working sites.
4) Unstable placement of tester may impact the result of tester.
5) Provides an analog display result.
6) Require very high care and safety during use of the same.

A.C. BRIDGES

A.C. Bridges

- **AC bridges** are similar to Wheatstone bridge(D.C. bridge) in which D.C. source is replaced by an A.C. source and galvanometer with head phone/null detector.
- These are used to measure Resistance,Inductance,Capacitance,impedance and frequency etc.

AC BRIDGES

Self- Inductance

- Maxwell's Bridge
  - Maxwell inductance bridge
  - Maxwell inductance capacitance bridge
- Hay's Bridge
- Anderson bridge
- Owen's Bridge

Capacitance

- De Saulty Bridge
- Schering's bridge

Mutual Inductance

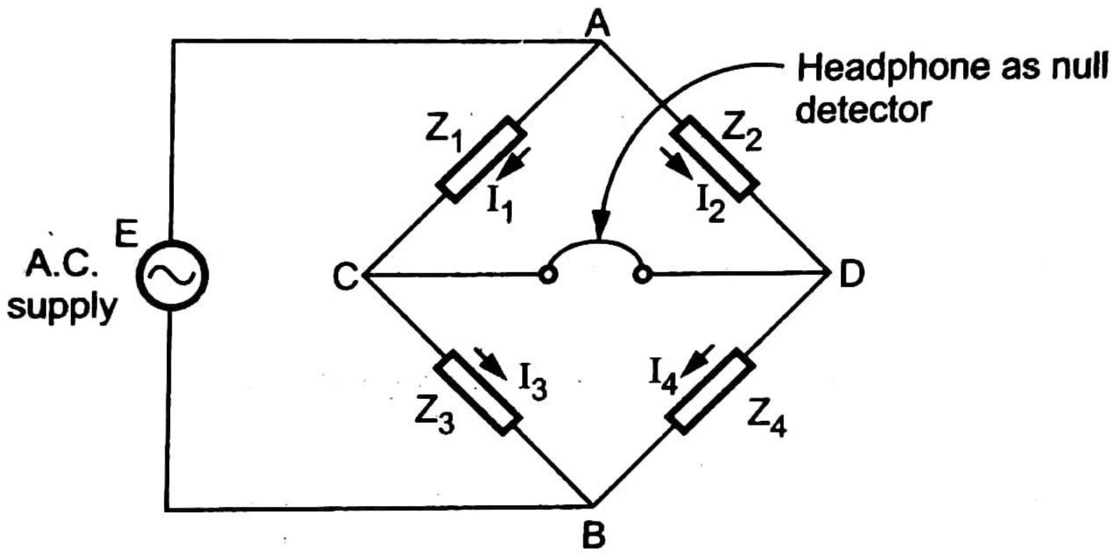
- Carey foster; Heydweiller bridge

Frequency

- Wien's Bridge

A.C. Bridges

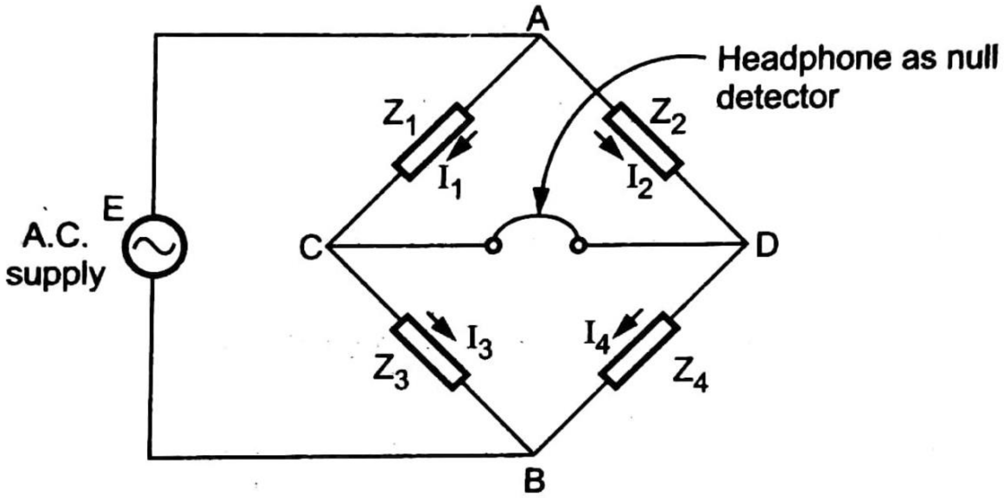
- An ac bridge in its basic form consists of four arms, a source of excitation and a balance detector. Each arm consists of an impedance. The source is an a.c. supply which supplies ac. voltage at the required frequency.
- For high frequencies, the electronic oscillators are used as the source. The balance detectors commonly used for a.c. bridges are head phones, tunable amplifier circuits or vibration galvanometers.
- The headphones are used as detectors at the frequencies of 250 Hz to 3 to 4 kHz. While working with single frequency a tuned detector is the most sensitive detector.



Fig(a)A.C. Wheatstone bridge

- The vibration galvanometers are useful for low audio frequency range from 5 Hz to 1000 Hz but are commonly used below 200 Hz. Tunable amplifier detectors are used for frequency range of 10 Hz to 100 Hz.
- This is similar to d.c. Wheatstone bridge. The bridge arms are impedances. The bridge is excited by a.c. supply and pair of headphones is used as a null detector. The null response is obtained by varying one of the bridge arms as shown in fig(a).

A.C. Bridge Balance Equation



For bridge balance, the potential of point C must be same as the potential of point D. These potentials must be equal interms of amplitude as well as phase.
Thus the drop from A to C must be equal to drop across A to D, in both magnitude and phase for the bridge balance.

$$\bar{E}_{AC} = \bar{E}_{AD} \qquad \ldots(1)$$

The vector notation indicates, both amplitude and phase to be considered.

$$\overline{I_1 Z_1} = \overline{I_2 Z_2} \qquad \ldots(2)$$

When the bridge is balanced, no current flows through the headphones

$$\bar{I}_3 = \bar{I}_1 \quad \text{and} \quad \bar{I}_4 = \bar{I}_2$$

Now

$$\bar{I}_1 = \frac{\bar{E}}{\bar{Z}_1 + \bar{Z}_3} \qquad \ldots(3)$$

and

$$\bar{I}_2 = \frac{\bar{E}}{\bar{Z}_2 + \bar{Z}_4} \qquad \ldots(4)$$

Substituting equation (3) and equation (4) into equation (2) we get,

$$\frac{\bar{E}\cdot\bar{Z}_1}{\bar{Z}_1 + \bar{Z}_3} = \frac{\bar{E}\cdot\bar{Z}_2}{\bar{Z}_2 + \bar{Z}_4}$$

$$\overline{Z_1 Z_2} + \overline{Z_1 Z_4} = \overline{Z_1 Z_2} + \overline{Z_2 Z_3}$$

$$\overline{Z_1 Z_4} = \overline{Z_2 Z_3} \qquad \ldots(5)$$

The equation (5) is the balancing equation in the impedance form.

A.C. Bridge Balance Equation

In the admittance form the condition can be expressed as,

$$\overline{Y_1 Y_4} = \overline{Y_2 Y_3} \qquad \dots (6)$$

The admittance is the reciprocal of the impedance.

Now in the polar form the impedances are expressed as,

$$\overline{Z_1} = Z_1 \angle\theta_1$$

$$\overline{Z_2} = Z_2 \angle\theta_2$$

$$\overline{Z_3} = Z_3 \angle\theta_3$$

$$\overline{Z_4} = Z_4 \angle\theta_4$$

where Z_1, Z_2, Z_3, Z_4 are the magnitudes and $\theta_1, \theta_2, \theta_3$ and θ_4 are the phase angles.

Note that the product of the impedances must be carried out in polar form where magnitudes get multiplied and phase angles get added.

Substituting in equation (5) we get,

$$Z_1 \angle\theta_1 \times Z_4 \angle\theta_4 = Z_2 \angle\theta_2 \times Z_3 \angle\theta_3$$

$$Z_1 Z_4 \angle\theta_1 + \theta_4 = Z_2 Z_3 \angle\theta_2 + \theta_3 \qquad \dots (7)$$

The equation (7) gives the two conditions to be satisfied for the bridge balance.

Equating magnitudes of both sides we get the magnitude condition as,

$$Z_1 Z_4 = Z_2 Z_3 \qquad \dots (8)$$

Equating phase angles we get,

$$\theta_1 + \theta_4 = \theta_2 + \theta_3 \qquad \dots (9)$$

- Thus the products of the magnitudes of the opposite arms must be equal while sum of the phase angles of the opposite arms must be equal.
- Thus the bridge must be balanced for both the conditions magnitude as well as phase.
- The phase angles depend on the components of the individual impedances.

A.C. Bridge Balance Equation

The phase angles are positive for the inductive impedances and negative for the capacitive impedances.

For inductive branch, $Z_L = R + jX_L = |Z_L| \angle +\theta$

For capacitive branch, $Z_C = R - jX_C = |Z_C| \angle -\theta$

where $X_L = 2\pi f L \, \Omega$ and $X_C = \dfrac{1}{2\pi f C} \Omega$

Maxwell's Bridge

Maxwell's Bridge

- Maxwell's bridge can be used to measure inductance by comparison either with a variable standard self inductance or with a standard variable capacitance. These two measurements can be done by using the Maxwell's bridge in two different forms.

Maxwell's Inductance Bridge

Using this bridge, we can measure inductance by comparing it with a standard variable self inductance arranged in bridge circuit as shown in Fig(a).

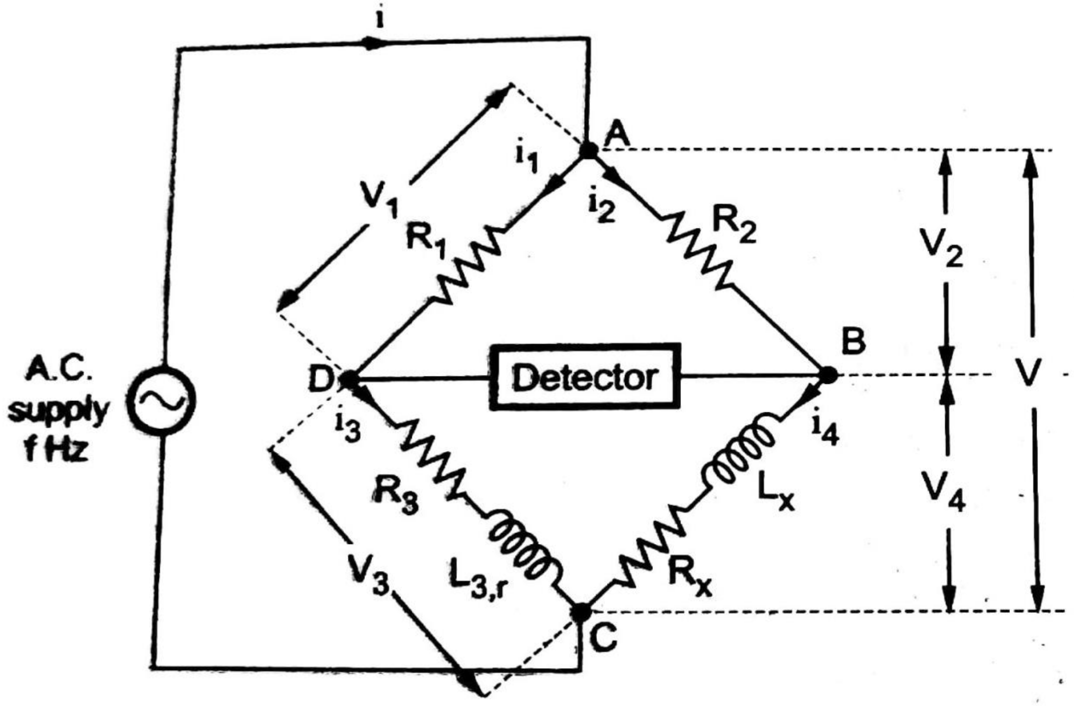
Consider Maxwell's inductance bridge as shown in the Fig(a). Two branches consist of non-inductive resistances R_1 and R_2. One of the arms consists variable inductance with series resistance r. The remaining arm consists unknown inductance L_X.

At balance, we get condition as

$$\frac{R_1}{[(R_3+r)+j\omega L_3]} = \frac{R_2}{R_x+j\omega L_x} \quad \ldots(1)$$

$$R_1[R_x+j\omega L_x] = R_2[(R_3+r)+j\omega L_3]$$

$$R_1R_x+j\omega R_1L_x = R_2(R_3+r)+j\omega R_2L_3$$



(a) Circuit diagram

Equating imaginary terms, we can write

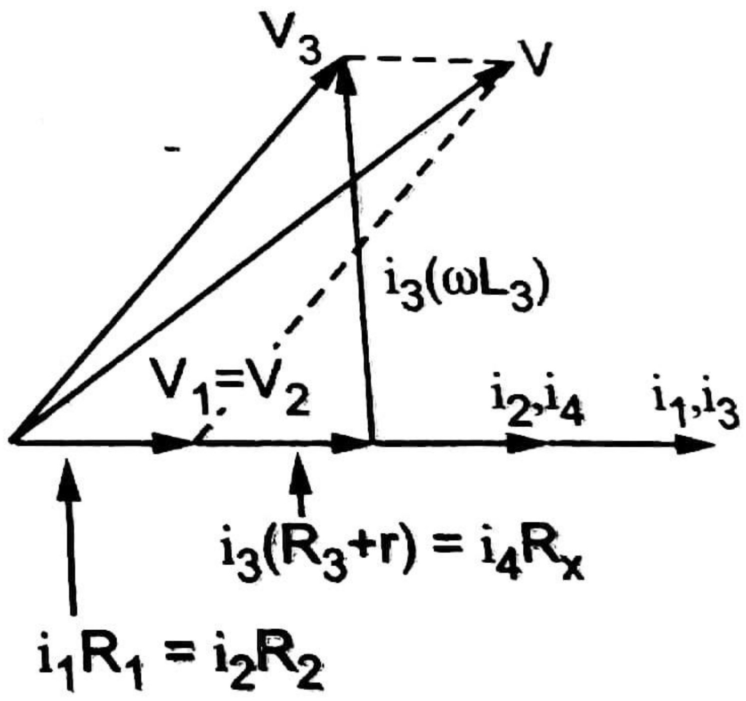
$$R_1 L_x = R_2 L_3$$

$$\boxed{L_x = \frac{R_2}{R_1} L_3} \qquad ...(2)$$

Equating real terms, we can write,

$$R_1 R_x = R_2(R_3 + r)$$

$$R_x = \frac{R_2}{R_1}(R_3 + r) \qquad ...(3)$$



Under the balanced condition, the vector diagram for Maxwell's inductance bridge is as shown in the Fig(b).

Fig(b). Phasor diagram

Fig(b).

Maxwell's Inductance Capacitance Bridge(Maxwell Wien bridge)

Using this bridge, we can measure inductance by comparing with a variable standard capacitor. The bridge circuit diagram is as shown in the Fig(c). One of the ratio arms consists of resistance and capacitance in parallel. Hence it is simple to write the bridge equations in the admittance form.

The general bridge balance equation is,

$$\overline{Z_1 Z_x} = \overline{Z_2 Z_3}$$

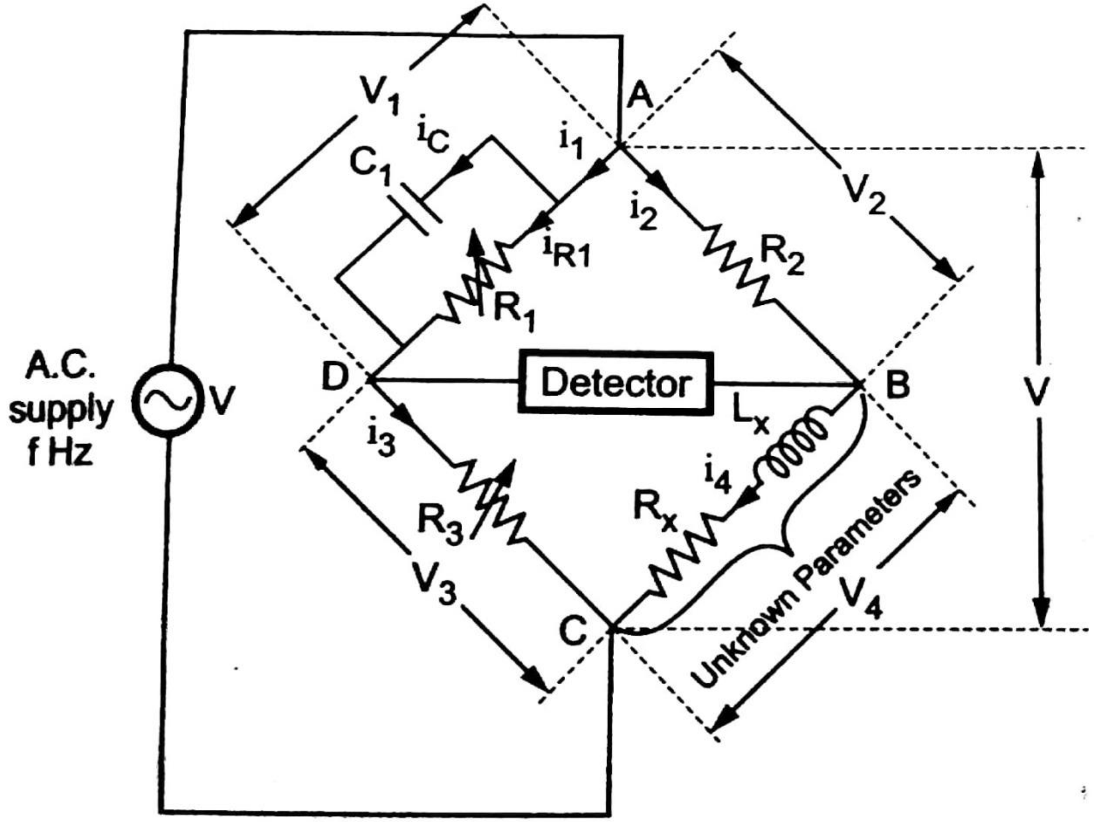
$$\overline{Z_x} = \frac{\overline{Z_2 Z_3}}{\overline{Z_1}} = \overline{Z_2 Z_3}\ \overline{Y_1} \qquad \text{...(4)}$$

where $\overline{Y_1} = \frac{1}{\overline{Z_1}}$ i.e. R_1 in parallel with C_1

$$\overline{Z_2} = R_2$$

$$\overline{Z_3} = R_3$$

$$\overline{Z_x} = R_x + j\omega L_x \text{, as } L_x \text{ in series with } R_x$$



Fig(c) Circuit diagram

Now $\overline{Y_1} = \frac{1}{R_1} + j\,\omega C_1$ .....(5)

as $\overline{Z_1} = R_1 \,||\, j\left(\frac{1}{\omega C_1}\right)$ as $\frac{1}{j} = -j$

Substituting all the values in equation (4) we get,

$$R_x + j\,\omega L_x = R_2\,R_3\left[\frac{1}{R_1} + j\omega C_1\right]$$

$$R_x + j\,\omega L_x = \frac{R_2\,R_3}{R_1} + j\,R_2\,R_3\,\omega C_1 \quad ...(6)$$

Equating real parts,

$$R_x = \frac{R_2\,R_3}{R_1} \quad ...(7)$$

Equating imaginary parts,

$$\omega L_x = R_2\,R_3\,\omega C_1$$

$$\boxed{L_x = R_2\,R_3\,C_1} \quad ...(8)$$

The resistances are expressed in ohms, the inductances in henries and capacitance in

The quality factor of the coil is given by,

$$Q = \frac{\omega L_x}{R_x} = \frac{\omega R_2\,R_3\,C_1}{\left(\frac{R_2\,R_3}{R_1}\right)}$$

$$\boxed{Q = \omega R_1\,C_1} \quad ...(9)$$

- The advantages of using standard known capacitor for measurement are,

1) The capacitors are less expensive than stable and accurate standard inductors.
2) The capacitors are almost lossless.
3) External fields have less effect on a capacitor. The standard inductor requires well shielding in order to eliminate the effect of stray magnetic fields.
4) The standard inductor will not present its rated value of inductance unless current flow through it is precisely adjusted.
5) The capacitors are smaller in size.

This bridge is also called as Maxwell Wien bridge

- Advantages of Maxwell Bridge

The advantages of the Maxwell bridge are,

1) The balance equation is independent of losses associated with inductance.
2) The balance equation is independent of frequency of measurement.
3) The scale of the resistance can be calibrated to read the inductance directly.
4) The scale of R_1 can be calibrated to read the Q value directly.
5) When the bridge is balanced, the only component in series with coil under test is resistance R_2. If R_2 is selected such that it can carry high current, then heavy current carrying capacity coils can be tested using this bridge.

Disadvantages of Maxwell Bridge

The disadvantages of the Maxwell bridge are,

1) It cannot be used for the measurement of high Q values. Its use is limited to the measurement of low Q values from 1 to 10. This can be proved from phase angle balance condition which says that, sum of the angles of one pair of opposite arms must be equal.θ_1 +θ_4 = θ_2+θ_3
   But θ_2 and θ_3 are zero, as the corresponding impedances are pure resistances. For high Q values, the angle θ_4 is almost 90°. Hence θ_1 must be -90°. But θ_1 gets decided by parallel combination of R_1 and C_1. To get θ_1 as almost -90°, the value of R_1 should be very very high. Practically, such high resistance is not possible.
   Hence high Q values cannot be measured.
2) There is an interaction between the resistance and reactance balances. Getting the balance adjustment is little difficult.
3) It is unsuited for the coils with low Q values, less than one, because of balance convergence problem.
4) The bridge balance equations are independent of frequency. But practically, the properties of coil under test vary with frequency which can cause error.
   Commercial Maxwell bridge measures the inductance from 1 - 1000 H, with $\pm$ 2 % error.

1Q) : The arms of an a.c. Maxwell's bridge are adjusted as:
Arm AB : non-reactive resistance of 700Ω
Arm CD: non-reactive resistance of 300 Ω
Arm AD : non-reactive resistance of 1200 Ω in parallel with capacitor of 0.5μF.
If the bridge is balanced under this condition, find the components of the branch BC.

From the bridge,

$$C_1 = 0.5\ \mu F, \quad R_1 = 1200\ \Omega$$

$$R_2 = 700\ \Omega, \quad R_3 = 300\ \Omega$$

From bridge balance equation,

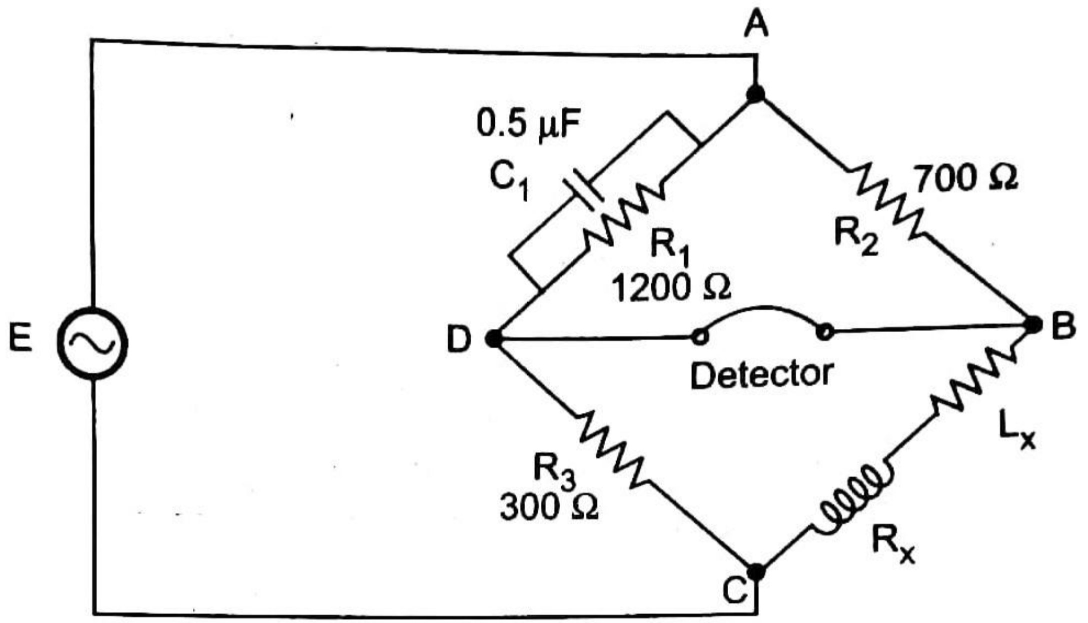
$$R_x = \frac{R_2 R_3}{R_1} = \frac{700 \times 300}{1200}$$

$$= 175\ \Omega$$

And

$$L_x = R_2 R_3 C_1 = 700 \times 300 \times 0.5 \times 10^{-6}$$

$$= 105\ mH$$

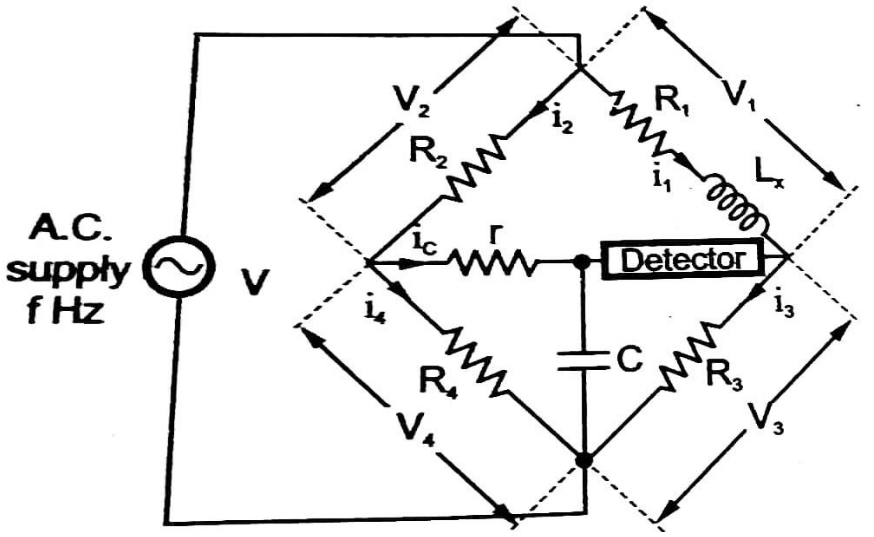


Key Point : If the branches are not given in standard form as they are assumed for deriving bridge balance equation, derive the bridge balance equation again from the basic condition $\overline{Z_1 Z_4} = \overline{Z_2 Z_3}$

Anderson Bridge

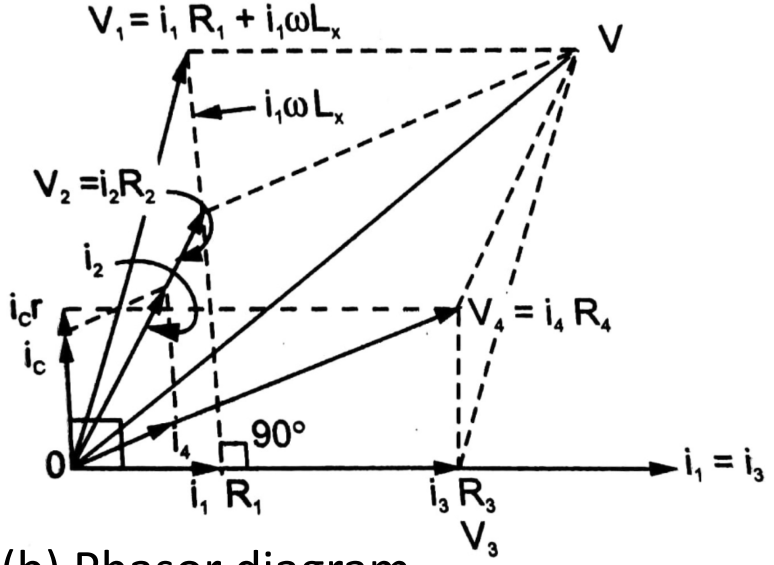
Anderson Bridge

- It is another important a.c. bridge used for the measurement of self inductance interms of a standard capacitor.
- Actually this bridge is nothing but modified Maxwell's bridge in which also the value of self inductance is obtained by comparing it with a standard capacitor. This bridge is basically used for the precise measurement of inductance over a wide range of value.



Fig(a).circuit diagram

- The Anderson bridge is as shown in the Fig(a).
  One arm of the bridge consists of unknown inductor L_x with known resistance in series with L_x. This resistance R_1 includes resistance of the inductor.
- C is the standard capacitor with r, R_2, R_3 and R_4 are non-inductive known resistances.



Fig(b) Phasor diagram

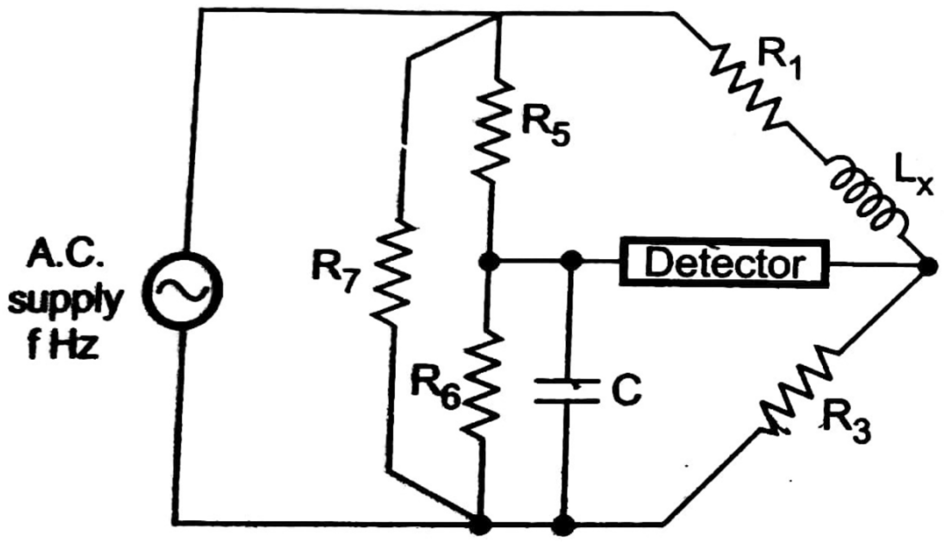
The bridge balance equations are,

$$i_1 = i_3,\ i_2 = i_4 + i_C,\ V_2 = i_2 R_2,\ V_3 = i_3 R_3$$

$$V_1 = V_2 + i_C r \text{ and } V_4 = V_3 + i_C r,\ V_1 = i_1 R_1 + i_1 \omega L_1,\ V_4 = i_4 R_4$$

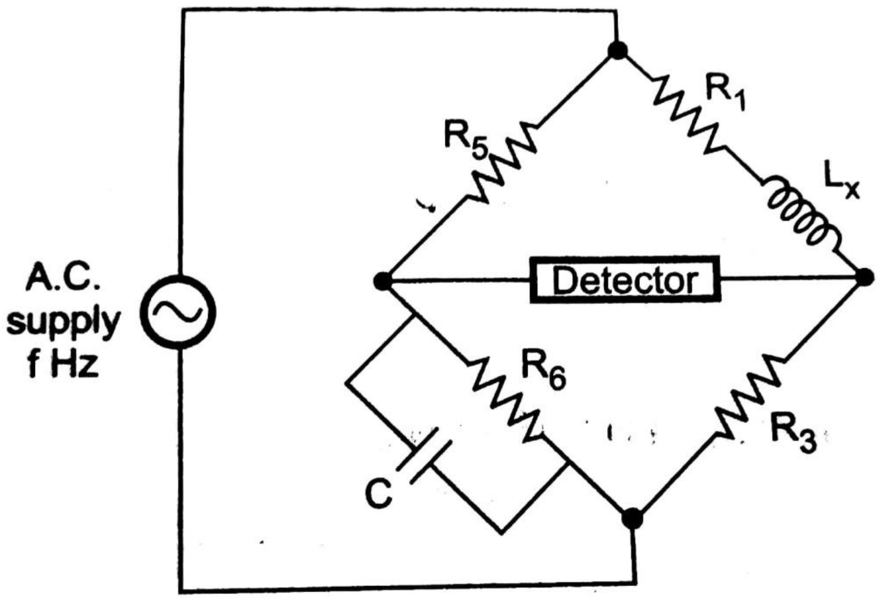
$$V = \overline{V}_2 + \overline{V}_4 = \overline{V}_1 + \overline{V}_3$$

To find balance equations transforming a star formed by R_2, R_4 and r into its equivalent delta as shown in the Fig(c) and (d).



Fig(c)

Fig Transformed Anderson bridge



Fig(d)

The elements in equivalent delta are given by,

$$R_5 = \frac{R_2 r + R_4 r + R_2 R_4}{R_4}$$

$$R_6 = \frac{R_2 r + R_4 r + R_2 R_4}{R_2}$$

$$R_7 = \frac{R_2 r + R_4 r + R_2 R_4}{r}$$

- Now R_7 shunts the source, hence it does not affect the balance condition. Thus by neglecting R_7 and rearranging a network as shown in the Fig(d), we get a Maxwell inductance bridge
  Thus, balance equations are given by,

$$L_x = CR_3R_5 \text{ and}$$

$$R_1 = R_3\frac{R_5}{R_6}$$

Substituting values of R_5 and R_6 ,we can write,

$$\boxed{L_x = \frac{CR_3}{R_4}[R_2r + R_4r + R_2R_4] \quad \text{and}}$$

$$\boxed{R_1 = \frac{R_2R_3}{R_4}}$$

- If the capacitor used is not perfect, the value of inductance remains unchanged, but the value of R_1 changes. This method can also be used to measure the capacitance of the capacitor C if a calibrated self inductance is available.

- **Advantages of Anderson Bridge**

  The advantages of Anderson's bridge are,

1) Can be used for accurate measurement of capacitance interms of inductance.
2) Other bridges require variable capacitor but a fixed capacitor can be used for Anderson's bridge.
3) The bridge is easy to balance from convergence point of view compared to Maxwell's bridge in case of low values of Q.

- **Disadvantages of Anderson Bridge**

  The disadvantages of Anderson's bridge are,

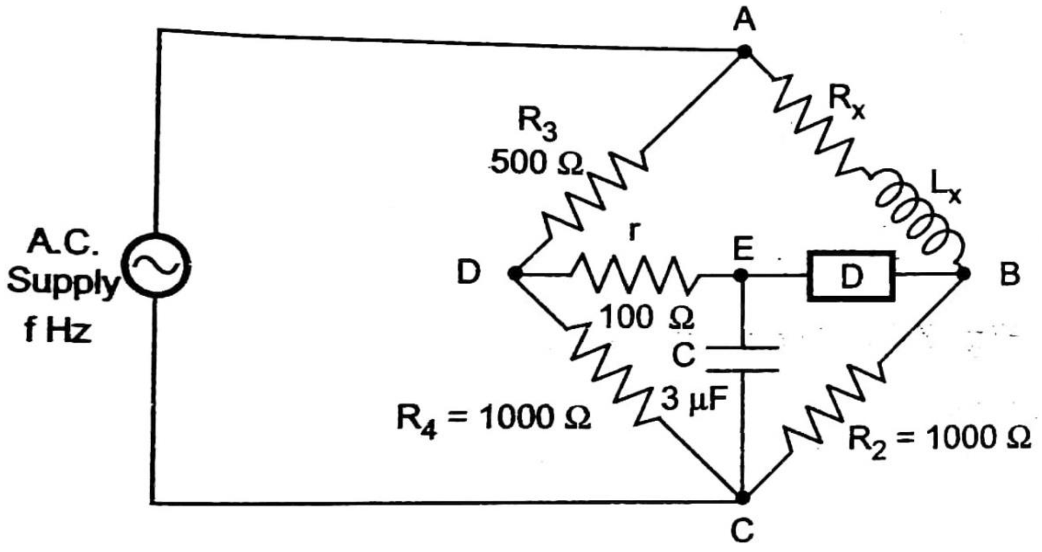
  1. It is more complicated than other bridges.

  2 Uses more number of components.

  3. Balance equations are also complicated to derive.

  4. Bridge cannot be easily shielded due to additional junction point, to avoid the effects of stray capacitances.

1) An Anderson a.c. bridge is as follows:
Arm AB : Unknown inductance R, and L, ,
Arm BC : Non-reactive resistance R2 = 1000***Ω***
Arm CD: Non-reactive resistance R4 = 1000 ***Ω***
Arm DA : Non-reactive resistance R3 = 500 ***Ω***
Arm DE : Resistance r = 100 ***Ω***
Arm EB : Detector and a.c. supply between AC
Arm EC : Capacitor C = 3 μF
State the expressions for L, and R, and find the values of them for given values of elements.



Solution : The bridge is shown in the Fig.

It can be noticed that the names of resistances branches AD and BC are reversed compared to what is assumed in the derivation earlier. Hence change R_2 and R_3 in the equations . And R_1 is denoted as R_x

$$R_x = \frac{R_2 R_3}{R_4} = \frac{1000 \times 500}{1000} = 500\ \Omega$$

$$L_x = \frac{C R_2}{R_4}[R_3 r + R_4 r + R_3 R_4]$$

$$= \frac{3\times10^{-6}\times1000}{1000}[500\times100+1000\times100+500\times1000]$$

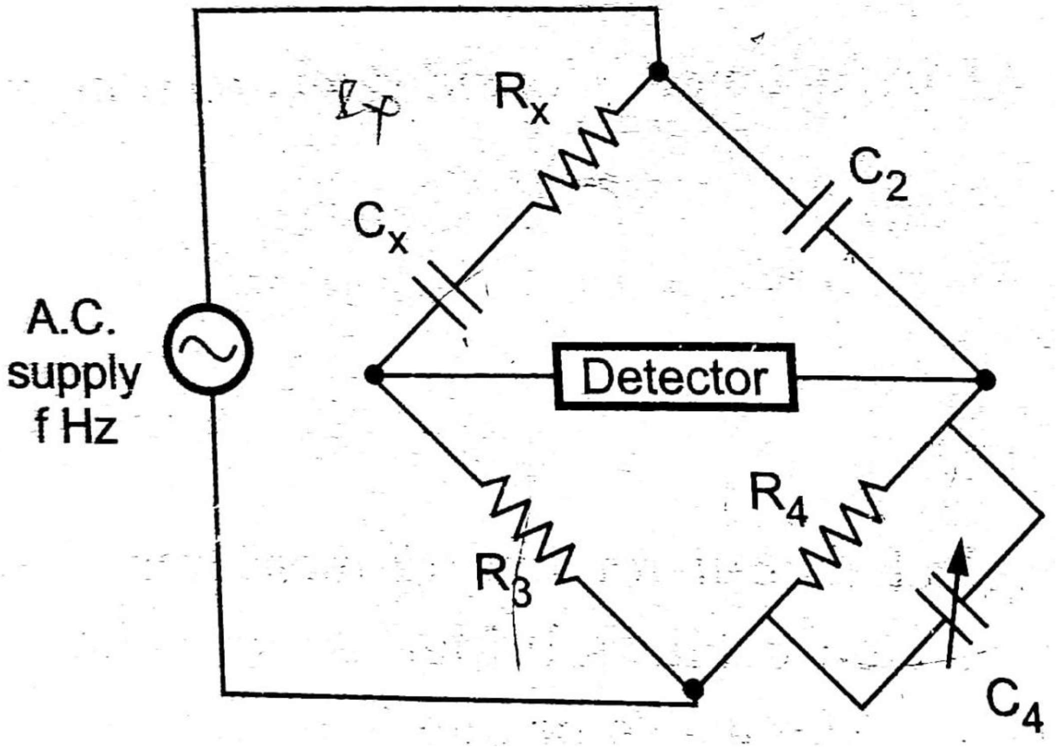
$$= 1.95\ \text{H}$$

SCHERING BRIDGE

Schering Bridge

- It is one of the most widely used a.c. bridges for the measurement of unknown capacitance, dielectric loss and power factor.

- The Fig(a) shows the connections of Schering bridge. It can be used for low voltages.
- The Cx is perfect capacitor to be measured. Rx is series resistance. C2 is standard air capacitor having very stable value. R3 and R4 non-inductive resistances while C4 is variable capacitor.



Fig(a).Schering bridge

From the general balance equation, $\overline{Z_1 Z_4} = \overline{Z_2 Z_3}$

Now

$$Z_1 = R_x - j\frac{1}{\omega C_x}$$

$$Z_2 = -\frac{j}{\omega C_2}$$

$$Z_3 = R_3$$

$$Z_4 = R_4 || \frac{-j}{\omega C_4} = \frac{R_4\left(-\frac{j}{\omega C_4}\right)}{\left(R_4 - j\frac{1}{\omega C_4}\right)}$$

Schering Bridge

$$Z_4 = \frac{-jR_4}{\omega R_4 C_4 - j} = \frac{-jR_4(\omega R_4 C_4 + j)}{(\omega R_4 C_4 - j)(\omega R_4 C_4 + j)} = \frac{R_4 - j\omega R_4^2 C_4}{\omega^2 R_4^2 C_4^2 + 1}$$

$$Z_1 = \frac{Z_2 Z_3}{Z_4} = \frac{\left(-\frac{j}{\omega C_2}\right)(R_3)}{\left(\frac{R_4 - j\omega R_4^2 C_4}{1+\omega^2 R_4^2 C_4^2}\right)} = \frac{(1+\omega^2 R_4^2 C_4^2) R_3 \left(-\frac{j}{\omega C_2}\right)}{(R_4 - j\omega R_4^2 C_4)}$$

$$\text{Rationalising, } Z_1 = R_3(1+\omega^2 R_4^2 C_4^2)\left\{\frac{-\frac{j}{\omega C_2}(R_4 + j\omega R_4^2 C_4)}{R_4^2 + \omega^2 R_4^4 C_4^2}\right\}$$

$$\therefore \quad R_x - j\frac{1}{\omega C_x} = \frac{R_3(1+\omega^2 R_4^2 C_4^2)}{R_4^2(1+\omega^2 R_4^2 C_4^2)}\left\{\frac{R_4^2 C_4}{C_2} - \frac{jR_4}{\omega C_2}\right\}$$

Equating real and imaginary parts,

$$\boxed{R_x = \frac{R_3}{R_4^2} \times \frac{R_4^2 C_4}{C_2} = \frac{R_3 C_4}{C_2}} \quad ...(1)$$

$$-j\frac{1}{\omega C_x} = -j\frac{R_3}{R_4^2} \times \frac{R_4}{\omega C_2} = -j\left[\frac{1}{\frac{R_4}{R_3}\omega C_2}\right]$$

$$\omega C_x = \frac{R_4}{R_3}\omega C_2$$

$$\boxed{C_x = \frac{R_4}{R_3} C_2} \quad ...(2)$$

The equations (1) and (2) gives the required values of C_x and R_x

➢ Power Factor and Loss Angle

(i).Power factor (p.f): The power factor of the series RC combination is defined as the cosine of the phase angle of the circuit. Thus,

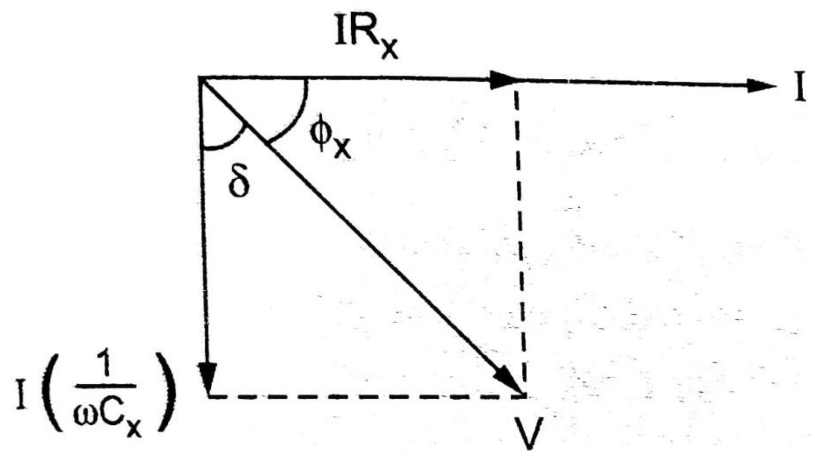
$$p.f. = \cos\phi_x = \frac{R_x}{Z_x}$$

For phase angles ray dose to 90°, the reactance is almost equal to the impedance,

$$p.f. = \frac{R_x}{X_x} = \frac{R_x}{\left(\frac{1}{\omega C_x}\right)}$$

$$\boxed{p.f. = \omega R_x C_x}$$

(ii) Loss angle (δ): For a series combination of $R_x$and $C_x$the angle between the voltage across the series combination and voltage across the capacitor C_x is called loss angle δ. This is shown in the Fig(b).



Fig(b)

$$\text{Now } \tan\delta = \frac{IR_x}{I\left(\frac{1}{\omega C_x}\right)} = \omega R_x C_x$$

$$\therefore \quad \tan\delta = \omega\left(\frac{R_3 C_4}{C_2}\right)\left(\frac{R_4}{R_3}C_2\right) = \omega R_4 C_4$$

Thus loss angle can be measured, knowing the values of $R_4$and C_4

iii) Dissipation factor (D): For Rx-Cx series circuit, it is cotangent of the phase angle

$$D = \cot\phi_x = \frac{1}{\tan\phi_x} = \frac{1}{\left[\dfrac{I\left(\dfrac{1}{\omega C_x}\right)}{I R_x}\right]} = \omega R_x C_x = \omega R_4 C_4$$

- The quality factor Q = X/R = 1/ω CR hence dissipation factor is reciprocal of quality factor Q and gives the information about quality of the capacitor.
- Thus if the resistance R4 is fixed, then the dial of capacitor C_4 can be directly calibrated to give dissipation factor D i.e. quality of the capacitor. As the term ω is present in the equation, the calibration of C_4 dial holds good for only one particular frequency.
- The different frequency can be used but a correction should be made to multiply the C_4 dial reading by the ratio of the two frequencies.
- Similarly, if the resistance ratio is maintained at fixed value, the dial of C_3 can be graduated in terms of direct readings of C_x .
- Commercial Schering bridge measures the capacitors from 100 pF - 1 μF, with $\pm$2% accuracy.
- The bridge is widely used for testing small capacitors at low voltages with very high precision.

The phasor diagram is shown in fig(c) at the balanced condition.

I_1 is chosen reference. Now V_1 is drop across R_x and C_x .

Thus OA= $I_1 R_x$ and OB = $I_1 \left(\frac{1}{\omega C_x}\right)$ and I_1 leads capacitor drop by 90°. Thus OC = V_1 is $\overline{\text{OA}}$+ $\overline{\text{O}B}$.

Now

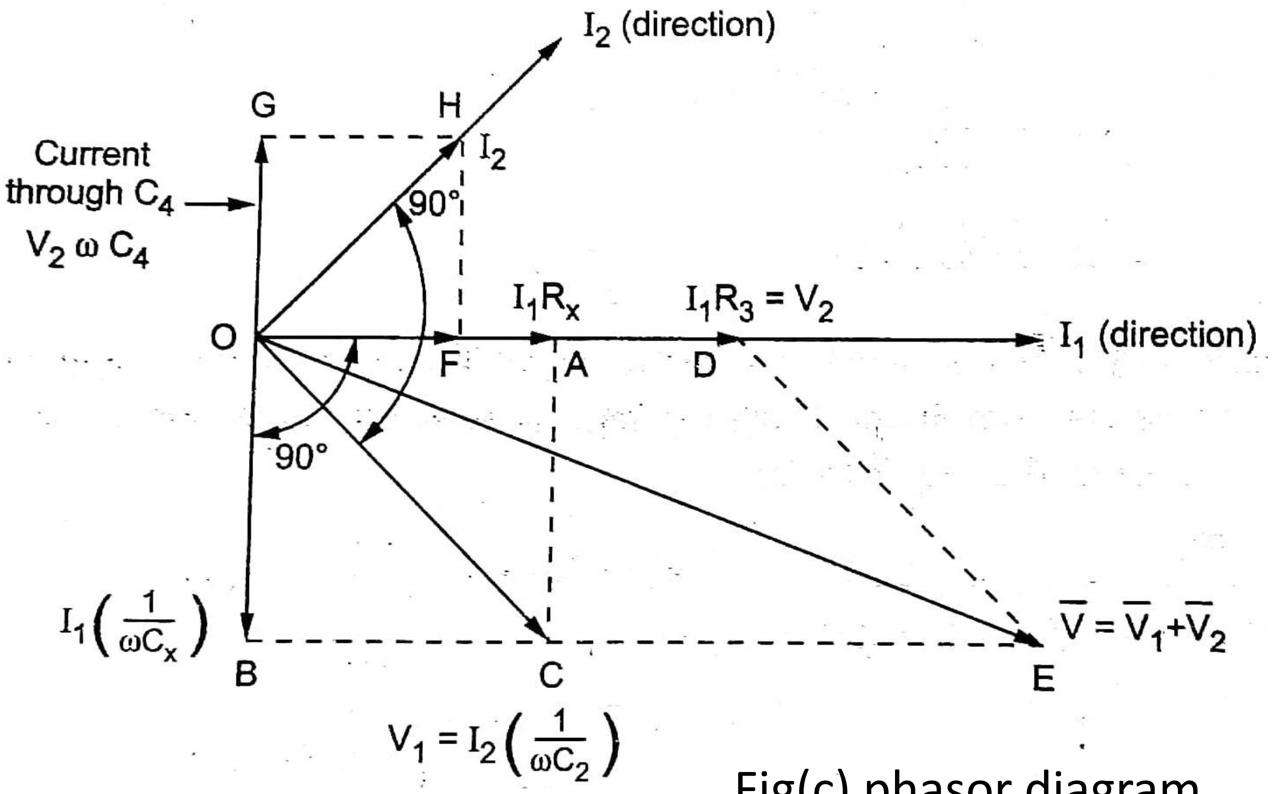
V_1 = $I_2 \left(\frac{1}{\omega C_2}\right)$ i.e. drop across C_2.

So current I_2 leads V_2 by 90°.

Then V_2 = I_1 R_3 which is OD in phase with I_1.

And $\overline{V_1}$+ $\overline{V_2}$= $\overline{\text{V}}$. is supply voltage OE.

V_2 is also drop across R_4 and C_4



Fig(c) phasor diagram

Let OF is current through R_4 thus OF = V_2 / R_4 in phase with V_2 while OG is current through C4 which is V_2 ωC_4, and is leading V_2 by 90°.

The OH = I_2 which is vector sum of OF and OG.

(1Q).The Schering bridge has the following constants :

Arm AB - capacitor of 1 μF in parallel with 1.2 kΩ resistance

Arm AD - resistance of 4.7 kΩ

Arm BC - capacitor of 1 μF

Arm CD - unknown capacitor C_x and R_x

The frequency of supply is 0.5 kHz. Calculate the unknown capacitance and its dissipation factor

Solution :

From the given information,

$$R_1 = 1.2 \text{ k}\Omega \qquad C_1 = 1 \ \mu\text{F}$$

$$R_2 = 4.7 \text{ k}\Omega \qquad C_3 = 1 \ \mu\text{F}$$

From the balanced equations,

$$R_x = \frac{R_2 C_1}{C_3} = \frac{4.7 \times 10^3 \times 1 \times 10^{-6}}{1 \times 10^{-6}}$$

$$= 4.7 \text{ k}\Omega$$

$$C_x = \frac{R_1 C_3}{R_2} = \frac{1.2 \times 10^3 \times 1 \times 10^{-6}}{4.7 \times 10^3}$$

$$= 0.255 \ \mu\text{F}$$

The dissipation factor,

$$D = \omega C_x R_x = 2\pi f C_x R_x$$

$$= 2\pi \times 0.5 \times 10^3 \times 0.255 \times 10^{-6} \times 4.7 \times 10^3 = 3.765$$

WIEN BRIDGE

Wien Bridge

- Basically the bridge is used for the frequency measurement but it is also used for the measurement of the unknown capacitor with great accuracy.
- Its one ratio arm consists of a series RC A.C. circuit i.e. $R_1$and C_1. The second ratio arm consists of a resistance R_2.
- The third arm consists of the parallel combination of resistance and capacitor i.e. R_3 and C_3. The circuit of the Wien bridge is shown in the Fig.(a)

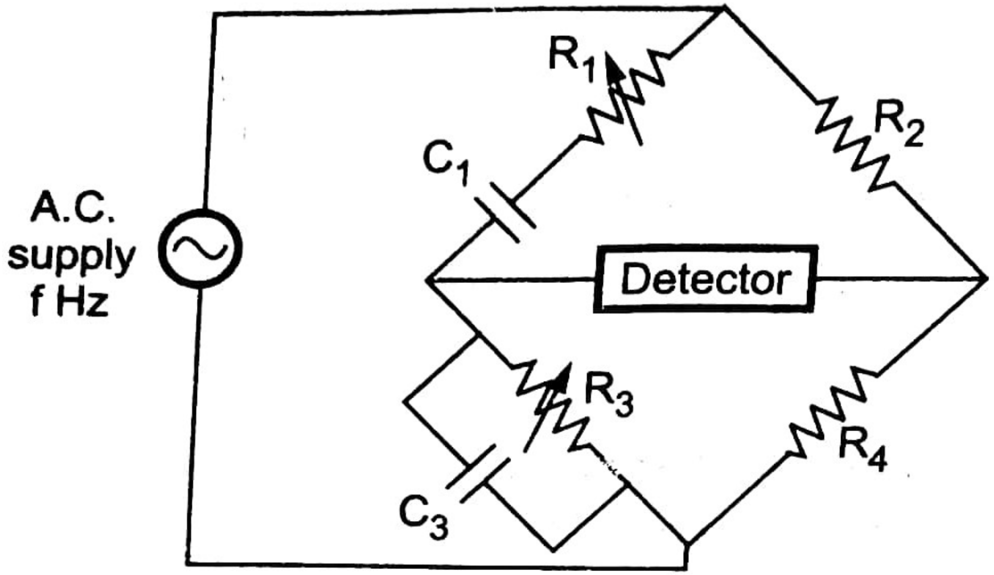


Fig.(a) Wien bridge

From the Fig.(a) can write

$$Z_1 = R_1 - j\left(\frac{1}{\omega C_1}\right)$$

$$Z_2 = R_2$$

$$Z_3 = R_3 \parallel C_3$$

$$Y_3 = \frac{1}{R_3} + j\omega C_3$$

and $Z_4 = R_4$

The balance condition is

$$\overline{Z_1 Z_4} = \overline{Z_2 Z_3}$$

$$\overline{Z_2} = \frac{\overline{Z_1 Z_4}}{\overline{Z_3}} = Z_1 \overline{Z_4 Y_3}$$

$$R_2 = \left[R_1 - j\left(\frac{1}{\omega C_1}\right)\right] R_4 \left[\frac{1}{R_3} + j\omega C_3\right]$$

$$R_2 = R_4 \left[\frac{R_1}{R_3} + j\omega R_1 C_3 - j\frac{1}{\omega C_1 R_3} + \frac{C_3}{C_1}\right]$$

$$R_2 = R_4 \left[\frac{R_1}{R_3} + \frac{C_3}{C_1}\right] + jR_4 \left[\omega R_1 C_3 - \frac{1}{\omega C_1 R_3}\right]$$

Equating real parts of both sides

$$R_2 = \frac{R_4 R_1}{R_3} + \frac{C_3 R_4}{C_1}$$

$$\boxed{\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1}} \quad \text{...(1)}$$

Equating imaginary parts of both sides,

$$\omega R_1 C_3 - \frac{1}{\omega C_1 R_3} = 0$$

$$\omega^2 = \frac{1}{R_1 R_3 C_1 C_3}$$

$$\omega = \frac{1}{\sqrt{R_1 C_1 R_3 C_3}} \quad \text{...(2)}$$

$$\boxed{f = \frac{1}{2\pi\sqrt{R_1 C_1 R_3 C_3}}} \quad \text{...(3)}$$

The equation (1) gives the resistance ratio while the equation (3) gives the frequency of applied voltage

Generally, in Wien bridge, the selection of the components is such that

$$R_1 = R_3 = R$$

and

$$C_1 = C_3 = C$$

$$\frac{R_2}{R_4} = 2 \quad \text{...(4)}$$

and

$$\boxed{f = \frac{1}{2\pi RC}} \quad \text{...(5)}$$

The equation (5) is the general equation for the frequency of the bridge circuit.

➢ Applications

- The bridge is used to measure the frequency in audio range. The audio range is [20-200-2k - 20 k] Hz. The resistances are used for the range changing while the capacitors are used for fine frequency control.
- The bridge can be used for capacitance measurement if the operating frequency is known.
- The bridge is also used in a harmonic distortion analyzer, as a notch filter and in audio frequency and radio frequency oscillators as a frequency determining element. The accuracy of 0.5 % -1% can be readily obtained using this bridge.

(1Q).Find the equivalent parallel resistance and capacitance that causes a Wien bridge to null with the following component values.

$$R_1 = 2.7\ \text{k}\Omega$$
$$C_1 = 5\ \mu\text{F}$$
$$R_2 = 22\ \text{k}\Omega$$
$$R_4 = 100\ \text{k}\Omega$$

The operating frequency is 2.2kHz

Solution: From the bridge balance equations

$$\omega^2 = \frac{1}{R_1 C_1 R_3 C_3}$$

$$C_3 = \frac{1}{\omega^2 R_1 C_1 R_3} \qquad \ldots(1)$$

Now $$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1}$$

Substituting value of C_3 *in the above equation*

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{1}{\omega^2 R_1 C_1^2 R_3}$$

$$\frac{22\times10^3}{100\times10^3} = \frac{2.7\times10^3}{R_3} + \frac{1}{[2\pi\times2.2\times10^3]^2\times(2.7\times10^3)\times(5\times10^{-6})^2\times R_3}$$

$$0.22 = \frac{2.7\times10^3}{R_3} + \frac{0.07753}{R_3}$$

$$0.22 = \frac{1}{R_3}(2700.0775)$$

$$R_3 = 12.273\ \text{k}\Omega$$

Substituting in equation (1)

$$C_3 = \frac{1}{(2\pi\times2.2\times10^3)^2\times(2.7\times10^3)\times(5\times10^{-6})\times(12.27\times10^3)}$$

$$= 31.59\ pF$$

WAGNER EARTHING DEVICE

Wagner Earthing Device

Definition:

- The Wagner earthing device is used for removing the earth capacitance from the bridges.

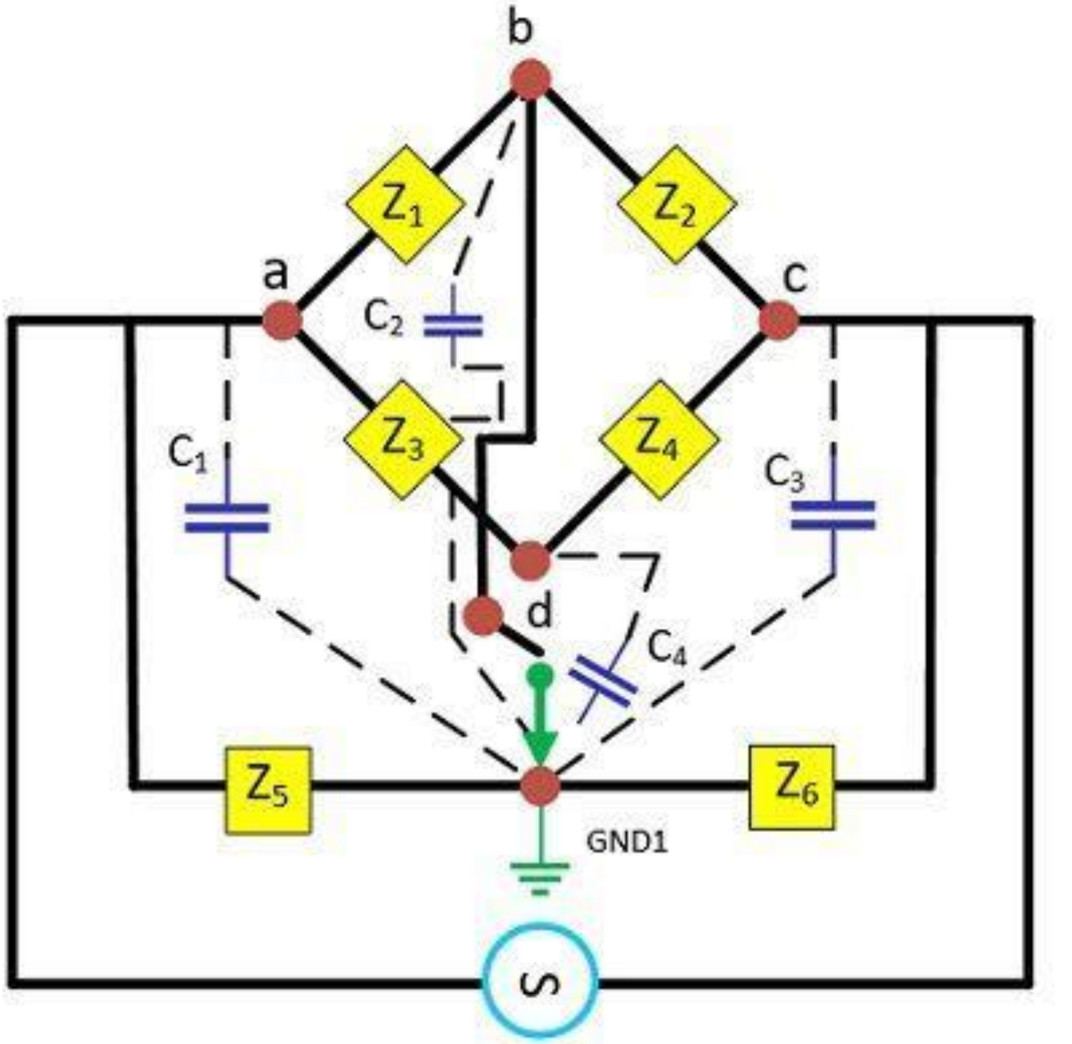
- It is a type of voltage divider circuit used to reduces the error which occurs because of stray capacitance. The Wagner Earth device provides high accuracy to the bridge.

- At high frequency, stray capacitance is induced between the bridge elements, ground and between the arms of the bridge. This stray element causes the error in the measurement.

- One of the way, of controlling these capacitances is too enclosed the bridge elements into the shield. Another way of eliminating these stray capacitance is to places the Wagner Earth device between the elements of the bridge.

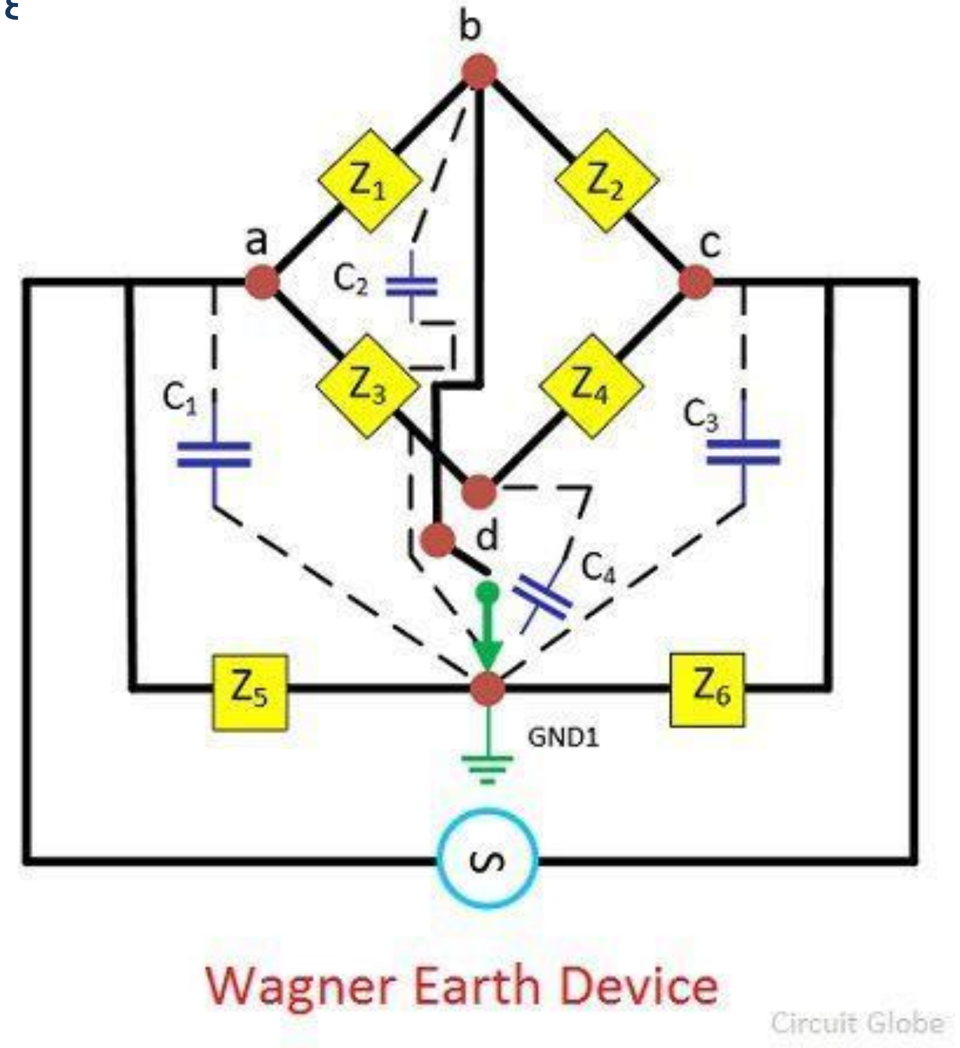
➢ Construction of Wagner Earthing Device

- The circuit diagram of the Wagner Earth Device is shown in the figure.
- Consider the Z_1, Z_2, Z_3, and Z_4 are the impedances arm of the bridge.
- The Z_5 and the Z_6 are the two variable impedances of the Wagner Earth Device.
- The centre point of the Wagner earth device is earthed. The impedance of Wagner device arms is similar to the arms of the bridge.
- The impedance of the arm consists the resistance and capacitances.
- The Wagner impedance placed in such a way so that they make the bridge balance with Z_1, Z_3 and Z_2, Z_4. The C_1, C_2 C_3 and C_4 show the stray capacitances of the bridges. The D is the detector of the bridge.



Wagner Earth Device

Construction of Wagner Earthing Device



Wagner Earth Device

- The bridge comes in the balance condition by adjusting the impedances of arms Z_1 and Z_4.
- The stray capacitance prevents bridge to comes in the balanced condition. When the S is not thrown on 'e' then the D is connected between the point p and q.
- But when S is thrown on 'e' then the detector D is connected between the terminal b and earth.
- The impedance Z_4 and Z_5 are adjusted until the minimum sound is obtained. The headphones again connected between the point b and d for obtaining the minimum sound.
- The headphones are reconnected between the point b and d, Z_4 and Z_5 are adjusted for obtaining the minimum sound. The process is continuously repeated for obtaining the silent sound.
- The point b, d, and e all are in the same potential. And the capacitance C_1, C_2, C_3, C_4 all are eliminated from the bridge circuit along with the impedances Z_5 and Z_6.

Thank you

ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

By

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ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

Syllabus

UNIT–I	Analog Ammeter and Voltmeters
UNIT–II	Analog Watt meters and Power Factor Meters
UNIT–III	Measurements of Electrical parameters
UNIT–IV	Transducers
UNIT–V	Digital meters

Text Books:

- Electrical & Electronic Measurement & Instruments by A.K.Sawhney Dhanpat Rai & Co.Publications
- Electrical and Electronic Measurements and instrumentation by R.K.Rajput, S.Chand
- Electrical Measurements and measuring Instruments by E.W. Golding and F.C.Widdis, fifth Edition, Wheeler Publishing
- Electrical Measurements by Buckingham and Price, Prentice – Hall

ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

UNIT-IV

TRANSDUCERS

UNIT–IV

TRANSDUCERS

Topics:

- Definition
- Classification
- Resistive, Inductive and Capacitive Transducer
- LVDT
- Strain Gauge
- Thermistors
- Thermocouples
- Piezo electric and Photo Diode Transducers,
- Digital shaft encoders, Hall effect sensors- Numerical Problems.

TRANSDUCERS

- Introduction

- The physical quantity under measurement makes its first contact in the time of measurement with a sensor. A sensor is a device that measures a physical quantity and converts it into a signal which can be read by an observer or by an instrument
- For example, a mercury thermometer converts the measured temperature into expansion and contraction of a liquid which can be read on a calibrated glass tube.
- A thermocouple converts temperature to an output voltage which can be read by a voltmeter. For accuracy, all sensors need to be calibrated against known standards.
- In everyday life, sensors are used everywhere such as touch sensitive mobile phones, laptop's touch pad, touch controller light, etc. People use so many applications of sensors in their everyday lifestyle that even they are not aware about it.
- Examples of such applications are in the field of medicine, machines, cars, aerospace, robotics and manufacturing plants.


Types of Transducer
INFO
MENU
ASHCROFT
Model A – 10
Transducer
setra

Transducers

Definition:

- The device which converts one form of energy into another form of energy is known as **transducer**. The process of conversion is known as **transduction.** The conversion is done by sensing and transducing the physical quantities like temperature, pressure(of water,gas,liquid etc.), sound, etc.
- The electrical transducer converts the mechanical (eg.pressure,displacement) or non-mechanical (eg.heat,light) energy(signal) into an electric signal. The electrical signal may be voltage, current and frequency. The production of the signal depends on the resistive, inductive and capacitive effects of the physical input.

Need of Transducer :

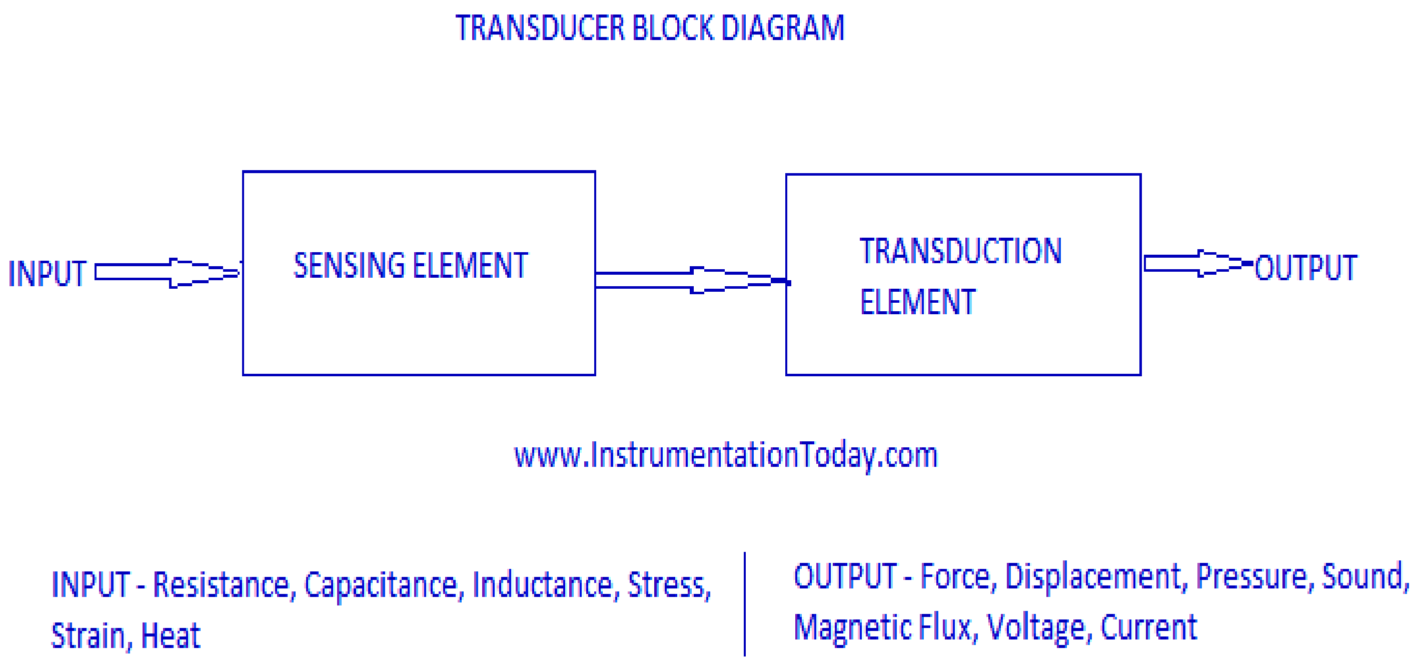
- The quantities which cannot be measured directly, such as temperature, pressure, displacement, humidity, fluid flow etc. But if these quantities are converted into an electrical signal, then their value is easily measured with the help of the meter.
- The transducers convert the physical forces into an electrical signal which can easily be handled and transmitted for measurement.

Transducers

The following are the advantages of converting the physical quantity into an electrical signal:

1) The attenuation and amplification of the electrical signals are very easy.
2) The electrical signal produces less friction error.
3) The small power is required for controlling the electrical systems.
4) The electrical signals are easily transmitted and processed for measurement.
5) The component used for measuring the electrical signal is very compact and accurate.
6) The electrical signals are used in telemetry.

The block diagram of a transducer is given below.



1. Sensing or Detector Element

- It is the part of the transducers which give the response to the physical sensation.
- The response of the sensing element depends on the physical phenomenon.

2. Transduction Element

- The transduction element converts the output of the sensing element into an electrical signal.
- This element is also called the secondary transducer.

Factors Influencing the Choice of Transducer :

- The choice of the transducers used for measuring the physical quantity depends on the following factors.

1) Operating Principle – The transducers are selected by their operating principles. The operating principle may be resistive, inductive, capacitive, optoelectronic, piezoelectric, etc.
2) Sensitivity – The sensitivity of the transducer is high enough for inducing the detectable output.
3) Operating Range – The transducer must have wide operating ranges so that it does not break during working.
4) Accuracy – The transducers gives accuracy after calibration. It has a small value for repeatability which is essentials for the industrial applications.
5) Cross Sensitivity – The transducer gives variable measured value for the different planes because of the sensitivity. Hence, for the accurate measurement, the cross sensitivity is essential.
6) Errors – The errors are avoided by taking the input output relations which is obtained by the transfer function.

7) Loading Effect – The transducers have high input impedance and low output impedance for avoiding the errors.

8) Environmental Compatibility – The transducers should be able to work in any specified environments like in a corrosive environment. It should be able to work under high pressure and shocks.

9) Insensitivity to Unwanted Signals – The transducer should be sensitive enough for ignoring the unwanted signals and highly sensitive to desired signals.

10)Usage and Ruggedness – The durability, size and weight of the transducer must be known before selecting it.

11) Stability and Reliability – The stability of the transducers should be high enough for the operation and their reliability should be good in case of failure of the transducer.

12)Static characteristic – The transducer should have a high linearity and resolution, but it has low hysteresis. The transducer is always free from the load and temperature effects.

ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

Applications of TRANSDUCERS

Applications of the Transducer:

1) It is used for detecting the movement of muscles which is called acceleromyograph.
2) The transducer measures the load on the engines.
3) It is used as a sensor for knowing the engine knock.
4) The transducers measure the pressure of the gas and liquid by converting it into an electrical signal.
5) It converts the temperature of the devices into an electrical signal or mechanical work.
6) The transducer is used in the ultrasound machine. It receives the sound waves of the patient by emitting their sound waves and pass the signal to the CPU.
7) The transducer is used in the speaker for converting the electrical signal into acoustic sound.
8) It is used in the antenna for converting the electromagnetic waves into an electrical signal.
9) The classification of the transducers depend on the various factors like by transduction, the converting electrical signal from AC or DC, etc.

❑ Individual Applications of the Transducer:

- There are a variety of transducer types like pressure transducer, piezoelectric transducer, ultrasonic transducer, temperature transducer, and so on. Let us discuss the use of different types of transducers in practical applications.

Applications of Ultrasonic Transducer

- This transducer can be used to measure the distance of the sound based on reflection such that the distance is measured and displayed on an LCD display. Here, the display is interfaced with a microcontroller. The ultrasonic transducer produces 40kHz frequency waves.

Fig.Ultrasonic Transducer

Applications of Temperature Transducer

- A temperature transducer is used to measure the temperature of the air such that to control the temperature of several control systems like air-conditioning, heating, ventilation, and so on.
- Based on the temperature measured a pulse width modulation o/p will be generated by the program of an Arduino board. The output of this is used to control the DC fan through the motor driver IC.

Applications of Piezoelectric Transducer

1) This transducer is mainly used to detect the stick drummer's impact on electronic drum pads. And also used to detect the movement of the muscle, which can be named acceleromyography.

2) The load of the engine can be determined by calculating diverse absolute pressure, which can be done by using these transducers as the MAP sensor in fuel injection systems.

3) This sensor can be used as a knock sensor in automotive engine management systems for noticing the knock of the engine.

Fig.Piezoelectric Transducer

Applications of Pressure Transducer

1) The applications of pressure transducer mainly involve altitude sensing, pressure sensing, level or depth sensing, flow sensing, and leak testing.

2) These transducers can be used for generating electrical power under the speed breakers on the highways or roads where the force of the vehicles can be converted into electrical energy.

❑ Transducer Characteristics:

➢The characteristics of a transducer are given below that are determined by examining the output response of a transducer to a variety of input signals. Test conditions create definite operating conditions as closely as possible. The methods of computational and standard statistical can be applied to the test data.

1. Accuracy: It is defined as the closeness with which the reading approaches an accepted standard value or ideal value or true value, of the variable being measured.
2. Ruggedness: The transducer should be mechanically rugged to withstand overloads. It should have overload protection.
3. Linearity: The output of the transducer should be linearly proportional to the input quantity under measurement. It should have linear input - output characteristic.
4. Repeatability: The output of the transducer must be exactly the same, under same environmental conditions, when the same quantity is applied at the input repeatedly.
5. High output: The transducer should give reasonably high output signal so that it can be easily processed and measured. The output must be much larger than noise.

Now-a-days, digital output is preferred in many applications.

6. High Stability and Reliability: The output of the transducer should be highly stable and reliable so that there will be minimum error in measurement. The output must remain unaffected by environmental conditions such as change in temperature, pressure, etc.

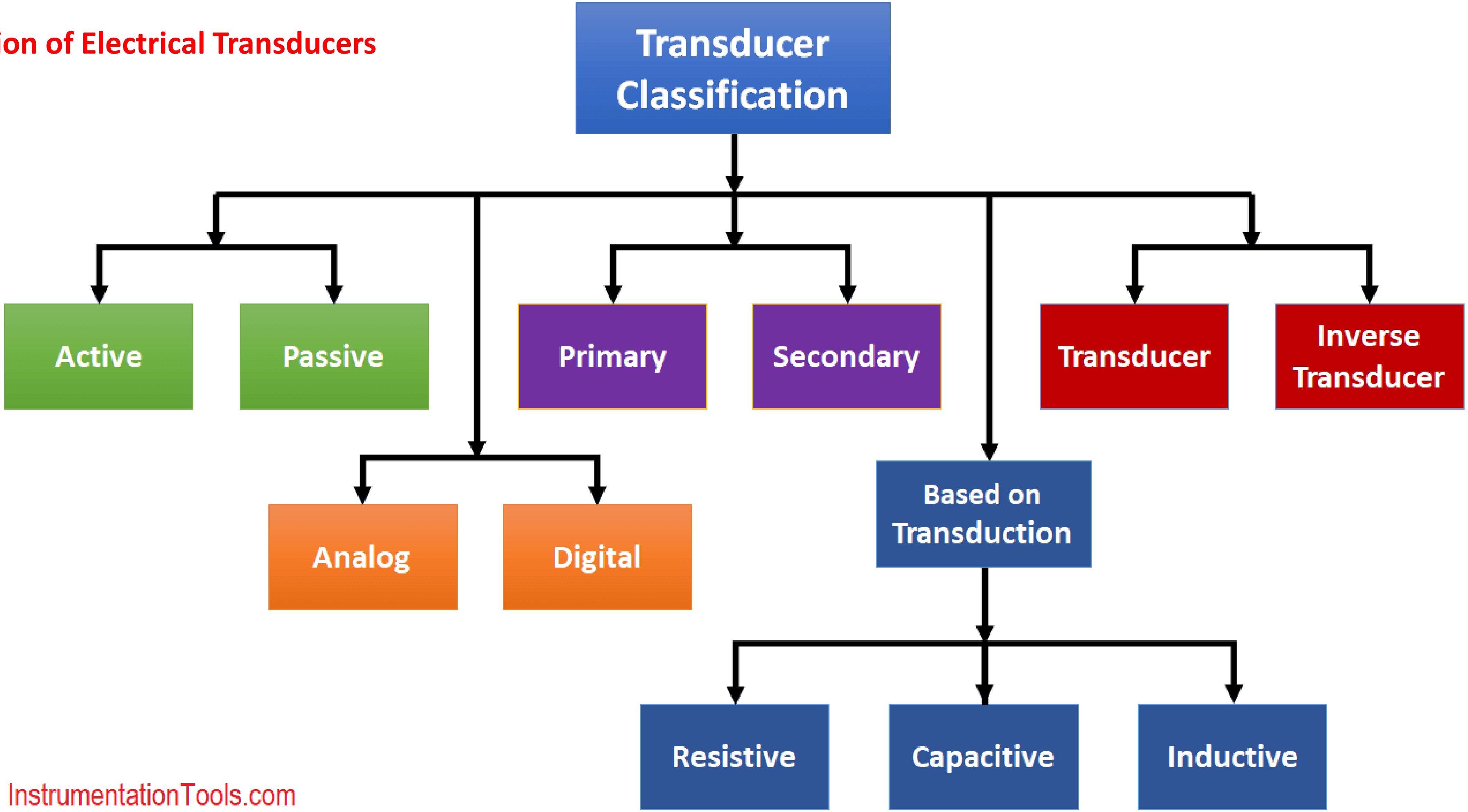
7. Sensitivity: The sensitivity of the electrical transducer is defined as the electrical output obtained per unit change in the physical parameter of the input quantity. For example, for a transducer used for temperature measurement, sensitivity will be expressed in $mV/^0$ C. A high sensitivity is always desirable for a given transducer.

8. Dynamic Range: For a transducer, the operating range should be wide, so that it can be used over a wide range of measurement conditions.

9. Size: The transducer should have smallest possible size and shape with minimal weight and volume. This will make the measurement system very compact.

10. Speed of Response: It is the rapidity with which the transducer responds to changes in the measured quantity. The speed of response of the transducer should be as high as possible.

➢ Classification of Electrical Transducers



Classification of Electrical Transducers

➢In general, electrical transducers are classified according to their structures, application area, method of energy conversion, output signal nature etc., Thus the electrical transducers are classified as:

1) Active & Passive Transducers
2) On the basis of transduction principle used
3) Analog & Digital transducers
4) Primary & Secondary transducers
5) Transducer & Inverse transducer

Active & Passive Transducers:

➢ Active Transducers

- These are self generating type of transducers. These transducers develop an electrical parameter(voltage or current) which is proportional to the quantity under measurement.
- These transducers do not require any external source or power for their operation. They can be subdivided as shown in Fig(a).

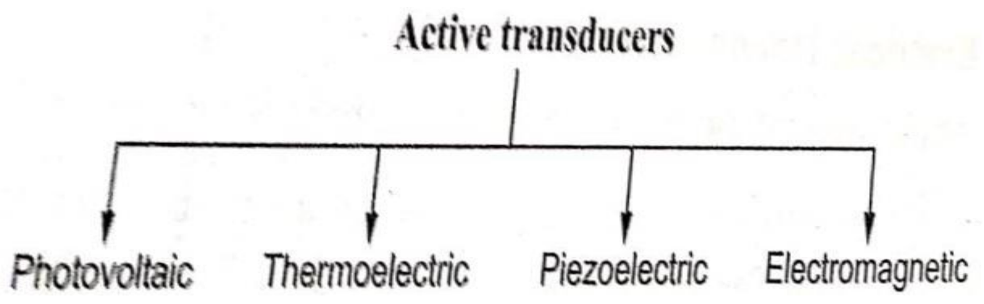


Fig.(a)

Passive Transducers:

- Passive transducers do not generate any electrical signal by themselves. To obtain an electrical signal from such transducers, an external source of power is essential.
- Passive transducers depend upon the change in an electrical parameter(R,L orC). They are also known as externally power driven transducers.
- They can be subdivided as shown in Fig(b).

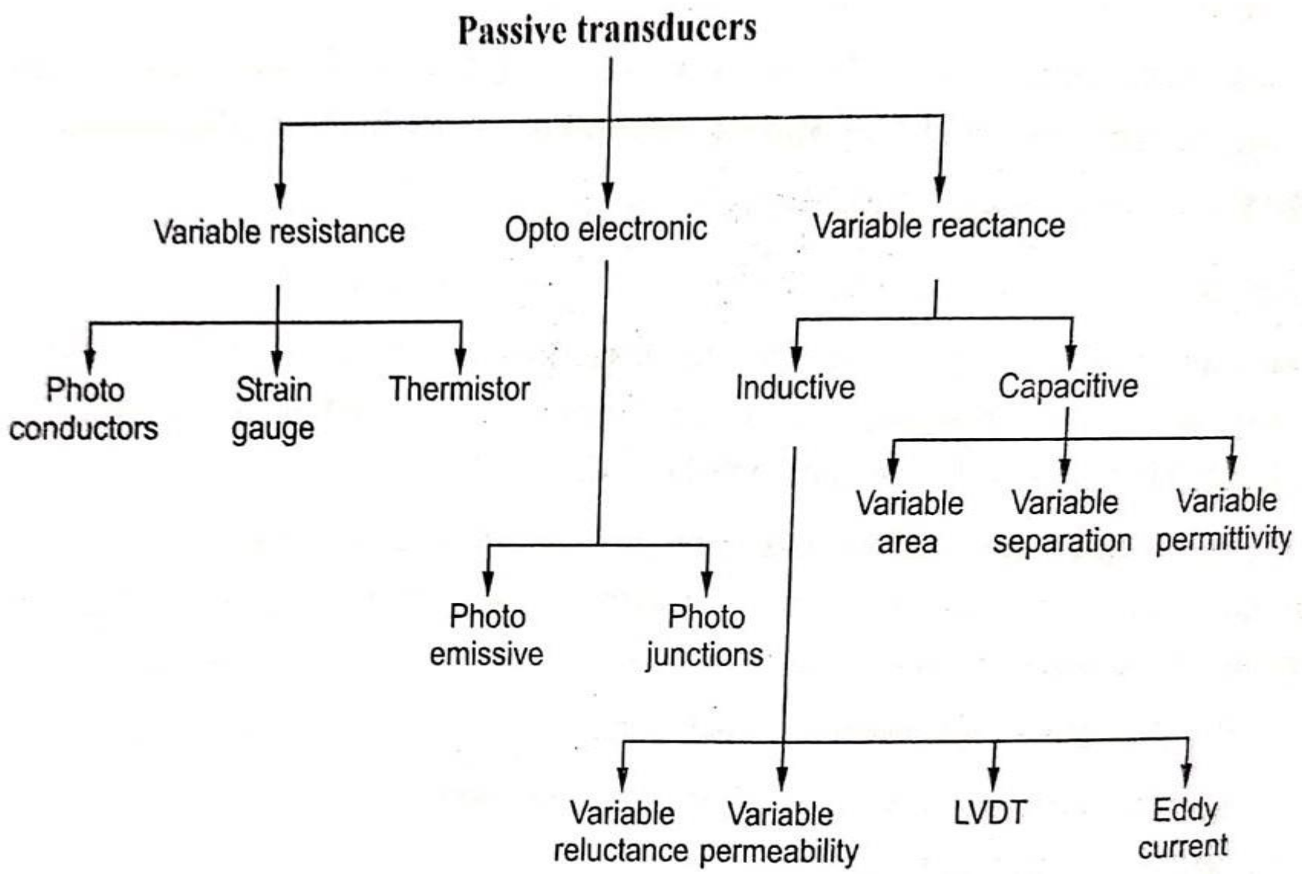


Fig.(b)

According to transduction principle:

- The transducers can be classified according to principle used in transduction. Let us discuss few of them.

Capacitive Transduction:

- In this measurand is converted into a change in capacitence. A capacitor basically consists of two conductors separated by an insulator(dielectric).
- A change in the capacitor occurs either by changing the distance between two plates or by a change in the dielectric shown in Fig(c).

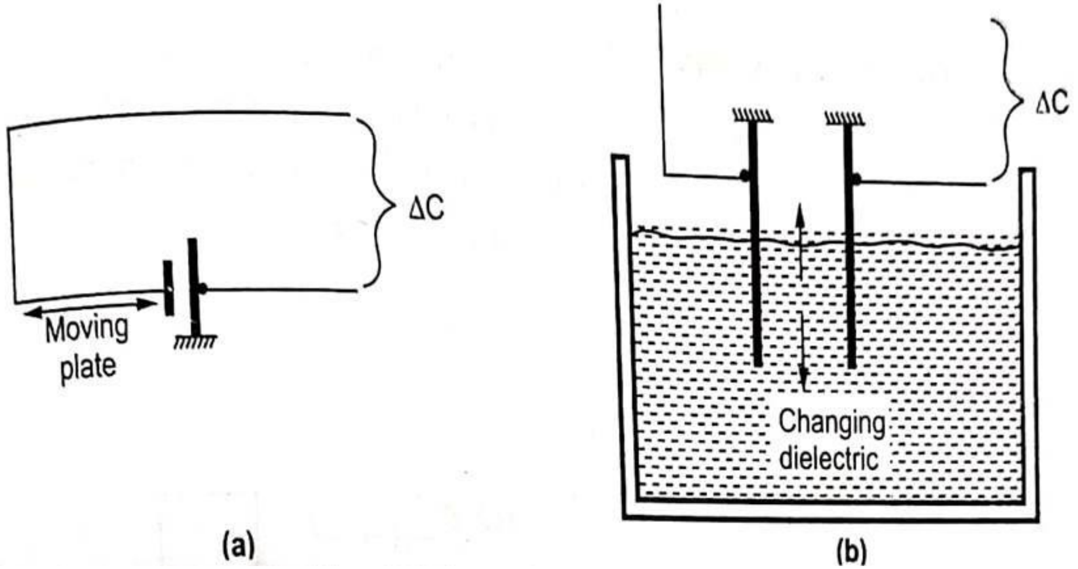
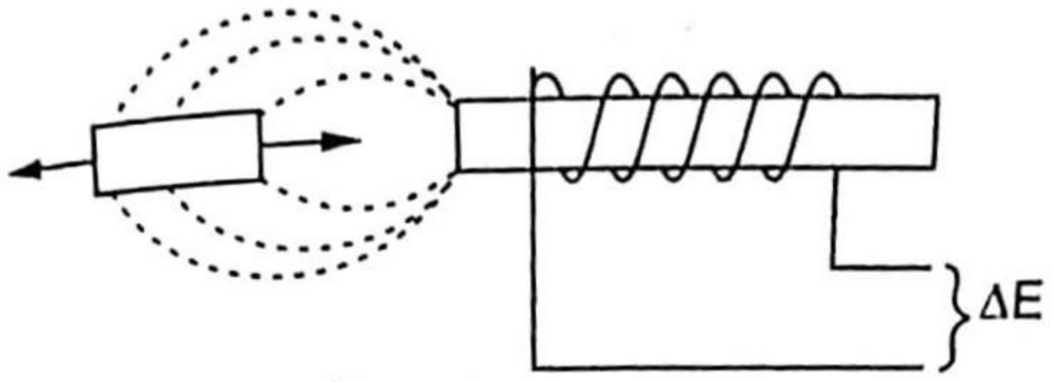


Fig.(c)Capacitive Transduction

Electromagnetic transduction:

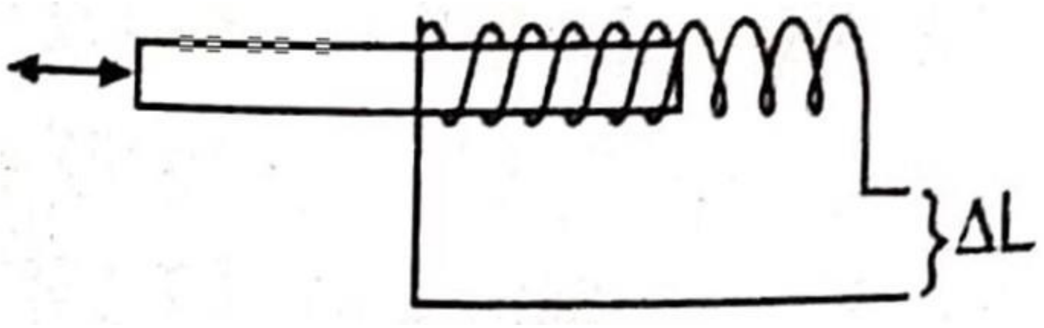
- In this measurand is converted into an emf(voltage) induced in a conductor by change in the magnetic flux, in the absence of excitation.
- These types of transducers are self generating active type transducers. The relative motion between a magnet or a piece of magnetic material and an electromagnet brings out the change in the magnetic flux shown in Fig(d)



Fig(d)Electromagnetic transduction

Inductive Transduction:

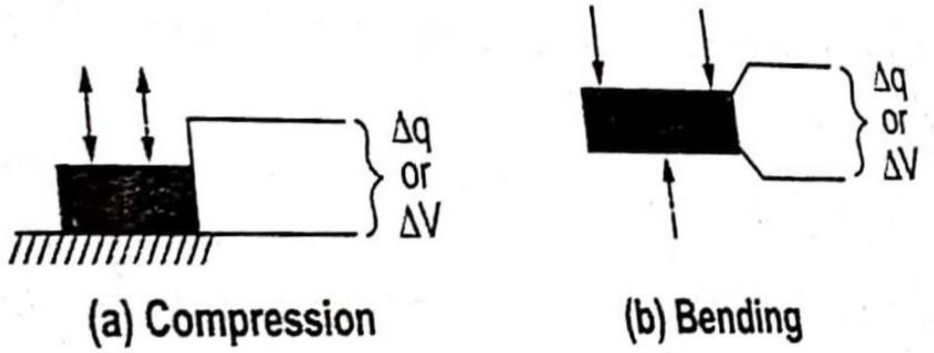
- In this the measurand is converted into a change in the self inductance of a single coil.
- This is accomplished by displacing the coil's core which is linked or attached to a mechanical sensing element as shown in fig (e).



Fig(e).Inductive transduction

Piezoelectric Transduction:

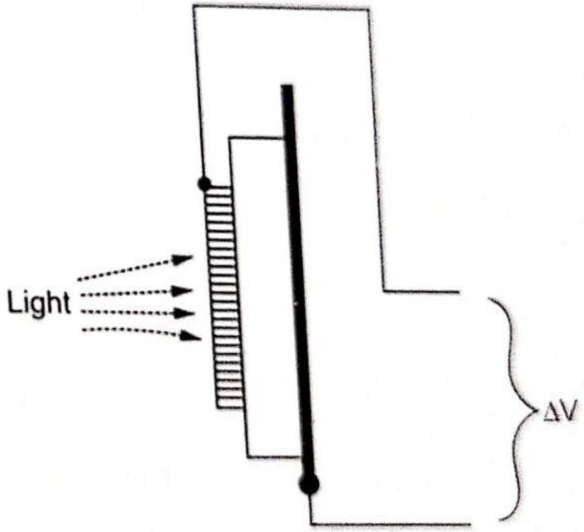
- In this, measurand is converted into a change in electrostatic charge, q or voltage, V generated by crystals when mechanically stressed as shown in fig(f).



Fig(f). Piezoelectric transduction

Photovoltaic Transduction:

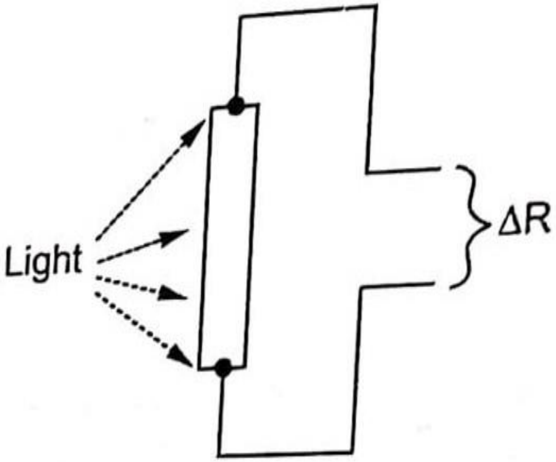
- photovoltaic current is generated as a result of the absorption of photons of a voltage difference across a p-n junction (generation of voltage).
- In this the measurand is converted in the voltage generated when a junction between dissimilar materials is illuminated as shown in fig (g).



Fig(g). Photovoltaic transduction

Photoconductive Transduction:

- photoconductive detectors use the increase in electrical conductivity resulting from increases in the number of free carriers generated when photons are absorbed (generation of current)
- In this, measurand is converted into a change in resistance(conductance) of a semiconductor material by a change in the amount of illumination incident on the material shown in fig (h).



Fig(h). Photoconductive transductio

Analog and Digital Transducers:

- These transducers can be classified on the basis of the output which may be a continuous function of time or may be in discrete steps.

Analog Transducers:

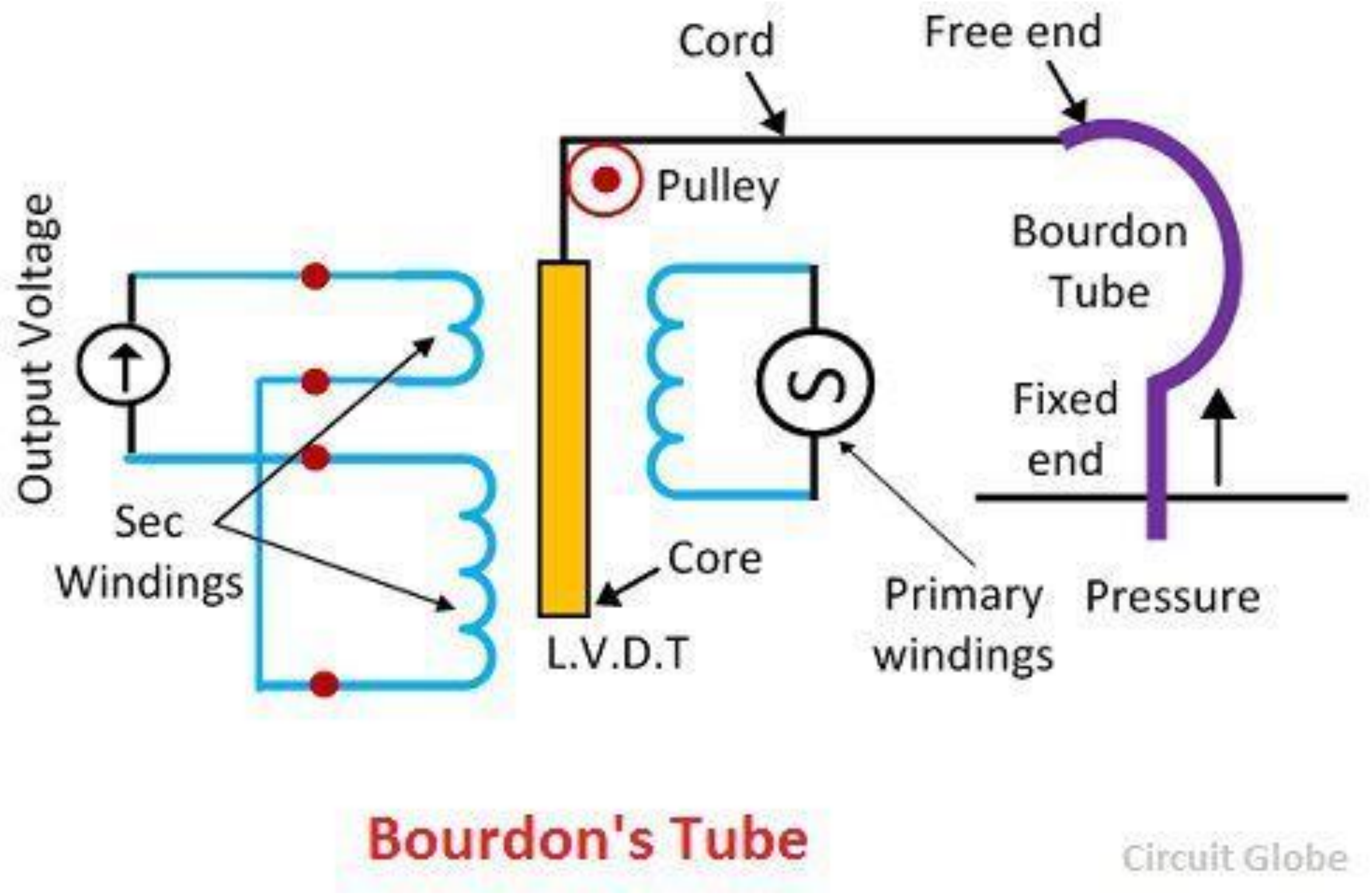
- These transducers convert the input quantity into an analog output which is a continuous function of time. Strain guage, LVDT, thermocouples or thermistors are called analog transducers as they produce an output which is a continuous function of time.

Digital Transducers:

- Digital transducers produce an electrical output in the form of pulses which forms an unique code. Unique code is generated for each discrete value sensed.

Primary or Secondary Transducers:

- Some transducers consist of mechanical device along with the electrical device.
- In such transducers mechanical device acts as a primary transducer and converts physical quantity into mechanical signal.
- The electrical device then converts mechanical signal produced by primary transducer into an electrical signal. Therefore, electrical device acts as a secondary transducer.
- For example, in pressure measurement Bourdon's tube acts as a primary transducer which converts a pressure into displacement and LVDT acts as a secondary transducer which converts this displacement into an equivalent electrical signal.



Fig(f).Pressure measurement
(e.g. of Primary or Secondary Transducers)

Transducer and Inverse Transducer:

- Transducers convert non-electrical quantity into electrical quantity whereas Inverse Transducer converts electrical quantity into non-electrical quantity. For example, microphone is a transducer which converts sound signal into an electrical signal whereas loudspeaker is an inverse transducer which converts electrical signal into sound signal.

➢ The classification of transducer-based on physical quantity includes the following.

1) Flow transducer like flow Meter
2) Acceleration transducer like accelerometer
3) Temperature transducer like thermocouple
4) Level transducer like torque Tube
5) Pressure transducer like Bourdon Gauge
6) Displacement transducer like Linear Variable Differential Transformer (LVDT)
7) Force Transducer like dynamometer

Resistive Transducers

Resistive Transducer

- **Definition:** The transducer whose resistance varies because of the environmental effects such type of transducer is known as the resistive transducer. The change in resistance is measured by the ac or dc measuring devices.

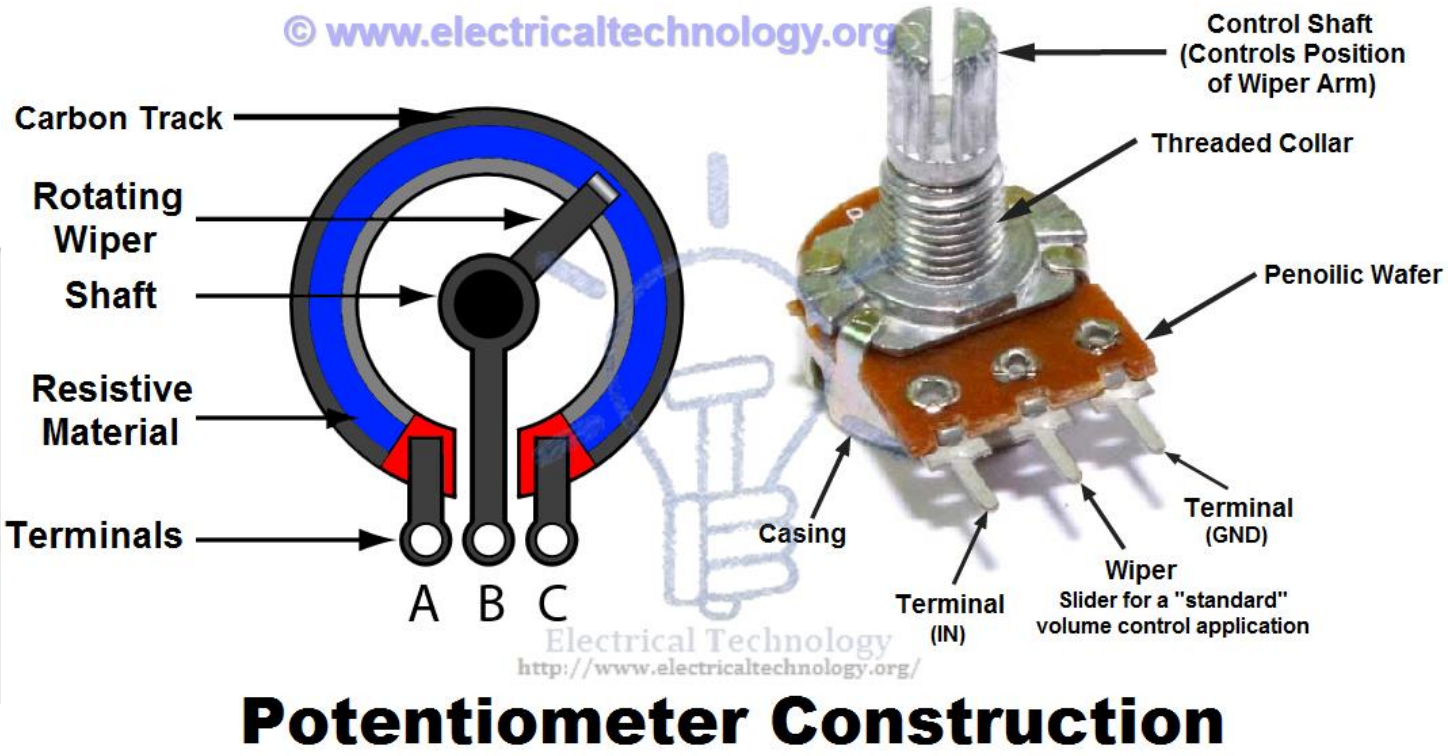
- The resistive transducer is used for measuring the physical quantities like temperature, displacement, vibration etc. The measurement of the physical quantity is quite difficult.

- The resistive transducer converts the physical quantities into variable resistance which is easily measured by the meters. The process of variation in resistance is widely used in the industrial applications.

- The resistive transducer can work both as the primary as well as the secondary transducer. The primary transducer changes the physical quantities into a mechanical signal, and secondary transducer directly transforms it into an electrical signal. The displacement of the slider is converted into an electrical signal.

Resistive Transducers

Fig. Linear (Translational) Motion Resistive Transducer With Circular Body



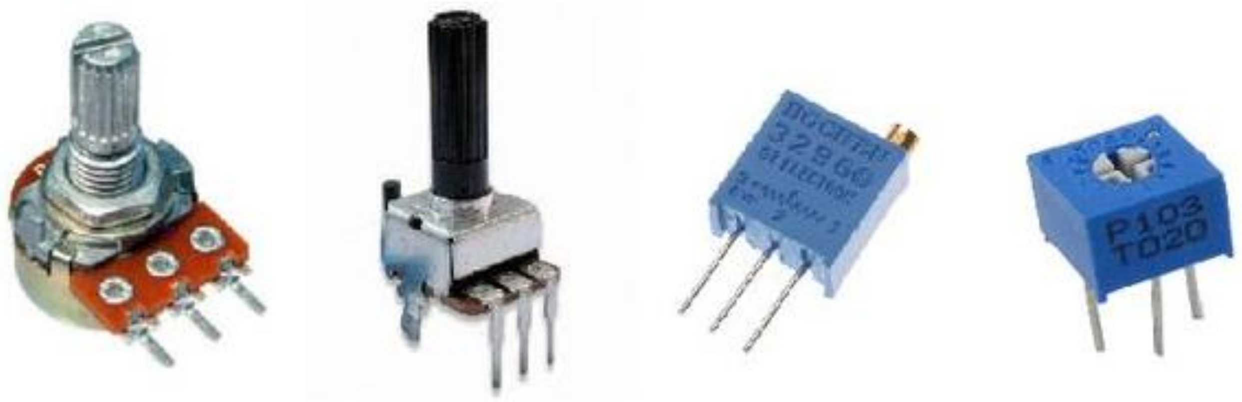


Fig. Rotary Motion Resistive Transducers

Working Principle of Resistive Transducer :

- The resistive transducer element works on the principle that the resistance of the element is directly proportional to the length of the conductor and inversely proportional to the area of the conductor.

$$R = \rho L / A$$

Where R – resistance in ohms.
A – cross-section area of the conductor in meter square.
L – Length of the conductor in meter.
ρ – the resistivity of the conductor in materials in ohm meter.

- The resistive transducer is designed by considering the variation of the length, area and resistivity of the metal. Basically a resistance potentiometer, or simply a POT, (a resistive potentiometer used for the purposes of voltage division is called a POT) consists of a resistive element provided with a sliding contact.
- This sliding contact is called a wiper. The motion of the sliding contact may be translatory or rotational. A linear pot and a rotary pot are shown in below Figure. Some POTS use the combination of the two motions, i.e.; translational as well as rotational. These POTS have their resistive element in the form of a helix and therefore, they are called helipots.

Translational motion

Let us confine, our discussion of dc excited potentiometers.
Consider a translational potentiometer as shown in Fig(a).
Let,

$\mathbf{e_i}$ & $\mathbf{e_o}$ = input and output voltages respectively; V,

$\mathbf{x_t}$ = total length of translational pot; m,

$\mathbf{x}_i$ = displacement of wiper from its zero position; m,

R_p = total resistance of the potentiometer; $\boldsymbol{\Omega}$

If the distribution of the resistance with respect to translational movement is linear, the resistance per unit length is $\boldsymbol{R_p/x_t}$

The output voltage under ideal conditions is :

$$e_0 = \left(\frac{\text{resistance at the output terminals}}{\text{resistance at the input terminals}}\right) \times \text{input voltage}$$

$$= \left[\frac{R_p(x_i/x_t)}{R_p}\right] e_i = \frac{x_i}{x_t} \times e_i$$

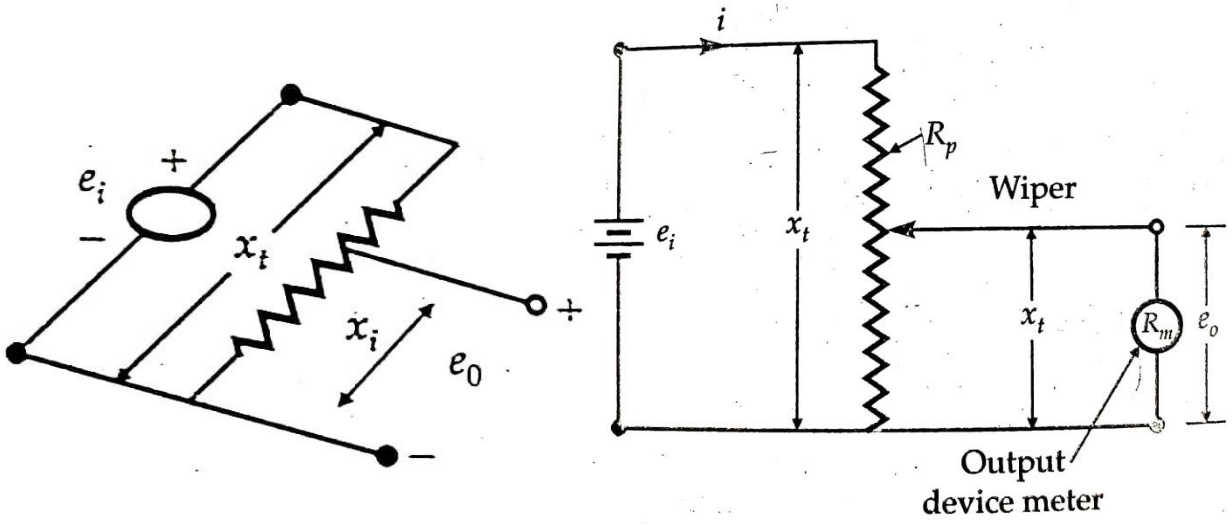


Fig.(a) translational potentiometer

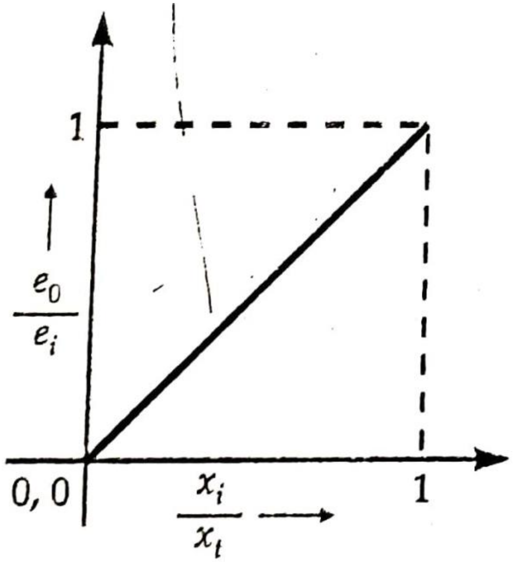


Fig.(b) Characteristics of potentiometers

- The under ideal circumstances, the output voltage varies linearly with displacement as shown in Fig(b).
- The translational resistive elements are straight devices and have a stroke of 2 mm to 0.5 m.

Sensitivity $S = \frac{output}{input}$

$$= \frac{\mathbf{e_o}}{x_i} = \frac{e_i}{x_t}$$

Thus under ideal conditions the sensitivity is constant and the output is faithfully reproduced and has a linear relationship with input. The same is true of rotational motion

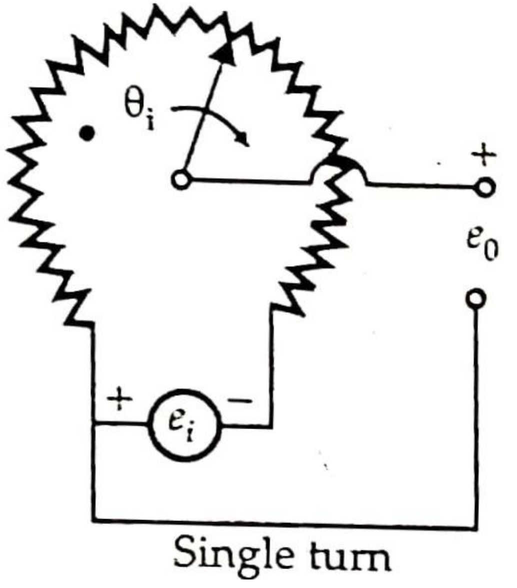


Fig.(c) Rotational motion

- Rotational motion

Let θ_i = input angular displacement in degrees,
and θ_t =total travel of the wiper in degrees as shown in Fig(c)

Output voltage $e_o = e_i\left(\frac{\theta_i}{\theta_t}\right)$

This is true of single turn potentiometers only.

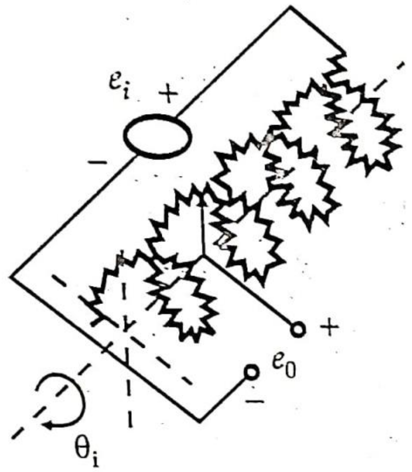


Fig.(d) Helipot

- The rotational devices are circular in shape and are used for measurement of angular displacement.
- They may have a full scale angular displacement from 10^0 to 60^0 full single turns.
- Multi turn potentiometers may measure up to 360^0 of rotation through use of helipots as shown in Fig(d).

Advantages of Resistive Transducer :

The following are the advantages of the resistive transducer

1) Both the AC and DC, current or voltage is appropriate for the measurement of variable resistance.
2) The resistive transducer gives the fast response.
3) It is available in various sizes and having a high range of resistance.

Disadvantages:

1) In linear potentiometers, large force is required to move wiper.
2) Suffer from mechanical wear and misalignment of wiper.
3) Limited resolution and high electronic noise in output.
4) The output of potentiometer is insensitive to the variations in displacement of wiper between two consecutive turns of wire.

Applications of Resistive Transducer:

The following are the applications of the resistive transducer.

1) Potentiometer – The translation and rotary potentiometer are the examples of the resistive transducers. The resistance of their conductor varies with the variation in their lengths which is used for the measurement of displacement.
2) Strain gauges – The resistance of their semiconductor material changes when the strain occurs on it. This property of metals is used for the measurement of the pressure, force-displacement etc.
3) Resistance Thermometer – The resistance of the metals changes because of changes in temperature. This property of conductor is used for measuring the temperature.
4) Thermistor – It works on the principle that the temperature coefficient of the thermistor material varies with the temperature. The thermistor has the Negative Temperature Coefficient. The Negative temperature coefficient means the Temperature is inversely proportional to Resistance. There are a number of ways because of which the resistance of the metal changes with the changed in the physical phenomenon. And this property of conductors is used for measuring the physical quantities of material.

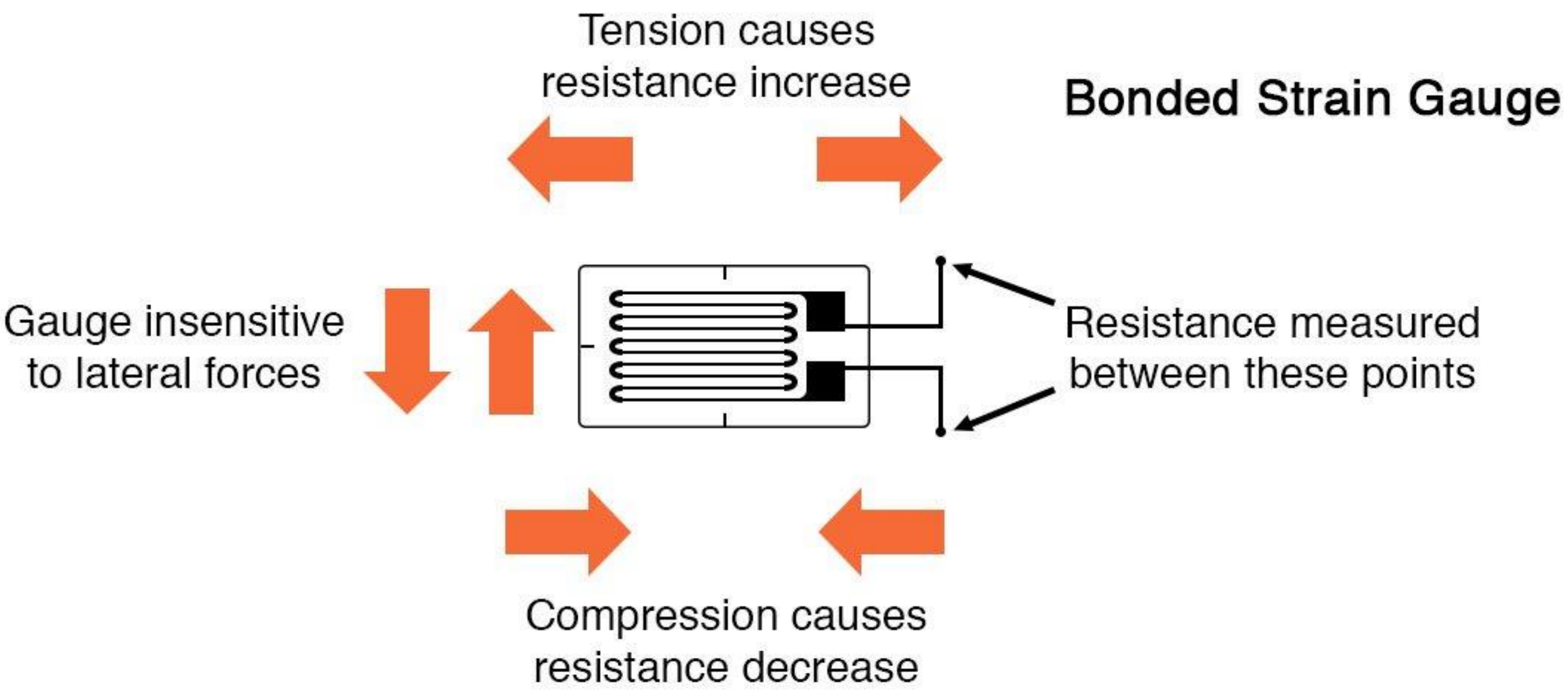
STRAIN GAUGE

STRAIN GAUGE

❑ What is Strain Gauge?

- The strain gauge is a passive, resistive transducer which converts the mechanical elongation and compression into a resistance change (used for the measurement of strain and stress, displacement, force, and pressure).

- This change in resistance takes places due to variation in length and cross sectional area of the gauge wire, when an external force acts on it.



TYPES

Based on principle of working :

- Mechanical
- Electrical
- Piezoelectric

Based on construction :

- Foil strain gauge
- Semiconductor strain gauge
- Photoelectric Strain gauge

Based on mounting :

- Bonded strain gauge
- Unbonded strain gauge

ELECTRICAL STRAIN GAUGE

- When an electrical wire is stretched within the limits of its elasticity such that it does not break or permanently deform, it will become narrower and longer, changes that increase its electrical resistance end-to-end.
- Strain can be inferred by measuring change in resistance.

MECHANICAL STRAIN GAUGE

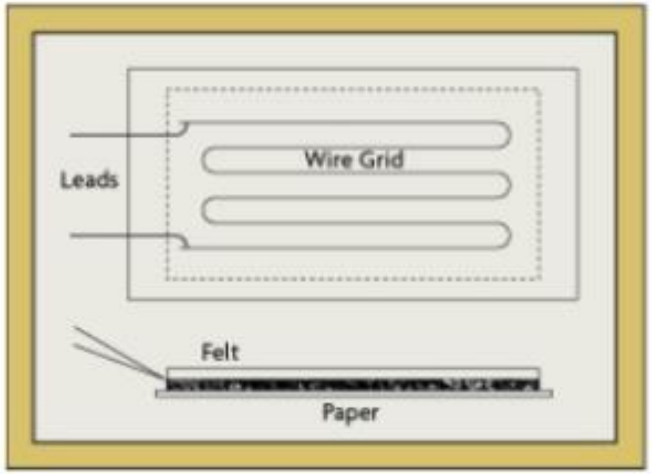
- It is made up of two separate plastic layers. The bottom layer has a ruled scale on it and the top layer has a red arrow or pointer. One layer is glued to one side of the crack and one layer to the other. As the crack opens, the layers slide very slowly past one another and the pointer moves over the scale. The red crosshairs move on the scale as the crack widens.

PIEZOELECTRIC STRAIN GAUGE

- Piezoelectric generate electric voltage when strain is applied over it. Strain can be calculated from voltage. Piezoelectric strain gauges are the most sensitive and reliable devices.

BONDED STRAIN GAUGE

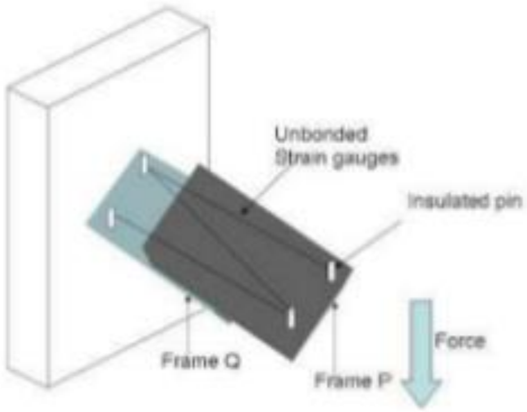
- A bonded strain-gage element, consisting of a metallic wire, etched foil, vacuum-deposited film, or semiconductor bar, is cemented to the strained surface.



UNBONDED STRAIN GAUGE

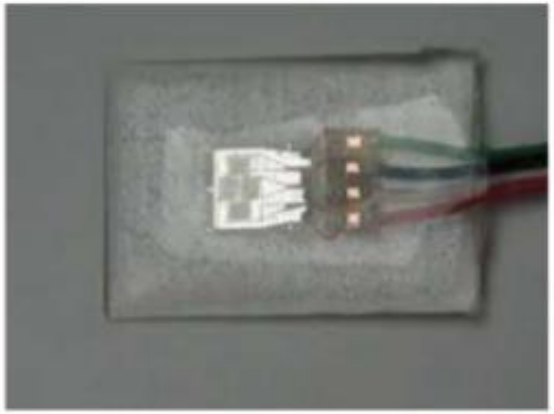
- The unbonded strain gage consists of a wire stretched between two points in an insulating medium such as air. One end of the wire is fixed and the other end is attached to a movable element.

Unbonded Strain Gauge



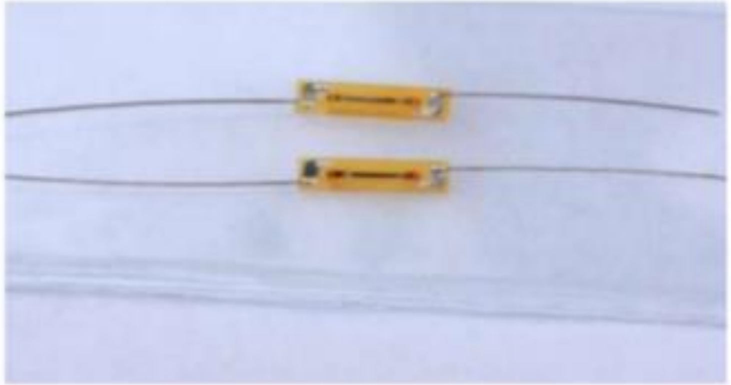
FOIL STRAIN GAUGE

- The foil strain gage has metal foil photo-etched in a grid pattern on the electric insulator of the thin resin and gage leads attached,



SEMICONDUCTOR STRAIN GAUGE

- For measurements of small strain, semiconductor strain gauges, so called piezoresistors, are often preferred over foil gauges. Semiconductor strain gauges depend on the piezoresistive effects of silicon or germanium and measure the change in resistance with stress as opposed to strain.

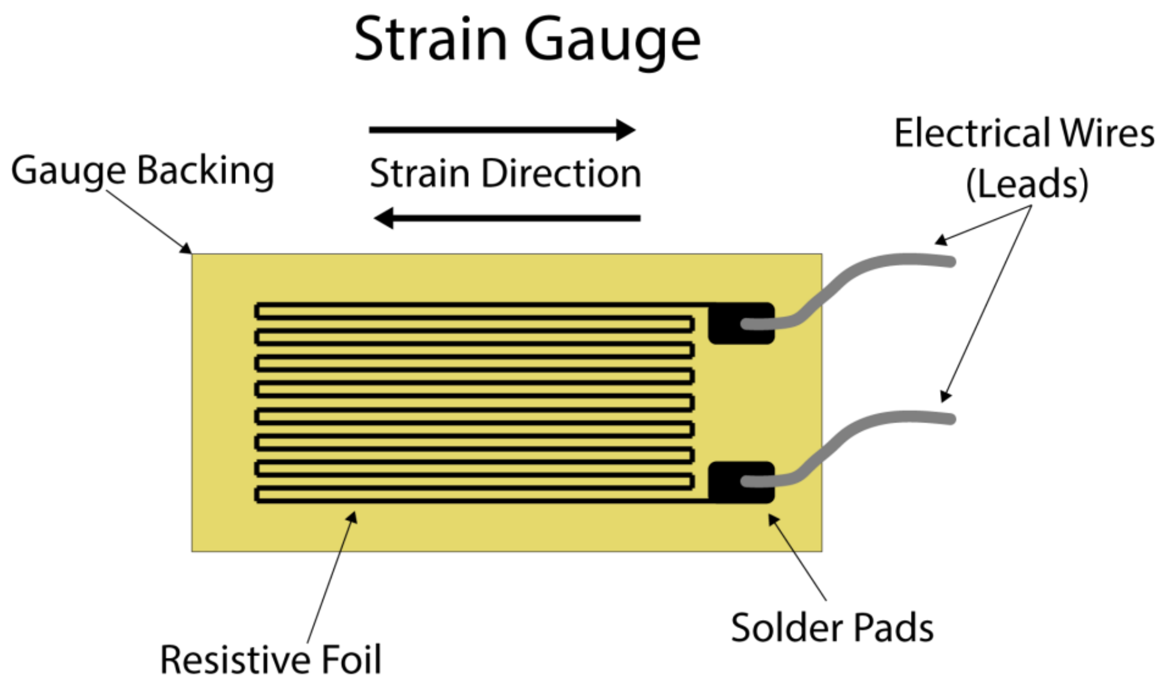


PHOTOELECTRIC STRAIN GAUGE

- The photoelectric gauge uses a light beam, two fine gratings, and a photocell detector to generate an electrical current that is proportional to strain. The gage length of these devices can be as short as 1/16 inch, but they are costly and delicate.

STRAIN GAUGE

- The gauge is attached to an object by using an adhesive under stress.
- A strain gauge is a thin,wafer-like device that can be attached to a variety of materials to measure applied strain.
- The majority of strain gauge are foil types,avalable in a wide choice of shapes and sizes to suit a variety of applications.They consist of a pattern of resistive foil which is mounted on a backing material.
- They operate on the principle that as the foil is subjected to stress, the resistance of the foil changes in a defined way. It operates on the "Piezoresistive Effect" principle.



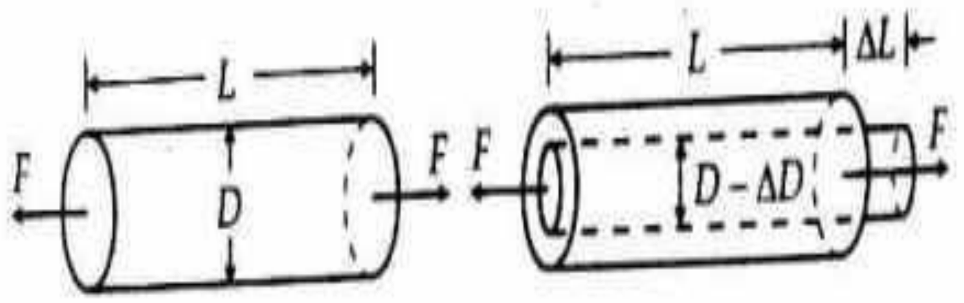
Strain Gauge:

- A strain gauge is used to measure the strain on object. When an external force is applied on an object, due to which there is a deformation occurs in the shape of the object.
- This deformation in the shape is both compressive or tensile is called strain, and it is measured by the strain gauge.
- When an object deforms within the limit of elasticity, either it becomes narrower and longer or it become shorter and broadens. As a result of it, there is a change in resistance end-to-end.
- The strain gauge is sensitive to that small changes occur in the geometry of an object.
- By measuring the change in resistance of an object, the amount of induced stress can be calculated.
- If a metal conductor is stretched or compressed, its resistance changes on account of the fact that both length and diameter of conductor change.
- Also there is a change in the value of resistivity of the conductor when it is strained and this property is called piezo resistive effect.
- Resistance strain gauges are also known as piezo resistive gauges.

Theory of Strain Gauge:

- The change in the value of resistance by straining the gauge may be partly explained by the normal dimensional behavior of elastic material.

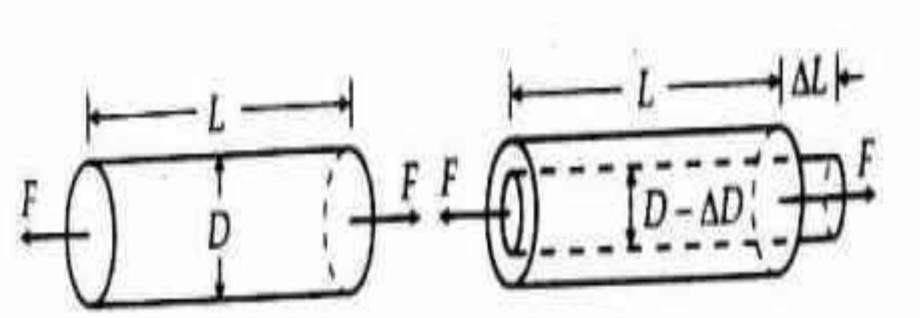
- If a strip of elastic material is subjected to tension, as shown in Fig(a) or in other words positively strained, its longitudinal dimension will increase while there will be a reduction in the lateral dimension.



Fig(a) Change in dimension of a strain gauge element when subjected to a tensile force

- So when a gauge is subjected to a positive strain, its length increases while its area of cross-section decreases as shown in Fig(a). Since the resistance of a conductor is proportional to its length and inversely proportional to its area of cross-section, the resistance of the gauge increases with positive strain.

- The change in the value resistance of strained conductor is more than what can be accounted for an increase in resistance due to dimensional changes. The extra change in the value of resistance is attributed to a change in the value of resistivity of a conductor when strained. This property is known as piezo resistive effect.



Fig(a) Change in dimension of a strain gauge element when subjected to a tensile force

Let us consider a strain gauge made of circular wire.

➢The wire has the dimensions:

length = L, area = A, diameter = D before being strained. The material of the wire has a resistivity ρ.

Resistance of unstrained gauge R = ρL/ A

➢Let a tensile stress S be applied to the wire. This produces a positive strain causing the length to increase and area to decrease as shown in Fig(a). Thus when the wire is strained there are changes in its dimensions.

➢Let ΔL = change in length, ΔA = change in area,

ΔD = change in diameter and ΔR = change in resistance.

➢In order to find how ΔR depends upon the material physical quantities, the expression for R is differentiated with respect to stress S.

Thus we get:

$$\frac{dR}{ds} = \frac{\rho}{A}\frac{\partial L}{\partial s} - \frac{\rho L}{A^2}\frac{\rho A}{\partial s} + \frac{L}{A}\frac{\partial \rho}{\partial s} \qquad \ldots(25.59)$$

Dividing Eqn. 25.59 throughout by resistance $R = \rho L / A$, we have

$$\frac{1}{R}\frac{dR}{ds} = \frac{1}{L}\frac{\partial L}{\partial s} - \frac{1}{A}\frac{\partial A}{\partial s} + \frac{1}{\rho}\frac{\partial \rho}{\partial s} \qquad \ldots(25.60)$$

It is evident from Eqn. 25.60, that the per unit change in resistance is due to :

(*i*) per unit change in length $= \Delta L / L$,

(*ii*) per unit change in area $= \Delta A / A$, and

(*iii*) per unit change in resistivity $= \Delta \rho / \rho$

Area $\quad A = \frac{\pi}{4} D^2 \quad \therefore \quad \frac{\partial A}{\partial s} = 2.\frac{\pi}{4} D.\frac{\partial D}{ds} \qquad \ldots(25.61)$

or

$$\frac{1}{A}\frac{dA}{ds} = \frac{(2\pi/4)D}{(\pi/4)D^2}\frac{\partial D}{\partial s} = \frac{2}{D}\frac{\partial D}{\partial s} \qquad \ldots(25.62)$$

$\therefore$ Equation 25.60 can be written as :

$$\frac{1}{R}\frac{dR}{ds} = \frac{1}{L}\frac{\partial L}{\partial s} - \frac{2}{D}\frac{\partial D}{\partial s} + \frac{1}{\rho}\frac{\partial \rho}{\partial s} \qquad \ldots(25.63)$$

$$R = \frac{\rho L}{A}$$

$$\frac{dR}{ds} = \frac{d}{ds}\left[\frac{\rho L}{A}\right]$$

$$= \frac{\rho}{A}\frac{\partial L}{\partial s} - \frac{\rho L}{A^2}\frac{\partial A}{\partial s} + \frac{L}{A}\frac{\partial \rho}{\partial s}$$

$$\frac{d(A^{-1})}{ds} = -1A^{-1-1} = -\frac{1}{A^2}$$

$$\frac{1}{R}\frac{dR}{ds} = \frac{\rho}{AR}\frac{\partial L}{\partial s} - \frac{\rho L}{A^2 R}\frac{\partial A}{\partial s} + \frac{L}{AR}\frac{\partial \rho}{\partial s}$$

$$= \frac{\cancel{\rho}}{\cancel{A}\cdot\frac{\cancel{\rho} L}{\cancel{A}}}\frac{\partial L}{\partial s} - \frac{\cancel{\rho L}}{A^{\cancel{2}}\cdot\frac{\cancel{\rho L}}{\cancel{A}}}\frac{\partial A}{\partial s} + \frac{\cancel{L}}{\cancel{A}\cdot\frac{\rho \cancel{L}}{\cancel{A}}}\frac{\partial \rho}{\partial s}$$

$$= \frac{1}{L}\frac{\partial L}{\partial s} - \frac{1}{A}\frac{\partial A}{\partial s} + \frac{1}{\rho}\cdot\frac{\partial \rho}{\partial s}$$

Now, Poisson's ratio

$$\nu = \frac{\text{lateral strain}}{\text{longitudinal strain}} = -\frac{\partial D / D}{\partial L / L} \quad \ldots(25.64)$$

or $\partial D / D = -\nu \times \partial L / L$

$$\therefore \quad \frac{1}{R}\frac{dR}{ds} = \frac{1}{L}\frac{\partial L}{\partial s} + \nu\frac{2}{L}\frac{\partial L}{\partial s} + \frac{1}{\rho}\frac{\partial \rho}{\partial s} \quad \ldots(25.65)$$

For small variations, the above relationship can be written as :

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} + 2\nu\frac{\Delta L}{L} + \frac{\Delta \rho}{\rho} \quad \ldots(25.66)$$

The gauge factor is defined as the ratio of per unit change in resistance to per unit change in length.

Gauge factor $\quad G_f = \dfrac{\Delta R / R}{\Delta L / L} \quad \ldots(25.67)$

or $\quad \dfrac{\Delta R}{R} = G_f \dfrac{\Delta L}{L} = G_f \times \varepsilon \quad \ldots(25.68)$

where ε = strain $= \dfrac{\Delta L}{L}$

$$\frac{1}{R}\frac{dR}{ds} = \frac{1}{L}\frac{\partial L}{\partial s} - \frac{2}{D}\frac{\partial D}{\partial s} + \frac{1}{\rho}\cdot\frac{\partial \rho}{\partial s}$$

$$= \frac{1}{L}\frac{\partial L}{\partial s} - 2\cdot\frac{-\nu\cdot\partial L}{\partial s . L} + \frac{1}{\rho}\cdot\frac{\partial \rho}{\partial s}$$

$$= \frac{1}{L}\frac{\partial L}{\partial s} + \frac{2\nu\partial L}{L\partial s} + \frac{1}{\rho}\cdot\frac{\partial \rho}{\partial s}$$

$$\frac{\Delta R/R}{\Delta L/L} = \frac{\Delta L/L}{\Delta L/L} + 2\gamma\frac{\Delta L/L}{\Delta L/L} + \frac{\Delta\rho/\rho}{\Delta L/L}$$

$$G_f = 1 + 2\gamma + \frac{\Delta\rho/\rho}{\Delta L/L}$$

The gauge factor can be written as :

$$= 1 + 2\nu + \frac{\Delta\rho/\rho}{\varepsilon} \qquad \text{...(25.69)}$$

$$= \underbrace{1}_{\substack{\text{Resistance}\\ \text{change due to}\\ \text{change of length}}} + \underbrace{2\nu}_{\substack{\text{Resistance}\\ \text{change due to}\\ \text{change in area}}} + \underbrace{\frac{\Delta\rho/\rho}{\varepsilon}}_{\substack{\text{Resistance}\\ \text{change due to}\\ \text{piezoresistive effect}}}$$

$$G_f = \frac{\Delta R/R}{\Delta L/L} = 1 + 2\nu + \frac{\Delta\rho/\rho}{\Delta L/L}$$

The strain is usually expressed in terms of microstrain. 1 microstrain = 1 μm / m.

If the change in the value of resistivity of a material when strained is neglected, the gauge factor is :

$$G_f = 1 + 2\nu \qquad \text{...(25.70)}$$

- Equation 25.70 is valid only when Piezo resistive Effect i.e., change in resistivity due to strain is almost negligible.
- The Poisson's ratio for all metals is between 0 and 0.5.
- This gives a gauge factor of approximately, 2.
- The common value for Poisson's ratio for wires is 0.3.
- This gives a value of 1.6 for wire wound strain gauges.

➢ . The below table gives the value of gauge factors for the various materials

Material	***Gauge***	***Material***	***Gauge Factor***
Nickel	– 12.1	Platinum	+ 4.8
Manganin	+ 0.47	Carbon	+ 20
Nichrome	+ 2.0	Doped Crystals	100 – 5000
Constantan	+ 2.1		
Soft iron	+ 4.2		

- Whenever temperature changes, the resistance will change in the same proportion in the both arms of the rheostat, and the bridge remains in the state of balance. Effect of temperature get nullifies. It is good to keep voltage low so that the self-heating of strain gauge could be evaded.
- Self-heating of gauge depends upon its mechanical behavior.

Applications of Strain Gauge:

1) In the field of mechanical engineering development.
2) To measure the stress generated by machinery.
3) In the field of component testing of aircraft like; linkages, structural dimensions etc.
4) Used in load cells, torque meters, diaphragm type pressure gauges, temperature sensors, accelerometers and flow meters and for vibration measurement, residual stress, compression and tension measurement

Advantages

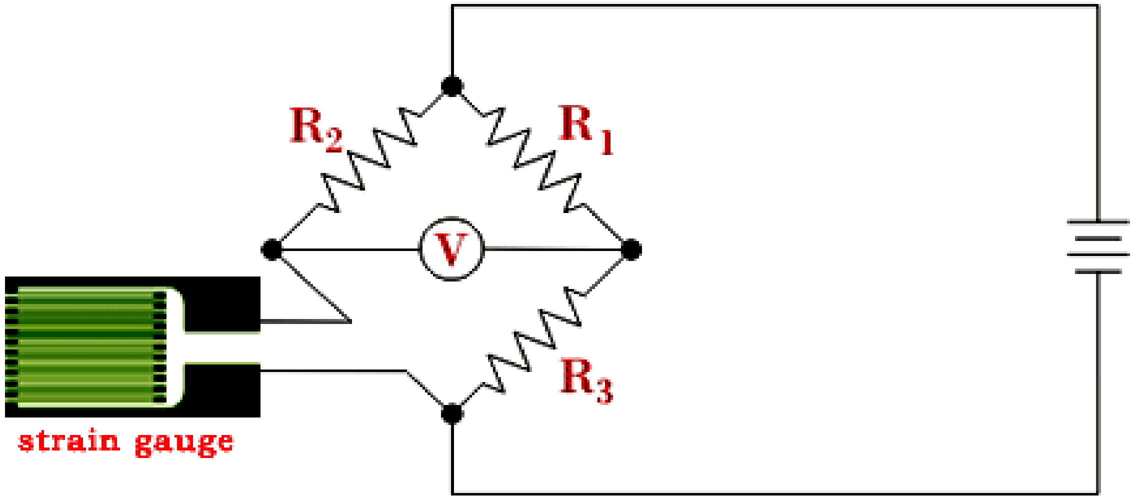
1) There is no moving parts
2) It is small and inexpensive

Disadvantages

1) It is non-linear
2) It needs to be calibrated

➢ Strain Gauge Bridge Circuit:

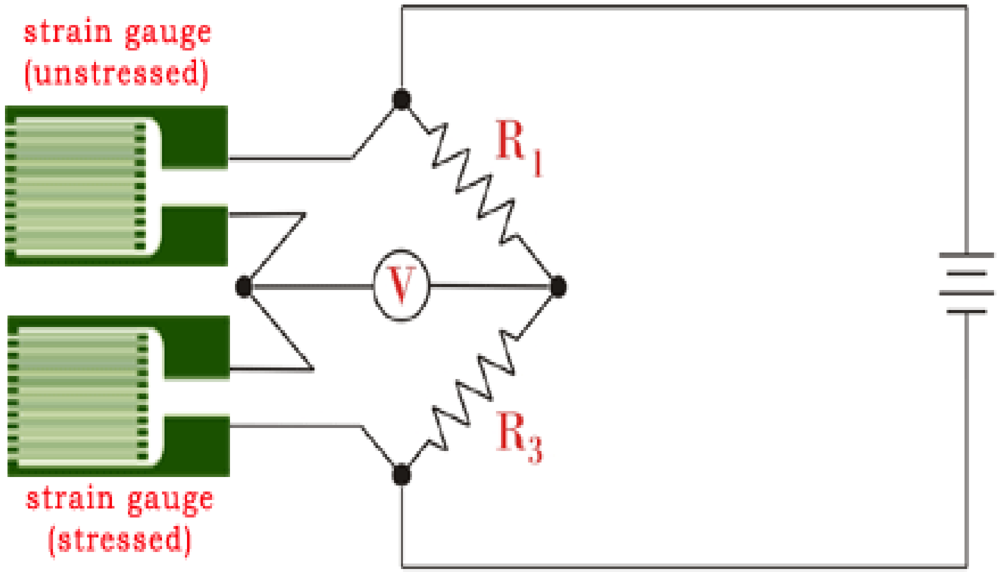
- Strain gauge bridge circuit shows the measured stress by the degree of discrepancy, and uses a voltmeter in the center of the bridge to provide an accurate measurement of that imbalance.
- In this circuit, R1 and R3are the ratio arms equal to each other, and R2 is the rheostat arm has a value equal to the strain gage resistance.
- In this condition i.e.,When the gauge is unstrained, the bridge(Wheatstone bridge) is balanced, and voltmeter shows zero value.



Quarter-bridge strain gauge circuit

- As there is a change in resistance of strain gauge, the bridge gets unbalanced and producing an indication at the voltmeter.
- The output voltage from the bridge can be amplified further by a differential amplifier.

- **Variation of Temperature of Strain Gauge:**
- One more factor that affects the resistance of the gauge is temperature.
- If the temperature is more resistance will be more and if the temperature is less the resistance will be less.
- This is a common property of all the conductors. We can overcome this problem by using strain gauges that are self-temperature-compensated or by a dummy strain gauge technique.
- Most of the strain gauges are made of constantan (55% copper-45% nickel) alloy which cancel out the effect of temperature on the resistance. But some strain gauges are not of an isoelastic alloy.
- In such cases, dummy gauge is used in the place of R_2 in the quarter bridge strain gauge circuit which acts as a temperature compensation device.



Quarter-bridge strain gauge circuit with temperature compensation

STRAIN GAUGE

STRAIN GAUGE SELECTION CRITERIA:

- Gauge Length
- Number of Gauges in Gauge Pattern
- Arrangement of Gauges in Gauge Pattern
- Grid Resistance
- temperature sensitivity
- Carrier Material
- Gauge Width
- Availability
- low cost

Inductive Transducers

Inductive Transducer

- Inductive Transducer may be either of the self generating or the passive type. The self generating type utilizes the basic electrical generator principle, i.e. a motion between a conductor and magnetic field induces a voltage in the conductor (generator action).
- This relative motion between the field and the conductor is supplied by changes in the measured. An inductive electromechanical transducer is a device that converts physical motion (position change) into a change in inductance.

Let us consider the case of a general inductive transducer. If Inductive Transducer has N turns and a reluctance R. When a current i is passed through it, the flux is

$$\phi = \frac{Ni}{R}$$

$$\frac{d\phi}{dt} = \frac{N}{2} \times \frac{di}{dt} - \frac{Ni}{R^2} \times \frac{dR}{dt}$$

If the current varies very rapidly,

$$\frac{d\phi}{dt} = \frac{N}{2} \times \frac{di}{dt}$$

But emf induced in the coil is given by

$$e = N \times d\phi/dt$$

$$e = N \times \frac{N}{2} \times \frac{di}{dt} = \frac{N^2}{R} \times \frac{di}{dt}$$

Also the self inductance is given by

$$L = \frac{e}{di/dt} = \frac{N^2}{R}$$

➢Therefore, the output from an inductive transducer can be in the form of either a change in voltage or a change in inductance.

VARIABLE INDUCTANCE TRANSDUCERS

The variable inductance transducers work generally, upon one of the following three principles:

1) **change of self inductance,**
2) **change of mutual inductance, and**
3) **production of eddy currents,**

Fig. Inductive Transducer

Transducers working on principle of **change of Self Inductance**

The self inductance of a coil L= N^2/R

where N=number of turns, and

R = reluctance of the magnetic circuit.

The reluctance of the magnetic circuit R =l/μA

∴ Inductance L=$N^2\mu$(A/l)= $N^2\mu$G ....(1)

where μ = effective permeability of the medium in and around the coil ; H/m

G = A/l = geometric form factor

A= area of cross-section of coil ; m

and l = length of coil; m

It is clear from Eqn.(1) that the variation in inductance may be caused by

i. change in number of turns, N,
ii. change in geometric configurations, G, and
iii. change in permeability, μ .

Inductive transducers are mainly used for measurement of displacement. The displacement to be measured is arranged to cause variation of any of variables in Eqn.(1) and thus alter the self inductance L by ΔL

- The displacement to be measured is arranged to cause variation in any of three variables
  1. Change in Number of turns (N)
  2. Change in Permeability of the magnetic material or magnetic circuits (μ)
  3. Change in geometric configuration (G)

CHANGE IN SELF INDUCTANCE WITH NUMBERS OF TURNS:

- The output may be caused by a change in the number of turns. Figures 13.14(a) and (b) are transducers used for, the measurement of displacement of linear and angular movement respectively

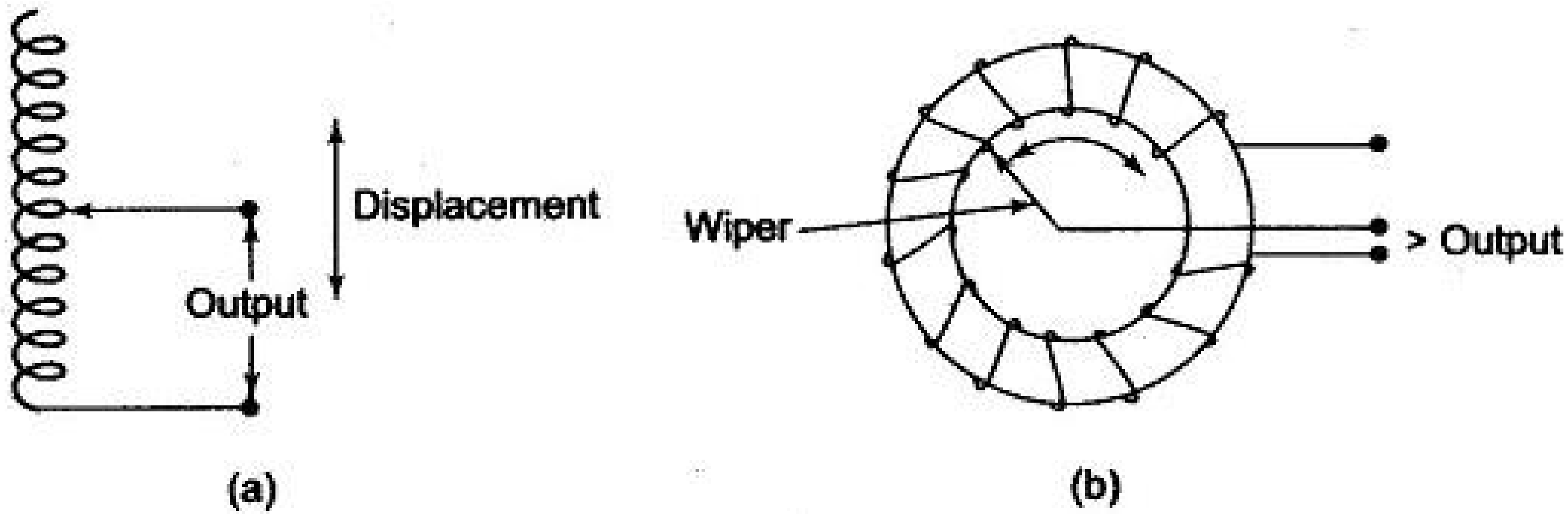


Fig. 13.14 (a) Linear Inductive Transducer (Using Air Core) (b) Angular Inductive Transducer (Using Ferrite Core)

Figure 13.14(a) is an air cored transducer for measurement of linear displacement.
Figure 13.14(b) is an iron cored coil used for the measurement of angular displacement.
In both cases, as the number of turns are changed, the self inductance and hence the output voltage also changes.

- **CHANGE IN SELF INDUCTANCE WITH CHANGE IN PERMEABILITY :**

- Figure 13.15 shows an Inductive Transducer which works on the principle of the variation of permeability causing a change in self inductance.
- The iron core is surrounded by a winding. If the iron core is inside the winding, its permeability is increased, and so is the inductance.
- When the iron core is moved out of the winding, the permeability decreases, resulting in a reduction of the self inductance of the coil.
- This transducer can be used for measuring displacement.

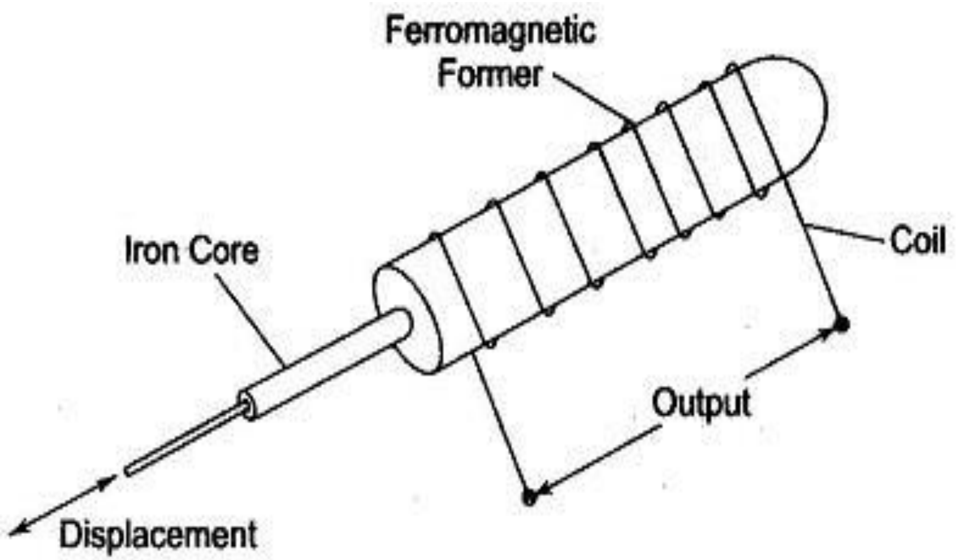


Fig. 13.15 Inductive Transducer Working on the Principle of Variation of Permeability

➢ VARIABLE RELUCTANCE TYPE TRANSDUCER:

- A transducer of the variable type consists of a coil wound on a ferromagnetic core. The displacement which is to be measured is applied to a ferromagnetic target.
- The target does not have any physical contact with the core on which it is mounted. The core and the target are separated by an air gap, as shown in Fig. 13.16(a) .
- The reluctance of the magnetic path is determined by the size of the air gap. The inductance of the coil depends upon the reluctance of the magnetic circuits.
- The self inductance of the coil is given by;

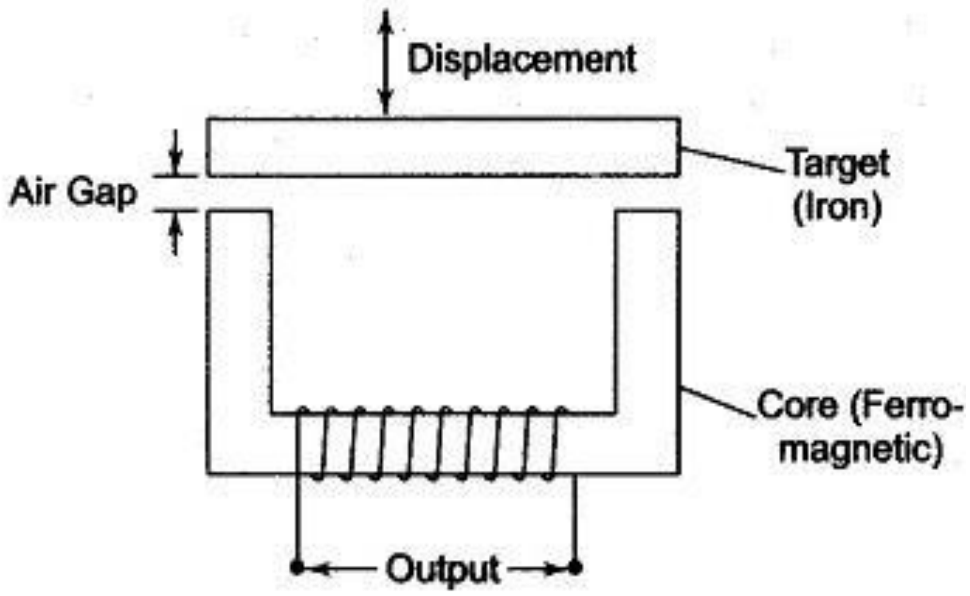


Fig. 13.16(a) Variable Reluctance Transducer

$$L = \frac{N^2}{R_i + R_g}$$

where N = number of turns

R_i = reluctance of iron parts

R_g = reluctance of air gap

The reluctance of the iron part is negligible compared to that of the air gap. Therefore

$$L = N^2/R_g$$

But reluctance of the air gap is given by → $$R_g = \frac{l_g}{\mu_o \times A_g}$$

where

l_g = length of the air gap

A_g = area of the flux path through air

μ_o = permeability

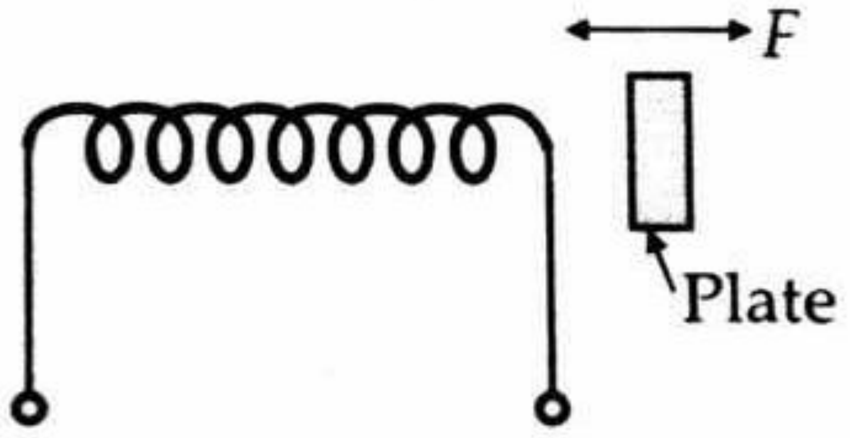
R_g is proportional to l_g, as μ_o and A_g are constants.

Hence L is proportional to $1/l_g$, as $L = N^2/R_g$

i.e. the self inductance of the coil is inversely proportional to the length of the air gap.

- When the target is near the core, the length is small and therefore the self inductance large. But when the target is away from the core the reluctance is large, resulting in a smaller self inductance value.
- Hence the inductance of the coil is a function of the distance of the target from the core, i.e. the length of the air gap.

➢ Transducers working on principle of production of eddy currents:



- These inductive transducers work on the principle that if a conducting plate is placed near a coil carrying alternating current, eddy currents are produced in the conducting plate.
- The conducting plate acts as a short-circuited secondary winding of a transformer.

- The eddy currents flowing in the plate produce a magnetic field of their own which acts the magnetic field produced by the coil.
- This results in reduction of flux and thus the inductance of the coil is reduced. The nearer is the plate to the coil, the higher are the eddy currents and thus higher is the reduction in the inductance of the coil.
- Thus the inductance of the coil alters with variation of distance between the plate and the coil.

- A number of arrangements are possible. The plate may be at right angle to the axis of the coil. The displacement of the plate causes a change in the inductance of the coil. The change in inductance is a measure of displacement.

➢ Transducer based on variation of mutual inductance:

➢The mutual inductance between the two coils is given by: $M = k\sqrt{L_1.L_2}$

Where M=Mutual inductance between two coils
k= coefficient of coupling

- The mutual inductance between two coils can be varied by varying either self inductances of the coils or coefficient of coupling.

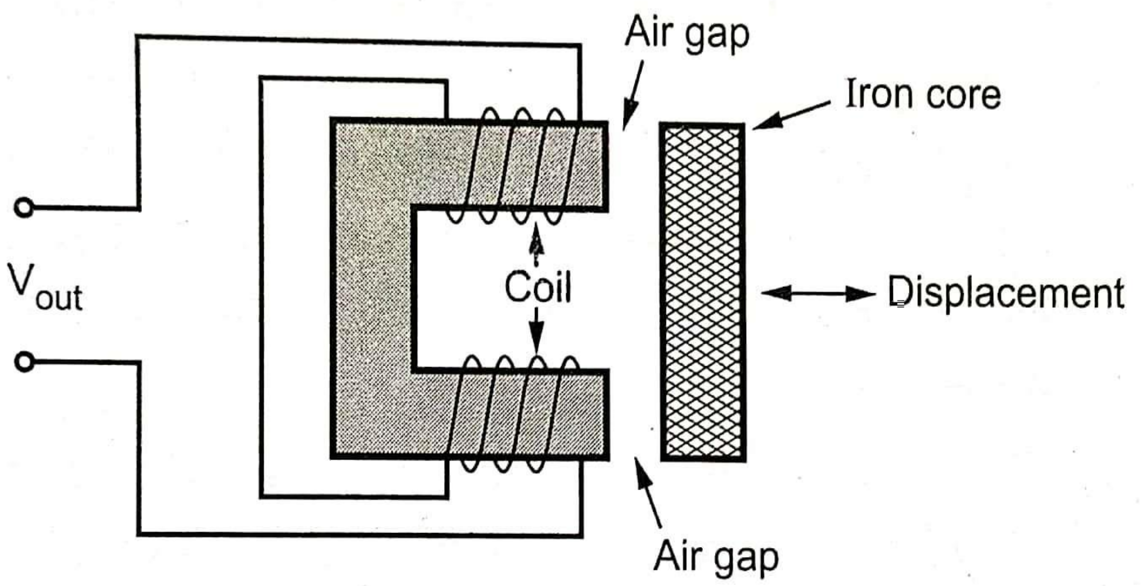


Fig. 6.22 Transducer based on mutual inductance variation

- The transducers based on variation of mutual inductance of two or more coils are used and the variation in measured is applied to the transducers.
- The output is measured in reference with variation in mutual inductance between the coils. The measurement of linear displacement based on the variation of mutual inductance is shown in fig.6.22
- The coils are connected in series type of connection. It may be series aiding or series opposing. Depending upon it, the total equivalent inductance varies from (L_1+L_2-2M) to (L_1+L_2+2M)

- When linear displacement is applied to iron core, the core moves towards or away from the circuit. This results in variation in air gap and thus the magnetic flux linking with two coils changes which results in variation of a mutual inductance between two coils.
- The change in the value of mutual inductance is sensed by the bridge circuit.
- The different types of inductive transducers for measurement of translational and rotary displacements are shown in the below figure:

Differential output:

- Normally the change in self inductance ΔL is adequate for detection of subsequent stages of instrumentation system. However, if the succeeding instrumentation responds to ΔL, rather than L+ ΔL, the sensitivity and accuracy will be much higher.
- The transducer can be designed to provide two outputs one of which is an increase of self inductance and the other is a decrease in self inductance. The succeeding stages of instrumentation system measure the difference between these outputs. This is known as the differential output.

➢ **The advantages of differential outputs are:**

1) The sensitivity and accuracy are increased.
2) The output is less affected by external magnetic fields.
3) The effective variations due to temperature changes are reduced.
4) The effects of changes in supply voltage and frequency are reduced.

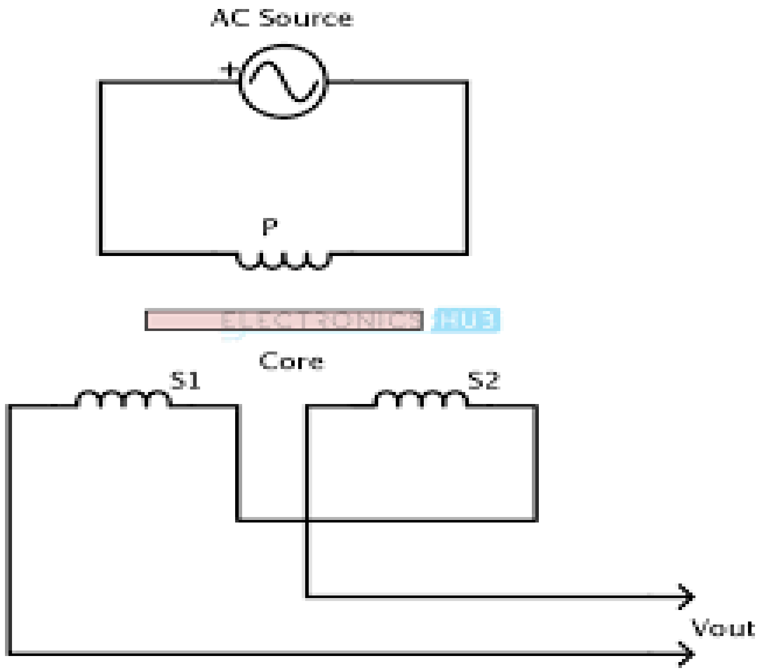
LINEAR VARIABLE DIFFERENTIAL TRANSFORMER (LVDT)

LINEAR VARIABLE DIFFERENTIAL TRANSFORMER (LVDT)

- Definition of LVDT: The term LVDT stands for the **L**inear **V**ariable **D**ifferential **T**ransformer. It is the most widely used inductive transducer(passive type) that coverts the linear motion into the electrical signal.

Main Features of Construction :

- The transformer consists of a primary winding P and two secondary windings S1 and S2 wound on a cylindrical former (which is hollow in nature and contains the ferromagnetic core).
- Both the secondary windings have an equal number of turns, and we place them on either side of primary winding
- The primary winding is connected to an AC source(generally1-10V) which produces a flux in the air gap and voltages are induced in secondary windings.



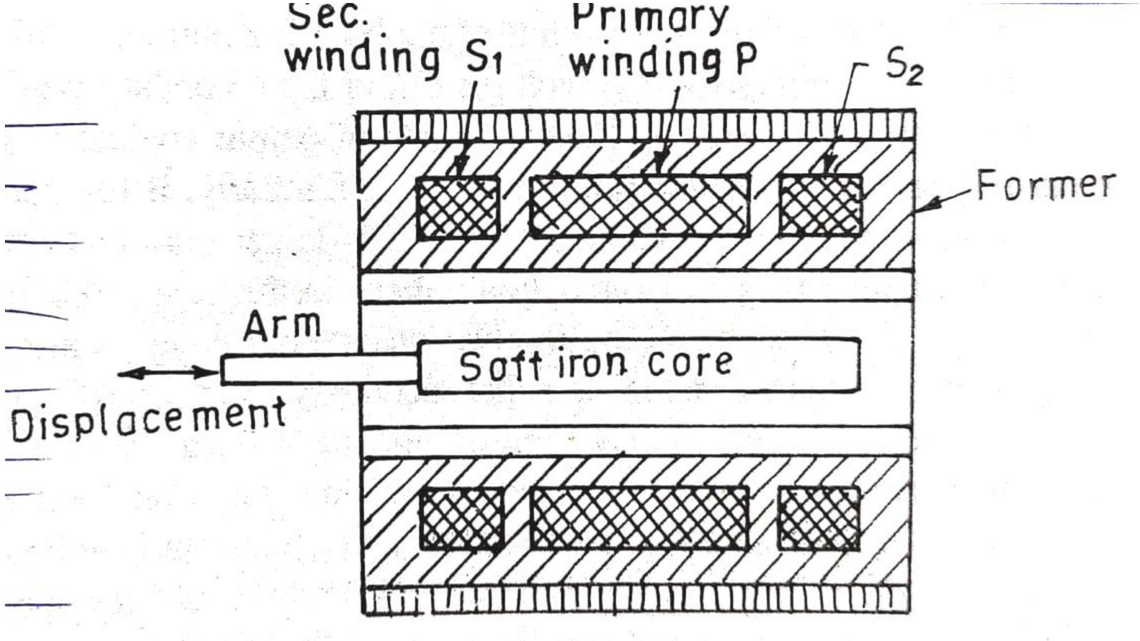


Fig. 25.88. Linear variable differential transformer (L.V.D.T.).

- A movable soft iron core is placed inside the former and displacement to be measured is connected to the iron core.
- The iron core is generally of high permeability which helps in reducing harmonics and high sensitivity of LVDT.
- The LVDT is placed inside a stainless steel housing because it will provide electrostatic and electromagnetic shielding.
- Both the secondary windings are connected in such a way that resulted output is the difference between the voltages of two windings.

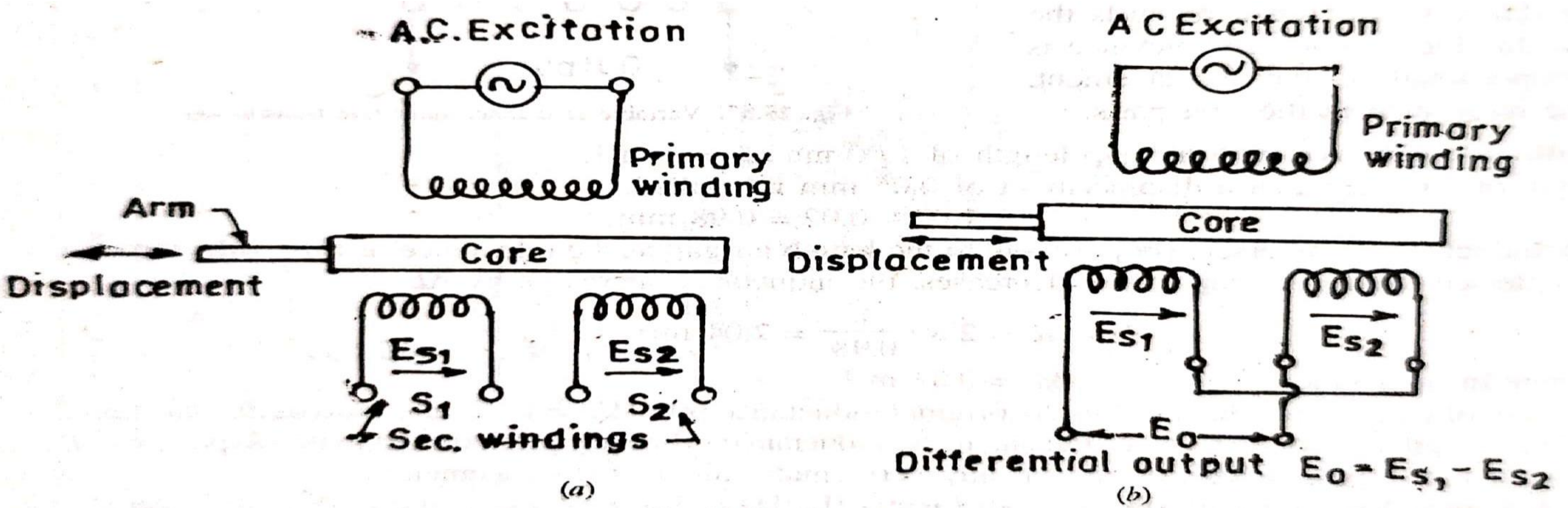
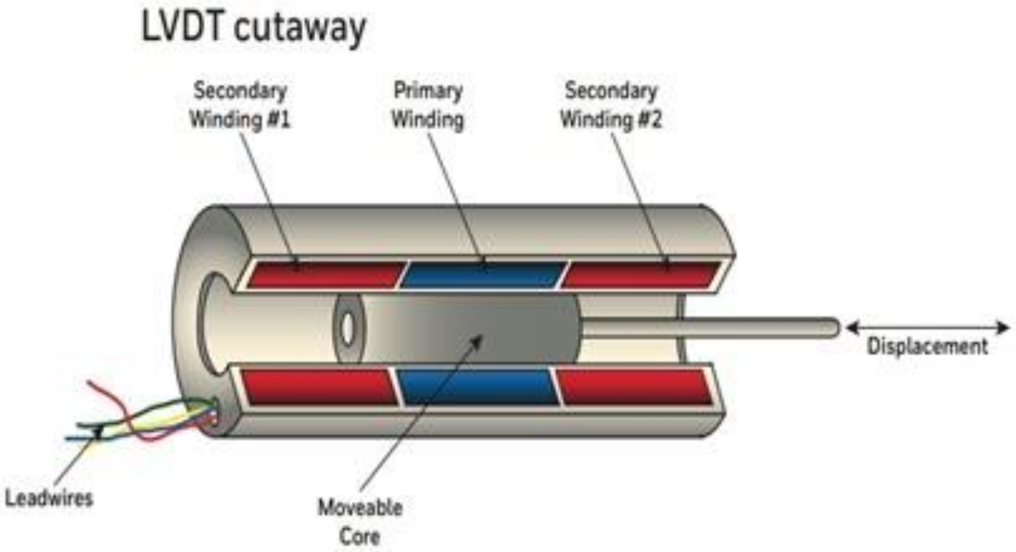
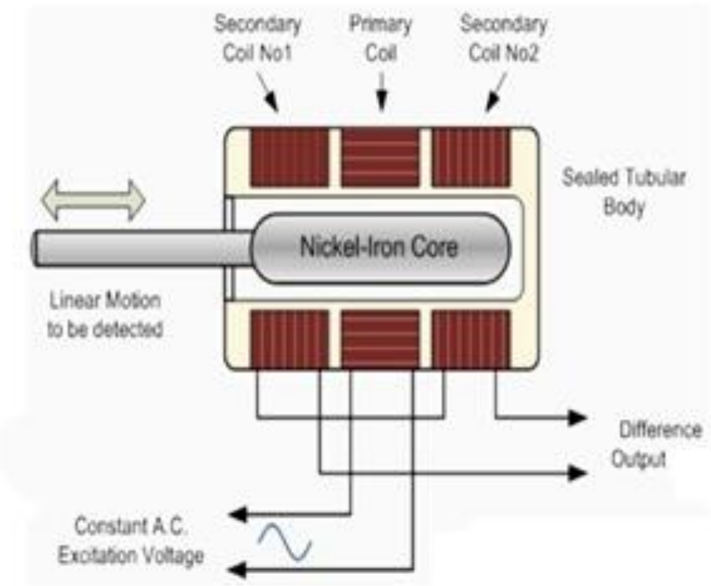


Fig. 25.89. Circuits of an LVDT.





LVDT

Principle of Operation and Working:

- As the primary is connected to an AC source so alternating current and voltages are produced in the secondary of the LVDT. The output in secondary S_1 is E_1 and in the secondary S_2 is E_2. So the differential output is, $E_{out} = E_1 - E_2$
- This equation explains the principle of Operation of LVDT.
- Now three cases(positions) arise according to the locations of core which explains the working of LVDT are discussed below as,

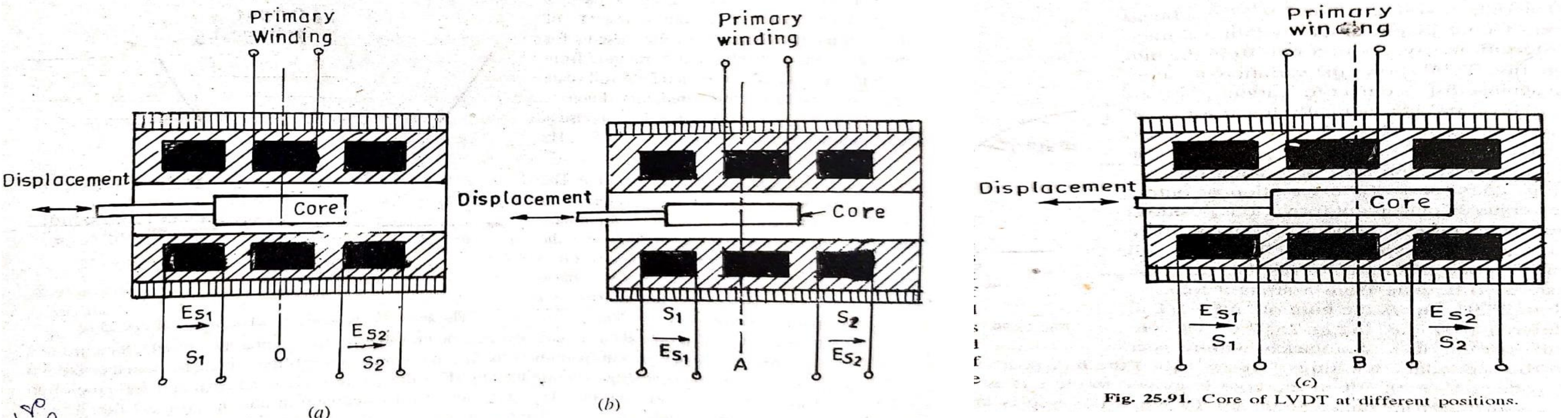


Fig. 25.91. LVDT at different positions.

Fig. 25.91. Core of LVDT at different positions.

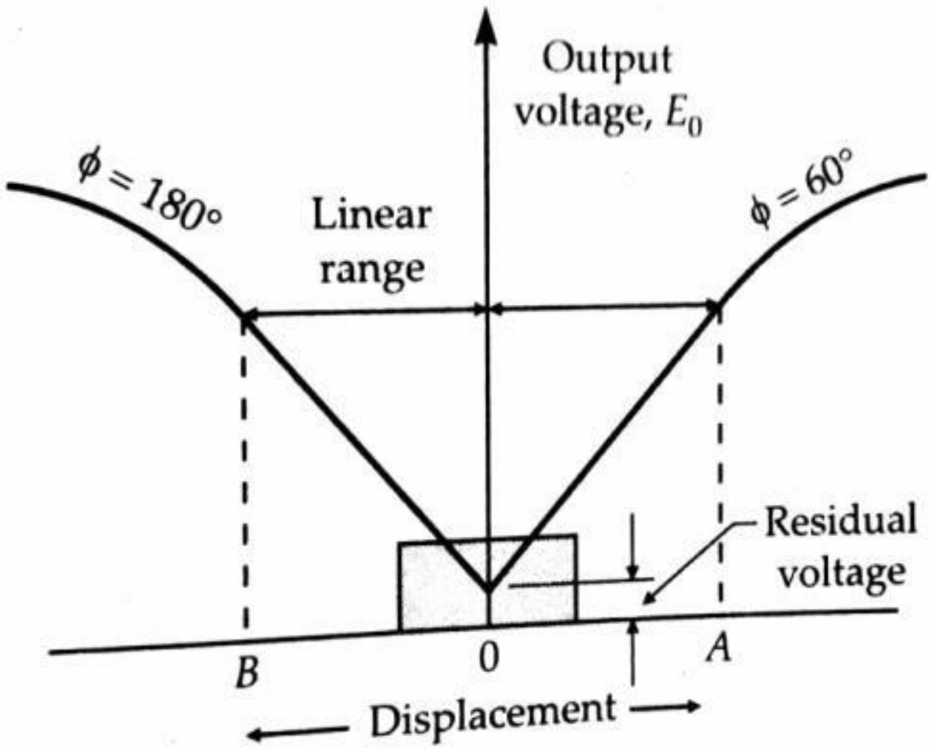
(1)Core at center $E_{out} = E_1 - E_2 = 0V$ as $E_1 = E_2$ (2)Core moves towards coil S1, $E_1 > E_2$ (3)Core moves towards coil S2, $E_1 < E_2$

Output VS Core Displacement:

- A linear curve shows that output voltage varies linearly with displacement of core.

- The output voltage of an LVDT is a linear function of core displacement within a limited range of motion, say, about 5 mm from the null position. The figure shows the variation of output voltage against displacement for various positions of core.
- The curve is practically linear for small displacements (upto about 5 mm as mentioned above).
- Beyond this range of displacement, the curve starts to deviate from a straight line.
- The characteristics are linear up to 0-A and 0-B but after then they become non-linear as shown in Fig.(d).

- Ideally the output voltage at the null position should be equal to zero.



Fig(d) Output voltage Vs Displacement

- However, in actual practice there exists a small voltage at the null position. This may be on account of presence of harmonics in the input supply voltage and also due to harmonics produced in the output voltage on account of using iron core.
- There may be either an incomplete magnetic or electrical unbalance or both which result in a finite output voltage at the null position.
- This finite residual voltage is generally less than 1% of the maximum output voltage in the linear range. Other causes of residual voltage are stray magnetic fields and temperature effects. The residual voltage is shown in Fig. (d).
- However, with improved technological methods and with the use of better AC Sources, the residual voltage can be reduced to almost a negligible value.

Some important points about magnitude and sign of voltage induced in LVDT

1) The amount of change in voltage either negative or positive is proportional to the amount of movement of core and indicates amount of linear motion.
2) By noting the output voltage increasing or decreasing the direction of motion can be determined
3) The output voltage of an LVDT is linear function of core displacement.

❑ Advantages of LVDT:

1) High Range - The LVDTs have a very high range for measurement of displacement. They can used for measurement of displacements ranging from 0.25 mm to 250 mm
2) No Frictional Losses - As the core moves inside a hollow former so there is no loss of displacement input as frictional loss so it makes LVDT as very accurate device.
3) High Input and High Sensitivity - The output of LVDT is so high that it doesn’t need any amplification. The transducer possesses a high sensitivity which is typically about 40V/mm.
4) Low Hysteresis - LVDTs show a low hysteresis and hence repeatability is excellent under all conditions
5) Low Power Consumption - The power is about 1W which is less as compared to other transducers.
6) Direct Conversion to Electrical Signals - They convert the linear displacement to electrical voltage which are easy to process.

Disadvantages of LVDT:

1) LVDT is sensitive to stray magnetic fields so it always requires a setup to protect them from stray magnetic fields.
2) LVDT gets affected by vibrations and temperature.
3) Relatively large displacements are required for appreciable differential output.

Applications of LVDT:

- We use LVDT in the applications where displacements to be measured are ranging from a fraction of mm to few cms. The LVDT acting as a primary transducer converts the displacement into electrical signal directly.

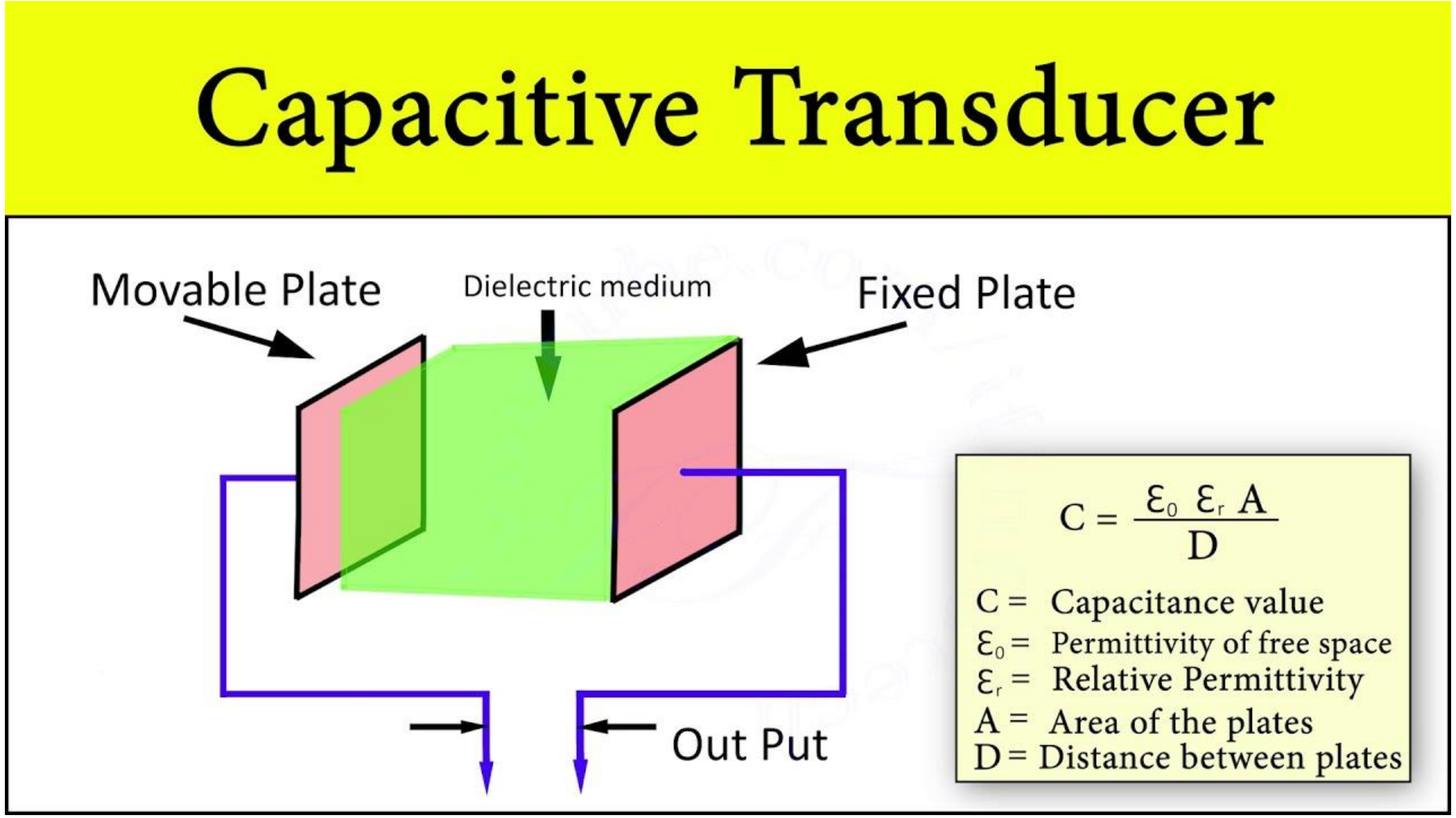
- The LVDT can also act as a secondary transducer. E.g. the Bourbon tube which acts as a primary transducer and it converts pressure into linear displacement and then LVDT coverts this displacement into an electrical signal which after calibration gives the readings of the pressure of fluid.

Capacitive Transducers

Capacitive Transducer:

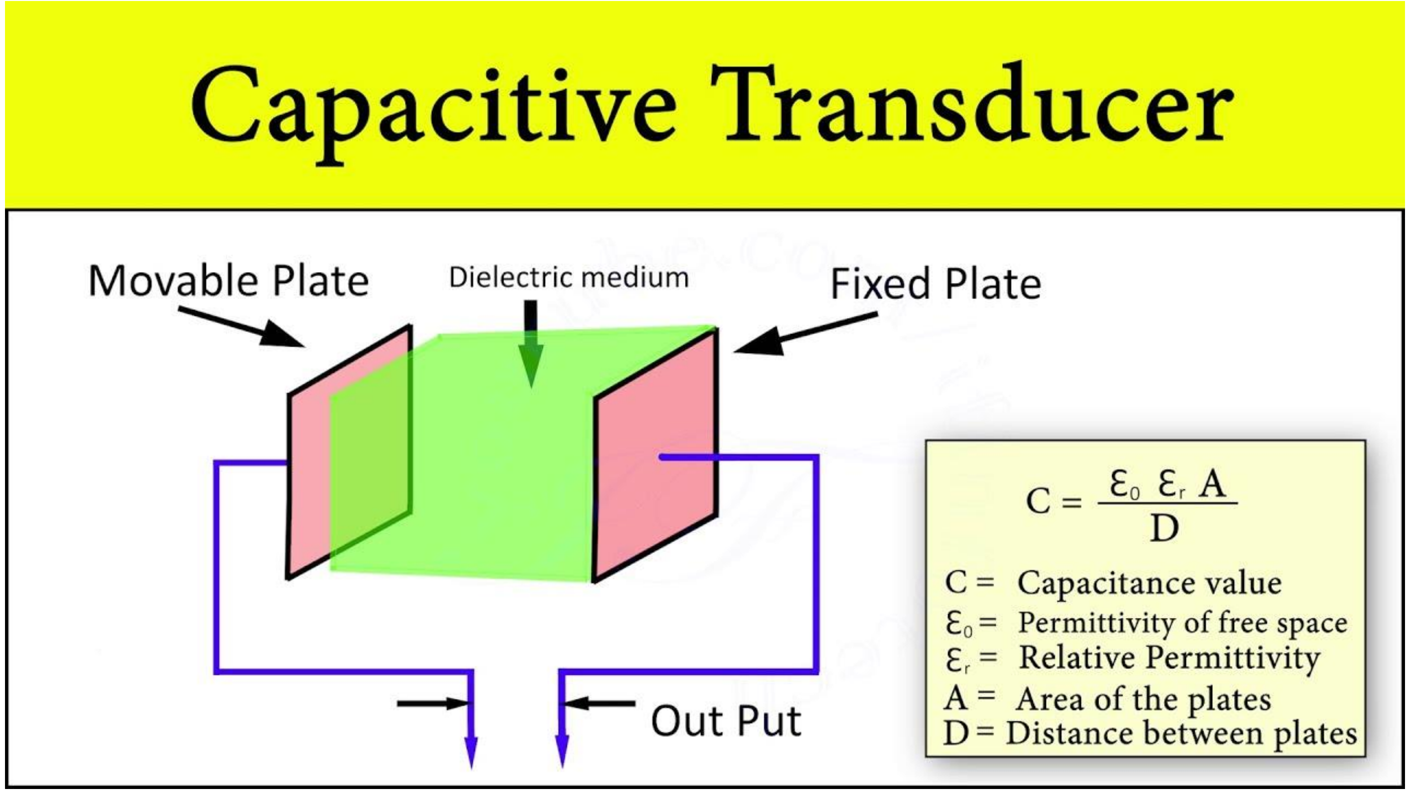
Definition:

The capacitive transducer is used for measuring the displacement, pressure and other physical quantities. It is a passive transducer that means it requires external power for operation. The capacitive transducer works on the principle of variable capacitances.



- The capacitance of the capacitive transducer changes because of many reasons like overlapping of plates, change in distance between the plates and dielectric constant. The capacitive transducer contains two parallel metal plates.
- These plates are separated by the dielectric medium which is either air, material(paper,mica etc), gas or liquid. In the normal capacitor the distance between the plates are fixed, but in capacitive transducer the distance between them are varied.

- The capacitive transducer uses the electrical quantity of capacitance for converting the mechanical movement into an electrical signal.
- The input quantity causes the change of the capacitance which is directly measured by the capacitive transducer.
- The capacitors measure both the static and dynamic changes. The displacement is also measured directly by connecting the measurable devices to the movable plate of the capacitor.
- It works on with both the contacting and non-contacting modes.



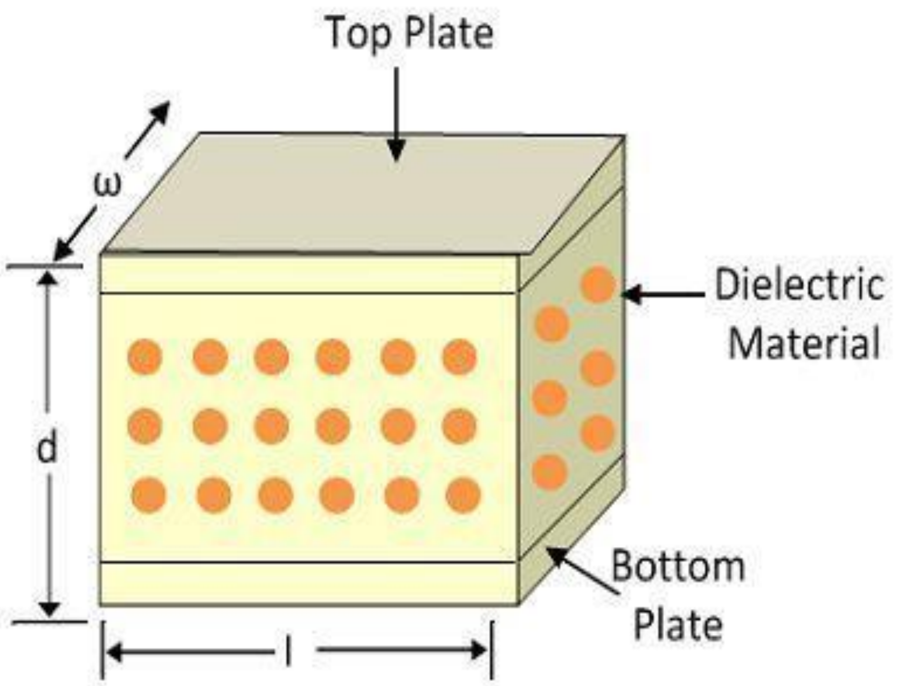
Principle of Operation:

- The equations below express the capacitance between the plates of a capacitor :

$$C = \varepsilon A/d$$

$$C = \varepsilon_r \varepsilon_0 A/d$$

Where A – overlapping area of plates in m^2
d – the distance between two plates in meter
ε – permittivity of the medium in F/m
ε_r – relative permittivity (for air $\varepsilon_r = 1$)
ε_0 – the permittivity of free space $= 8.85 \times 10^{-12}\ F/m$



Parallel Plate Capacitive Transducer

- The schematic diagram of a parallel plate capacitive transducer is shown in the above figure: The change in capacitance occurs because of the physicals variables like displacement, force, pressure, etc.
- The capacitance of the transducer also changes by the variation in their dielectric constant which is usually because of the measurement of liquid or gas level.

➢The capacitance of the transducer is measured with the bridge circuit. The output impedance of transducer is given as:

$$X_c = 1/2\pi f c$$

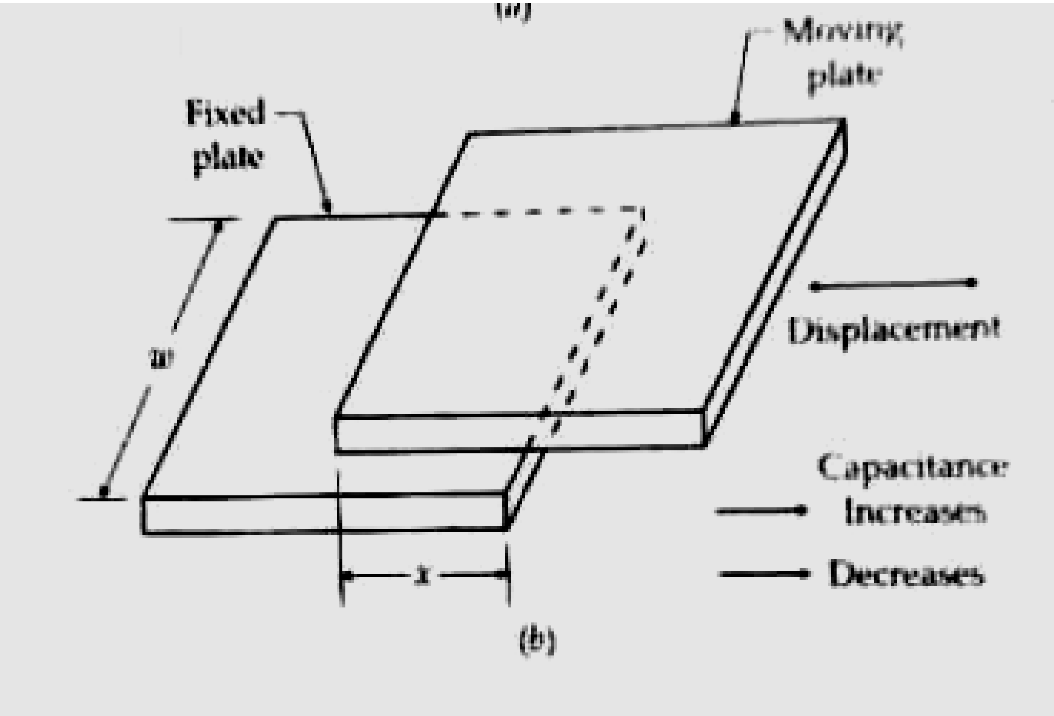
Where, C – capacitance

f – frequency of excitation in Hz.

➢The capacitive transducer is mainly used for measurement of linear displacement.

➢The capacitive transducer uses the following three effects.

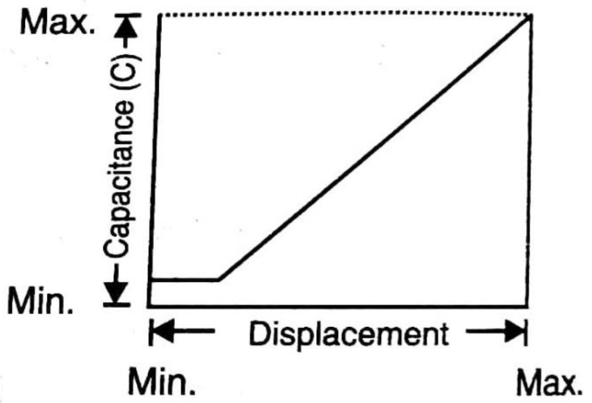
1. Variation in capacitance of transducer is because of the overlapping of capacitor plates.
2. The change in capacitance is because of the change in distances between the plates.
3. The capacitance changes because of dielectric constant.



- The following methods are used for the measuring displacement.

1.Transducer Using The Change In The Area Of Plates:

- The equation below shows that the capacitance is directly proportional to the area of the plates.
- The capacitance changes correspondingly with the change in the position of the plates. The capacitive transducers are used for measuring the large displacement approximately from 1mm to several cms.



- The area of the capacitive transducer changes linearly with the capacitance and the displacement. Initially, the nonlinearity occurs in the system because of the edges. Otherwise, it gives the linear response.
- The capacitance of the parallel plates is given as :

where x – the length of overlapping part of plates

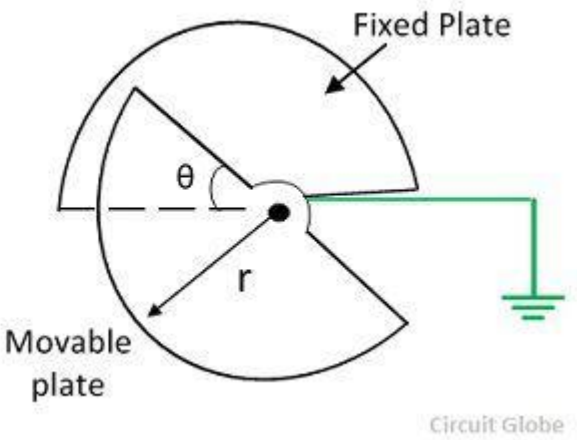
ω – the width of overlapping part of plates.

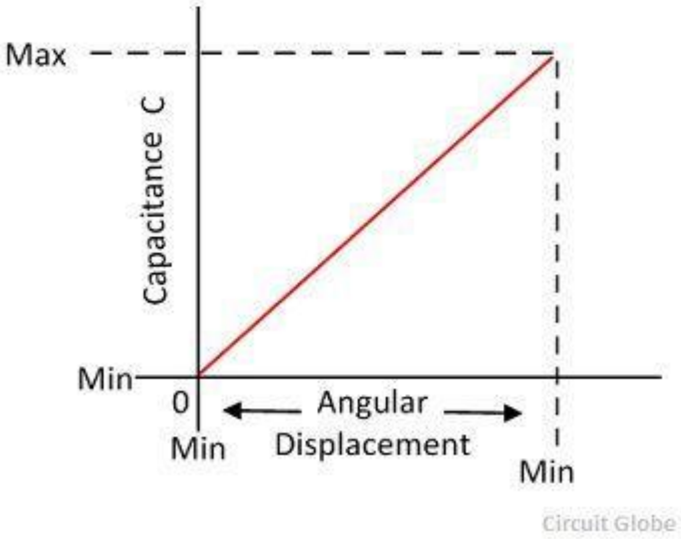
$$C = \frac{\varepsilon A}{d} = \frac{\varepsilon x \omega}{d} F$$

- The sensitivity of the displacement is constant, and therefore it gives the linear relation between the capacitance and displacement.

$$S = \frac{\partial C}{\partial x} = \varepsilon \frac{\omega}{d} F/m$$

- The capacitive transducer is used for measuring the angular displacement. It is measured by the movable plates shown below. One of the plates of the transducer is fixed, and the other is movable. The phasor diagram of the transducer is shown in the figure below.
- The angular movement changes the capacitance of the transducers. The capacitance between them is maximum when these plates overlap each other. The maximum value of capacitance is expressed as





$$C_{max} = \frac{\varepsilon A}{d} = \frac{\pi \varepsilon r^2}{2d}$$

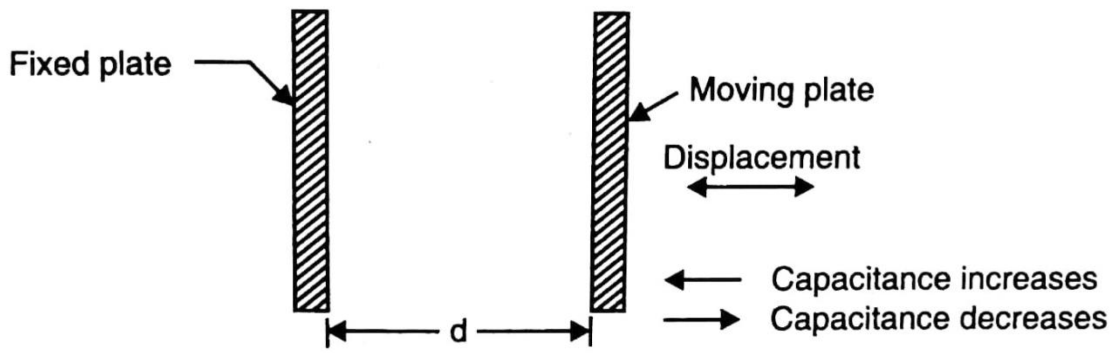
The capacitance at angle θ is given expressed as,
θ – angular displacement in radian.
The sensitivity for the change in capacitance is given as:

$$C = \frac{\varepsilon \theta r^2}{2d}$$

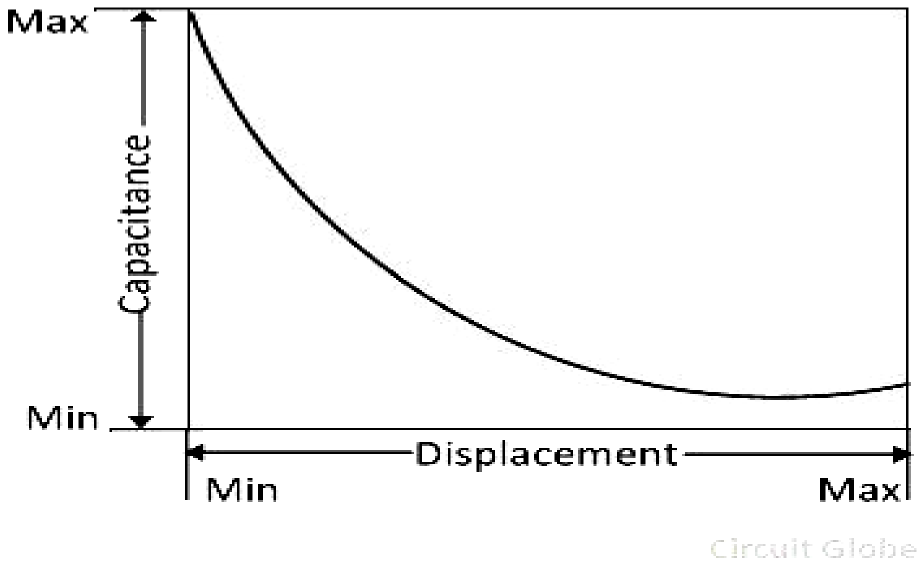
$$S = \frac{\partial C}{\partial \theta} = \frac{\varepsilon r^2}{2d}$$

2.The Transducer Using The Change In Distance Between The Plates:

- The capacitance of the transducer is inversely proportional to the distance between the plates.
- The one plate of the transducer is fixed, and the other is movable.
- The displacement which is to be measured links to the movable plates.

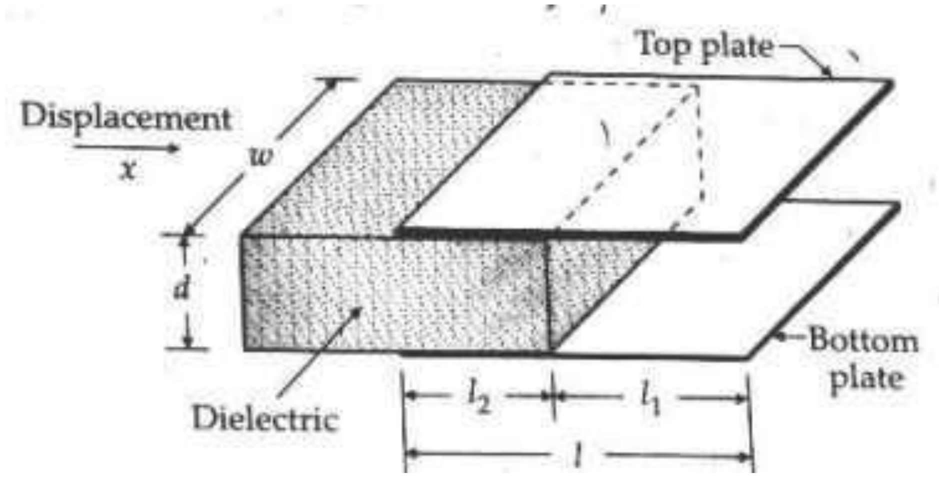


- The capacitance is inversely proportional to the distance because of which the capacitor shows the nonlinear response. Such type of transducer is used for measuring the small displacement. The phasor diagram of the capacitor is shown in beside figure.
- The sensitivity of the transducer is not constant and varies from places to places.



3.The Capacitance Changes Because Of Dielectric Constant:

- The third principle used in capacitive transducers is the variation of capacitance due to change in dielectric constant.
- Beside Figure shows a capacitive transducer for measurement of linear displacement working on the above mentioned principle. It has a dielectric of relative permittivity.



Initial capacitance of transducer

$$= C = \varepsilon_0 \frac{wl_1}{d} + \varepsilon_0 \varepsilon_r \frac{wl_2}{d}$$

$$= \varepsilon_0 \frac{w}{d}[l_1 + \varepsilon_r l_2]$$

Let the dielectric be moved through a distance x in the direction indicated. The capacitance changes from C to C+ ΔC

$$C + \Delta C = \varepsilon_0 \frac{w}{d}(l_1 - x) + \varepsilon_0 \varepsilon_r \frac{w}{d}(l_2 + x)$$

$$= \varepsilon_0 \frac{w}{d}[l_1 - x + \varepsilon_r (l_2 + x)]$$

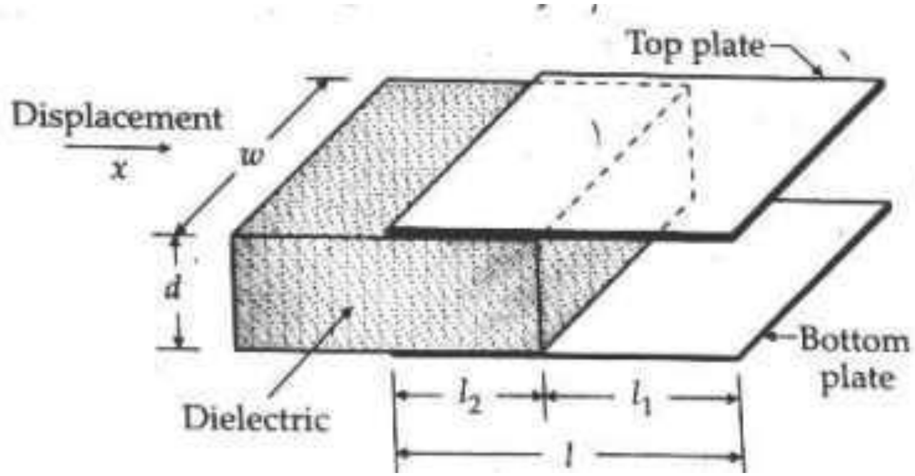
$$= \varepsilon_0 \frac{w}{d}(l_1 + \varepsilon_r l_2) + \varepsilon_0 \frac{wx}{d}(\varepsilon_r - 1)$$

$$= C + \varepsilon_0 \frac{wx}{d}(\varepsilon_r - 1)$$

Change in capacitance

$$\Delta C = \varepsilon_0 \frac{wx}{d}(\varepsilon_r - 1)$$

Hence the change in capacitance is proportional to displacement



➢ Advantage of Capacitive Transducer :

The following are the major advantages of capacitive transducers.

1) It requires an external force for operation and hence very useful for small systems.
2) The capacitive transducer is very sensitive.
3) It gives good frequency response because of which it is used for the dynamic study.
4) The transducer has high input impedance hence they have a small loading effect.
5) It requires small output power for operation.

Disadvantages of capacitive Transducer:

The main disadvantages of the transducer are as follows.

1) The metallic parts of the transducers require insulation.
2) The frame of the capacitor requires earthing for reducing the effect of the stray magnetic field.
3) Sometimes the transducer shows the nonlinear behaviors because of the edge effect which is controlled by using the guard ring.
4) The cable connecting across the transducer causes an error.

Uses(applications) of Capacitive Transducer:

The following are the uses of capacitive transducer.

1) The capacitive transducer uses for measurement of both the linear and angular displacement. It is extremely sensitive and used for the measurement of very small distance.
2) It is used for the measurement of the force and pressures. The force or pressure, which is to be measured is first converted into a displacement, and then the displacement changes the capacitances of the transducer.

- **Capacitive Transducer is used as a pressure transducer** in some cases, where the dielectric constant of the transducer changes with the pressure.

- The humidity in gases is measured through the capacitive transducer.
- The transducer uses the mechanical modifier for measuring the volume, density, weight etc.
- The accuracy of the transducer depends on the variation of temperature to the high level.
- In Capacitive pressure transducer diaphragm acts as one of the plates of a two plate capacitor while other plate is fixed.
- The fixed plate and the diaphragm are separated by a dielectric material. When a force is applied to the diaphragm, it changes its position from initial static position obtained with no force applied.
- Due to this, the distance of separation between the fixed plate and the diaphragm changes, hence the capacitance also changes
- The change in the capacitance can be measured by using any simple ac bridge.

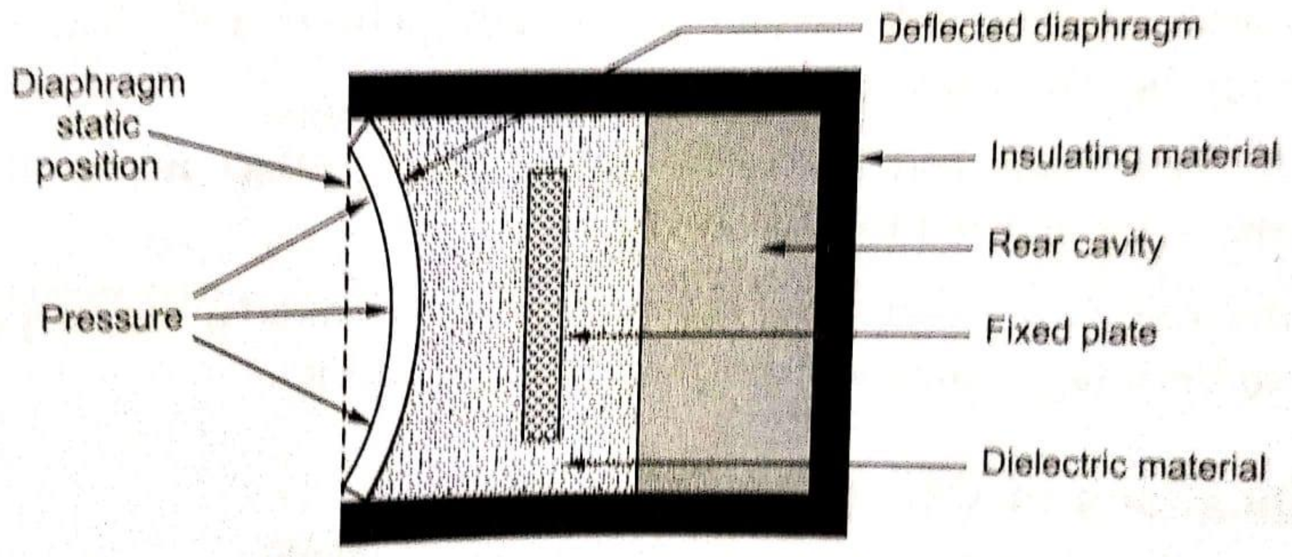


Fig. 6.43 Capacitive pressure transducer

- But practically the change in capacitance is measured using an oscillator circuit where capacitive transducer is part of that circuit. In this way, by using capacitive transducer, applied force can be measured in terms of change in the capacitance.

Temperature sensors:

- Temperature is one of the fundamental parameters indicating the physical condition of matter i,e. expressing the degree of hotness or coldness.

  Whenever a body is heated, various effects are observed. They include:
  - Change in the physical state,
  - change in physical dimensions,
  - changes in electrical properties,
  - mainly the change in resistance,
  - Generation of emf at the junction of two dissimilar metals.

- One of these effects can be employed for temperature measurement purposes. Electrical methods are the most convenient and accurate methods of temperature measurement. These methods are based on change in resistance with temperature and generation of thermal emf.
- The change in resistance with temperature may be positive or negative. According to that there are two types:

  Resistance Thermometer- Positive Temperature Coefficient

  Thermistor- Negative Temperature Coefficient

Resistance Temperature Detector(RTD):

- Resistance Temperature Detector is a primary electrical transducer which is used to measure the change in the temperature. It is commonly known as Resistance Thermometer.
- The resistance thermometers are based on the principle that the resistance of the conductor changes when the temperature changes.

The relationship between temperature and resistance of conductor is given by:

$$R_t = R_{ref} [1+\alpha \Delta t]$$

Where R_t : resistance of the conductor at temperature at t^oC

R_{ref} : resistance of the conductor at the reference temperature usually 0^oC

α : Temperature coefficient of resistance

Δt: Difference between the temperature to be measured & reference temperature

- Almost all metallic conductors have a positive temperature coefficient so that their resistance increases with an increase in temperature.
- A high value of α is desirable in a temperature sensing element so that a substantial change in resistance occurs for a relatively small change in temperature.
- This change in resistance ΔR can be measured with a wheat stone bridge, the output of which can be directly calibrated to indicate the temperature which caused the change in resistance.

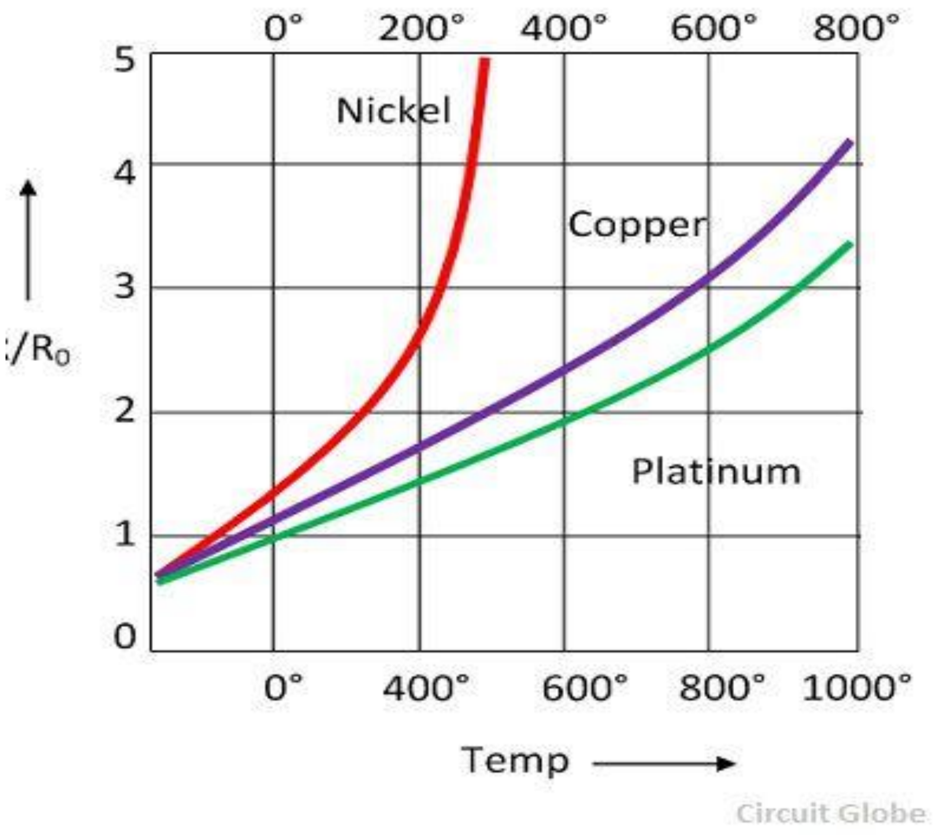
metal	Resistance temperature coefficient / °C	Temperature Range ° C	Melting point ° C
Platinum	0.39	-260 to 1110	1773
Copper	0.39	0 to 180	1083
Nickel	0.62	-220 to 300	1435
Tungsten	0.45	-200 to 1000	3370

Table: Metals used for resistance thermometers

The following are the requirements of the conductor used in the RTDs:

- The resistivity of the material is high so that the minimum volume of conductor is used for construction.
- The change in resistance of the material concerning temperature should be as high as possible.
- The resistance of the material depends on the temperature.

The resistance versus temperature curve is shown in the beside figure . The curves are nearly linear, and for small temperature range, it is very evident.

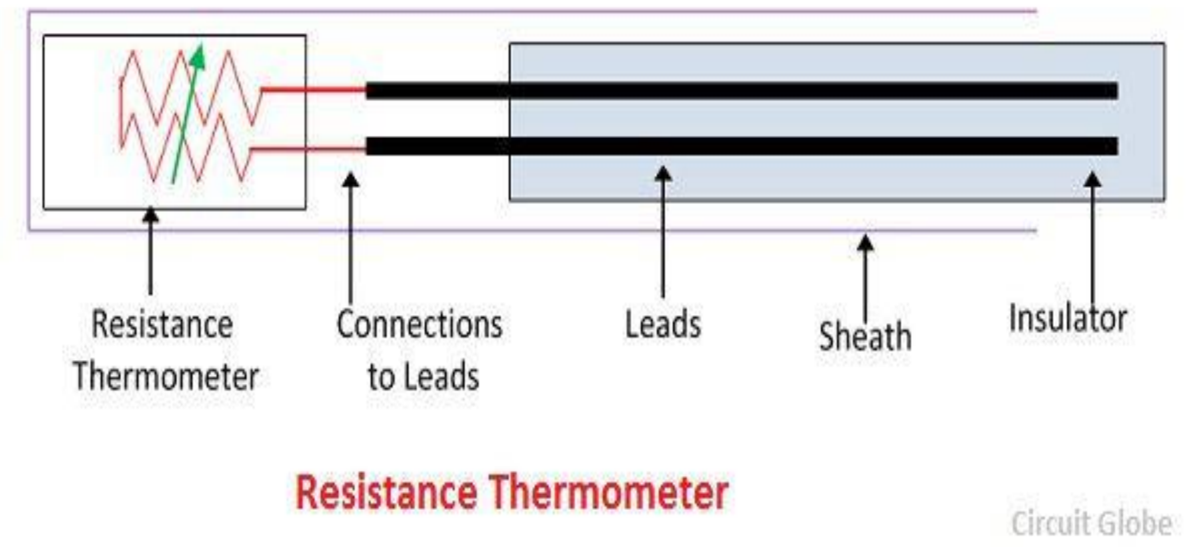


Applications of RTD

- RTD sensor is used in automotive to measure the engine temperature, an oil level sensor, intake air temperature sensors. In communication and instrumentation for sensing the over the temperature of amplifiers, transistor gain stabilizers, etc…
- RTD is used in power electronics, computer, consumer electronics, food handling and processing, industrial electronics, medical electronics, military, and aerospace.

Construction of Resistive Thermometer:

- The resistance thermometer is placed inside the protective tube for providing the protection against damage. The resistive element is formed by placing the platinum wire on the ceramic bobbin.
- This resistance element is placed inside the tube which is made up of stainless steel or copper steel.
- The lead wire is used for connecting the resistance element with the external lead.
- The lead wire is covered by the insulated tube which protects it from short circuit.
- The ceramic material is used as an insulator for high-temperature material and for low-temperature fiber or glass is used.



$$R_0(1+\alpha_1 T+\alpha_2 T^2+\cdots+\alpha_n T^n+\cdots)\ldots\ldots equ(1)$$

Where, R_0 – resistance at temperature T = 0 and α_1, α_2, α_3……..α_n are constants.

Operation of Resistance Thermometer:

- The tip of the resistance thermometer is placed near the measurand heat source. The heat is uniformly distributed across the resistive element. The changes in the resistance vary the temperature of the element. The final resistance is measured. The beside mention equations measure the variation in temperature.

Linear Approximation:

The linear approximation is the way of estimating the resistance versus temperature curve in the form of the linear equation.

$$R_\theta = R_{\theta_0}(1 + a_{\theta_0}\Delta\theta)$$

where R_θ –approximation resistance at θ°C
$R_{\theta 0}$ –approximation resistance at $\theta_0{}^0C$
Δθ – change in temperature ºC and the
$\alpha_{\theta 0}$ – resistance temperature coefficient at θ_0ºC

Quadratic Approximation:

The quadratic approximation gives the accurate approximation of the resistance temperature curve. The approximation is expressed in the form of the quadratic equation.

$$R_\theta = R_{\theta_0}[1 + \alpha_1\Delta\theta + \alpha_2(\Delta\theta)^2]$$

α_1 – linear fractional change in resistance
α_2 – quadratic function change in resistance.

Advantages of RTD:

1. The measurement is accurate.
2. Indicators, recorders can be directly operated.
3. The temperature sensor can be easily installed and replaced.
4. Measurement of differential temperature is possible.
5. They are smaller in size.
6. They have stability over long periods of time.

Disadvantages of RTD:

1. A bridge circuit with external power source is necessary for their operation.
2. They are comparatively costly.
3. There is a possibility of self heating.

THERMISTORS

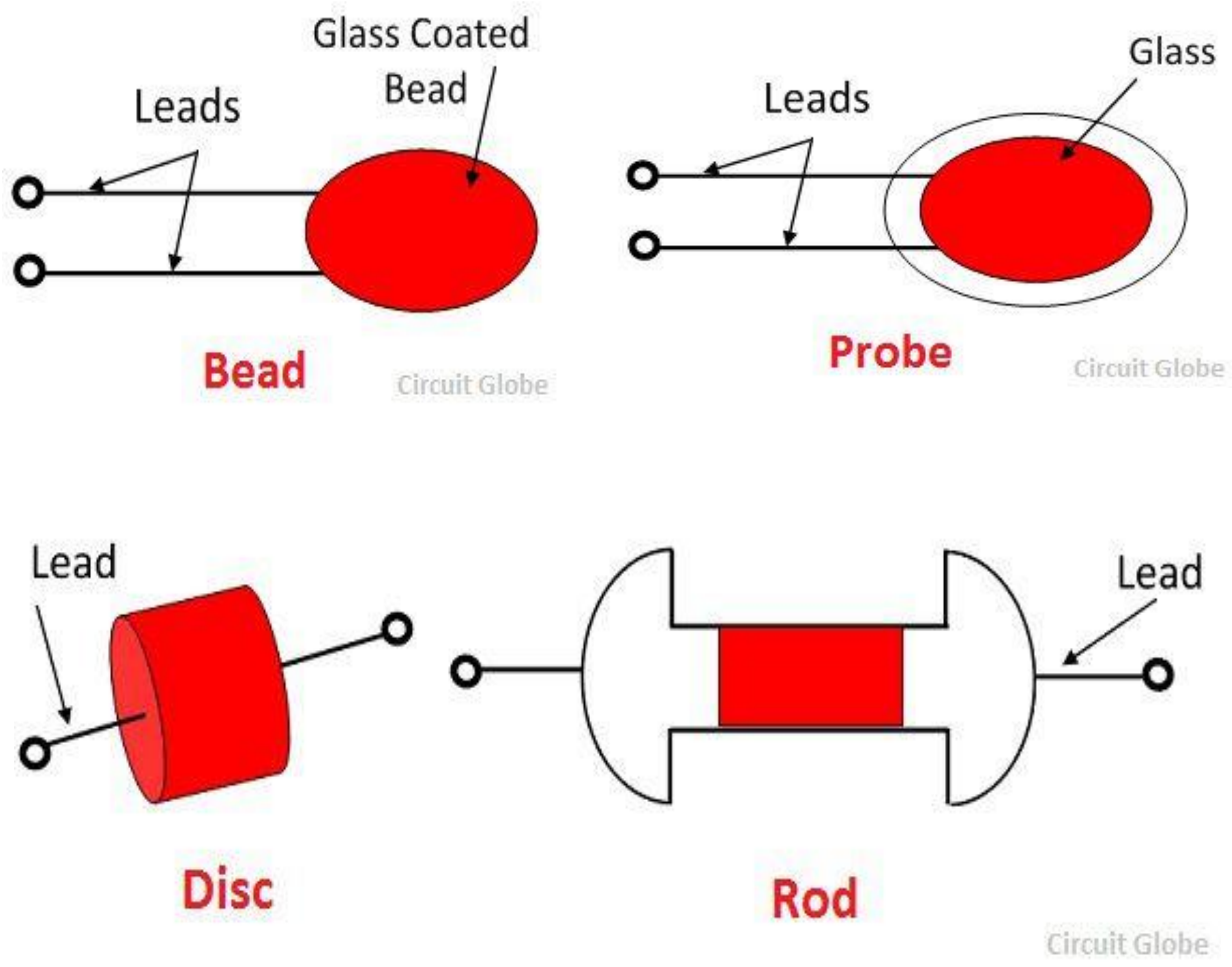
➢ THERMISTORS:

- Coined from the words **"THERMally controlled resISTOR",** thermistor is a temperature controlled resistor. Thermistors are one of the most commonly used devices for the measurement of temperature.
- The thermistors are resistors whose resistance changes with the temperature.
- While for most of the metals the resistance increases with temperature, the thermistors respond negatively to the temperature and their resistance decreases with the increase in temperature.
- Since the resistance of thermistors is dependent on the temperature, they can be connected in the electrical circuit to measure the temperature of the body.
- In some cases the resistance of the thermistors at room temperature may decrease as much as 5 percent for each 1^{o}C rise in temperature.
- This high sensitivity to temperature changes makes thermistors extremely useful for precision temperature measurements control and compensation.
- The thermistor is made of the semiconductor material that means their resistance lies between the conductor and the insulator.
- The variation in the thermistor resistance shows that either conduction or power dissipation occurs in the thermistor.

➢ Materials used for Thermistors and their Forms:

- The thermistors are made up of ceramic like semiconducting materials. They are mostly composed of oxides of manganese, nickel and cobalt having the resistivities of about 100 to 450,000 ohm-cm.
- Since the resistivity of the thermistors is very high the resistance of the circuit in which they are connected for measurement of temperature can be measured easily.
- This resistance is calibrated against, the input quantity, which is the temperature, and its value can be obtained easily.
- Thermistors are available in various shapes like disc, rod, washer, bead etc.
- They are of small size and they all can be fitted easily to the body whose temperature has to be measured and also can be connected to the circuit easily. Most of the thermistors are quite cheap.

➢ The different types of the thermistors are shown in the figure below



- The bead form of the thermistor is smallest in shape, and it is enclosed inside the solid glass rod to form probes.
- The disc shape is made by pressing material under high pressure with diameter range from 2.5 mm to 25mm.

Principle of Working of Thermistors:

- The Thermistor's temperature can change either due to external factors or due to internal factors. The most important internal factor is the current flowing through the device.
- As the current through it increases, it starts self heating its elements.
- This causes a rise in temperature of the Thermistor.

- Depending on the type of Thermistor (whether NTC or PTC), its resistance changes with respect to this change in temperature. Externally the Thermistor temperature can be changed by changing the ambient temperature.
- The resistance and temperature relationship can be approximated by the following equation:

$$R = R_0\, e^{\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)} \qquad \text{.....}\ (1)$$

Resistance – Temperature Relation Equation of Thermistor

Where,

R = Resistance of Thermistor at the temperature T (in K)

R0 = Resistance at given temperature T_0 (in K)

β = Material specific-constant

In terms of temperature coefficient of resistance this equation can be defined as:

R = Ro [1+α(T-To)] ....(2)

β is a constant whose value ranges from 3500 to 4500^0K depending on the material used for the thermistors and its composition.

- The thermistor acts as the temperature sensor and it is placed on the body whose temperature is to be measured.
- It is also connected in the electric circuit. When the temperature of the body changes, the resistance of the thermistor also changes, which is indicated by the circuit directly as the temperature since resistance is calibrated against the temperature.
- The thermistor can also be used for some control which is dependent on the temperature.

➢ **Types of Thermistors:**

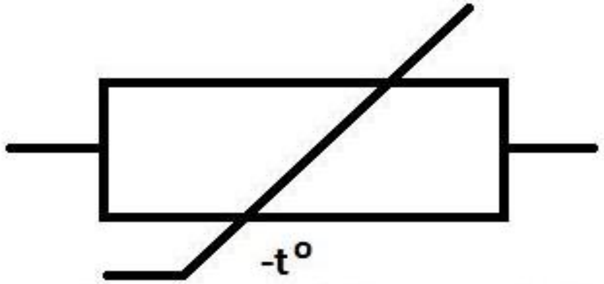
The thermistor is classified into two types. They are

1) the negative temperature coefficient and
2) the positive temperature coefficient thermistor.

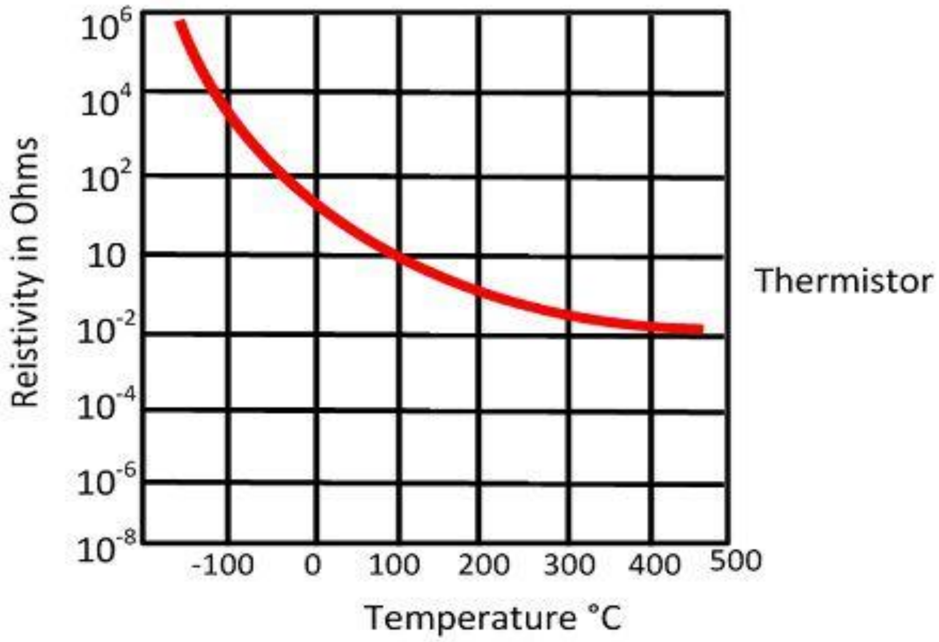
❑ **(1) NTC Thermistor:**

- ***Definition – NTC or negative temperature coefficient thermistor*** is a device whose resistance decreases with increase in temperature. These types of resistor usually exhibit a large, precise and predictable decrease in resistance with increase in temperature.
- ***Material Used for Construction –*** Unlike other resistors (fixed or variable), these are made of ceramics and polymers, which composed of metal oxides that are dried and sintered to obtain a desired form factor. In case of NTC thermistor, cobalt, nickel, iron and copper oxides are preferred.

- ***NTC Thermistor Symbol –*** *The symbol for NTC thermistor is given as:*



- ***Characteristic Curve –***
- The resistance temperature coefficient of the thermistor is shown in the figure beside. The graph below shows that the thermistor has a negative temperature coefficient, i.e., the temperature is inversely proportional to the resistance.
- The resistance of the thermistor changes from 10^5 to 10^{-2} at the temperature between -100C to 400C. From the figure we can say that they have a steep resistance temperature curve, denoting good temperature sensitivity.
- However due to the nonlinear relationship between resistance and temperature, some approximations are utilized to design practical system.
- Out of all the approximations, the simplest one is:
- $\Delta R = k\Delta T$, where k is the negative temperature coefficient of the Thermistor.



Resistance Temmperature Characteristic

(2)PTC Thermistor:

Definition – *PTC or Positive temperature coefficient Thermistors are those resistors whose* resistance increases with increase in ambient temperature.

Types of PTC Thermistors – *PTC Thermistors are grouped according to*

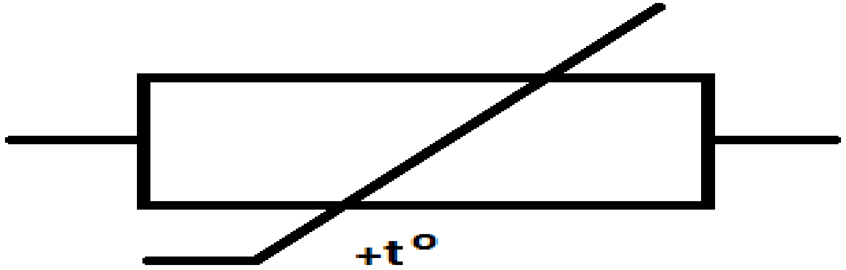
i. *their structure,*

ii. *materials* used and

iii. their manufacturing process.

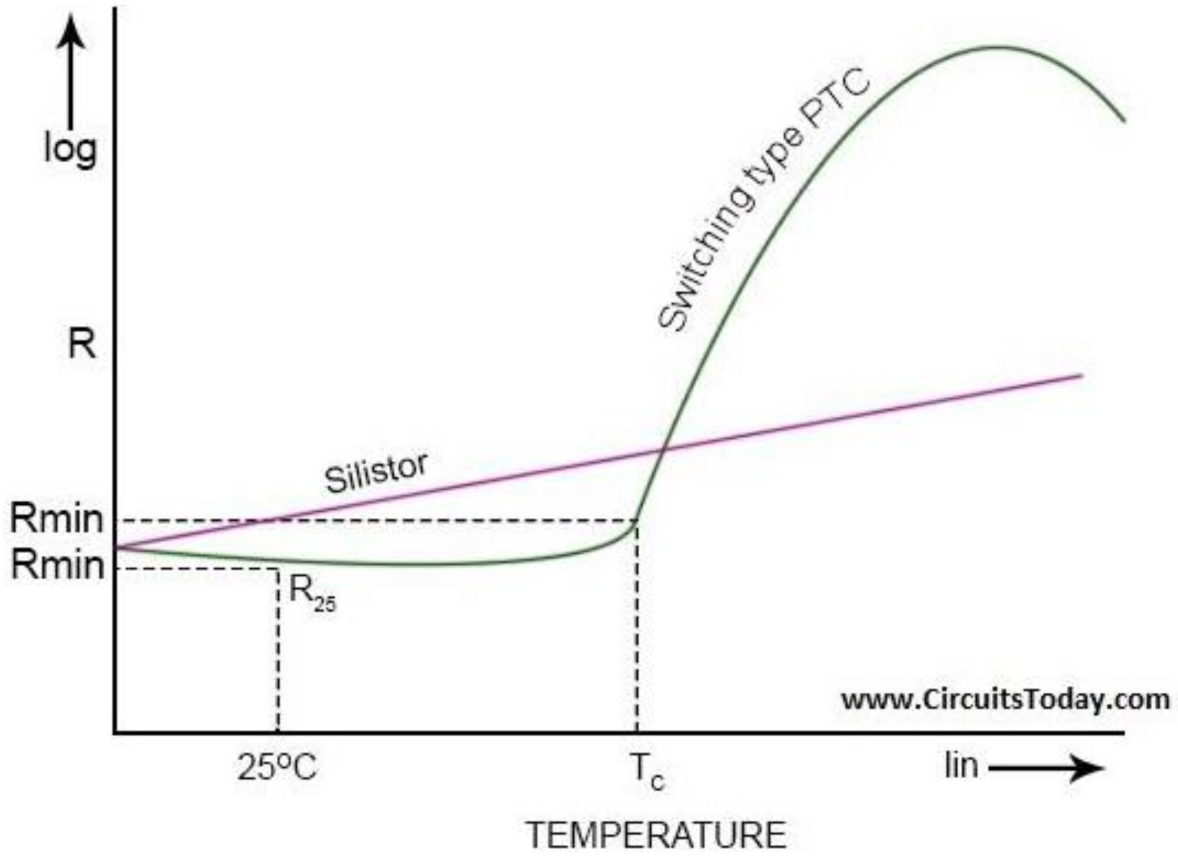
- Silistors, are PTC Thermistors that belong to the first group(according to material used and structure). They use silicon as the semiconductor and have linear characteristic.
- Switching type PTC Thermistors belong to the second category (according to the manufacturing process).
- This Thermistor has a non linear characteristic curve. As the switching type PTC Thermistor gets heated, initially the resistance starts to decrease, up to a certain critical temperature, after which as the heat is increased, the resistance increases dramatically.

***PTC Thermistor Symbol** – The following figure shows the symbol used for PTC Thermistors in a* circuit diagram.



PTC Thermistor Symbol - IEC Standard
www.CircuitsToday.com

- **Characteristic Curve –**
- *The following figure shows the characteristic curve of a Silistor and a* switching type PTC Thermistor.
- We see that, a silistor PTC has a linear characteristic. This means that this PTC Thermistor is quite sensitive to the change in temperature. Its resistance increases linearly with increase in temperature.
- The switching type PTC however, is different.
- Due to its poly-crystalline ceramic body, has a nonlinear characteristic curve.
- We see from the figure that upto a certain temperature, lets call it a threshold temperature, the resistance decreases with increase in temperature much like a NTC Thermistor.
- As the temperature increases beyond the threshold temperature, the resistance starts to increase dramatically with increase in temperature.



Silistor V/S PTC Thermistor - Characteristic Curve

Advantages of Thermistor:

The following are the advantages of the thermistor:

1) The thermistor is compact, long durable and less expensive.
2) The properly aged thermistor has good stability.
3) The response time of the thermistor changes from seconds to minutes. Their response time depends on the detecting mass and the thermal capacity of the thermistor.
4) The upper thermistor limit of the temperature depends on the physical variation of the material, and the lower temperature depends on the resistance reaching a large value.
5) The self-heating of the thermistor is avoided by minimising the current passes through it.
6) The thermistor is installed at the distance of the measuring circuit. Thus the reading is free from the error caused by the resistance of the lead.

Limitations:

1) The resistance vs Temperature characteristics is highly non-linear.
2) Not suitable over a wide temperature range.
3) Because of high resistance of thermistor, shielded cables have to be used to minimize interference.

Applications of Thermistors:

1) Useful as temperature transducers.
2) To compensate the effects of temperature on circuit performance due to NTC.
3) For pressure, flow, liquid level measurements.
4) For measurement of thermal conductivity.
5) For Vaccum measurements.
6) For measurement of composition of gasess.

Key Differences Between RTD and Thermistor:

The following are the key differences between the RTD and the Thermistor:

- The RTD is a type of instrument used for measuring the temperature, whereas, the thermistor is the thermal resistor whose resistance changes with temperature.
- The RTD is made of the metals having a positive temperature coefficient whereas the thermistor is made of the semiconductor materials.
- The accuracy of the RTD is low as compared to the thermistor. The thermistor has a negative temperature coefficient because of which they can detect even the smallest change in temperature.

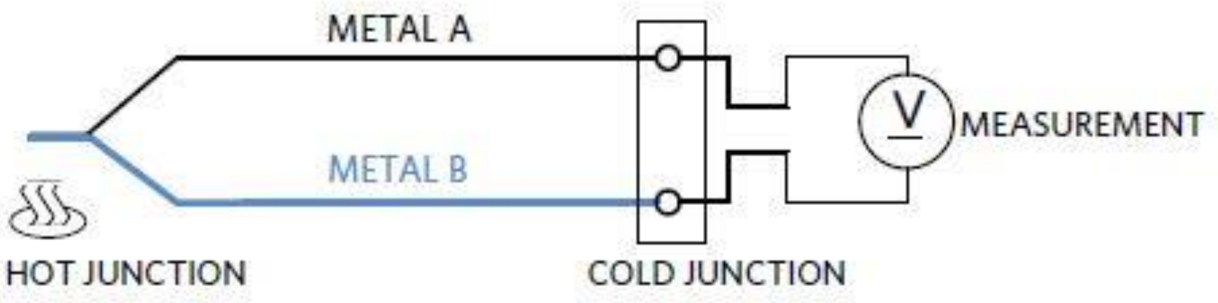
- The thermistor gives a quick response to the small changes, whereas the response time of the RTD is slow.
- The thermistor is used for measuring the small range of temperature up to 130ºC whereas the RTD measures the temperature up to 660ºC.
- The characteristic graph between resistance and temperature of the RTD is linear whereas that of thermistor it is nonlinear.
- The RTD is less sensitive as compared to the thermistor.
- The size of the RTD is much larger as compared to the thermistor.
- The cost of the thermistor is much higher than the RTD.
- The resistivity of the RTD is less as compared to the thermistor.
- The hysteresis effect in the thermistor is much high as compared to the RTD.
- The RTD is used in the industries for measuring the high temperature, whereas the thermistor is used in home appliances for measuring the small temperature.

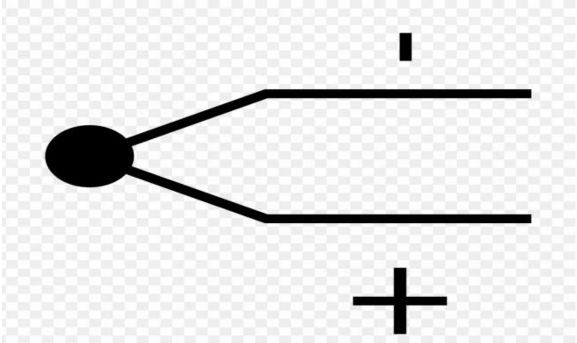
THERMOCOUPLE

Thermocouple:

➢ Definition:

- The thermocouple is a temperature measuring device. It uses for measuring the temperature at one particular point. In other words,
- it is a type of sensor used for measuring the temperature in the form of an electric current or the EMF.
- The thermocouple consists two wires of different metals which are welded together at the ends.
- The welded portion was creating the junction where the temperature is used to be measured.
- The variation in temperature of the wire induces the voltages.





Symbol

THERMOCOUPLE

Working Principle of Thermocouple:

The working principle of the thermocouple depends on the three effects.

1).Seebeck Effect –

- The Seebeck effect occurs between two different metals. When the heat provides to any one of the metal, the electrons start flowing from hot metal to cold metal. Thus, direct current induces in the circuit.
- In short, it is a phenomenon in which the temperature difference between the two different metals induces the potential differences between them. The Seeback effect produces small voltages per Kelvin of temperature.

2).Peltier Effect –

- The Peltier effect is the inverse of the Seeback effect. The Peltier effect state that "the temperature difference can be created between any two different conductors by applying the potential difference between them".

3).Thompson Effect –

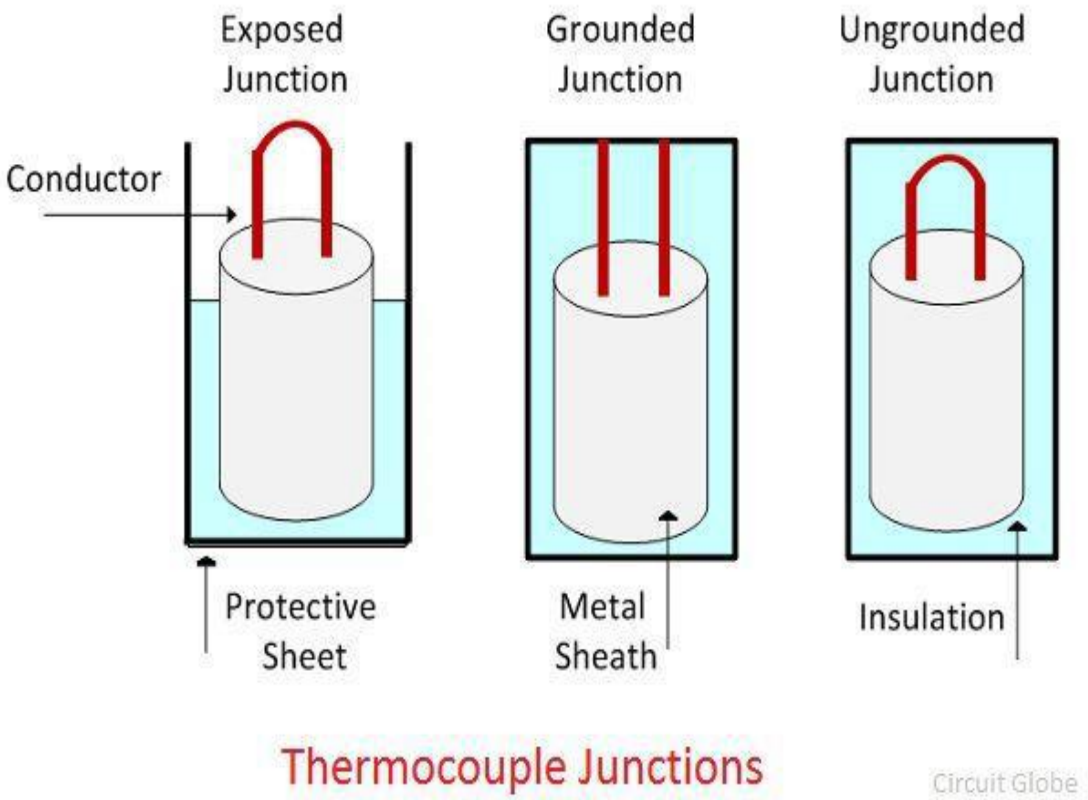
- The Thompson effect state that "when two dissimilar metals join together and if they create two junctions then the voltage induces the entire length of the conductor because of the temperature gradient".
- The temperature gradient is a physical term which shows the direction and rate of change of temperature at a particular location.

➢ Construction of Thermocouple:

- The thermocouple consists two dissimilar metals. These metals are welded together at the junction point.
- This junction considers as the measuring point.
- The junction point categorizes into three types.

1. Ungrounded Junction –

- In ungrounded junction, the conductors are entirely isolated from the protective sheath.
- It is used for high-pressure application works.
- The major advantage of using such type of junction is that it reduces the effect of the stray magnetic field.



Thermocouple Junctions

2.Grounded Junction –

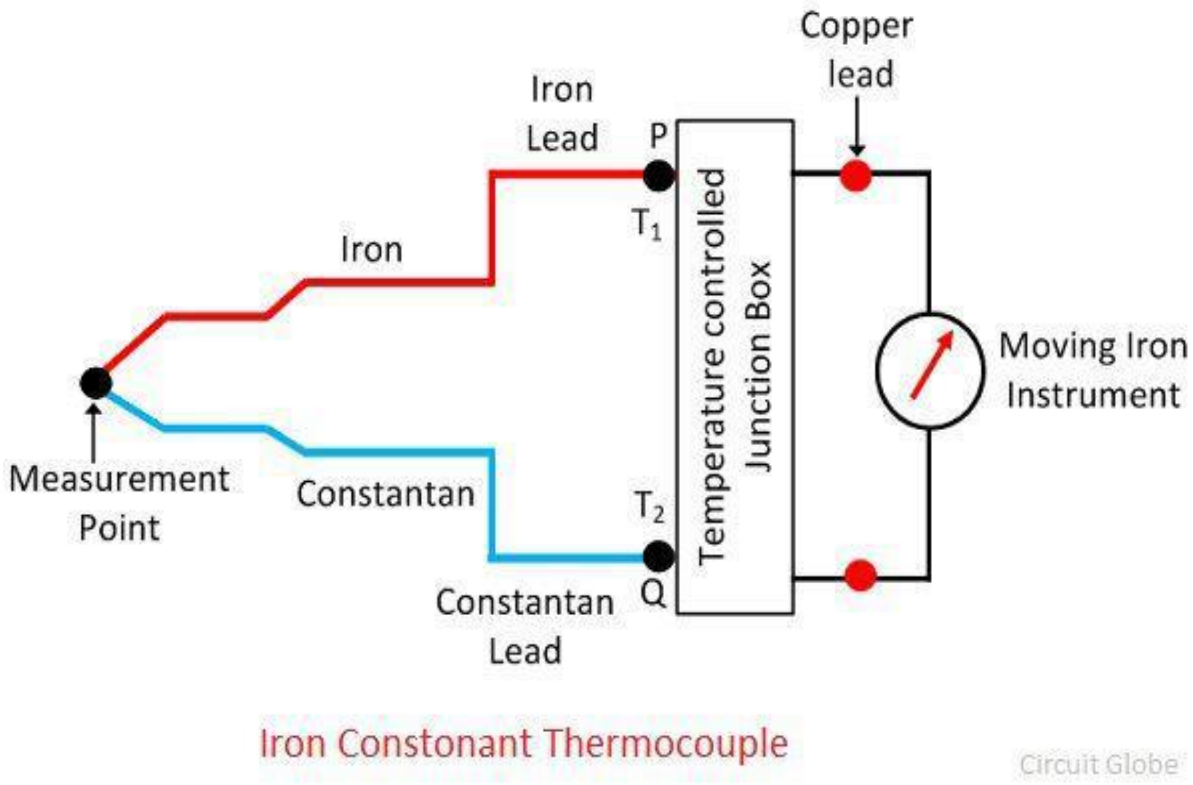
- In such type of junction the metals and protective sheath are welded together. The grounded junction use for measuring the temperature in the corrosive environment. This junction provides resistance to the noise.

3. Exposed Junction –

- Such type of junction uses in the places where fast response requires. The exposed junction is used for measuring the temperature of the gas.
- The material uses for making the thermocouple depend on the measuring range of temperature.

Working of Thermocouple:

- The circuit of the thermocouple is shown in the figure.
- The circuit consists two dissimilar metals.
- These metals are joined together in such a manner that they are creating two junctions.
- The metals are bounded to the junction through welding.



Iron Constonant Thermocouple

- Let the P and Q are the two junctions of the thermocouples.
- The T1 and T2 are the temperatures at the junctions. As the temperature of the junctions is different from each other, the EMF generates in the circuit.
- Thermocouples are used for measurement of temperature up to 1400^0C. If the temperature at the junction becomes equal, the equal and opposite EMF generates in the circuit, and the zero current flows through it.

- If the temperatures of the junction become unequal, the potential difference induces in the circuit.
- The magnitude of the EMF induces in the circuit depends on the types of material used for making the thermocouple. The total current flowing through the circuit is measured through the measuring devices.
- The EMF induces in the thermocouple circuit is given by the equation:

$$E = a(\Delta\theta) + b(\Delta\theta)^2$$

Where Δθ – temperature difference between the hot thermocouple junction and the reference thermocouple junction. a, b – constants

➢ Measurement of Thermocouple Output:

The output EMF obtained from the thermocouples can be measured through the following methods.

1. Multimeter –

- It is a simpler method of measuring the output EMF of the thermocouple. The multi meter is connected to the cold junctions of the thermocouple. The deflection of the multi meter pointer is equal to the current flowing through the meter.

2. Potentiometer –

- The output of the thermocouple can also be measured with the help of the DC potentiometer.

3. Amplifier with Output Devices –

- The output obtains from the thermocouples is amplified through an amplifier and then feed to the recording or indicating instrument.

Desirable characteristics of Thermocouple:

- The temperature – emf relationship should be reasonably linear.
- The thermocouple should generate sufficient thermo emf per degree of temperature changes to facilitate detection and measurement.
- The cost should be reasonable.
- The material should be physically able to withstand sustained high temperature, rapid temperature change and the effects of corrosive atmosphere.

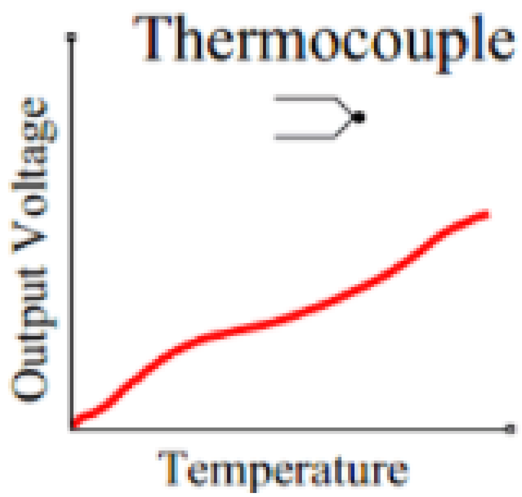


Fig.Characteristics curve

Materials used for thermocouples:

Materials used	Type of thermocouple	Temp: range
Copper-Constantan	T	-250 to 400
Iron-Constantan	J	-200 to 850
Chromel-Alumel	K	-200 to 110
Chromel-Constantan	E	-200 to 850
Platinum-Rhodium	S	0 to 1400
Tungsten-Molybdenum	--	0 to 2700
Tungsten-Rhenium	--	0 to 2600

Beside is the table illustrating the ranges of temperature measurements in the thermocouples of different materials

➢ Advantages of Thermocouples:

The following are the advantages of the thermocouples.

1) The thermocouple is comparatively cheaper in cost.
2) The thermocouple has the fast response time.
3) It has a wide temperature range from -270°C to 2700°C
4) The thermocouple offers good reproducibility
5) Measurement accuracy is quite satisfactory
6) The calibration can be easily checked

➢ Disadvantages of the Thermocouples:

1) The emf induced versus temperature characteristics is somewhat nonlinear.
2) For accurate temperature measurements, cold junction compensation is necessary.
3) In many applications, amplification of signal is required.
4) They are less suitable for applications where smaller temperature differences need to be measured with high accuracy, for example the range 0–100 °C with 0.1°C accuracy.For such applications thermistors, silicon bandgap temperature sensors and resistance thermometers are more suitable.

Applications

Some of the **applications of thermocouples** are

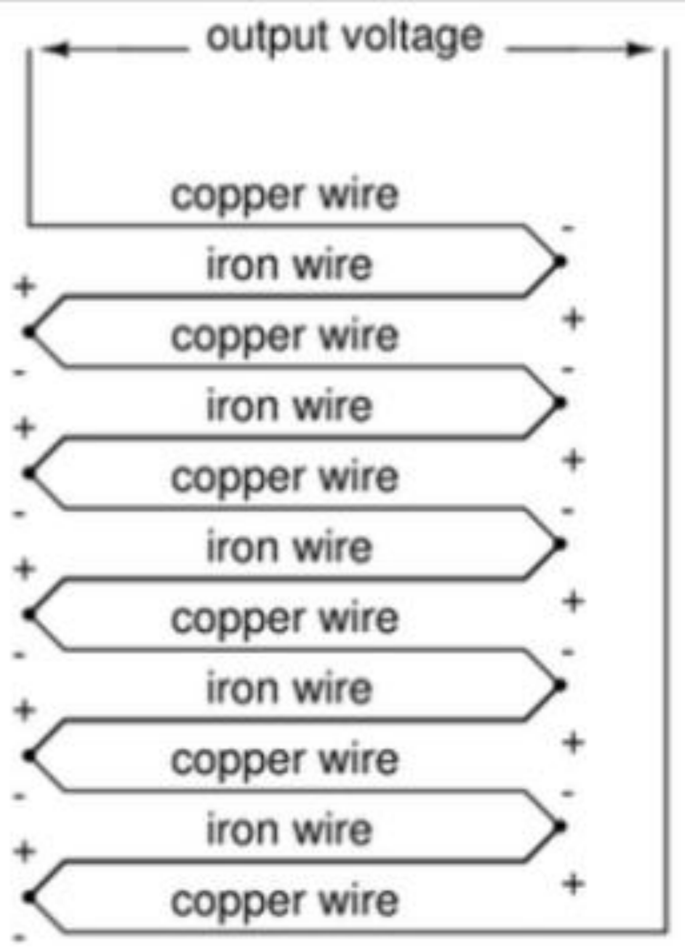
1) These are used as the temperature sensors in thermostats in offices, homes, offices & businesses.
2) These are used in industries for monitoring temperatures of metals in iron, aluminum, and steel industries
3) These are used in the food industry for cryogenic and Low-temperature applications.
4) Thermocouples are used as heat pumps for performing thermoelectric cooling.
5) These are used to test temperature in chemical plants, petroleum plants.
6) These are used in gas machines for detecting the pilot flame.
7) Electric arc furnaces and Thermocouples are widely used in science and industry. Applications include temperature measurement for kilns, gas turbine exhaust, diesel engines, and other industrial processes.

THERMOPILES

- Thermopile is a device that converts thermal energy into an electrical signal. A thermopile is composed of several thermocouples connected usually in series or rarely in parallel.

- A series of thermocouples connected together is called a thermopile, shown in the beside fig:
- Using thermopile, we can get more sensitive element as compared to single thermocouple. In Thermopile, all measuring junctions are at same temperature while all the reference junctions are at another temperature as shown in fig(a).

- The care must be taken to ensure accurate reading that individual thermocouples are electrically insulated from each other.
- The chromel-constantan thermopile provides 1mV/°C. It consists of 25 thermocouples. By using this thermopile, the temperature change as small as 0.0001°F can be detected.

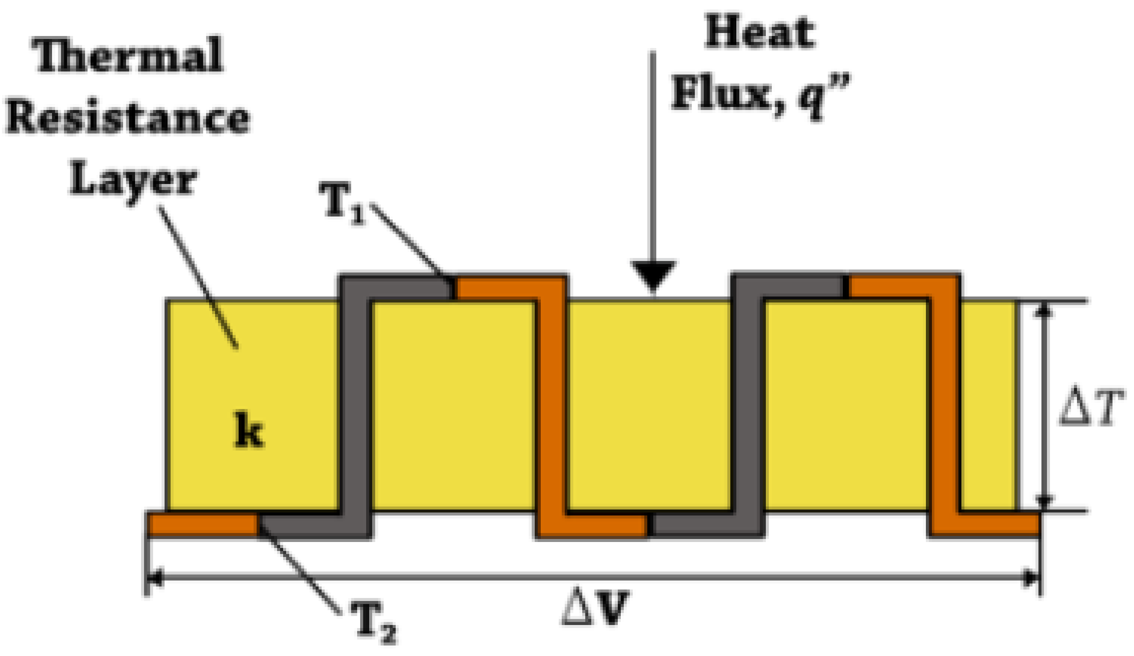


Fig(a) Thermopile connection

➢ Thermopile connections:

- A thermopile is a series of thermocouples interconnected in series, each of which consists of two different materials with a large thermoelectric power and opposite polarities.

- The thermocouples are placed through the hot and cold regions of a structure and the hot junctions are thermally isolated from the cold junctions.



Fig(b) Thermopile connections

- With a couple of thermocouples connected in series. The two top thermocouples are a connected junction are at temperature T1 while the two bottom thermocouple junction is at temperature T2.

- The output voltage from the thermopile delta V is directly proportional to the difference in temperature or T1-T2, across the thermal resistance layer and the number of thermocouple junctions pairs as shown in Fig(b).

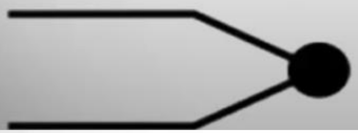
➢ Applications of Thermopiles:

1) Non-contact temperature measurement in the process industry
2) Handheld non-contact temperature measurement
3) Thermal line scanner
4) Commercial building HVAC and lighting control
5) Security human presence and detection
6) Black ice detection and early warning
7) Blood glucose monitoring
8) Automatic HVAC control
9) Fire detection in transportation tunnel
10) Aircraft flame and fire detection
11) Automatic exhaust gas analysis

Key Differences between Thermocouple and Thermistor

- The thermocouple is the temperature measuring devices and thermistor is a type of thermal resistor.
- The thermocouples are made from metal or alloys of metals. While the thermistor is made by the semiconductor or by the oxides of magnesium, nickel or cobalt.
- The thermocouple has high accuracy as compared to the thermistor. The thermistor has lead whose resistance reduces their accuracy.
- The temperature measuring ranges of the thermistor is -50°C to 250°C whereas that of the thermocouple is -200°C to 2000°C.
- The thermistor gives the quick response as compared to the thermocouples. The response time of both the sensors depends on their size.
- In thermocouples, the variation in temperature is determined by the voltage induces at their junction. The resistance of the thermistor changes when their surrounding temperature varies.
- The characteristic curve of the thermocouple between voltage and current is linear. The curve shows that the voltage of the thermocouple increases concerning the temperature. While in the thermistor, the characteristic curve between the resistance and temperature is non-linear when the thermistor has a negative temperature coefficient. The non-linear curve of thermistor shows that their resistance decreases with the increases in temperature.
- The thermocouple is more expensive as compared to the thermistor.
- The thermistor and thermocouple both are used for controlling and measuring temperature.
- The thermocouples are used in large industries, while the thermistors are used in home appliances.

Comparison of Thermocouple and Thermistor

Thermocouple	Thermistor
The thermocouple is a type of device used for measuring the temperature.	Thermistor is the thermal resistor whose resistance changes with the temperature.
The thermocouples are made from metal or alloys of metals such as Copper, iron, chromium and platinum etc.	The thermistor is made by the semiconductor or oxides of magnesium, nickel or cobalt.
Symbol of Thermocouple. 	Symbol of Thermistor. 
The thermocouple has high accuracy.	The thermistor has very low accuracy.
The temperature measuring ranges of the thermocouple is -200°C to 1250°C.	The temperature measuring ranges of the thermistor is -50°C to 250°C.
In thermocouples, the variation in temperature is determined by the voltage induces at their junction.	The resistance of the thermistor changes when their surrounding temperature varies.
The thermocouple is expensive as compared to the thermistor.	The thermistor is Cheap.

PIEZOELECTRIC TRANSDUCER

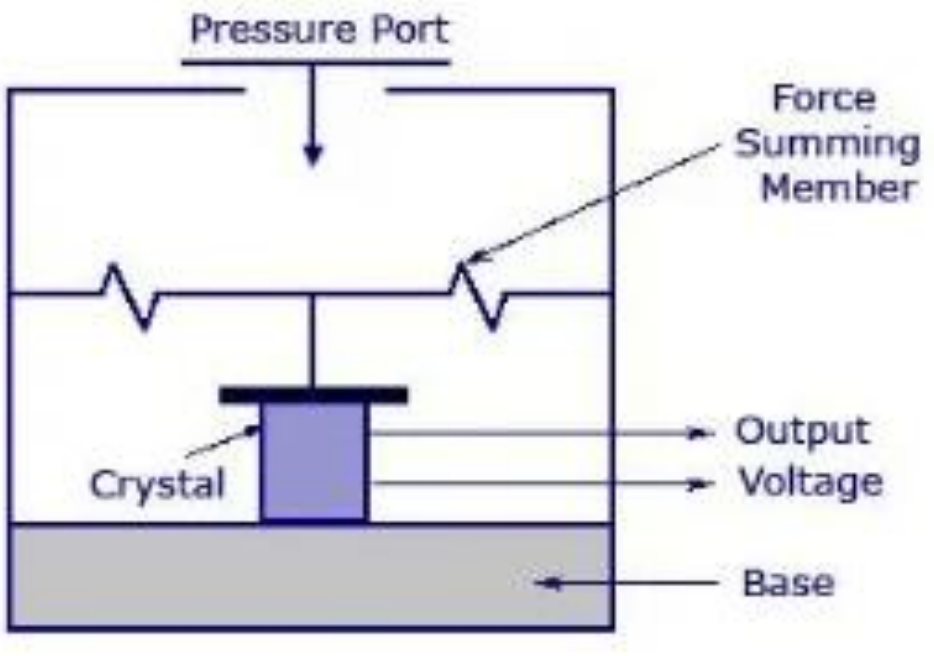
Piezo-Electric Transducer

- **What is Piezoelectric Transducer?**

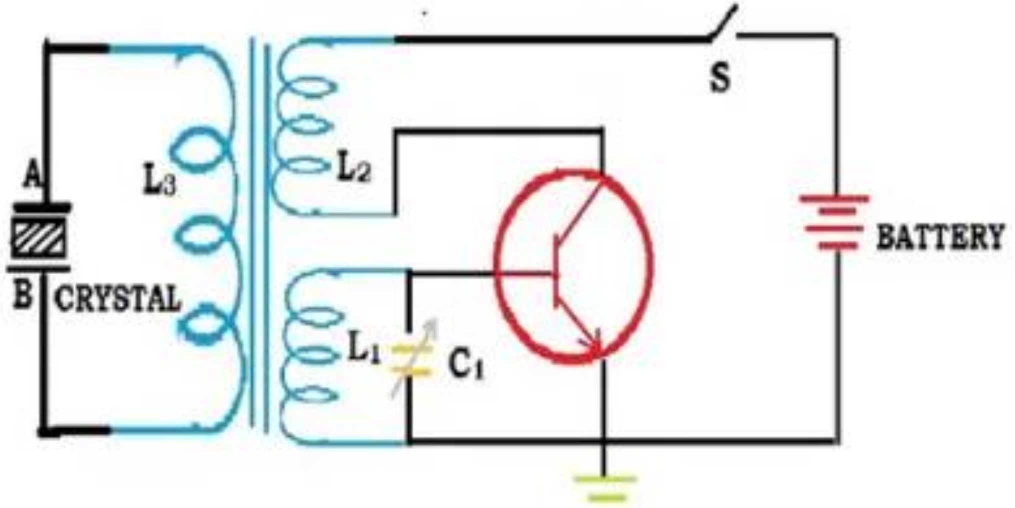
- A **piezoelectric transducer** (also known as a piezoelectric sensor) is a device that uses the piezoelectric effect to measure changes in acceleration, pressure, strain, temperature or force by converting this energy into an electrical charge.

- A transducer can be anything that converts one form of energy to another. The piezoelectric material is one kind of transducers.

- When we squeeze this piezoelectric material or apply any force or pressure, the transducer converts this energy into voltage. This voltage is a function of the force or pressure applied to it.

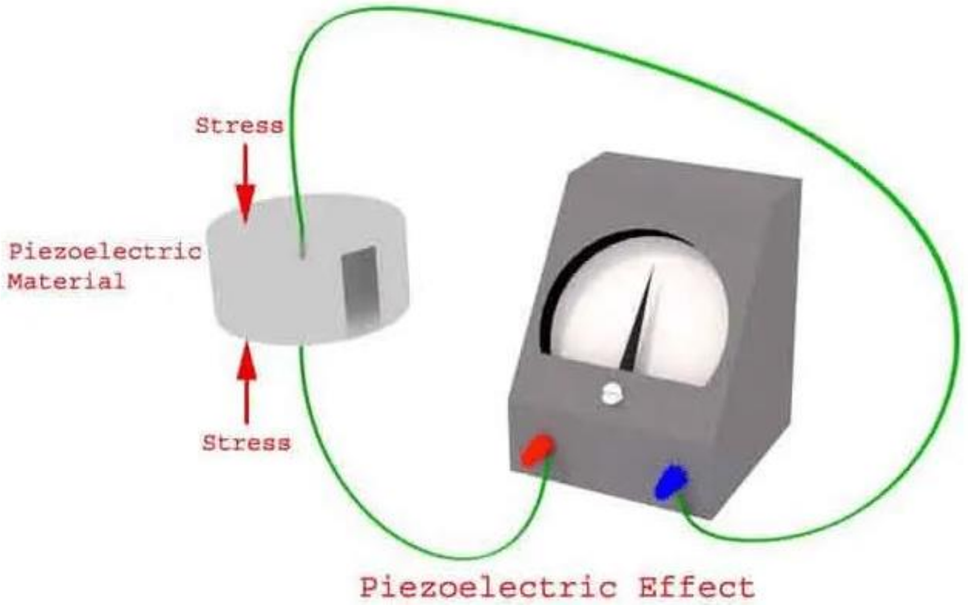


Piezo-Electric Transducer



Piezo-Electric Transducer

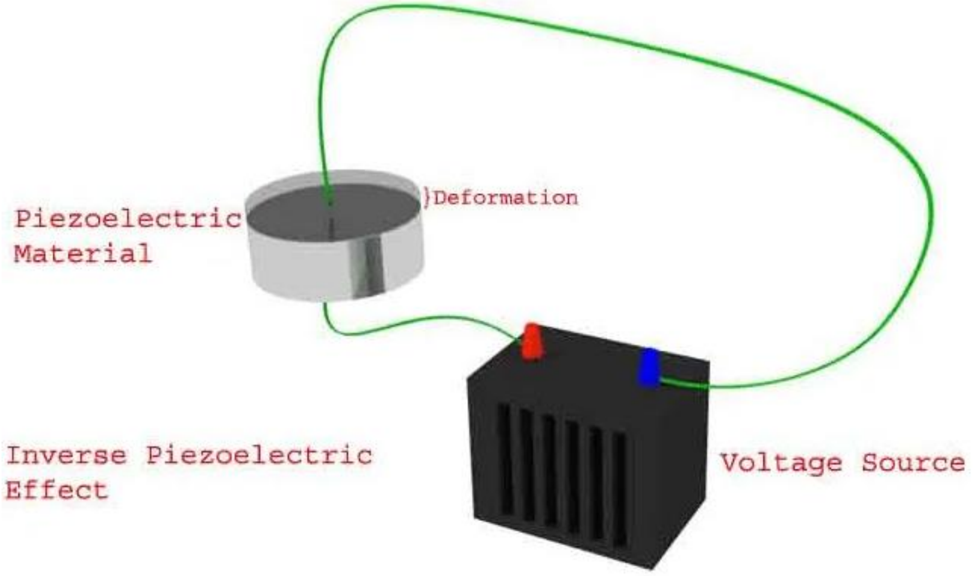
- The electric voltage produced by a piezoelectric transducer can be easily measured by the voltage measuring instruments. Since this voltage will be a function of the force or pressure applied to it.In this way, physical quantities like mechanical stress or force can be measured directly by using a piezoelectric transducer.



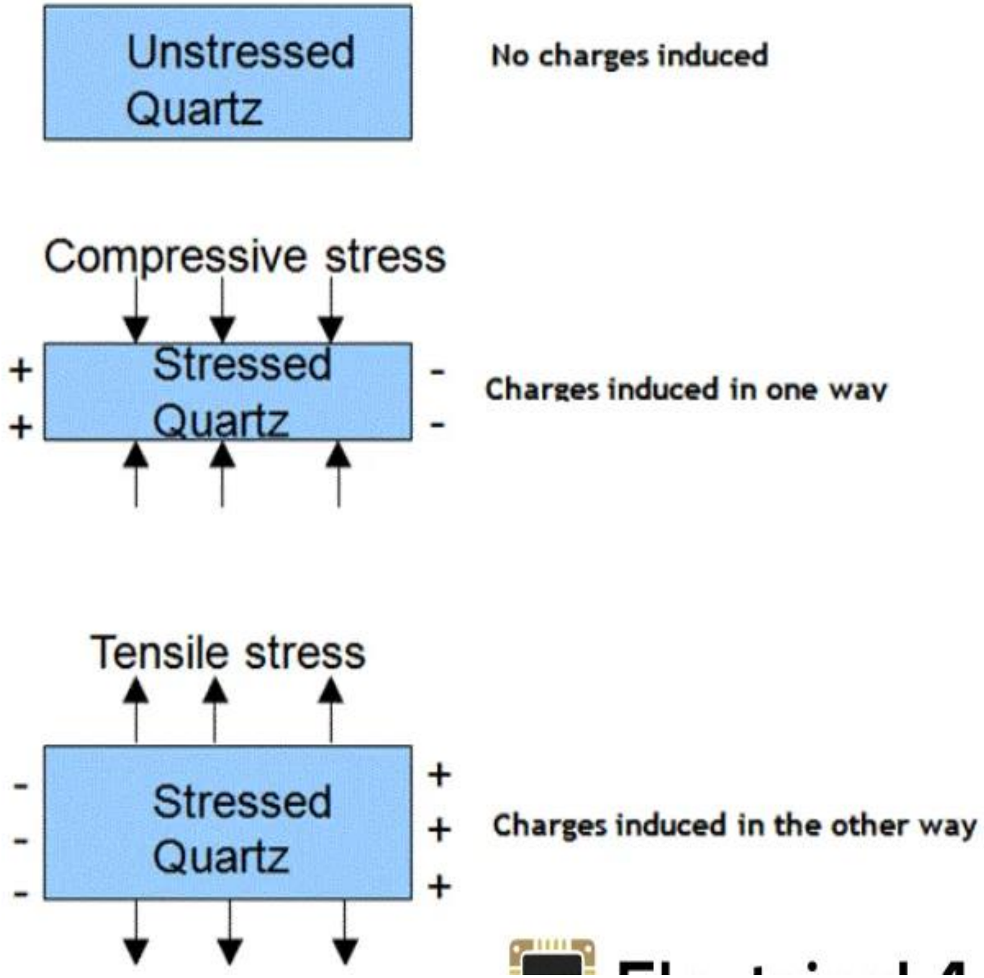
Piezoelectric Actuator

- A piezoelectric actuator behaves in the reverse manner of the piezoelectric sensor. It is the one in which the electric effect will cause the material to deform i.e. stretch or bend.

- That means in a piezoelectric sensor, when force is applied to stretch or bend it, an electric potential is generated and in opposite when on a piezoelectric actuator, an electric potential is applied it is deformed i.e. stretched or bend.



Piezo-Electric Transducer



- A piezoelectric transducer consists of quartz crystal which is made from silicon and oxygen arranged in crystalline structure (SiO_2).
- Generally, unit cell (basic repeating unit) of all crystal is symmetrical but in piezoelectric quartz crystal, it is not. Piezoelectric crystals are electrically neutral.
- The atoms inside them may not be symmetrically arranged but their electrical charges are balanced means positive charges cancel out negative charge.
- The quartz crystal has the unique property of generating electrical polarity when mechanical stress applied to it along a certain plane.
- Basically, There are two types of stress. One is compressive stress and the other is tensile stress.
- When there is unstressed quartz no charges induce on it. In the case of compressive stress, positive charges are induced on one side and negative charges are induced in the opposite side.
- The crystal size gets thinner and longer due to compressive stress. In the case of tensile stress, charges are induced in reverse as compare to compressive stress and quartz crystal gets shorter and fatter.

Piezo-Electric Transducer

- It is a self-generating transducer. It does not require an electric voltage source for operation. The electric voltage produced by the piezoelectric transducer is linearly varied to applied stress or force as shown in Fig(a).
- The piezoelectric transducer has high sensitivity and has excellent frequency of response.

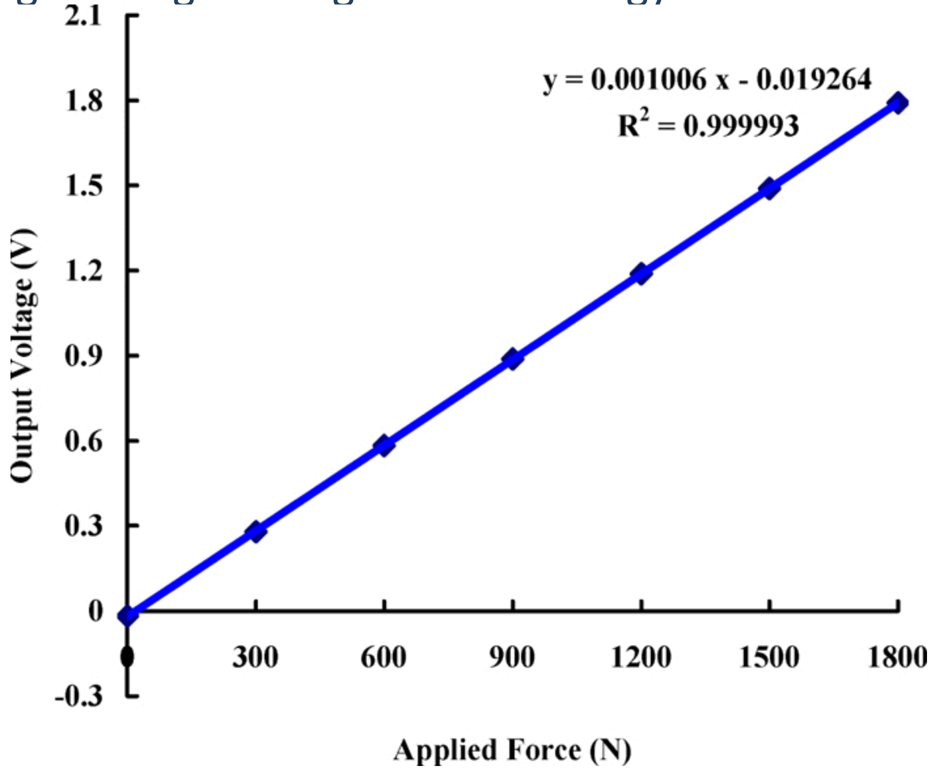


Fig.(a)Characteristics of Piezo-Electric Transducer

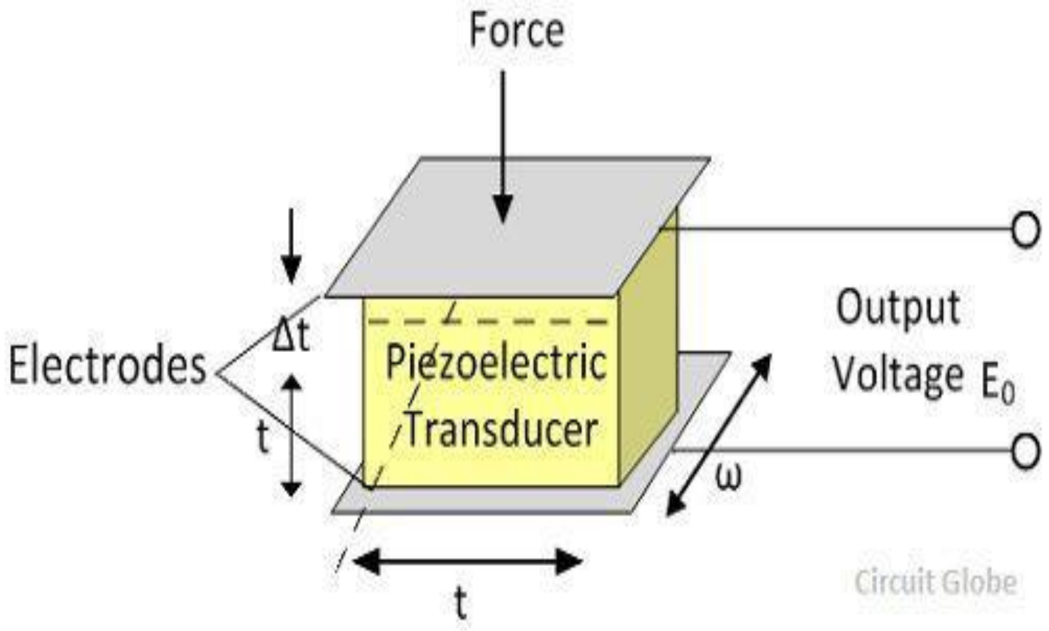
Materials used for piezoelectric transducer

- The word piezoelectric means the electricity produces by the pressure. The Quartz is the examples of the natural piezoelectric crystals, whereas the Rochelle salts, ammonium dehydration, phosphate, lithium sulphate, dipotassium tartrate are the examples of the man made crystals.
- The ceramic material is also used for piezoelectric transducer. The ceramic material has several advantages. It is available in different shapes and sizes. The material has the capability of working at low voltages, and also it can operate at the temperature more than 3000ºC

➢ Piezoelectric Effect:

- A piezoelectric transducer is based on the principle of the piezoelectric effect. The word piezoelectric is derived from the Greek word piezen, which means to squeeze or press. The piezoelectric effect states that when mechanical stress or forces are applied on quartz crystal, produce electrical charges on the quartz crystal surface.
- The piezoelectric effect is discovered by Pierre and Jacques Curie. The rate of charge produced will be proportional to the rate of change of mechanical stress applied to it. Higher will be stress higher will be voltage.

➢ Theory of Piezo-Electric Transducer: A piezoelectric crystal is shown in the figure below:



The polarity of the charge depends on the direction of the applies forces.

$$Charge\ Q = d \times F\ Coulomb$$

Where, d – charge sensitivity of the crystals
F – applied force in Newton
The force changes the thickness of the crystals.

$$F = \frac{AE}{t} \Delta t \ Netwon$$

Where A – area of crystals in meter square

t – the thickness of crystals in meter

E – Young's modulus N/m2

The young modulus is,

$$E = \frac{stress}{strain} = \left(\frac{F}{A}\right) . \frac{1}{\Delta t / t}$$

$$E = \frac{Ft}{A \Delta t} N/m^2$$

$$A = \omega l$$

where ω – width of crystals in meter

l – the length of crystals in meter

On substituting the value of force in the equation of charge, we get

$$Q = dAE(\frac{\Delta t}{t})$$

The output voltage is obtained because of the electrode charges.

$$E_o = \frac{Q}{C_p} = \frac{dF}{\varepsilon_r \varepsilon_0 A/t}$$

$$E_0 = \frac{d}{\varepsilon_r \varepsilon_0} tP$$

$$E_0 = gtP$$

$$g = \frac{d}{\varepsilon_r \varepsilon_0}$$

The g is the voltage sensitivity of the crystals.

$$g = \frac{E_0}{tP} = \frac{E_0/t}{P}$$

Where E_0 – electric field strength, V/m

The voltage sensitivity of the crystals is expressed by the ratio of the electric field intensity and pressure.

- When the mechanical deformation occurs in the crystals, it generates charges. And this charge develops the voltages across the electrodes.
- The Piezoelectric crystal is direction sensitive. The polarity of the voltage depends on the direction of the force which is either tensile or compressive.
- The magnitude and the polarity of the charges depend on the magnitude and the direction of the applied force.

Properties of Piezo-electric Crystals:

- The desirable properties of piezo-electric materials are stability, high output insensitivity to temperature and humidity and the ability to be formed into most desirable shape.
- Quartz is the most stable piezo-electric material. However, its output is quite small. On the other hand, Rochelle salt provides the highest output but it can be worked over a limited humidity range and has to be protected against moisture.
- The highest temperature is limited to 45^0C. Barium titanate has the advantage that it can be formed into a variety of shapes and sizes since it is polycrystalline.
- It has also a higher dielectric constant. Natural crystals possess the advantages that they have higher mechanical and thermal stability, can withstand higher stresses, and have a good frequency response.

➢ Advantages of Piezoelectric transducer:

1) Rugged construction and small size.
2) High output with negligible phase shift.
3) Excellent frequency response.
4) No need for an external force

➢ The disadvantages of piezoelectric transducers are:

1) Piezoelectric crystals are water soluble. Hence in a high immunity environment gets dissolved.
2) Temperature sensitive i.e.,It is affected by temperatures
3) It can be used for dynamic measurements only. It is not suitable for measurement in static condition
4) The output is low so some external circuit is attached to it
5) It is very difficult to give the desired shape to this material and also desired strength

Applications of Piezoelectric Transducer

Using piezoelectric materials, piezoelectric transducers can be used in a variety of applications,

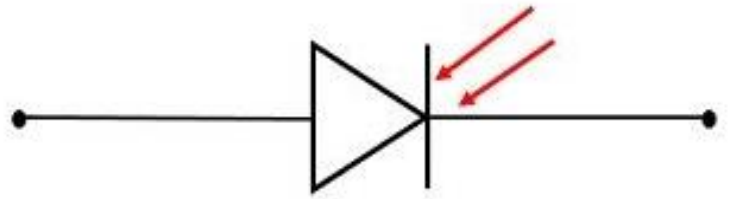
1) It is used for measurement of non-electrical quantities such as acceleration, vibration, sound intensity and dynamic pressures.
2) It is widely used in aerodynamics, supersonic wind tunnels, in the study of extremely high speed phenomena such as explosions, bomb blasts etc.
3) It is used in ultrasonic, ultrasonic flow meters, micro motion actuators.
4) It is also used in spark ignition engines and electrostatic dust filters.
5) In microphones, the sound pressure is converted into an electric signal and this signal is ultimately amplified to produce a louder sound.
6) Automobile seat belts lock in response to a rapid deceleration is also done using a piezoelectric material.
7) It is also used in medical diagnostics.
8) It is used in electric lighter used in kitchens. The pressure made on piezoelectric sensor creates an electric signal which ultimately causes the flash to fire up.
9) They are used for studying high-speed shock waves and blast waves.
10) Used in Inkjet printers
11) It is also used in restaurants or airports where when a person steps near the door and the door opens automatically. In this, the concept used is when a person is near the door pressure is exerted person weight on the sensors due to which the electric effect is produced and the door opens automatically.

PHOTODIODE TRANSDUCER

PHOTODIODE TRANSDUCER

Photodiode Basics

- Photodiodes basically perform the opposite effect to LEDs and laser diodes. Instead of using electric current to cause electrons and holes to combine to create photons, photodiodes absorb light energy (photons) to generate electron-hole pairs, so creating an electric current flow.
- Definition : A special type of PN junction device that generates current when exposed to light is known as Photodiode. It is also known as photo detector or photo sensor. It operates in reverse biased mode and converts light energy into electrical energy. The beside figure shows the symbolic representation of a photodiode:

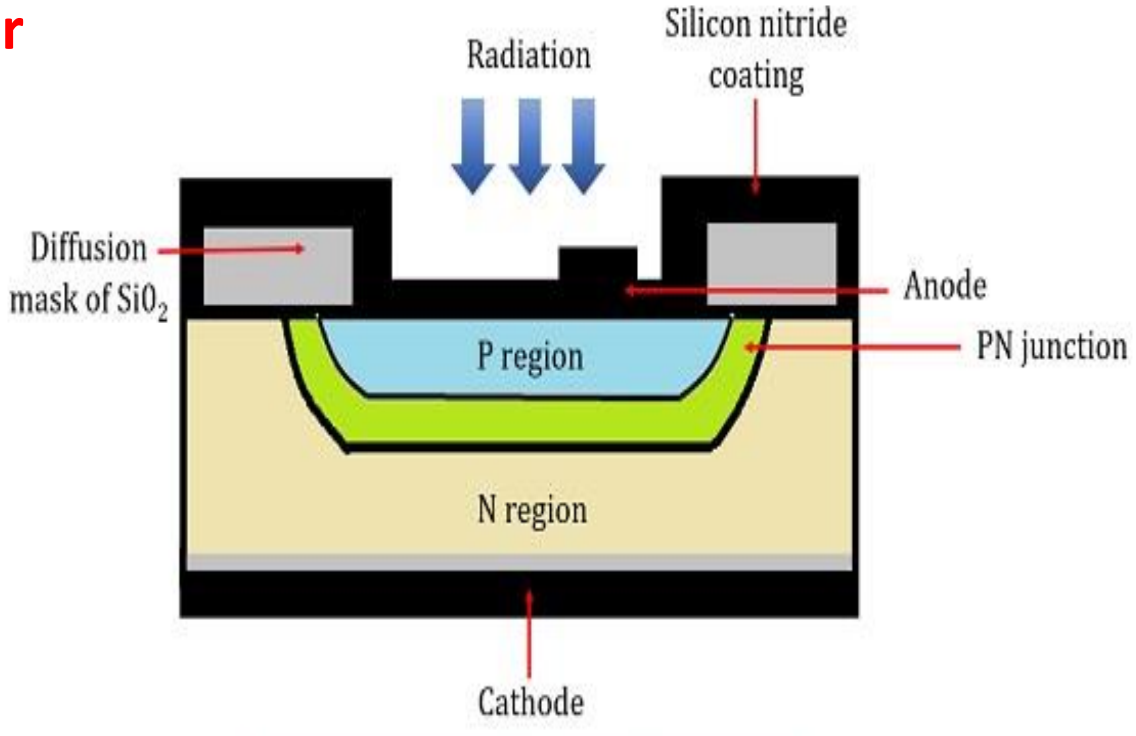


Symbolic representation of Photodiode

- **Principle of Photodiode Transducer:**
  - It works on the principle of Photoelectric effect.
  - The operating principle of the photodiode is such that when the junction of this two-terminal semiconductor device is illuminated then the electric current starts flowing through it.
  - Only minority current flows through the device when the certain reverse potential is applied to it.

Construction of Photodiode Transducer

- The figure below shows the constructional detail of a photodiode transducer.
- The PN junction of the device placed inside a glass material. This is done to order to allow the light energy to pass through it. As only the junction is exposed to radiation, thus, the other portion of the glass material is painted black or is metalized.
- The overall unit is of very small dimension nearly about 2.5 mm. It is noteworthy that the current flowing through the device is in micro-ampere and is measured through an ammeter.



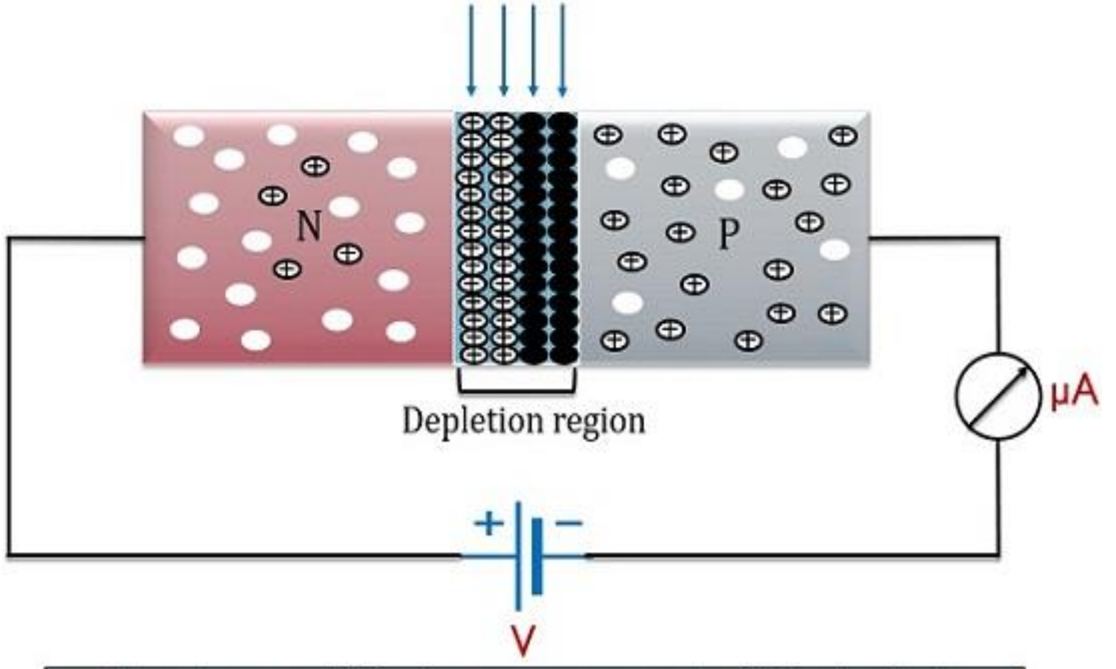
Structure of Photodiode

- Two basic methods for generating electricity from light, using photodiodes are photovoltaic and photoconductive operation. Both methods use light sensitive semiconductor diodes:

- **Photovoltaic mode:** It is also known as zero-bias mode because no external reverse potential is provided to the device. However, the flow of minority carrier will take place when the device is exposed to light.

- **Photoconductive mode:** When a certain reverse potential is applied to the device then it behaves as a photoconductive device. Here, an increase in depletion width is seen with the corresponding change in reverse voltage.

Let us now understand the detailed circuit arrangement and working of the photodiode transducer..



Structure and biasing arrangement of Photodiode

Working of Photodiode transducer:

- In the photodiode, a very small reverse current flows through the device that is termed as dark current.
- It is called so because this current is totally the result of the flow of minority carriers and is thus flows when the device is not exposed to radiation.
- The electrons present in the p side and holes present in n side are the minority carriers. When a certain reverse-biased voltage is applied then minority carrier, holes from n-side experiences repulsive force from the positive potential of the battery.
- Similarly, the electrons present in the p side experience repulsion from the negative potential of the battery. Due to this movement electron and hole recombine at the junction resultantly generating depletion region at the junction. Due to this movement, a very small reverse current flows through the device known as dark current.

- The combination of electron and hole at the junction generates neutral atom at the depletion. Due to which any further flow of current is restricted.

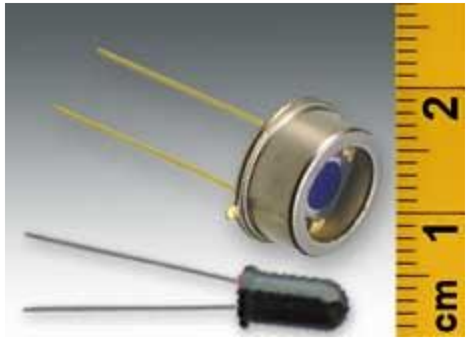
- Now, the junction of the device is illuminated with light. As the light falls on the surface of the junction, then the temperature of the junction gets increased. This causes the electron and hole to get separated from each other.

- As the two gets separated then electrons from n side gets attracted towards the positive potential of the battery. Similarly, holes present in the p side get attracted to the negative potential of the battery. This movement then generates high reverse current through the device. With the rise in the light intensity, more charge carriers are generated and flow through the device. Thereby, producing a large electric current through the device. This current is then used to drive other circuits of the system.

- So, we can say the intensity of light energy is directly proportional to the current through the device. Only positive biased potential can put the device in no current condition in case of the photodiode.

➢ **Types of Photodiode transducers based on Materials used**

  - **Silicon Photodiode transducers**
  - **Germanium Photodiode transducers**
  - **Indium Gallium Arsenide Photodiode transducers etc.**



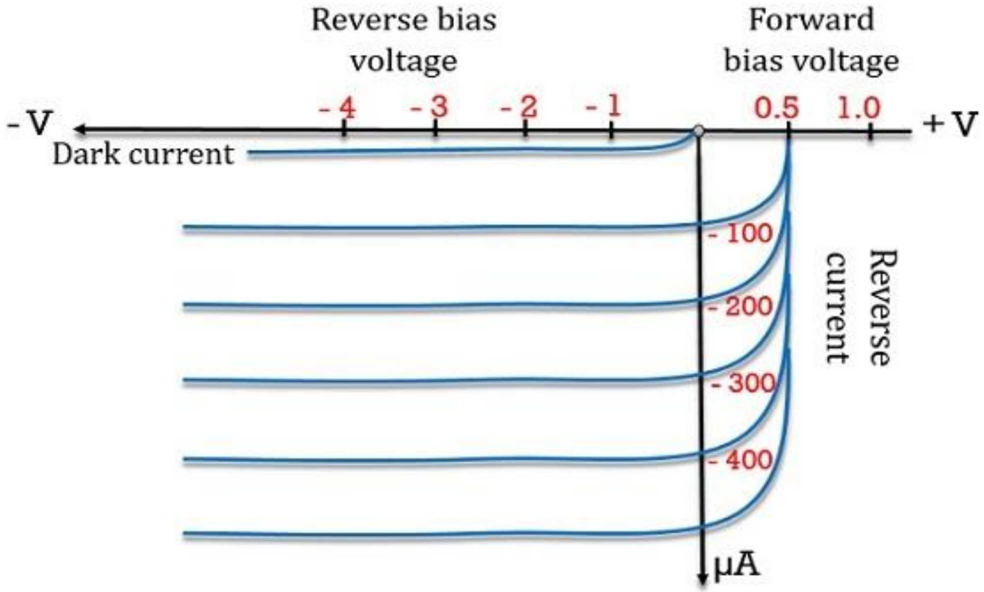
➢ CHARACTERISTICS OF PHOTODIODE TRANSDUCER:

- The beside figure shows the V-I characteristic curve of a photodiode:

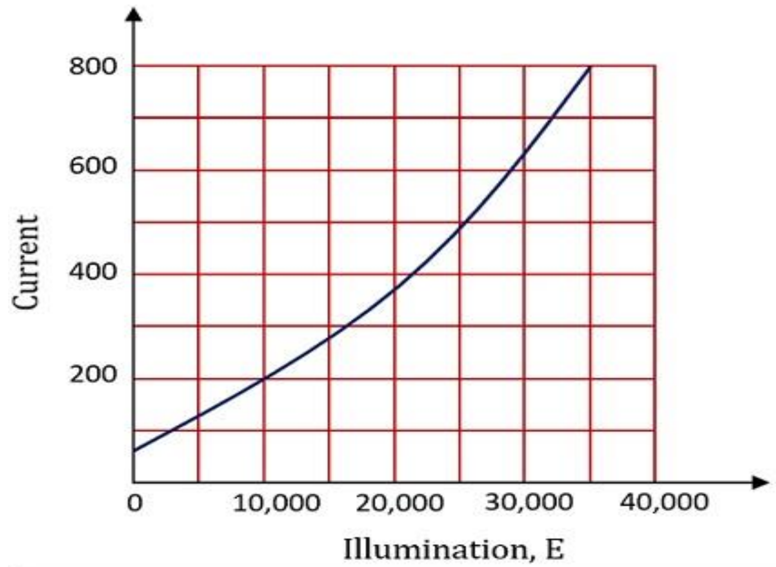
- Here, the vertical line represents the reverse current flowing through the device and the horizontal line represents the reverse-biased potential.

- The first curve represents the dark current that generates due to minority carriers in the absence of light. As we can see in the above figure that all the curve shows almost equal spacing in between them. This is so because current proportionally increases with the luminous flux.

- The beside figure shows the curve for current versus illumination:
- It is noteworthy here that, the reverse current does not show a significant increase with the increase in the reverse potential.



Characteristics curve of Photodiode



Illumination versus current curve

Advantages of Photodiode Transducers:

1. It shows a quick response when exposed to light.
2. Photodiode offers high operational speed.
3. It provides a linear response.
4. It is a low-cost device.

Disadvantages of Photodiode transducers:

1. It is a temperature-dependent device. And shows poor temperature stability.
2. When low illumination is provided, then amplification is necessary.

Applications of Photodiode transducers:

1) Photodiodes are used in electronic systems such as fibre optic communications systems.
2) Cameras use photodiodes to measure light, and to control the shutter, focus and flash. Medical uses include X-ray detection and pulse measurement.
3) Photoconductive diodes are the sensor of choice for many industrial systems where light needs to be measured, from bar code scanners and position sensors to smoke detectors and surveying instruments.
4) Photodiodes majorly find its use in counters and switching circuits.
5) Photovoltaic diodes by contrast are manufactured as very large size solar panels to maximize the efficiency of light collection.
6) It is widely used in burglar alarm systems. In such alarm systems, until exposure to radiation is not interrupted, the current flows. As the light energy fails to fall on the device, it sounds the alarm.

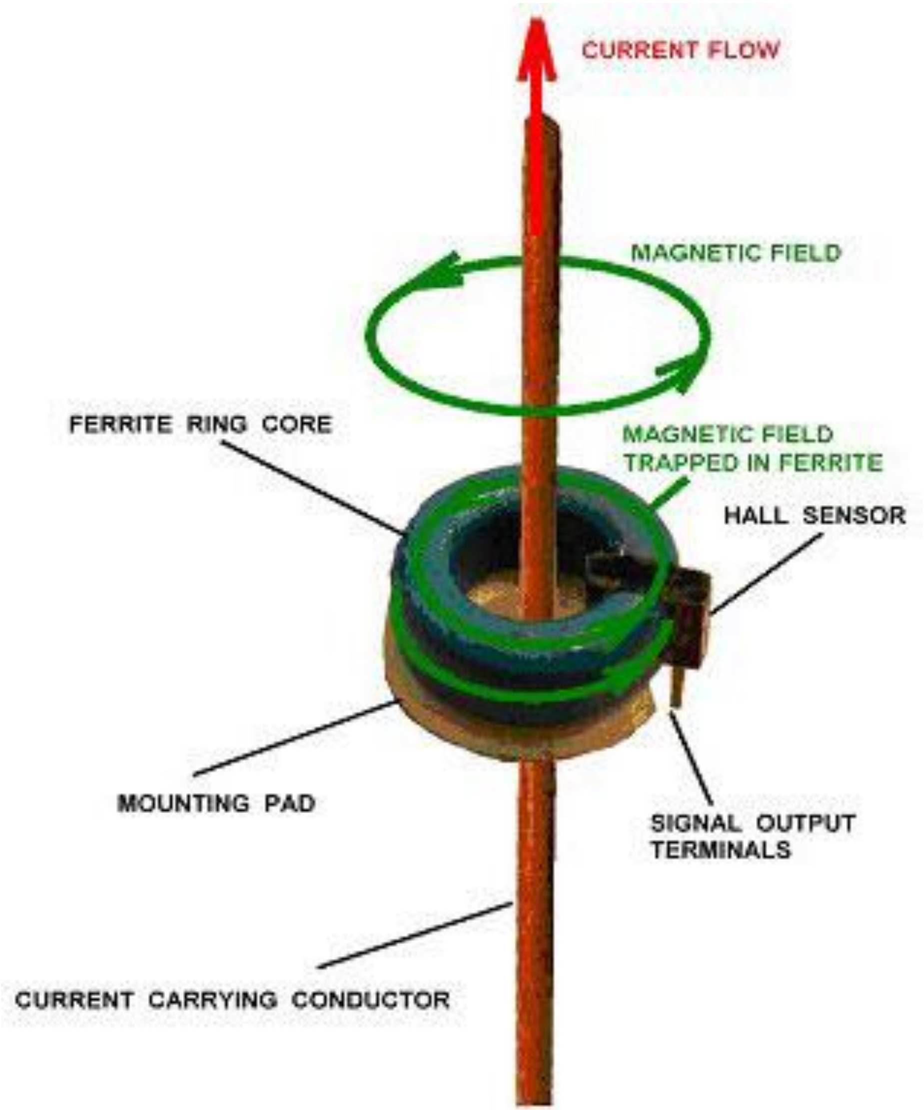
In case of a typical photodiode, the normal reverse current is in tens of microampere range.

Hall Effect Transducer

Hall Effect Transducer

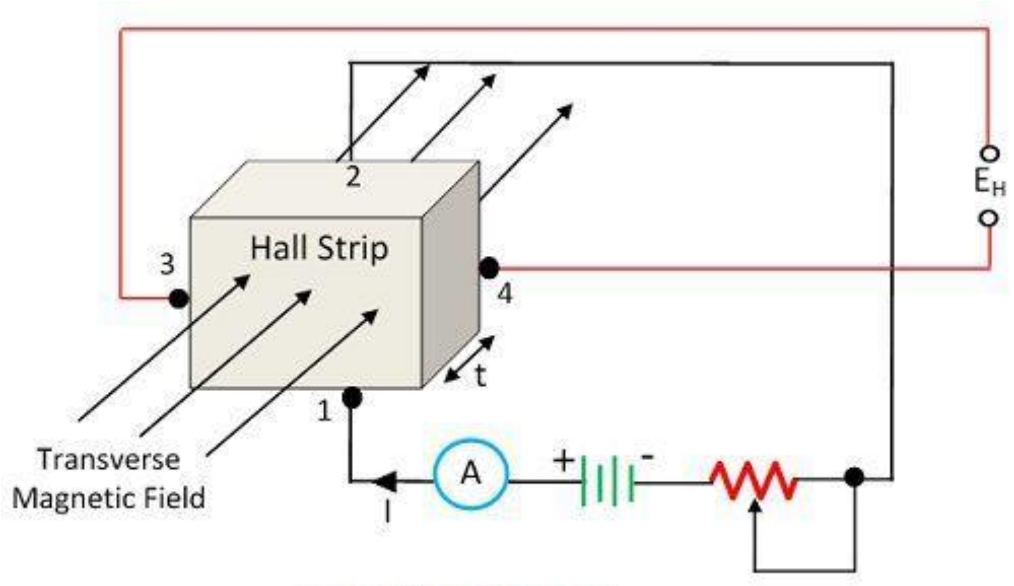
- **Definition:**

- The hall effect element is a type of transducer used for measuring the magnetic field by converting it into an emf. The direct measurement of the magnetic field is not possible. Thus the Hall Effect Transducer is used. The transducer converts the magnetic field into an electric quantity which is easily measured by the analogue and digital meters.



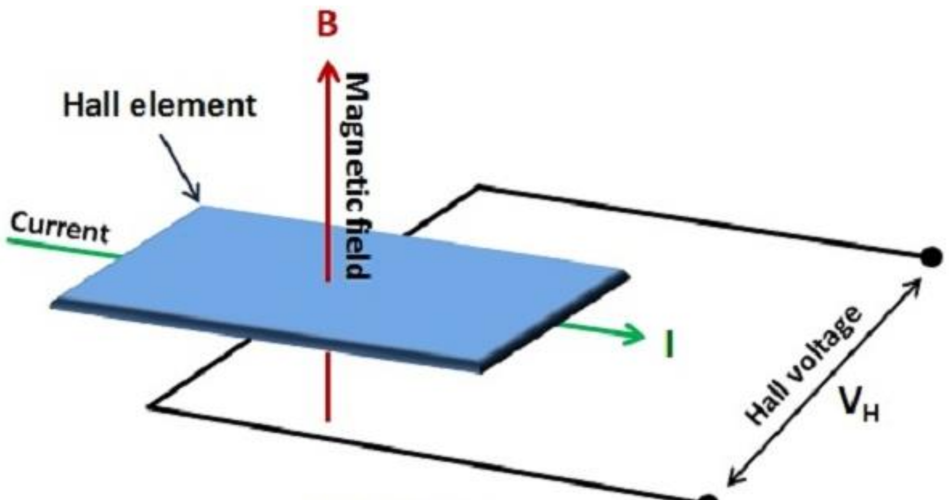
➢ Principle of Hall Effect Transducer:

- The principle of hall effect transducer is that if the current carrying strip of the conductor is placed in a transverse magnetic field, then the EMF develops on the edge of the conductor. The hall voltage is only produced when directions of the current and magnetic field are perpendicular to each other.
- The magnitude of the develop voltage depends on the density of flux, and this property of a conductor is called the Hall effect. The Hall effect element is mainly used for magnetic measurement and for sensing the current.
- The metal and the semiconductor has the property of hall effect which depends on the densities and the mobility of the electrons. Consider the hall effect element shown in the figure.



Hall Effect Element

- The current supply through the lead 1 and 2 and the output is obtained from the strip 3 and 4. The lead 3 and 4 are at same potential when no field is applied across the strip.
- When the magnetic field is applied to the strip, the output voltage develops across the output leads 3 and 4. The develops voltage is directly proportional to the strength of the material.



Hall effect

The output voltage is,

$$E_H = K_H IB/t$$

where,

$$K_H - Hall\ effect\ coefficient\ ; \frac{V - m}{A - Wbm^{-2}}$$

$$t - thickness\ of\ Strip\ ; m$$

The I is the current in ampere
and the B is magnetic flux density in Wb/m^2

- The emf produced is called as Hall voltage and depends upon the current, flux density and a property of conductor known as Hall Effect Coefficient (K_H). The strip of conducting material is called a Hall element.

- The current and magnetic field strength both can be measured with the help of the output voltages.
- The hall effect EMF is very small in conductors because of which it is difficult to measure.
- But semiconductors like germanium produces large EMF which is easily measured by the moving coil instrument.

➢ Applications of Hall Effect Transducer

The following are the application of the Hall effect Transducers.

1.Magnetic to Electric Transducer –

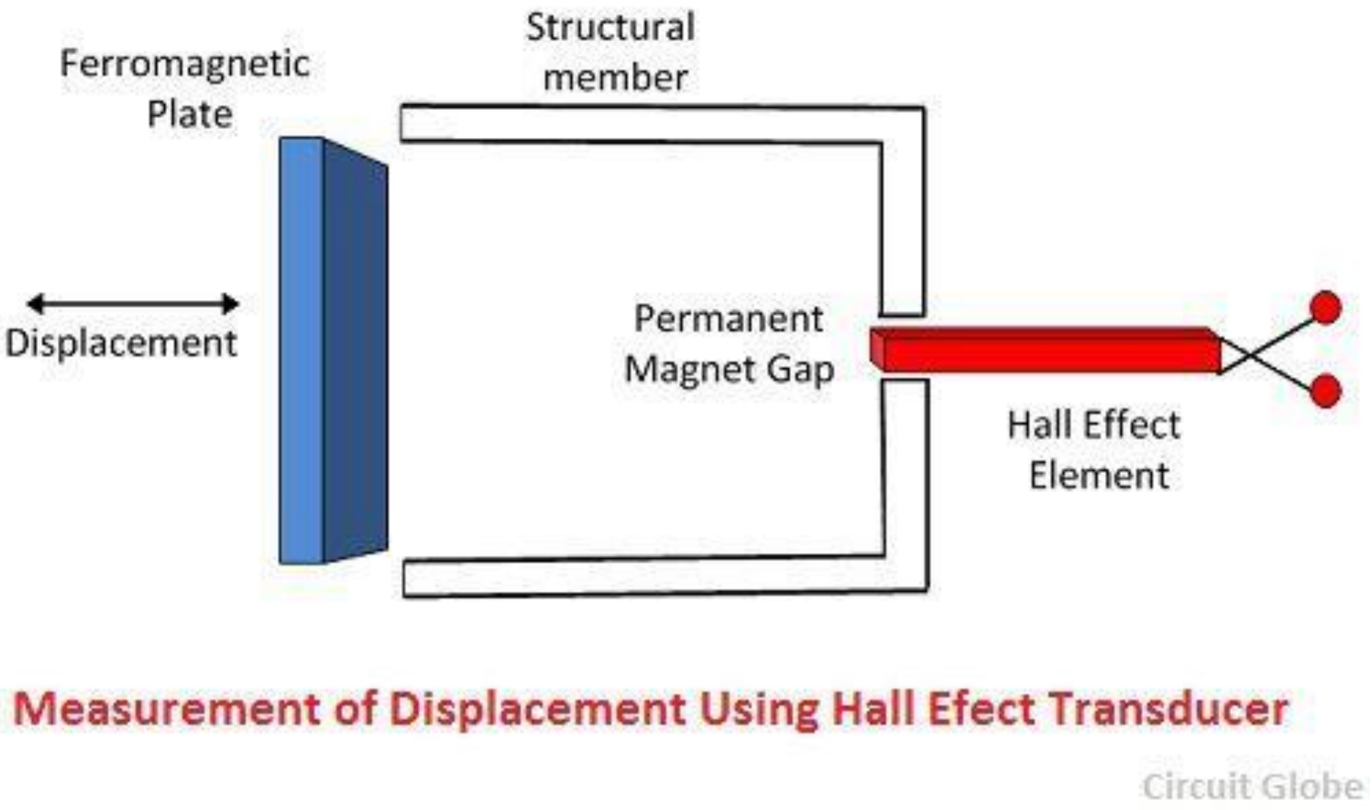
- The Hall effect element is used for converting the magnetic flux into an electric transducer. The magnetic fields are measured by placing the semiconductor material in the measurand magnetic field.
- The voltage develops at the end of the semiconductor strips, and this voltage is directly proportional to the magnetic field density.
- The Hall Effect transducer requires small space and also gives the continuous signal concerning the magnetic field strength.
- The only disadvantage of the transducer is that it is highly sensitive to temperature and thus calibration requires in each case.

Applications of Hall Effect Transducer

2. Measurement of Displacement –

- The Hall effect element measures the displacement of the structural element.
- **For example –** Consider the ferromagnetic structure which has a permanent magnet.

- The hall effect transducer placed between the poles of the permanent magnet. The magnetic field strength across the hall effect element changes by changing the position of the ferromagnetic field.



Measurement of Displacement Using Hall Efect Transducer

3. Measurement of Current –

- The hall effect transducer is also used for measuring the current without any physical connection between the conductor circuit and meter.
- The AC or DC is applied across the conductor for developing the magnetic field.
- The strength of the magnetic field is directly proportional to the applied current.
- The magnetic field develops the emf across the strips. And this EMF depends on the strength of the conductor.

4. Measurement of Power –

- The hall effect transducer is used for measuring the power of the conductor.
- The current is applied across the conductor, which develops the magnetic field.
- The intensity of the field depends on the current. The magnetic field induces the voltage across the strip.
- The output voltage of the multiplier is proportional to the power of the transducer.

➢ **Characteristics of Hall effect Transducer**

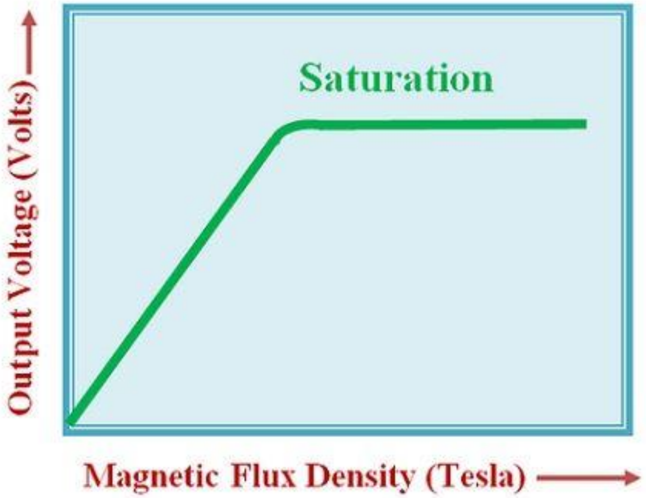


Fig. Variation of Hall Voltage with magnetic flux density

➢ Advantages of hall effect transducer

1) High speed operation over 100 KHz possible. Whereas at high frequencies the inductive or capacitive sensor output begins to distort.
2) Non contact operation so there is no wear and friction, hence unlimited number of operating cycles.
3) It has a long life. Hall Effect transducers are not affected by ambient conditions such as rain, dust, humidity, vibrations, etc.
4) It can be operated over a broad temperature range from -40 °C to +150°C.
5) It operates with stationary input.
6) Highly repeatable operation.
7) Capable of measuring large current.

➢ Disadvantages of hall Effect transducer

1) It may be affected by external interfering stray magnetic field.
2) Large temperature drift.
3) Large offset voltage.
4) Not capable of measuring a current flow at a distance greater than 10 cm.

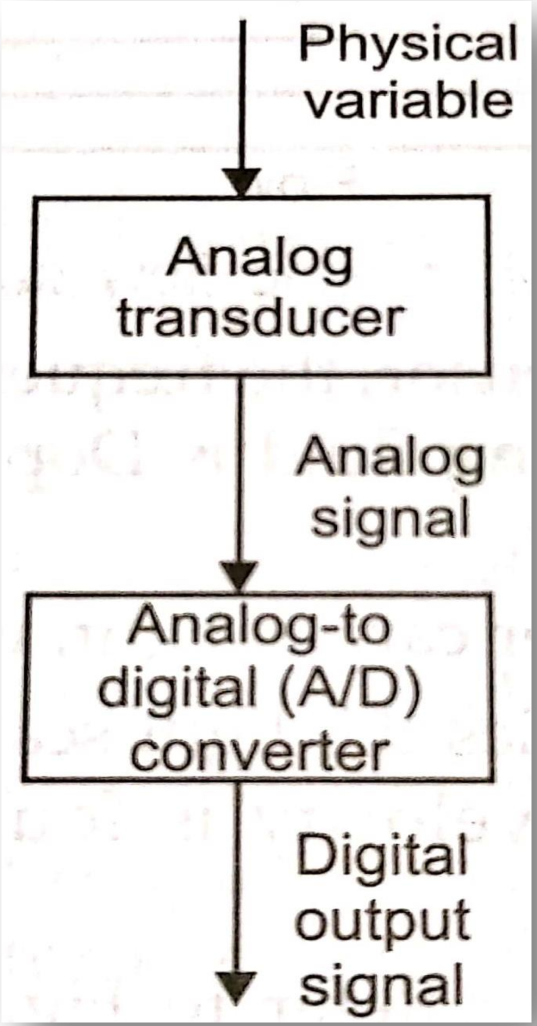
➢ Applications of Hall Effect Transducers

1) It is used as magnetic to an electric transducer to measure the magnetic field.
2) It is used for the measurement of displacement.
3) It is used for the measurement of a.c. or d.c. current.
4) It is used for the measurement of power.
5) Open/close detection of laptop flip screen. Hence conserve power while switching laptop to sleep.
6) Variable speed drives
7) Used in Flow meters
8) Used in Encoded switches
9) Used in Motor control protection/indicators
10) Used in Rotary encoders

DIGITAL ENCODING TRANSDUCERS

➢ Introduction

- Whereas in "analog transducers” the output varies continuously according to the input, the output in "digital transducers” is discrete and may give frequency type output or a digitally coded output, of binary or some other type.
- Digital transducers range from frequency generating types of transducers to digital encoders.
- Alternatively, an instrument may incorporate an analog transducer and an analog-to-digital (A/D) converter giving a digital output as shown in Figure.



DIGITAL ENCODING TRANSDUCERS

- The transducers often communicate with digital computers and therefore transducers which have a digital output are preferable as they are convenient to use since they can be directly interfaced with a digital computer.
- However, few transducers exist which can provide a direct digital output. In most of the situations in measurement systems, we come across transducers which provide only an analog output.
- Therefore, with such transducers we have to use an analog to digital (A/D) converters to realize the digital data which could be handled by digital computers.
- Digital transducers are called Encoders. They are available but they are normally in the form of linear or rotary displacement transducers.
- Digital encoding transducers, or Digitisers, enable a linear or rotary displacement to be directly converted into digital form without intermediate forms of analog to digital (A/D) conversion. Such digitisers may be known as digital encoders or linear digitisers, or for rotary applications, shaft digitisers or shaft encoders.
- A digitiser is perhaps the most elementary form of analog to digital (A/D) converter because it converts a continuous displacement an analog quantity to be defined incrementally in some binary or decimal code.
- There are several techniques used for achieving this conversion, each with its own advantages and limitations, which include cost, simplicity of associated circuitry and reliability etc

CLASSIFICATION OF ENCODERS

The Digital Displacement transducers can be classified into three major categories :

1) A tachometer transducers
2) incremental transducers
3) absolute transducers.

Tachometer Encoders

- A tachometer encoder has only a single output signal which consists of a pulse for each increment of displacement. This is shown in Fig.(a).
- If the motion were always in one direction, a digital counter could accumulate these pulses to determine the displacement from a known starting point.

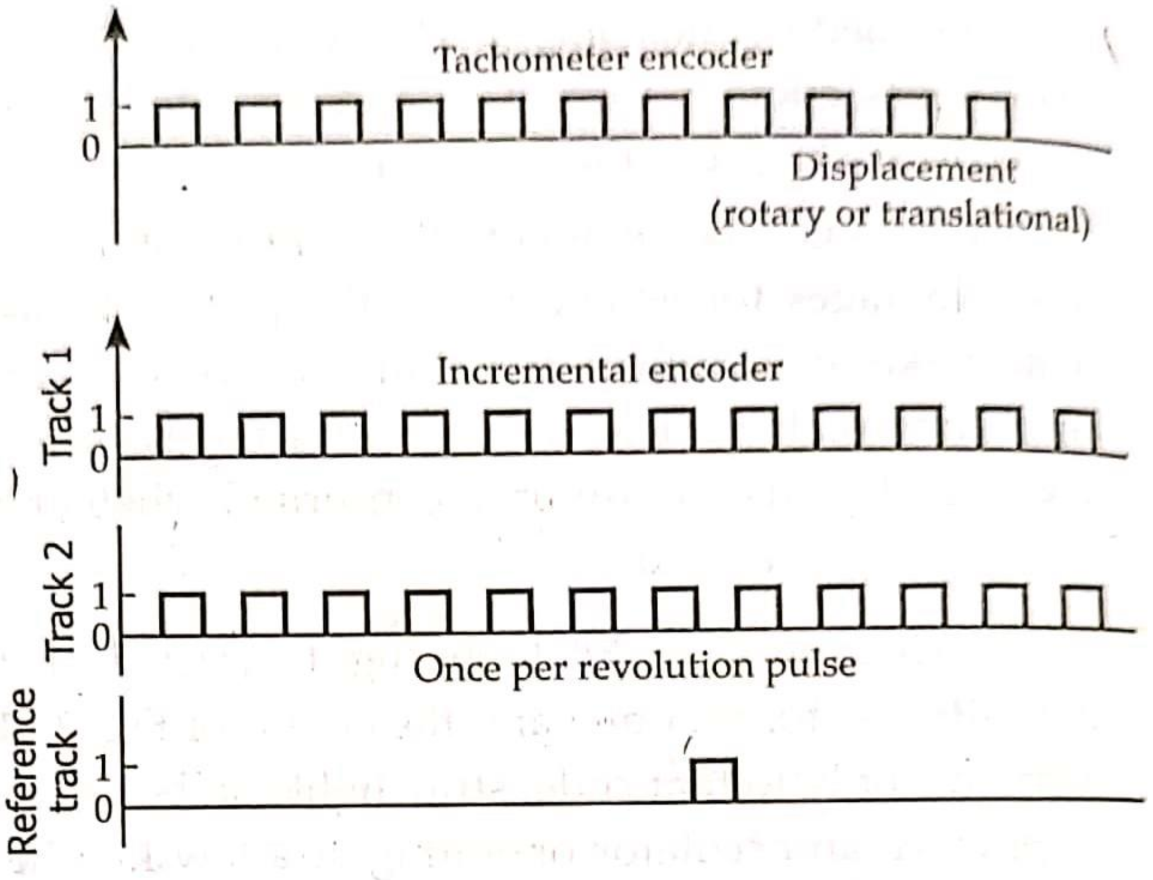


Fig.(a) Classification of Encoders.

- However, any motion in the opposite direction, would also produce identical pulses, which would produce errors.
- Therefore, this digital transducer, is usually used for measurement of speed, rather than for displacement, and in situations where the rotation never reverses.

Incremental Encoders

- The problems caused by reverse motion in the case tachometer encoder are solved by using air incremental encoder.
- The incremental encoder uses at least two (and sometimes a third) signal generating elements.
- The two tracks (the tachometer encoder uses only one track), in the case incremental encoder are mechanically shifted by 4 cycle relative to each other,This allows detection of motion which signal rises first.
- Thus an up down pulse counter can be used to subtract pulses whenever the motion reverses.

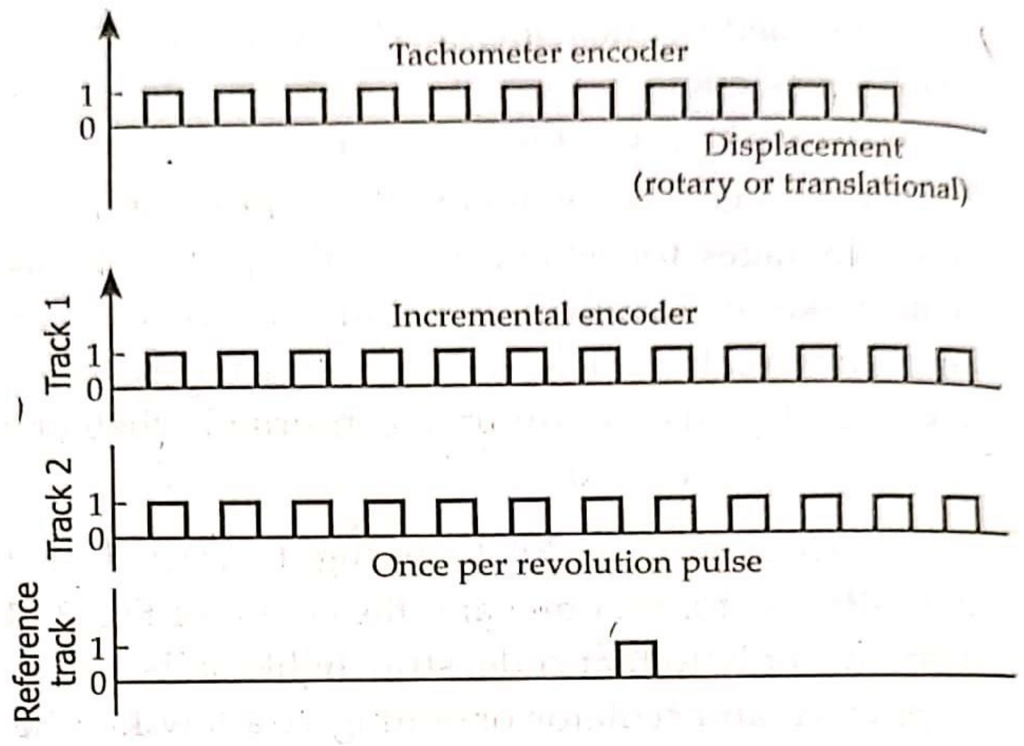
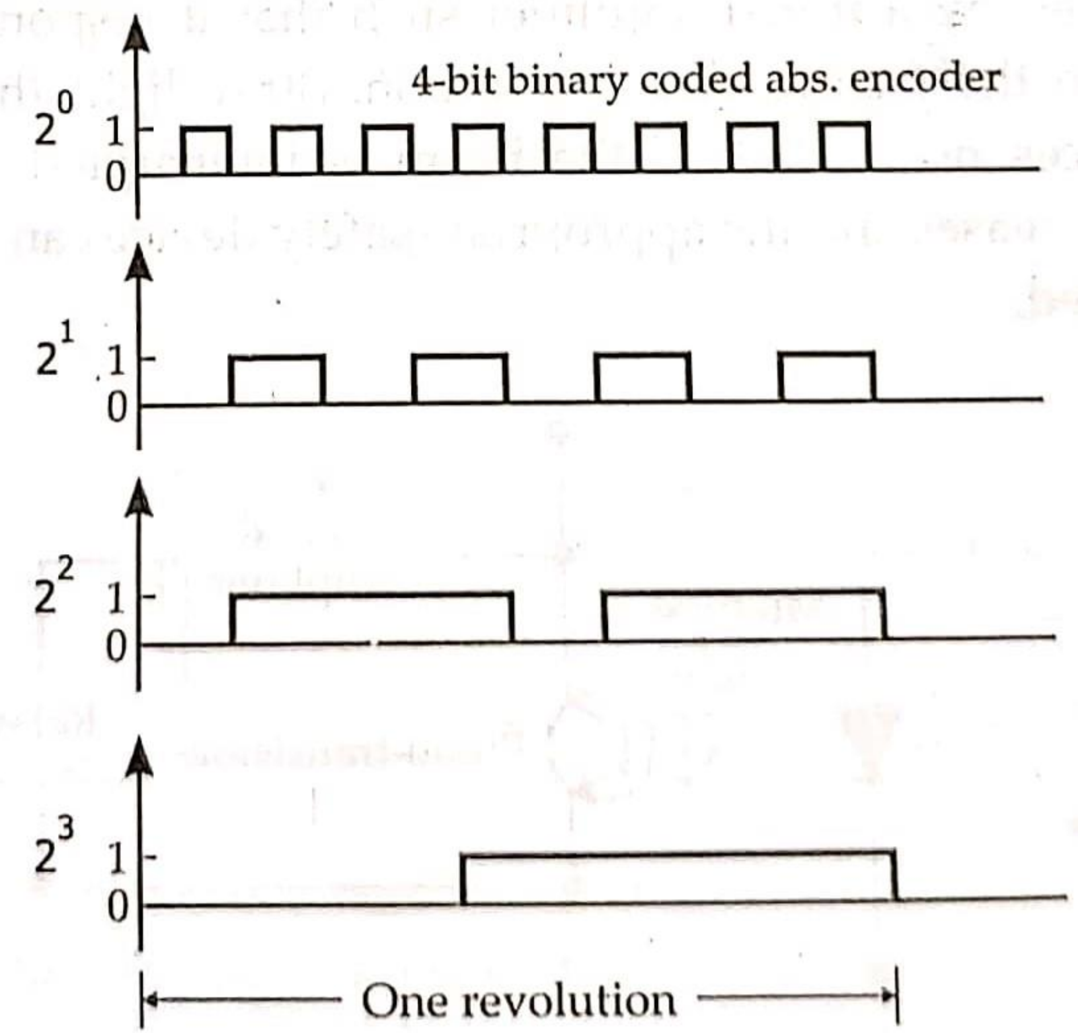


Fig. Classification of Encoders.

- A third output, which produces one pulse per revolution at a distinct point, is sometimes provided for zero reference.
- An incremental encoder has the advantage of being able to rotate through as many revolutions as the application requires. However, any false pulses resulting from electric noise will cause errors that persist even when the noise disappears. The failure of system power also causes total information about the position data which cannot be retrieved even after re-application of power .

❑ Absolute Encoders

- These are generally limited to measurement of a single revolution.
- They use multiple tracks and outputs, which are read out in parallel to produce a binary representation of the angular shaft input position.
- Since, there is a one-to-one correspondence between binary output, position data are recovered when power is restorted after an outage.
- The transient electric noise causes only transient measurement errors.



- **Applications:**
  1) Robotic manipulators,
  2) machine tools,
  3) data storage systems,
  4) plotters,
  5) Printers,
  6) other rotating machinery
- **Advantages:**
  1) High resolution (word size),
  2) high accuracy (noise immunity),
  3) relative ease of adaptation in digital control systems with reduction in cost
  4) It is possible to obtain accuracy in pulse count.
  5) It becomes easy to use digital computers along with the transducers, for data manipulation.
  6) It is easy to transmit digital signals without distortion and external noise.

- **Types of digital transducers**
- Frequency Domain Transducers: out put in these transducers is in the form of pulses
  – Electromagnetic frequency domain transducers
  – Opto-electrical frequency domain transducers
  – Vibrating string transducers
- Digital Encoders
  - Optical type encoder
  - Magnetic type encoder

Frequency Domain Transducers

- The output in these transducers is in the form of pulses or sinusoidal wave forms, the Frequency of which is a measure of the magnitude of the physical variable.
- The following three types of frequency domain transducer:

  1. Electromagnetic frequency domain transducer.
  2. Opto-electrical frequency domain transducer.
  3. Vibrating string transducer.

1. Electromagnetic frequency domain transducer :

Fig.(a) shows an electromagnetic frequency domain transducer which can employed for the measurement of speed.

- It consists of a permanent magnent on a solenoid.
- A gear of ferromagnetic material is mounted on the rotating shaft whose speed is to be measured.

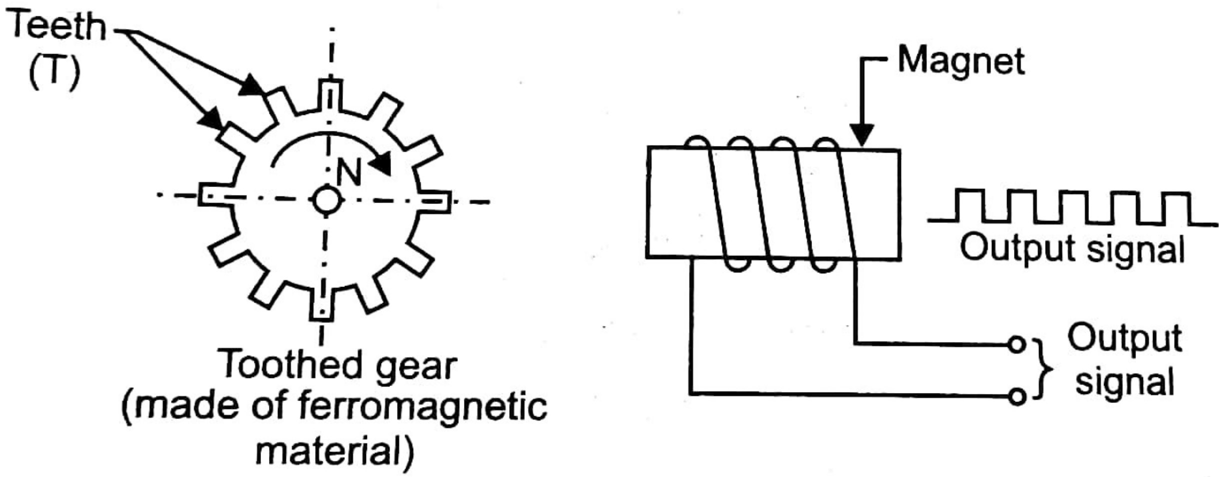


Fig.(a). Electromagnetic frequency domain transducer.

- As each gear tooth passes in front of the magnet, the gap length changes.
- Consequently there is a change in the flux density and a voltage pulse is induced in the coil.
- Pulse frequency equal speed N times the number of teeth T. The form of the output signal is shown in Fig.(a).

  Thus, pulse frequency is a measure of rotational speed.

2. Opto-electrical frequency domain transducer :

- Fig.(b), shows schematic arrangement of this type of transducer for **linear motion** :
- The arrangement includes a transparent scale with a grating
- The moving object is attached to the transparent scale.
- Light from a source passes through the scale and the slit and then falls on a photoelectric transducers.
- The slit width is such that a motion equal to the pitch of the grating produces one complete cycle of light and darkness at the photoelectric cell.
- Thus, a pulse output is obtained. From the number of pulses, the change in motion of the scale and the object attached to it can be determined.

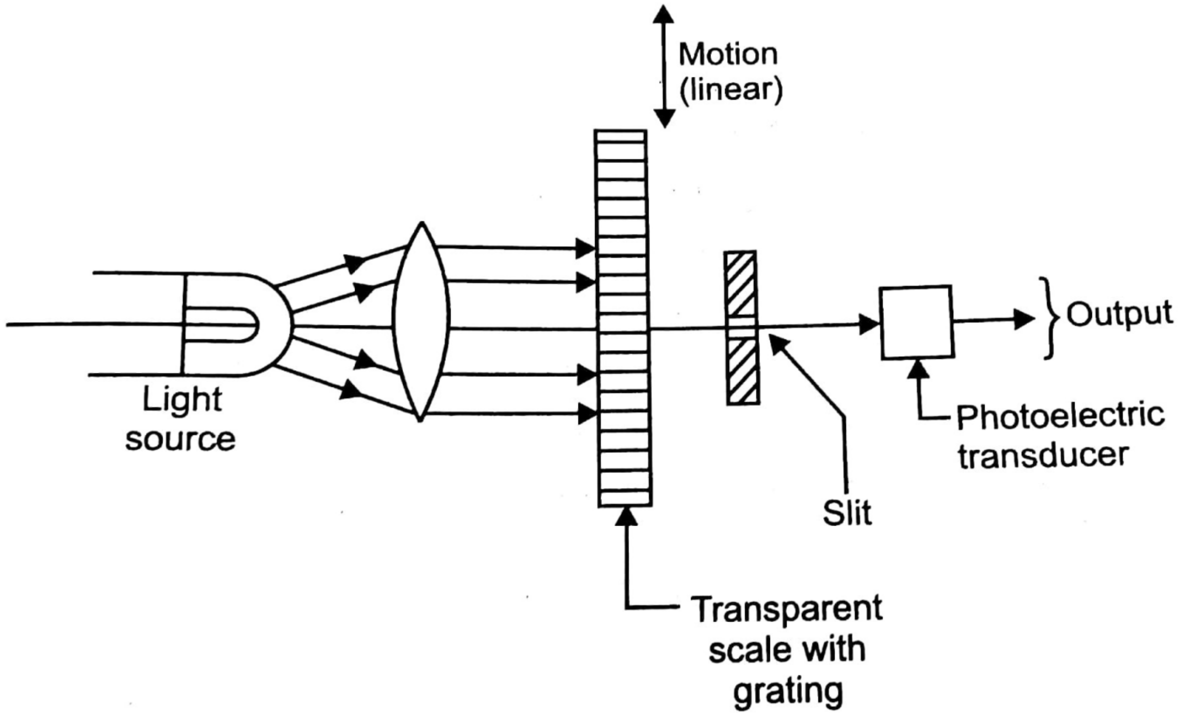


Fig.(b).Opto-electrical frequency domain transducer

- Fig.(c), shows opto electrical frequency domain transducer for measurement of **rotational speed** of a shaft :
- The shaft has half dark and half white or shining portion

- Every time the white/shining portion of the shaft is in front of the light source, the reflected light, falling on the photoelectric transducer, gives an electrical pulse output.
- The frequency of the pulses is thus a measure of speed of rotation of the shaft.

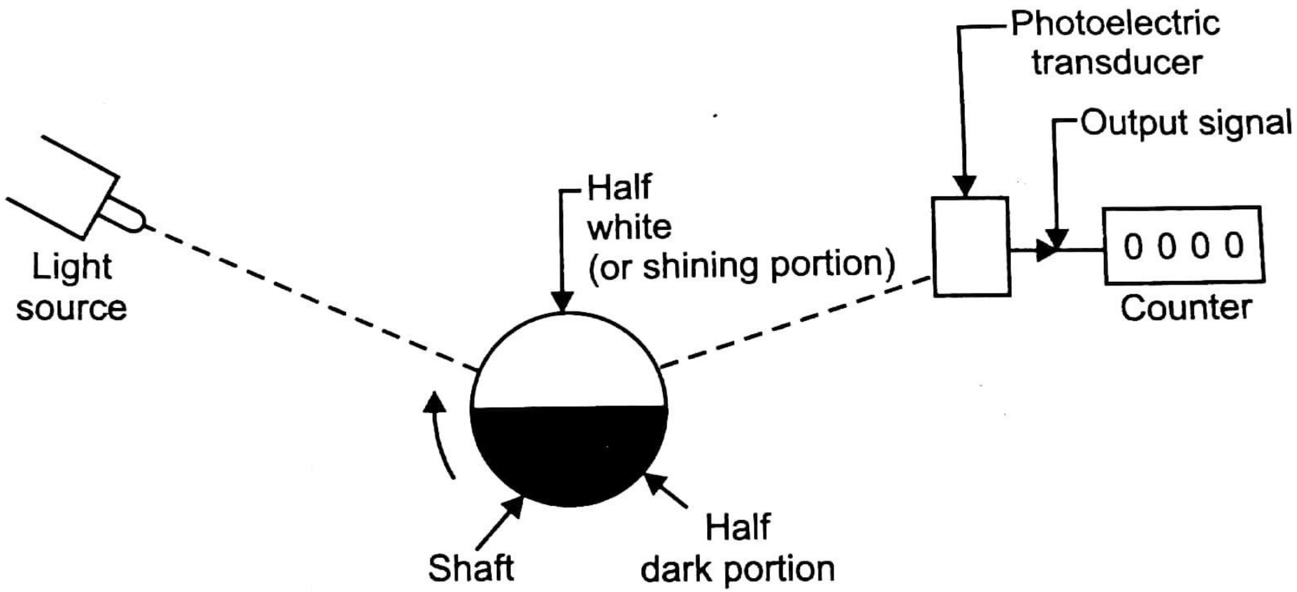


Fig.(c). Opto-electrical frequency domain transducer for rotary motion.

3.Vibrating string transducer

- This type of transducer is essentially employed to measure the force applied to a metal string, which is kept-vibrating, the frequency of which is dependent on the force applied.

The natural frequency of the string is given by :

$$f = \frac{1}{2l}\sqrt{\frac{P}{a\rho}}$$

f = Natural frequency of the string,
l = Length of the string,
P = Force applied,
a = Area of cross-section of the string, and
ρ = Mass density of the wire material.

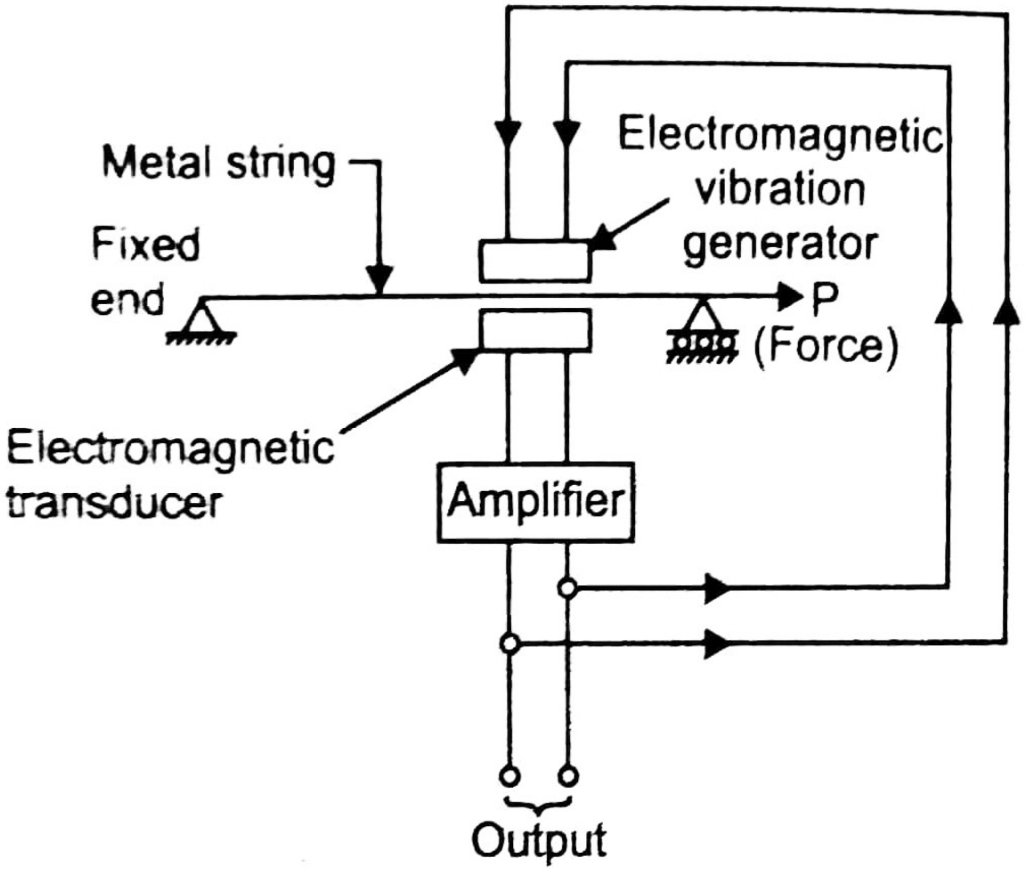


Fig.(d). Vibrating string transducer.

- Fig.(d) , shows the schematic arrangement of a vibrating string transducer
- One end of the string is fixed while the other can be moved relative to it, due to the force applied.

- The vibrations are picked up by the electromagnetic transducer. The output of is amplified (by the amplifier) and then fed to the electromagnetic vibration generator, which maintains the string vibration at its natural frequency f.
- Frequency f gets changed due to change in magnitude of force P.
- The frequency is measured by a frequency counter and is a measure of the force applied to the string.
- This transducer can be used for measurement of force and displacement.
- Vibrating diaphragm and cylinder types of transducers have been developed to measure pressure.

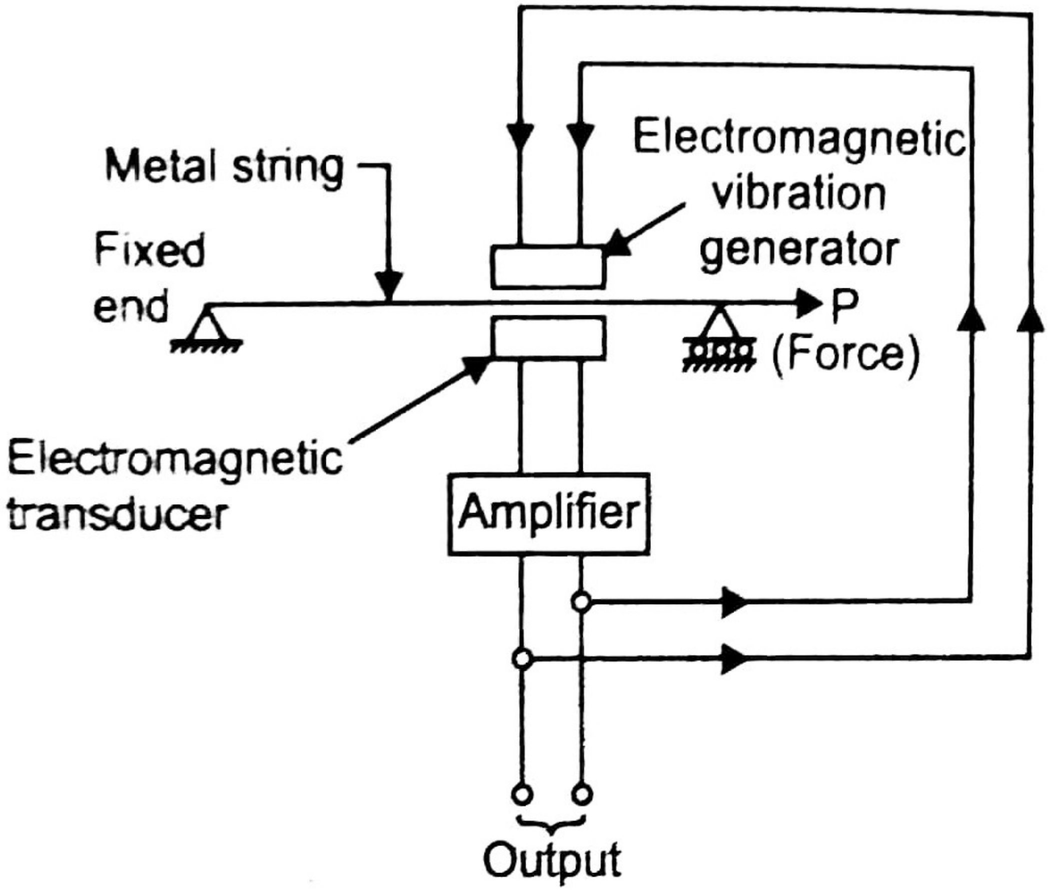


Fig.(d). Vibrating string transducer.

DIGITAL ENCODERS

In the contact type arrangement there is direct contact between the brushes and encoder disc.

- The disadvantage of such a system is the wear of brushes and disc (due to friction between the brushes and disc)

1. Optical type encoder:

Fig.(a) illustrates the arrangement of an optical type encoder :

- It is a non-contact encoder.
- The disc has transparent and opaque areas, corresponding to the conducting and non-conducting one respectively, in the contacting type encoder disc.

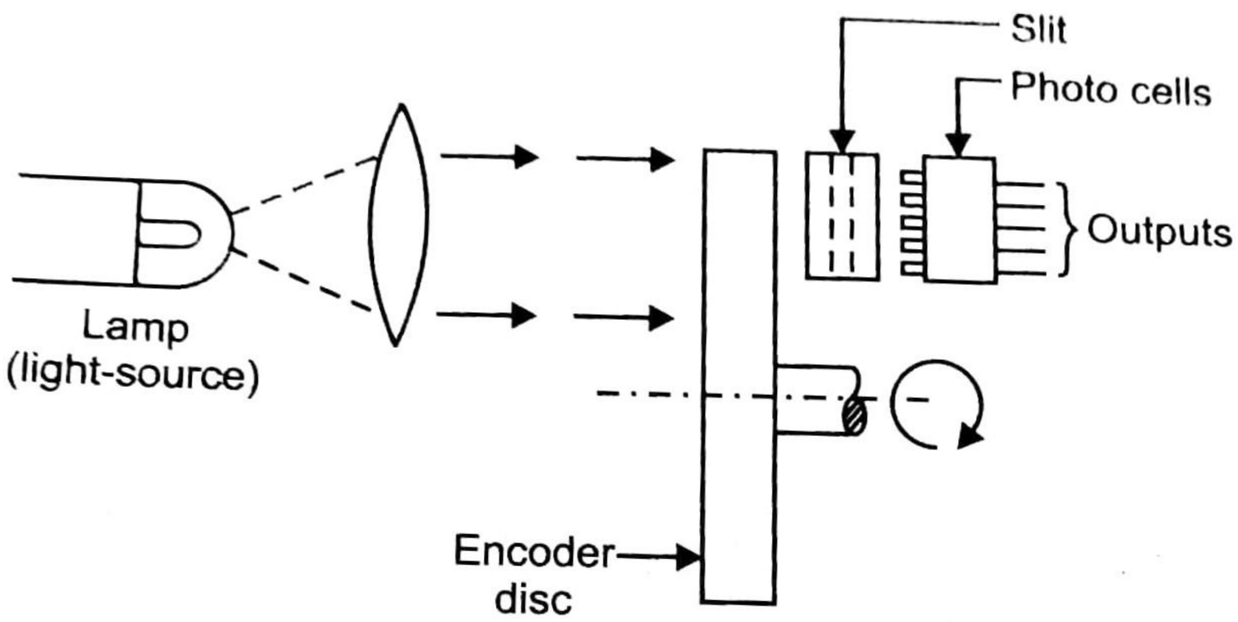


Fig.(a) Optical type encoder.

- A light source is used along with slits and photo cells. The photo cell, corresponding to a particular track, would produce an electrical output if the transparent portion is in front of the slit and light source, giving state ON (or 1)
- while no electrical output from a cell would correspond to OFF (or 0) state.

2 Magnetic type encoder :

- Fig.(b) shows a magnetic encoder circuit :
- It is also a non-contact type encoder.
- This encoder uses a number of small toroidal magnets with coils around them.
- The area corresponding to the conducting and non-conducting ones in the contact type encoder disc form the non-magnetic and magnetic areas on the disc.

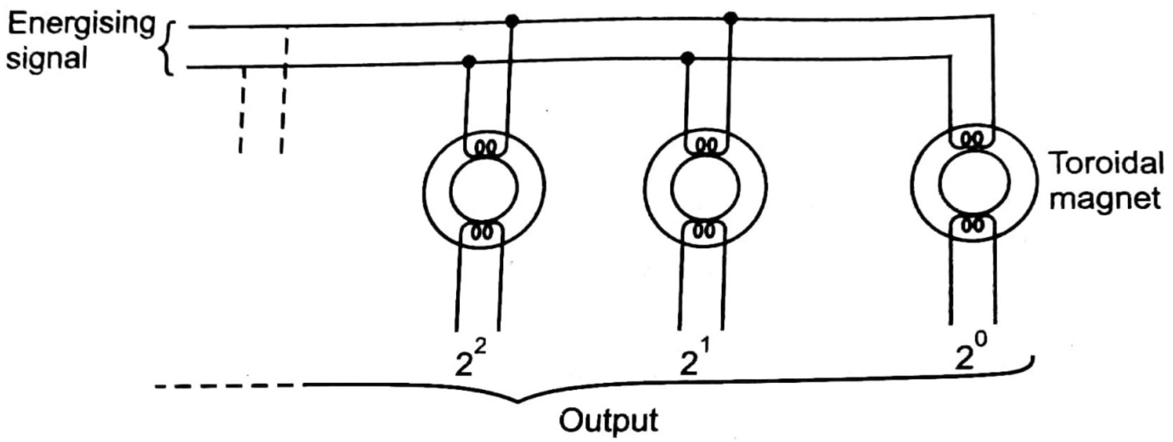


Fig. (b). Magnetic encoder circuit.

- The presence or absence of such areas is detected by coils which are in close proximity to each track on the disc.
- One of the coils in each toroidal magnet is energised with high frequency A.C. carrier signal. If the toroid is over a non-magnetic area of the encoder disc, a voltage due to transformer action is induced in the output coils (state 1) while a magnetic area would saturate the magnetic circuit and output is very small (state 0).
- The signals on the output windings are demodulated to remove the high frequency carrier signal.

Numerical Problems

Numerical Problems on LVDT

(1). In a linear voltage differential transformer (LVDT) the output voltage is 1.8 V at maximum displacement. At a certain load the deviation from linearity is maximum and it is $\pm$0.0045 V from a straight line through the origin. Find the linearity at the given load.

Solution: Given : The output voltage of LVDT at maximum displacement = 1.8 V

The deviation from a straight line through the origin = $\pm$ 0.0045 V

%age linearity = $\pm \frac{0.0045}{1.8} \times 100 = \pm\, 0.25\%$

(2). The output voltage of a LVDT is 1.5 V at maximum displacement. At a load of 0.5 MΩ, the deviation from linearity is maximum and it is $\pm$ 0.003 V from a straight line through origin.
Find the linearity at the given load.

Solution:

%age linearity = $\pm \frac{0.003}{1.5} \times 100 = \pm\, 0.2\%$

Numerical Problems on LVDT

(3). The output of a LVDT is connected to a 4 V voltmeter through an amplifier whose amplification factor is 500. An output of 1.8 mV appears across the terminals of LVDT when the core moves through a distance of 0.6 mm. If the milli voltmeter scale has 100 divisions and the scale can be read to $\frac{1}{4}$ of a division, calculate:

(i) The sensitivity of LVDT.

(ii) The resolution of the instrument in mm.

Solution. (*i*) **The sensitivity of LVDT :**

$$\text{The sensitivity of LVDT} = \frac{\text{Output voltage}}{\text{Displacement}} = \frac{1.8}{0.6} = \mathbf{3\ mV/mm\ (Ans.)}$$

(*ii*) **The resolution of the instrument :**

$$\text{Sensitivity of measurement} = \text{Amplification factor} \times \text{sensitivity of LVDT}$$

$$= 500 \times 3 = 1500\ \text{mV/mm}$$

$$1\ \text{scale division} = \frac{4}{100}\ \text{V} = 40\ \text{mV}$$

Minimum voltage that can be read on the voltmeter

$$= \frac{1}{4} \times 40 = 10\ \text{mV}$$

$\therefore$ Resolution of the instrument

$$= 10 \times \left(\frac{1}{1500}\right) = \mathbf{0.0067\ mm\ (Ans.)}$$

Numerical Problems on LVDT

(4). The output of an LVDT is connected to a 5 V voltmeter through an amplifier whose amplification factor is 250. An output of 2 mV appears across the terminals of LVDT when the core moves through a distance of 0.5 mm. Calculate the sensitivity of the LVDT and that of the whole set-up. The milli-voltmeter scale has 100 divisions. The scale can be read to 1/5 of a division. Calculate the resolution of the instrument in mm.

Solution:

Sensitivity of LVDT

$$= \frac{\text{output voltage}}{\text{displacement}}$$

$$= \frac{2\times 10^{-3}}{0.5} = 4\times 10^{-3}\ \text{V/mm}$$

$$= 4\ \text{mV/mm}$$

Sensitivity of instrument

$$= \text{amplification factor} \times \text{sensitivity of LVDT}$$

$$= 4\times 10^{-3}\times 250 = 1\ \text{V/mm} = 1000\ \text{mV/mm}$$

$$1\ \text{scale division} = 5/100\ \text{V} = 50\ \text{mV}$$

Minimum voltage that can be read on the voltmeter

$$= (1/5)\times 50 = 1\ \text{mV}$$

$\therefore$ Resolution of instrument

$$= 1\times(1/1000) = 1\times 10^{-3}\ \text{mm}$$

Numerical Problems on Strain gauge

(5). The gauge factor of a resistance wire strain gauge using a soft iron wire small diameter is 4.2. Neglecting the piezo-resistive effect, calculate the Poisson's ratio.

Solution. Given: $G_f = 4.2$

When the piczo-resistive effect is neglected, the gauge factor is given by:

$$G_f = 1 + 2\mu$$

$$4.2 = 1 + 2\mu$$

$$\mu = \frac{4.2-1}{2} = \mathbf{1.6}$$

Numerical Problems on Strain gauge

(6). A simple electrical strain gauge of resistance 120 Ω and having a gauge factor of 2 is bonded to steel having an elastic limit stress of 400 MN/m^2 and modulus of elasticity is 200 GN/m^2. Calculate the change in resistance,

(i) due to a change in stress equal to $\frac{1}{10}$ of the elastic range;

(ii) due to change of temperature of 20°C if the material is advance alloy. The resistance temperature coefficient of advance alloy is 20 x 10^{-6}/°C.

Solution. *Given:* $R = 120\ \Omega$; $G_f = 2$; Elastic limit stress = 400 MN/m^2; Modulus of elasticity = 200 GN/m^2; Resistance temperature coefficient, $\alpha_0 = 20 \times 10^{-6}/°C$.

Change in resistance:

(i) $$\text{Change in stress} = \frac{1}{10} \times 400\ MN/m^2 = 40 \times 10^6\ N/m^2$$

$$\text{Modulus of elasticity} = 200\ GN/m^2 = 200 \times 10^9\ N/m^2$$

$$\text{Strain, } e = \frac{\text{Stress}}{\text{Modulus of elasticity}} = \frac{40 \times 10^6}{200 \times 10^9} = \frac{1}{5} \times 10^{-3}$$

$$\text{Gauge factor } G_f = \frac{\text{Per unit change in resistance}}{\text{Per unit change in length}}$$

or, $$G_f = \frac{\Delta R / R}{e} \quad \text{or} \quad \Delta R = R\, G_f\, e$$

or, $$\Delta R = 120 \times 2 \times \frac{1}{5} \times 10^{-3} = 48 \times 10^{-3}\ \Omega = 45\ \text{m}\Omega \text{ (Ans.)}$$

(ii) $$R_{t2} = R_{t1}\,[1 + \alpha_0(t_2 - t_1)]$$

$\therefore$ Change in resistance $R_{t2} - R_{t1} = R_{t1}\,\alpha_0(t_2 - t_1)$

or, $$\Delta R = R_{t2} - R_{t1} = 120 \times 20 \times 10^{-6} \times (20)$$
$$= 48 \times 10^{-3}\ \Omega = 48\ \text{m}\Omega \text{ (Ans.)}$$

Numerical Problems on Strain gauge

(7). A strain gauge is bonded to a beam which is 12 cm long and has a cross- sectional area of 3.8 cm². The unstrained resistance and gauge factor of the strain gauge are 220 ***Ω*** and 2.2 respectively. On the application of load the resistance of the gauge changes by 0.015 ***Ω*** . If the modulus of elasticity for steel is 207 GN/m², calculate:

(i) The change in length of the steel beam.

(ii) The amount of force applied to the beam.

Solution. *Given:* $L = 12$ cm $= 0.12$ m; $A = 3.8$ cm^2 $= 3.8 \times 10^{-4}$ m^2; $R = 220\ \Omega$; $G_f = 2.2$; $\Delta R = 0.015\ \Omega$; $E = 207$ GN/m^2.

(*i*) **The Change in length of steel beam. ΔL:**

$$\text{Gauge factor, } G_f = \frac{\Delta R/R}{\Delta L/L}$$

$$\therefore \quad \Delta L = \frac{(\Delta R/R)L}{G_f} = \frac{(0.015/220)\times 0.12}{2.2} = \mathbf{3.72 \times 10^{-6}\ m} \text{ (Ans.)}$$

(*ii*) **The amount of force applied to the beam, *F*:**

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{e}$$

$$\therefore \quad \sigma = E \times e = E \times \frac{\Delta L}{L}$$

$$= (207 \times 10^9) \times \frac{3.72 \times 10^{-6}}{0.12} = 6.417 \times 10^6 \text{ N/m}^2$$

$$\therefore \quad \textit{Force, } F = \sigma \cdot A = 6.417 \times 10^6 \times 3.8 \times 10^{-4} \text{ N} = \mathbf{2.438\ kN} \text{ (Ans.)}$$

Numerical Problems on Strain gauge

(8). A resistance, wire strain gauge with a gauge factor of 2 is bonded to a steel structural member subjected to a stress of 100 MN/m^2. The modulus of elasticity of steel is 200 GN/m^2. Calculate the percentage change in the value of the gauge resistance due to the applied stress.

Solution.

Strain $\varepsilon = \dfrac{s}{E} = \dfrac{100 \times 10^6}{200 \times 10^9}$

$= 500 \times 10^{-6}$ (500 microstrain)

We have, $\dfrac{\Delta R}{R} = G_f \varepsilon = 2 \times 500 \times 10^{-6} = 0.001 = 0.1\%$

$\therefore$ The change in resistance is only 0.1%.

Numerical Problems on Strain gauge

(9). A single strain gauge having resistance of 120 Ω is mounted on a steel cantilever beam at a distance of 0.15 m from the free end. An unknown force F applied at the free end produces a deflection of 12.7 mm of the free end. The change in gauge resistance is found to be 0.152 Ω . The beam is 0.25 m long with a width of 20 mm and a depth of 3 mm. The Young's modulus for steel is 200 GN/m². Calculate the gauge factor.

Solution.

Moment of inertia of beam,

$$I = 1/12\,(bd^3) = 1/12 \times 0.02 \times (0.003)^3$$

$$= 45 \times 10^{-12} \text{ m}^4$$

Deflection $\quad x = \dfrac{Fl^3}{3\,EI}$

$\therefore$ Force $\quad F = \dfrac{3\,EIx}{l^3}$

$$= \frac{3 \times 200 \times 10^9 \times 45 \times 10^{-12} \times 12.7 \times 10^{-3}}{(0.25)^3}$$

$$= 22 \text{ N}$$

Bending moment at 0.15 m from free end

$$M = Fx = 22 \times 0.15 = 3.3 \text{ Nm}$$

Stress at 0.15 m from free end

$$s = \frac{M}{I} \cdot \frac{t}{2} = \frac{3.3}{45 \times 10^{-12}} \times \frac{0.003}{2} = 110 \text{ MN/m}^2$$

Strain $\quad \varepsilon = \dfrac{\Delta L}{L}$

$$= \frac{s}{E} = \frac{110 \times 10^6}{200 \times 10^9} = 0.55 \times 10^{-3}$$

$\therefore$ Gauge factor

$$= \frac{\Delta R / R}{\Delta L / L} = \frac{0.152 \times 120}{0.55 \times 10^{-3}} = 23$$

Numerical Problems on Strain gauge

(10). A steel cantilever is 0.25 m long, 20 mm wide and 4 mm thick.

(a) Calculate the value of deflection at the free end for the cantilever when a force of 25 N is applied at this end. The modulus of elasticity for steel is 200 GN/m².

(b) An LVDT with a sensitivity of 0.5 V/mm is used. The voltage is read on a 10 V voltmeter having 100 divisions. Two tenths of a division can be read with certainty.

(c) Calculate the minimum and maximum value of force that can be measured with this arrangement.

Solution: (a) Moment of area of cantilever

$$I = \frac{1}{12} bt^3$$

$$= \frac{1}{12} \times (0.02) \times (0.004)^3$$

$$= 0.107 \times 10^{-9}\ \text{m}^4$$

Deflection $\quad x = \dfrac{Fl^3}{3EI}$

$$= \frac{25 \times (0.25)^3}{3 \times 200 \times 10^9 \times 0.107 \times 10^{-9}}$$

$$= 6.08 \times 10^{-3}\ \text{m} = 6.08\ \text{mm}$$

(b) Deflection per unit force

$$\frac{x}{F} = \frac{6.08}{25}$$

$$= 0.2432\ \text{mm/N}$$

Overall sensitivity of measurement system :

$$= 0.2432 \frac{\text{mm}}{\text{N}} \times 0.5 \frac{\text{V}}{\text{mm}}$$

$$= 0.1216\ \text{V/N}$$

1 scale division = (10/1000) = 0.1 V

Since two tenths of a scale division can be read with certainty, resolution = (2 / 10) × 0.1 = 0.02 V

(c) Minimum force that can be measured

$$= 0.02/0.1216 = 0.1645\ \text{N}$$

Maximum force that can be measured

$$= (10/0.1216) = 82.2\ \text{N}$$

Numerical Problems on Strain gauge

(11). A strain gauge is bonded to a beam 0.1 m long and has a cross-sectional area 4 cm^2. Young's modulus for. steel is 207 GN/m^2. The strain gauge has an unstrained resistance of 240 $\boldsymbol{\Omega}$ and a gauge factor of 2.2. When a load is applied, the resistance of gauge changes by 0.013 $\boldsymbol{\Omega}$. Calculate the change in length of the steel beam and the amount of force applied to the beam.

Solution. We have gauge factor

$$G_f = \frac{\Delta R / R}{\Delta L / L}$$

and therefore change in length

$$\Delta L = \frac{\Delta R / R . L}{G_f}$$

$$= \frac{(0.013/240)}{2.2} \times 0.1 = 2.46 \times 10^{-6}\ \text{m}$$

Stress $\quad s = E\varepsilon$

$$= \frac{207 \times 10^9 \times 2.46 \times 10^{-6}}{0.1}$$

$$= 5.092 \times 10^6\ \text{N/m}^2$$

Force $\quad F = sA$

$$= 5.092 \times 10^6 \times 4 \times 10^{-4} = 2.037 \times 10^3\ \text{N}$$

Numerical Problems on Thermistor

(12).

(a) A thermistor has a resistance temperature coefficient of -5% over a temperature range of 25°C to 50°C. If the resistance of the thermistor is 100 W at 25° C, what is the resistance at 35°C?

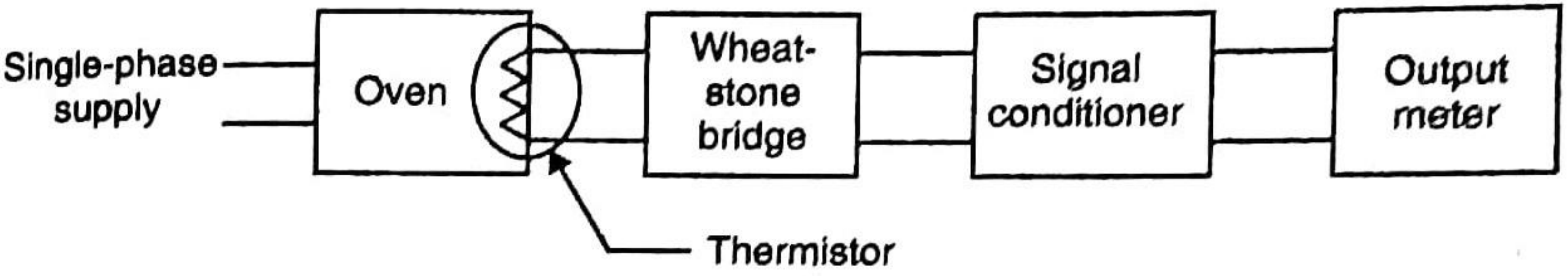
(b) Suggest a complete instrumentation scheme in block diagram form to measure the temperature in a closed oven with the help of thermistor.

Solution.

(a) R = Ro [1+α(T-To)]

$$R_{35} = R_{25}\ [1 + \alpha(35 - 25) = 100[1 – 0.05(35 – 25)] = 50\ \boldsymbol{\Omega}$$

(b) Figure shows the complete instrumentation scheme for the measurement of temperature with the help of a thermistor. Thermistor is mounted in the oven at a place where temperature is to be sensed. With the increase in temperature, resistance of the thermistor decreases causing imbalance in Wheatstone bridge circuit whose output balance voltage is amplified by signal conditioning device, the amplified output when connected to a suitable output device gives the value of the temperature of the oven.



Thank you

ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

By

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ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

Syllabus

UNIT–I	Analog Ammeter and Voltmeters
UNIT–II	Analog Watt meters and Power Factor Meters
UNIT–III	Measurements of Electrical parameters
UNIT–IV	Transducers
UNIT–V	Digital Meters

Text Books:

- Electrical & Electronic Measurement & Instruments by A.K.Sawhney Dhanpat Rai & Co.Publications
- Electrical and Electronic Measurements and instrumentation by R.K.Rajput, S.Chand
- Electrical Measurements and measuring Instruments by E.W. Golding and F.C.Widdis, fifth Edition, Wheeler Publishing
- Electrical Measurements by Buckingham and Price, Prentice – Hall

ELECTRICAL MEASUREMENTS AND INSTRUMENTATION

UNIT-V

DIGITAL METERS

UNIT–V
Digital Meters

Topics:

- Digital Voltmeter
  - Successive approximation DVM, Ramp type DVM and Integrating type DVM
- Digital Frequency Meter,
- Digital Multimeter,
- Digital Tachometer,
- Digital Energy Meter,
- Digital LCR Meter,
- Q Meter,
- Power Analyzer-Measurement of phase difference, Frequency, hysteresis loop using lissajious patterns in CRO
- Numerical Problems.

DIGITAL METERS

Introduction

- A digital instrument can be considered as a counter which counts the pulses in a predetermined time. Digital transducers whose output is in the form of pulses are used to monitor the desired parameter.
- "Accuracy" of a digital instrument is dependent on the number of pulses generated by the transducer because the fraction of pulse cannot be generated and in counting there can be ambiguity of only one pulse at start/stop.
- Hence more are pulses corresponding to a measure less the possibility of error corresponding to one pulse and more the accuracy.
- The digital instruments use logic circuits and techniques for carrying out measurements of quantities.
- The information in the electronic digital read-out (display) devices is presented as a series of digits on the tubes, screen, or printed on a piece of paper.

❑ The relevant characters (letters of alphabet from A to Z, numerals from 0 to 9, punctuation mark and other symbols in common use) can be generated by :

(i) Semiconductor light emitting diodes (LED).

(ii) Liquid crystal displays (LCD).

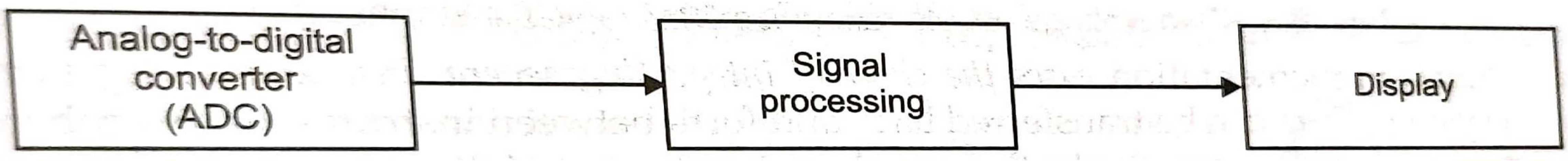
(iii) Numerical indicators tubes (NIT).

(iv) Hot filament or bar tubes.

DIGITAL METERS

- Digital instruments provide better resolution than analog instruments. A typical value of resolution of a digital instrument may be one part in 10^6 i.e., 1 μV can be read on the 1-V input range.
- The digital meters work on the principle of "quantization".

- Fig(a) shows the building block of a digital instrument. Any digital instrument invariably consists of a an analog-to-digital converter (ADC) in its input stage.
  The display block may be analog or digital in nature. If an analog read out is desired, it becomes necessary to include a stage involving digital-to-analog (D/A) conversion



Fig(a).Building block of a Digital Instrument

DIGITAL METERS

ANALOG AND DIGITAL SYSTEMS

- An analog system comprises devices that manipulate the physical quantities represented in analog form. Examples of analog systems are : Automobile speedometer, Magnetic tape recording and playback equipment etc.
- A digital system is a combination of devices designed for manipulating physical quantities or information represented in digital form.
- Examples of digital systems are : Digital computers, typewriters, digital watches etc.

Advantages of digital systems/techniques :

1. Easier to design.
2. Easy storage of information.
3. Greater accuracy and precision.
4. Operation is programmable.
5. Digital circuits are less affected by the noise

DIGITAL METERS

- Digital system may consist of the following components

(i) Resistors;
(ii) Capacitors;
(ii) Transistors;
(iv) Linear ICs;
(v) Digital ICs;
(vi) Digital devices;
(vii) Analog-to-digital
(viii) Digital-to-analog converters
(DAC).converters (ADC)

(ix)Binary counters
(x)Decimal counting unit(DCU)
(xi)Display devices
(xii)Frequency counters
(xiii)Period counters

- Data is digital form facilitates various operations that are normally required in signal processing.
- An increase in the availability and type of computer facilities and a decrease in the cost of various methods required for digital systems is accelerating the development of digital instrumentation for measurement and signal processing.

DIGITAL METERS

➢ Characteristics of Digital Meters

- Following are the few specifications which characterize digital meters:
  1. Resolution
  2. Sensitivity,
  3. Accuracy

1. Resolution: It is defined as the number of digit positions or simply the number of digits used in a meter.

If number of full digits is n, then resolution,

$$R = \frac{1}{10^n}$$

For n = 4, $R = \frac{1}{10^4} = 0.0001$ or 0.01%

A three-digit display on the digital meter for 0-1 V range will be able to indicate from 000 to 999 mV, with smallest increment (resolution) of 1 mv.

DIGITAL METERS

2 Sensitivity: It is the smallest change in input which a digital meter is able to detect. Thus, it is the full-scale value of the lowest voltage range multiplied by the resolution of the meter. In other words,

$$\text{Sensitivity, } S = (fs)_{\min} \times R$$

where,

$(fs)_{\min}$ = Lowest full-scale value of digital meter, and

R = Resolution is decimal.

3. Accuracy: In the accuracy specifications of digital meters, the following two quantities are included :

(i) A percentage of range.

(ii) A percentage of reading,

DIGITAL METERS

Example 8.3. *What is the resolution of a $4\frac{1}{2}$ digit display? How would 16.95 V be displayed on a 10 V range and 0.6564 V on a 1 V and 10 V ranges ?*

Solution. • Number of full digits on $4\frac{1}{2}$ digit display, $n = 4$

$$\therefore \qquad \text{Resolution, } R = \frac{1}{10^n} = \frac{1}{10^4} = \mathbf{0.0001} \quad \textbf{(Ans.)}$$

• There are 5 digit faces in $4\frac{1}{2}$ digit display, so *16.95 would be displayed as* **16.950** **(Ans.)**

Resolution on 1 V range is 0.000 × 1 = 0.0001 V.

Any reading upto the *4th decimal can be displayed.*

Hence, *0.6564 will be displayed as* **0.6564** **(Ans.)**

Resolution on 10 V range is 0.0001 × 10 = 0.001 V.

Hence, decimals upto *third decimal place can be displayed.*

Hence, *0.6564 will be displayed as* **0.656** **(Ans.)**

DIGITAL VOLTMETERS

DIGITAL VOLTMETERS

- **D**igital **V**olt**M**eters (**DVM**s) are measuring instruments that convert analog voltage signals into a digital or numeric readout. This digital readout can be displayed on the front panel and also used as an electrical digital output signal.

- Any DVM is capable of measuring analog dc voltages. However, with appropriate signal conditioners preceding the input of the DVM, quantities such as ac voltages, ohms, dc and ac currents etc can be measured. The common element in all these signal conditioners is the dc voltage, which is proportional to the level of the unknown quantity being measured. This dc output is then measured by the DVM.

- DVM s have various features such as speed, automation operation and programmability. There are several varieties of DVM which differ in the following ways:

  1. Number of digits
  2. Number of measurements
  3. Accuracy
  4. Speed of reading
  5. Digital output of several types.

DIGITAL VOLTMETERS

- The DVM displays ac and dc voltages as discrete numbers, rather than as a pointer on a continuous scale as in an analog voltmeter.
- A numerical readout is advantageous because it reduces human errors, eliminates parallax error, increases reading speed and often provides output in digital form suitable for further processing and recording.
- With the development of IC modules, the size, power requirements and cost of DVMs have been reduced, so that DVMs compete with analog voltmeters in portability and size.

- Two types of voltmeters are available for the purpose of voltage measurement i.e. analog and digital.
- Analog voltmeters generally contain a dial with a needle moving over it according to the measure and hence displaying the value of the same.
- With the passage of time analog voltmeters are replaced by digital voltmeters due to the same advantages associated with digital systems.
- Although analog voltmeters are not fully replaced by digital voltmeters, still there are many places where analog voltmeters are preferred over digital voltmeters.
- Digital voltmeters display the value of AC or DC voltage being measured directly as discrete numerical instead of a pointer deflection on a continuous scale as in analog instruments.

DIGITAL VOLTMETERS

➢ Advantages of Digital Voltmeters

1) Read out of DVMs is easy as it eliminates observational errors in measurement committed by operators.
2) Error on account of parallax and approximation is entirely eliminated.
3) Readings can be taken very fast.
4) Output can be fed to memory devices for storage and future computations.
5) Versatile and accurate
6) Compact and cheap
7) Low power requirements
8) Portability increased
9) Compatibility with other digital equipment for further processing and recording,

➢ Applications of DVMs

- DVMs are often used in "data processing systems" or "data logging systems".
  In such systems, a number of analog input signals are scanned sequentially by an electronic system and then each signal is converted to an equivalent digital value by the A/D (analog-to-digital) converter in the DVM.
- The digital value is then transmitted to a printer along with the information about the input line from which the signal has been derived.
- The whole data is then printed out.
- In this way, a large number of input signals can be automatically scanned or processed and their values either printed or logged.

DIGITAL VOLTMETERS

➢ Characteristic Features of DVMs

The characteristic features of digital voltmeters (DVMs) are:

1. Input range From $\pm$1.000 V to $\pm$1000 V with automatic range selection and overload indication

2. Absolute accuracy As high as $\pm$ 0.005% of the reading,

3. Resolution : 1 part in million (1μV reading can be read or measured on 1 V range)

4. Stability : Short-term, 0.002 % of the reading for a 24-hr period, long terms, 0.008% of the reading for a 6-month period.

5. Input resistance : Typically 10 MΩ;

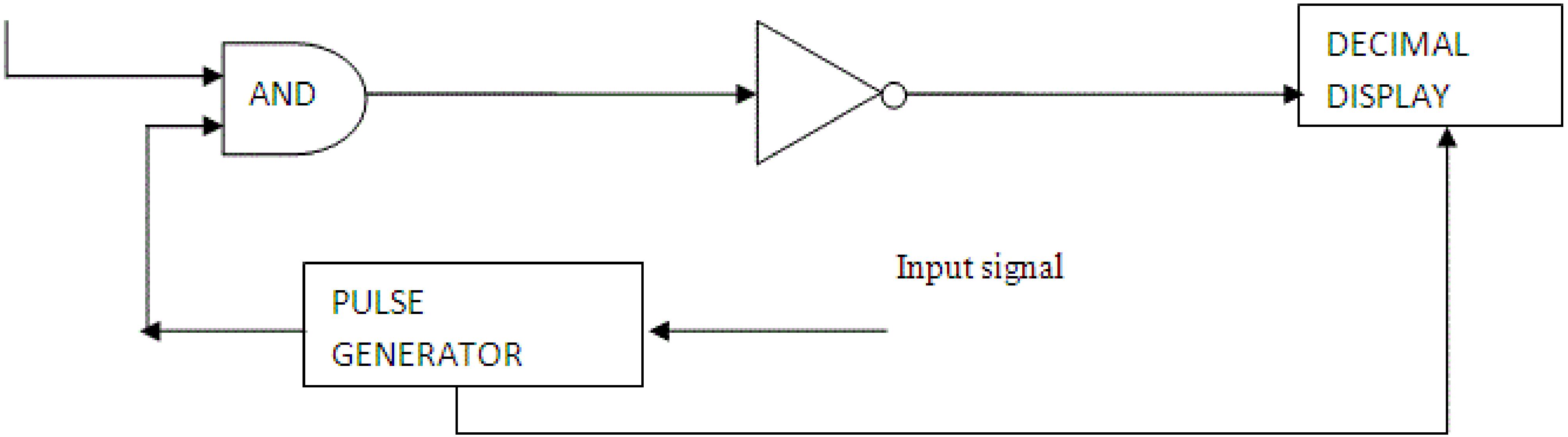
Input capacitance : 40 pF

6. Calibration Internally from stabilized reference sources, independent of measuring circuit

7. Output signals In BCD (Binary coded-decimal) form, for print output and further digital processing.

Optional features may include additional circuitry to measure current, ohms and voltage ratio.

➢ Working Principle of Digital Voltmeter:



The block diagram of a simple digital voltmeter is shown in above figure.

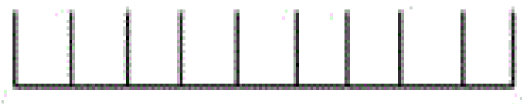
Explanation of various blocks:

- **Input signal:** It is basically the signal i.e. voltage to be measured.
- **Pulse generator:** Actually it is a voltage source. It uses digital, analog or both techniques to generate a rectangular pulse. The width and frequency of the rectangular pulse is controlled by the digital circuitry inside the generator while amplitude and rise & fall time is controlled by analog circuitry.

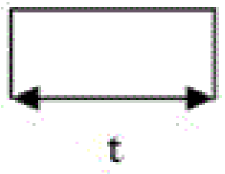
DIGITAL VOLTMETERS

AND gate: It gives high output only when both the inputs are high. When a train pulse is fed to it along with rectangular pulse, it provides us an output having train pulses with duration as same as the rectangular pulse from the pulse generator.

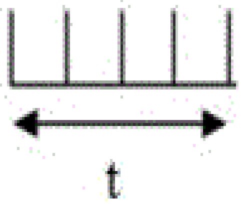
Train Pulse



Rectangular Pulse



Output of AND Gate



Decimal Display: It counts the numbers of pulses and hence the duration and display the value of voltage on LED or LCD display after calibrating it.

➢Now we are in situation to understand the working of a digital voltmeter as follows:

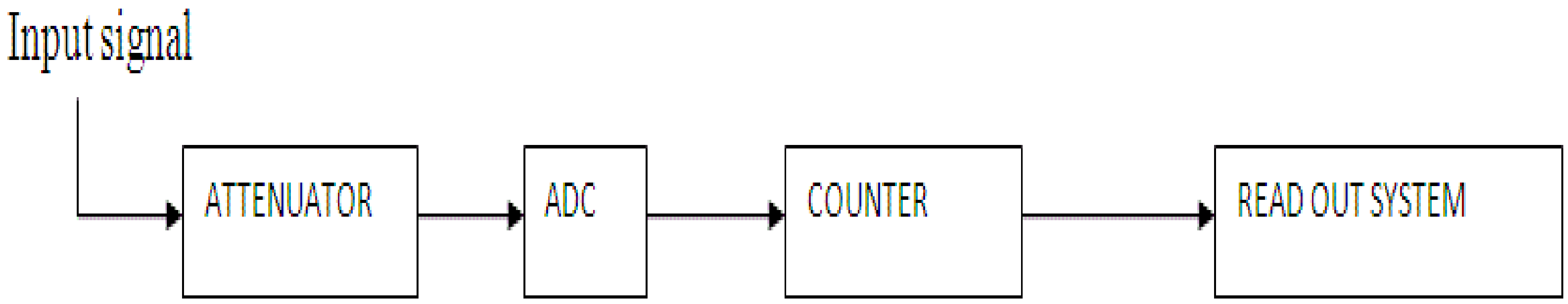
- Unknown voltage signal is fed to the pulse generator which generates a pulse whose width is proportional to the input signal.
- Output of pulse generator is fed to one leg of the AND gate.
- The input signal to the other leg of the AND gate is a train of pulses.
- Output of AND gate is positive triggered train of duration same as the width of the pulse generated by the pulse generator.

DIGITAL VOLTMETERS

This positive triggered train is fed to the inverter which converts it into a negative triggered train.

1) Output of the inverter is fed to a counter which counts the number of triggers in the duration which is proportional to the input signal i.e. voltage under measurement.
2) Thus, counter can be calibrated to indicate voltage in volts directly.

- We can see the working of digital voltmeter that it is nothing but an analog to digital converter which converts an analog signal into a train of pulses, the number of which is proportional to the input signal. So a digital voltmeter can be made by using any one of the A/D conversion methods.



Classification of Digital Voltmeters (DVMs)

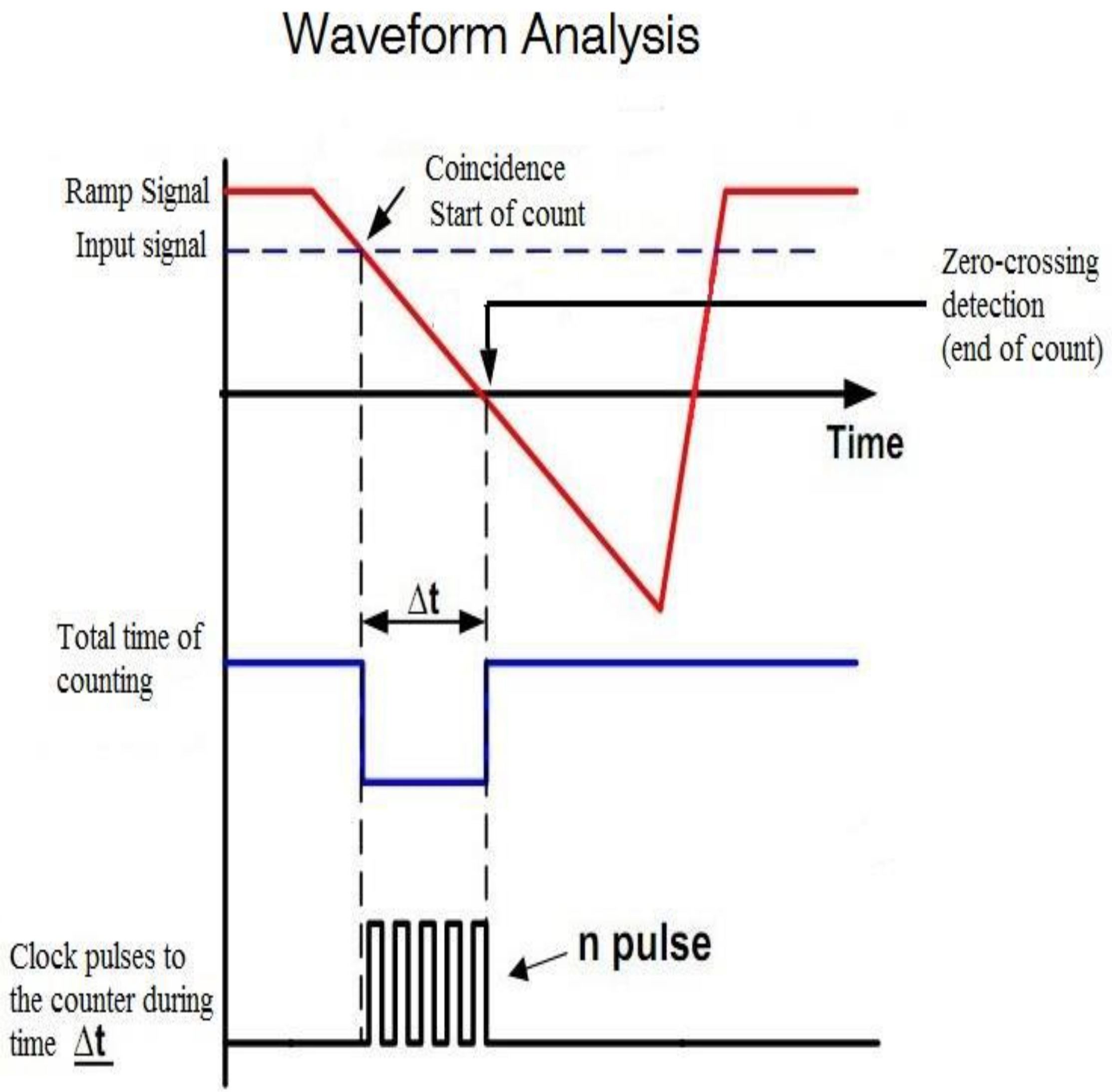
➢On the basis of A/D conversion method used digital voltmeters can be classified as:

1) Ramp type digital voltmeter
2) Dual slope integrating type DVM(Voltage to time conversion)
3) Integrating type DVM(Voltage to frequency conversion)
4) Successive approximation type DVM
5) Continuous balance type DVM or Servo balancing or Potentiometer type DVM

➢Now-a-days digital voltmeters are also replaced by digital multimeters due to its multitasking feature i.e. it can be used for measuring current, voltage and resistance.

➢ RAMP TYPE DVM

- The operating principle is to measure the time that a linear ramp takes to change the input level to the ground level, or vice-versa. This time period is measured with an electronic time-interval counter and the count is displayed as a number of digits on an indicating tube or display.
- The operating principle and block diagram of a ramp type DVM are shown in next figures.
- The ramp may be positive or negative; in this case a negative ramp has been selected.



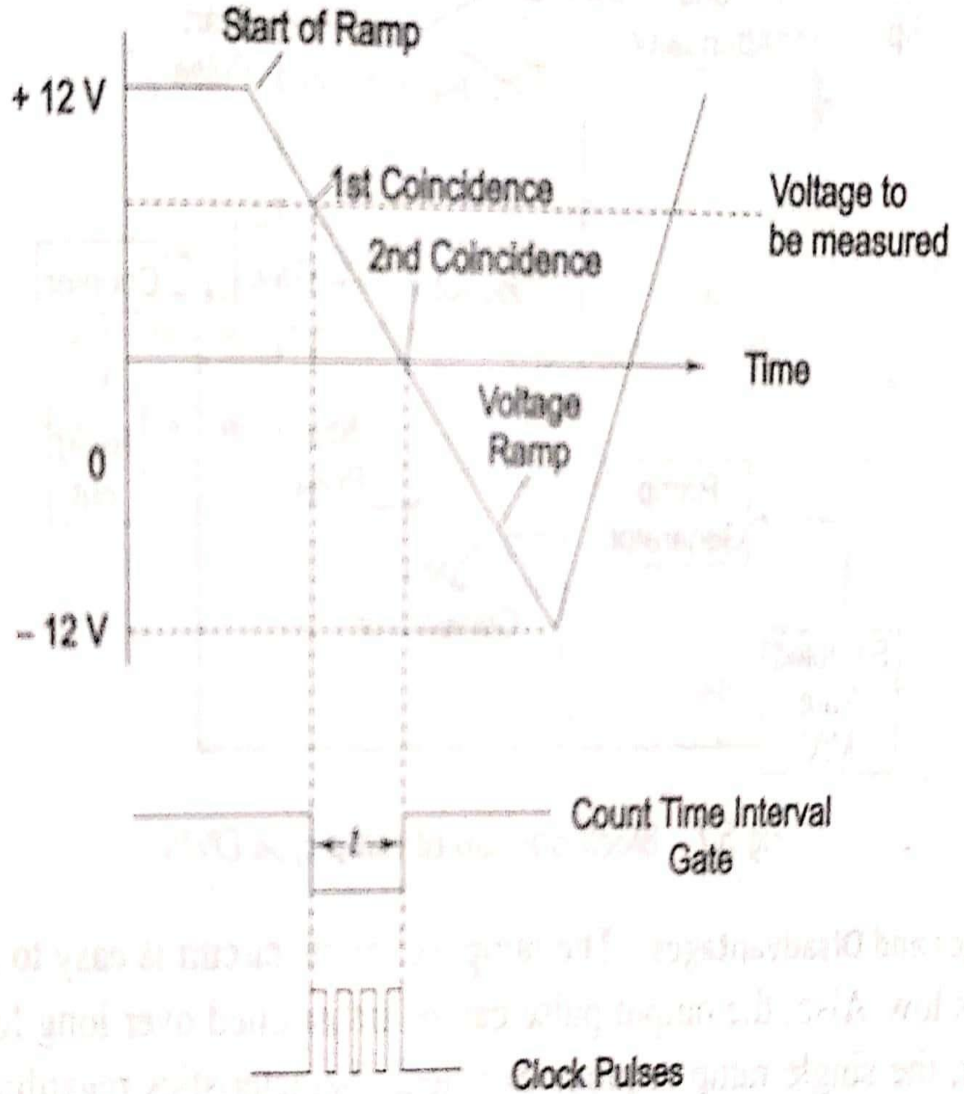


Fig. 5.1 Voltage to time conversion

DIGITAL VOLTMETERS

- At the start of the measurement a ramp voltage is initiated(counter is reset to 0 and sampled rate multi vibrator gives a pulse which initiates the ramp generator).

- The ramp voltage is continuously compare with the voltage that is being measured. At the instant these two voltages become equal, a coincidence circuit generates a pulse which opens a gate, i.e., the input comparator generates a start pulse.

- The ramp continues until the second comparator circuit senses that the ramp has reached zero value. The ground comparator compares the ramp with ground. When the ramp voltage equals zero or reaches ground potential, the ground comparator generates a stop pulse.

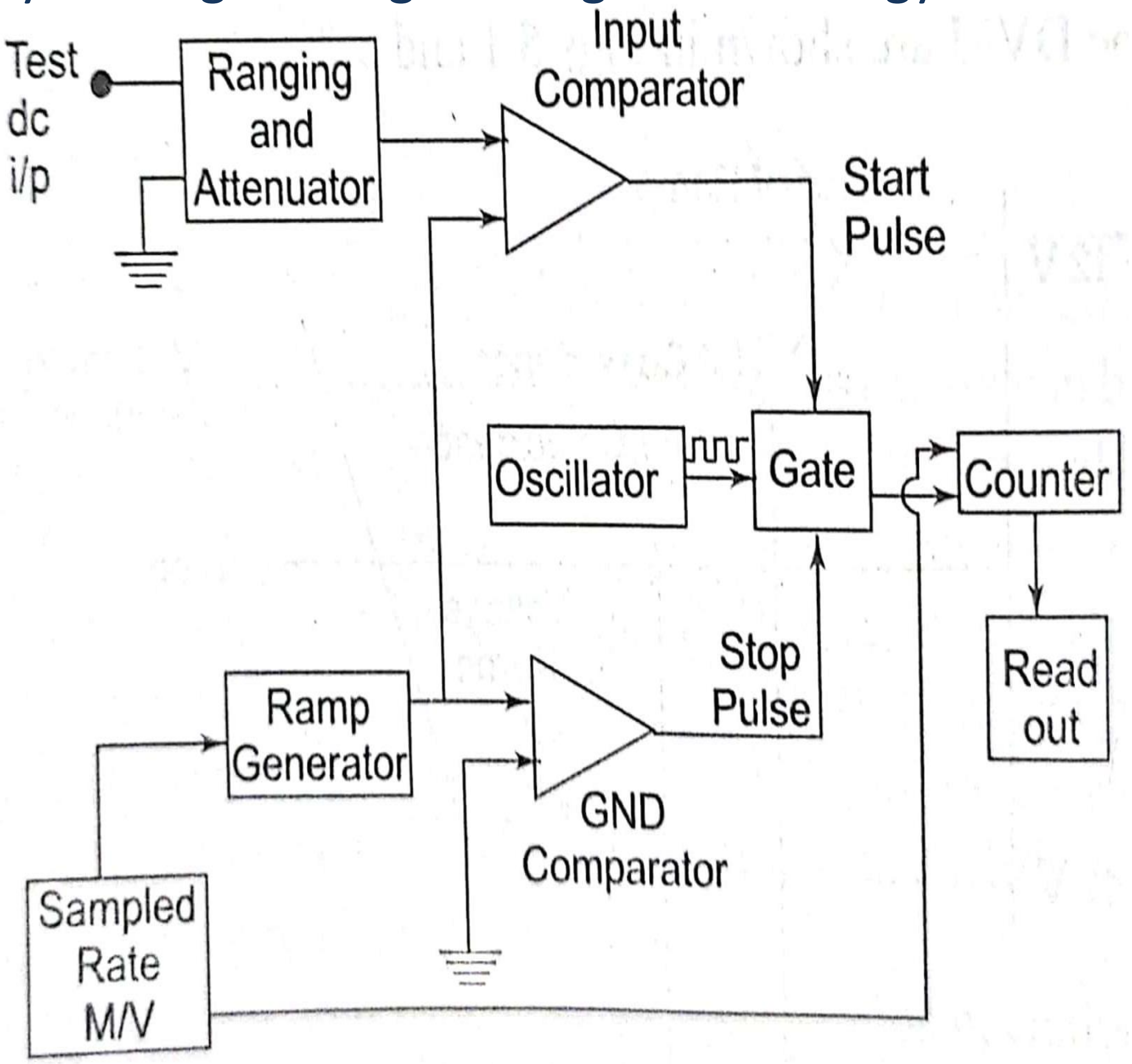


Fig. 5.2 Block diagram of ramp type DVM

DIGITAL VOLTMETERS

- The output pulse from this comparator closes the gate. The time duration of the gate opening is proportional to the input voltage value.
- In the time interval between the start and stop pulses, the gate opens and the oscillator circuit drives the counter. The magnitude of the count indicates the magnitude of the input voltage, which is displayed by the readout.
- Therefore, the voltage is converted into time and the time count represents the magnitude of the voltage. The sample rate multi vibrator determines the rate of cycle of measurement. A typical value is 5 measuring cycles per second, with an accuracy of ±0.005% of the reading.
- The sample rate circuit provides an initiating pulse for the ramp generator to start its next ramp voltage. At the same time a reset pulse is generated, which resets the counter to the zero state.

Any DVM has a fundamental cycle sequence which involves sampling, displaying and reset sequences.

Advantages and disadvantages:

1) The ramp technique circuit is easy to design and its cost is low. Also, the output pulse can be transmitted over long feeder lines. However, the single ramp requires excellent characteristics regarding linearity of the ramp and time measurement.
2) Large errors are possible when noise is superimposed on the input signal. Input filters are usually required with this type of converter.

INTEGRATING TYPE DVM:

- The principle of operation of an integrating type DVM is illustrated in fig 5.5. A constant input voltage is integrated and the slope of the output ramp is proportional to the input voltage. When the output reaches a certain value, it is discharged to 0 and another cycle begins. The frequency of the output waveform is proportional to the input voltage.

- The block diagram is illustrated in fig 5.6.
- The input voltage produces a charging current, e_i/R_1, that charges the capacitor 'C' to the reference voltage e_r. When e_r is reached, the comparator changes state, so as to trigger the precision pulse generator. The pulse generator produces a pulse of precision charge content that rapidly discharges the capacitor.

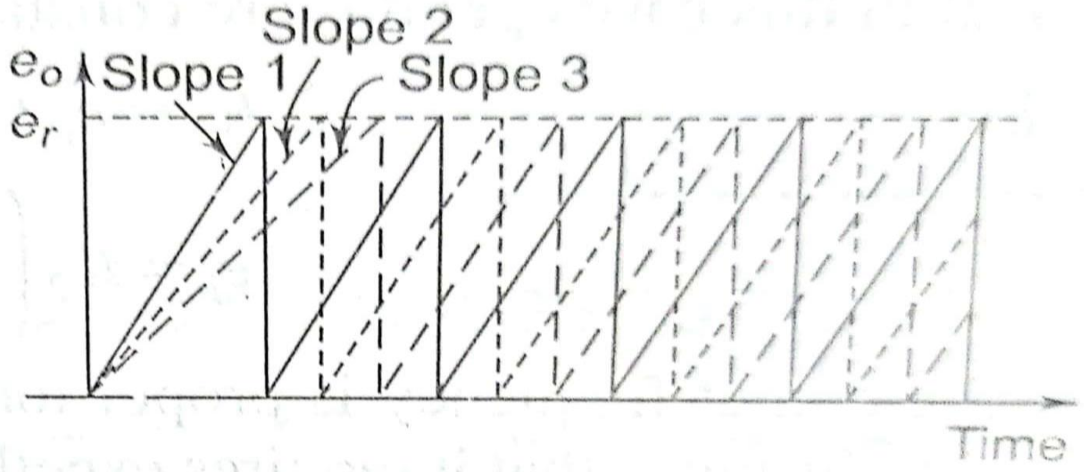


Fig. 5.5 Voltage to frequency conversion

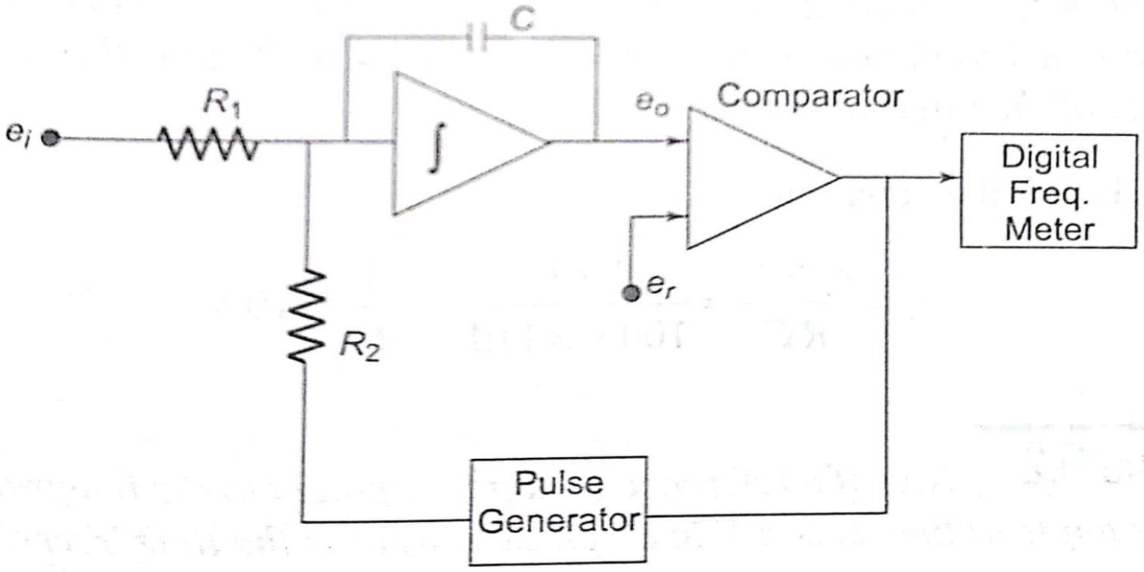


Fig. 5.6 Block diagram of an integrating type DVM

DIGITAL VOLTMETERS

- The rate of charging and discharging produces a signal frequency that is directly proportional to e_i.
- The voltage-frequency conversion can be considered to be a dual slope method, as shown in fig 5 .7.

Referring to eqn 5.3 we have;

$$e_i = \frac{e_r t_2}{t_1}$$

But in this case e_r and t_2 are constants.

Let $K_2 = e_r * t_2$

$e_i = K_2 (1/t_1) = K_2 (f_0)$

- The output frequency is proportional to the input voltage e_i.

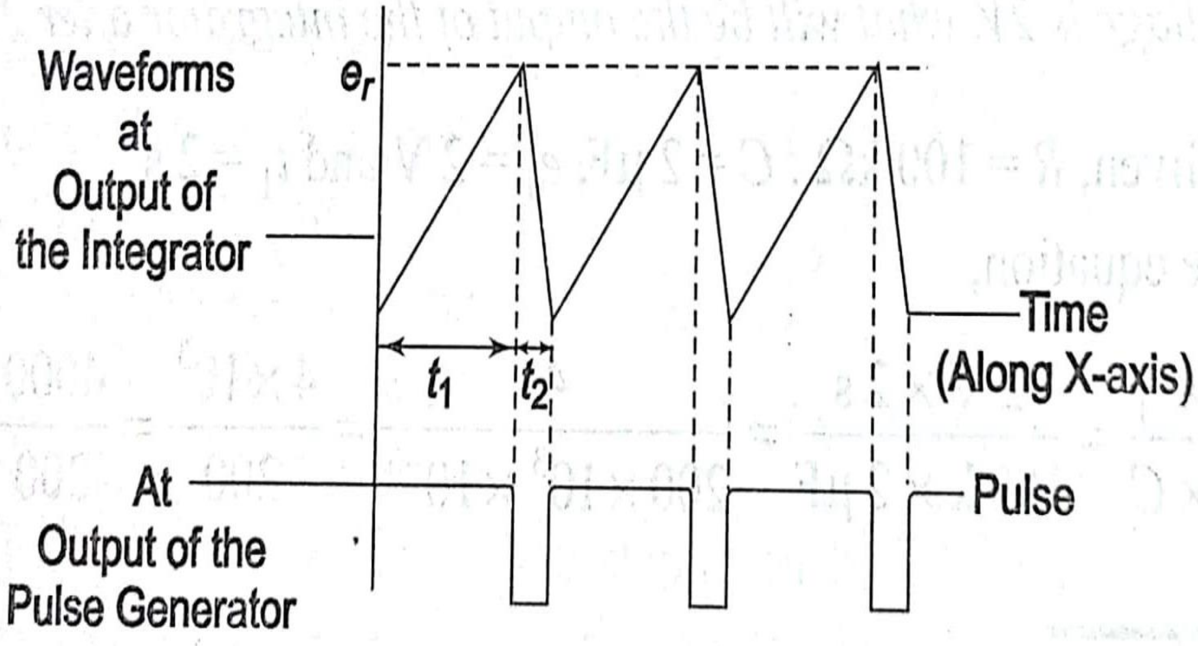


Fig. 5.7

- This DVM has the disadvantage that it requires excellent characteristics in linearity of the ramp. The ac noise and supply noise are averaged out.

➢ SUCCESSIVE APPROXIMATION TYPE DVM:

- The successive approximations principle can be easily understood using a simple example; the determination of the weight of an object. By using a balance and placing the object on one side and an approximate weight on the other side, the weight of the object is determined.
- If the weight placed is more than the unknown weight, the weight is removed and another weight of smaller value is placed and again the measurement is performed.
- Now if it is found that the weight placed is less than that of the object, another weight of smaller value is added to the weight already present, and the measurement is performed. If it is found to be greater than the unknown weight the added weight is removed and another weight of smaller value is added. In this manner by adding and removing the appropriate weight, the weight of the unknown object is determined.
- The successive approximation DVM works on the same principle. Its basic block diagram is shown in Fig. 5.10. When the start pulse signal activates the control circuit, the successive approximation register (SAR) is cleared. The output of the SAR is 00000000. V_{out} of the D/A converter is 0. Now, if $V_{in} > V_{out}$ the comparator output is positive.
- During the first clock pulse, the control circuit sets the D_7 to 1, and V_{out} jumps to the half reference voltage. The SAR output is 10000000. If V_{out} is greater than V_{in} the comparator output is negative and the control circuit resets D_7. However, if V_{in} is greater than V_{out} the comparator output is positive and the control circuits keep D_7 set.
- Similarly the rest of the bits beginning from D_7 to D_0 are set and tested.
- Therefore, the measurement is completed in 8 clock pulses.

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- At the beginning of the measurement cycle, a start pulse is applied to the start-stop multi vibrator. This sets a 1 in the MSB of the control register and a 0 in all bits (assuming an 8-bit control) its reading would be 10000000.
- This initial setting of the register causes the output of the D/A converter to be half the reference voltage, i.e. 1/2 V. This converter output is compared to the unknown input by the comparator.
- If the input voltage is greater than the converter reference voltage, the comparator output produces an output that causes the control register to retain the 1 setting in its MSB and the converter continues to supply its reference output voltage of 1/2 V_{ref}.

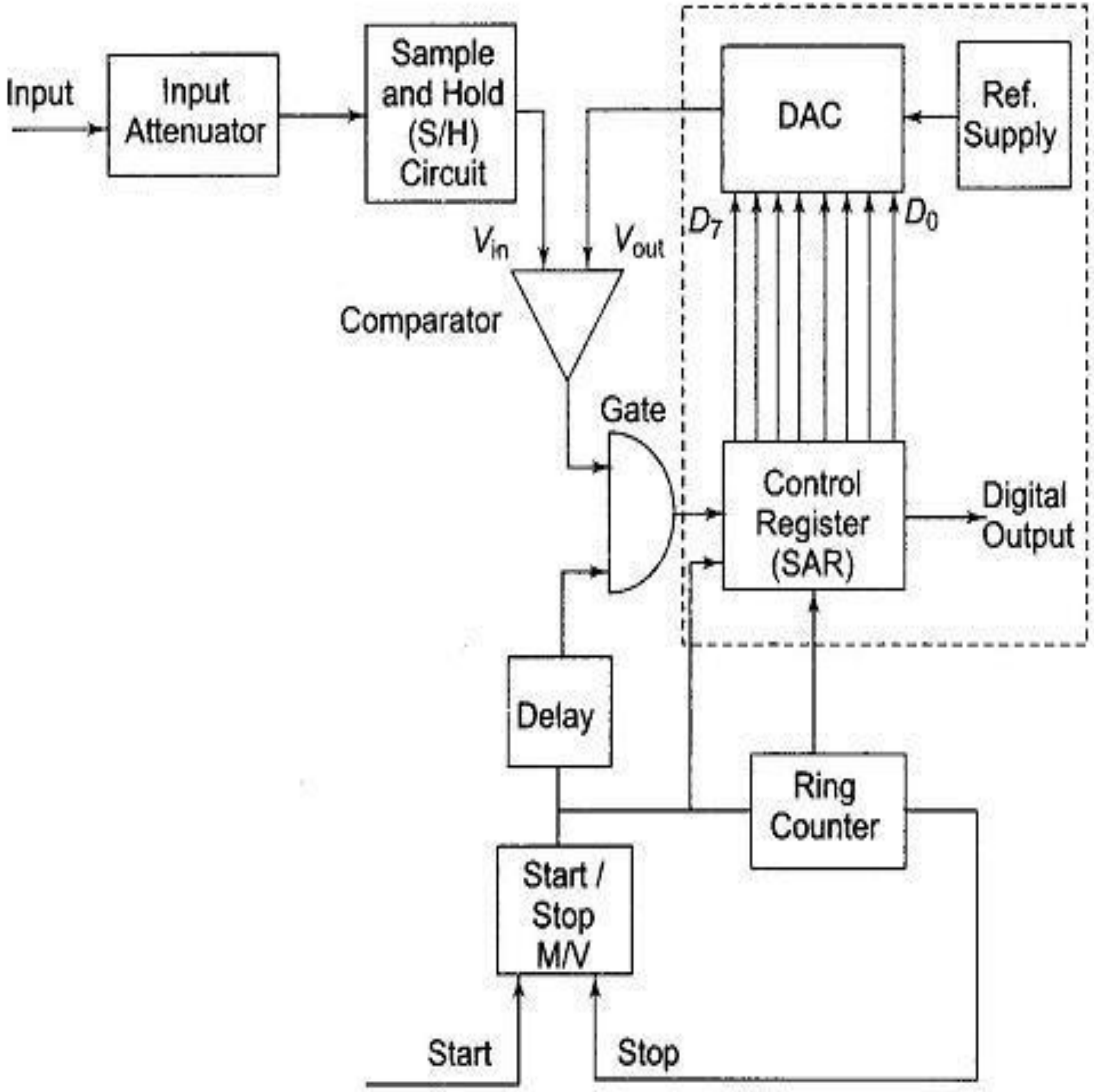


Fig. 5.10 Successive Approximation DVM

DIGITAL VOLTMETERS

- The ring counter then advances one count, shifting a 1 in the second MSB of the control register and its reading becomes 11000000. This causes the D/A converter to increase its reference output by 1 increment to 1/4 V, i.e. 1/2 V + 1/4 V, and again it is compared with the unknown input. If in this case the total reference voltage exceeds the unknown voltage, the comparator produces an output that causes the control register to reset its second MSB to 0. The converter output then returns to its previous value of 1/2 V and awaits another input from the SAR.

- When the ring counter advances by 1, the third MSB is set to 1 and the converter output rises by the next increment of 1/2 V + 1/8 V.

- The measurement cycle thus proceeds through a series of successive approximations. Finally, when the ring counter reaches its final count, the measurement cycle stops and the digital output of the control register represents the final approximation of the unknown input voltage.

DIGITAL FREQUENCY METER

DIGITAL FREQUENCY METER

- A Digital Frequency Meter is a general purpose ,basic, digital counter for measuring, setting, and monitoring frequencies, for counting random events, and for industrial counting applications.
- **Digital frequency meter** is a general purpose instrument that displays the frequency of a periodic electrical signal to an accuracy of three decimal places. It counts the number events occurring within the oscillations during a given interval of time.

Principle of Operation:

- A frequency meter has a small device which converts the sinusoidal voltage of the frequency into a train of unidirectional pulses. The frequency of input signal is the displayed count, averaged over a suitable counting interval out of 0.1, 1.0, or 10 seconds

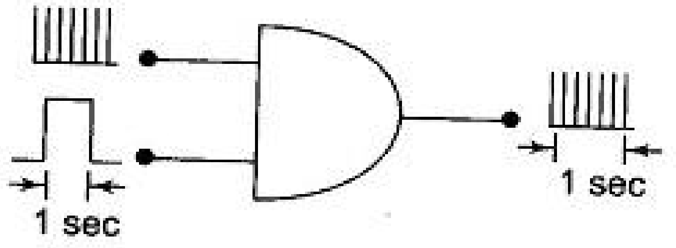


Fig. 6.4 Principle of Digital Frequency Measurement

- The signal waveform is converted to trigger pulses and applied continuously to an AND gate, as shown in Fig. 6.4. A pulse of 1 s is applied to the other terminal, and the number of pulses counted during this period indicates the frequency.
- The signal whose frequency is to be measured is converted into a train of pulses, one pulse for each cycle of the signal. The number of pulses occurring in a definite interval of time is then counted by an electronic counter.
- Since each pulse represents the cycle of the unknown signal, the number of counts is a direct indication of the frequency of the signal (unknown). Since electronic counters have a high speed of operation, high frequency signals can be measured.

➢ Basic Circuit of a Digital Frequency Meter:

The block diagram of a basic circuit of a digital frequency meter is shown in Fig. 6.5.

Fig. 6.5 Basic Circuit of a Digital Frequency Meter

- The signal may be amplified before being applied to the Schmitt trigger.
- The Schmitt trigger converts the input signal into a square wave with fast rise and fall times, which is then differentiated and clipped.
- As a result, the output from the Schmitt trigger is a train of pulses, one pulse for each cycle of the signal.

DIGITAL FREQUENCY METER

- The output pulses from the Schmitt trigger are fed to a START/STOP gate. When this gate is enabled, the input pulses pass through this gate and are fed directly to the electronic counter, which counts the number of pulses.
- When this gate is disabled, the counter stops counting the incoming pulses. The counter displays the number of pulses that have passed through it in the time interval between start and stop. If this interval is known, the unknown frequency can be measured.

Basic Circuit for Frequency Measurement:

- The basic circuit for frequency measurement is as shown in Fig. 6.6. The output of the unknown frequency is applied to a Schmitt trigger, producing positive pulses at the output.

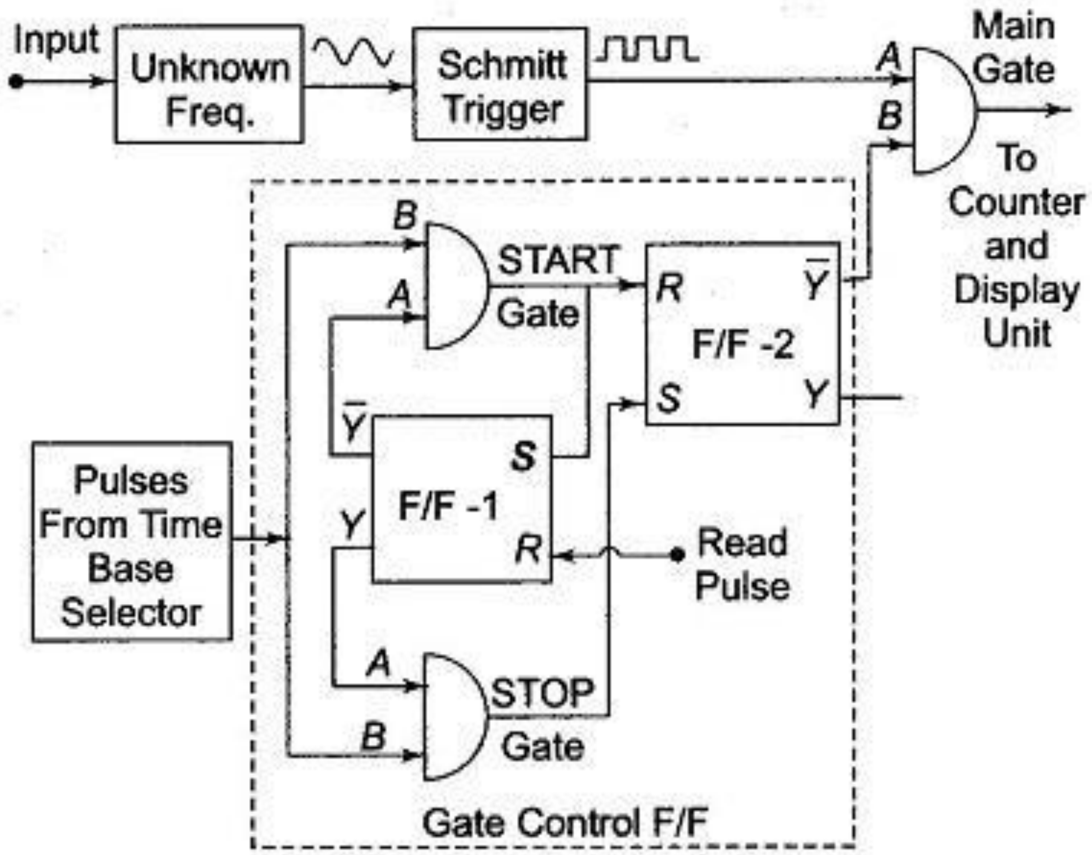


Fig. 6.6 Basic Circuit for Measurement of Frequency Showing Gate Control F/F

DIGITAL FREQUENCY METER

- These pulses are called the counter signals and are present at point A of the main gate.Positive pulses from the time base selector are present at point B of the START gate and at point B of the STOP gate.
- Initially the Flip-Flop (F/F-1) is at its logic 1 state. The resulting voltage from output Y is applied to point A of the STOP gate and enables this gate. The logic 0 stage at the output Y of the F/F-1 is applied to the input A of the START gate and disables the gate. As the STOP gate is enabled, the positive pulses from the time base pass through the STOP gate to the Set (S) input of the F/F-2 thereby setting F/F-2 to the 1 state and keeping it there. The resulting 0 output level from Y of F/F-2 is applied to terminal B of the main gate. Hence no pulses from the unknown frequency source can pass through the main gate.

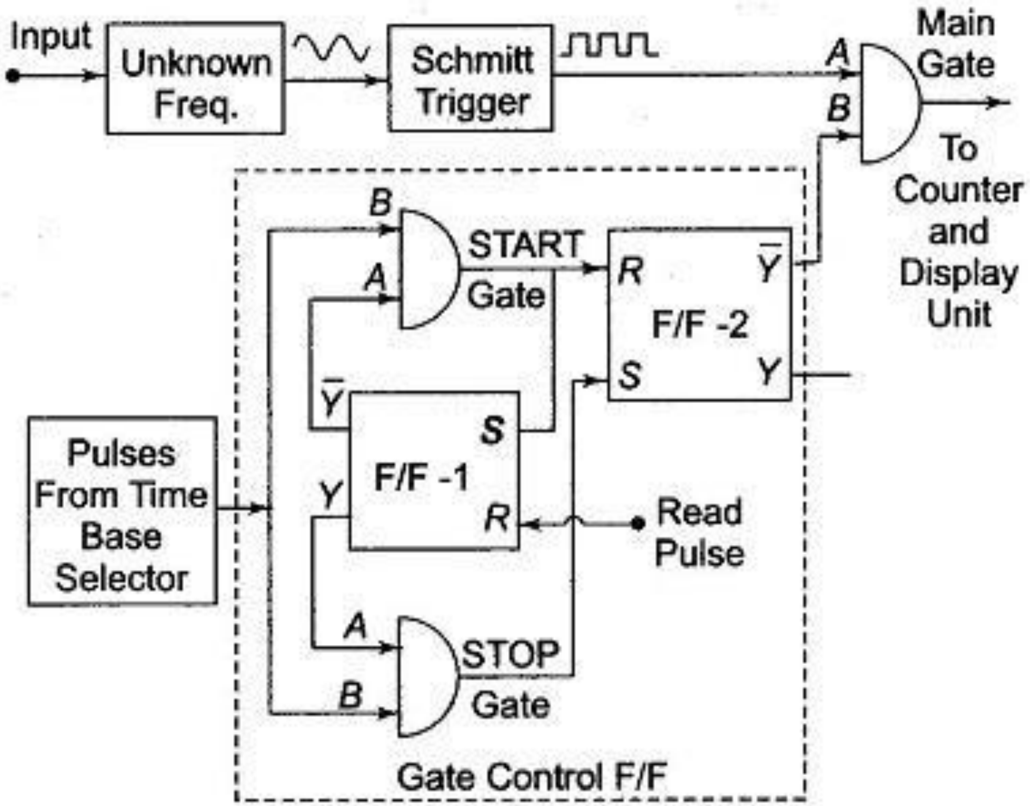


Fig. 6.6 Basic Circuit for Measurement of Frequency Showing Gate Control F/F

- In order to start the operation, a positive pulse is applied to (read input) reset input of F/F-1, thereby causing its state to change. Hence Y = 1, Y= 0, and as a result the STOP gate is disabled and the START gate enabled. This same read pulse is simultaneously applied to the reset input of all decade counters, so that they are reset to 0 and the counting can start.

DIGITAL FREQUENCY METER

When the next pulse from the time base arrives, it is able to pass through the START gate to reset F/F-2, therefore, the F/F-2 output changes state from 0 to 1, hence Y changes from 0 to 1. This resulting positive voltage from Y called the gating signal, is applied to input B of the main gate thereby enabling the gate.

➢Now the pulses from the unknown frequency source pass through the main gate to the counter and the counter starts counting. This same pulse from the START gate is applied to the set input of F/F-1, changing its state from 0 to 1. This disables the START gate and enables the STOP gate. However, till the main gate is enabled, pulses from the unknown frequency continue to pass through the main gate to the counter.

➢The next pulse from the time base selector passes through the enabled STOP gate to the set input terminal of F/F-2, changing its output back to 1 and f_i = 0. Therefore the main gate is disabled, disconnecting the unknown frequency signal from the counter.

➢The counter counts the number of pulses occurring between two successive pulses from the time base selector. If the time interval between this two successive pulses from the time base selector is 1 second, then the number of pulses counted within this interval is the frequency of the unknown frequency source, in Hertz.

DIGITAL FREQUENCY METER

➢The assembly consisting of two F/Fs and two gates is called a gate control F/F. The block diagram of a digital frequency meter is shown in Fig. 6.7.

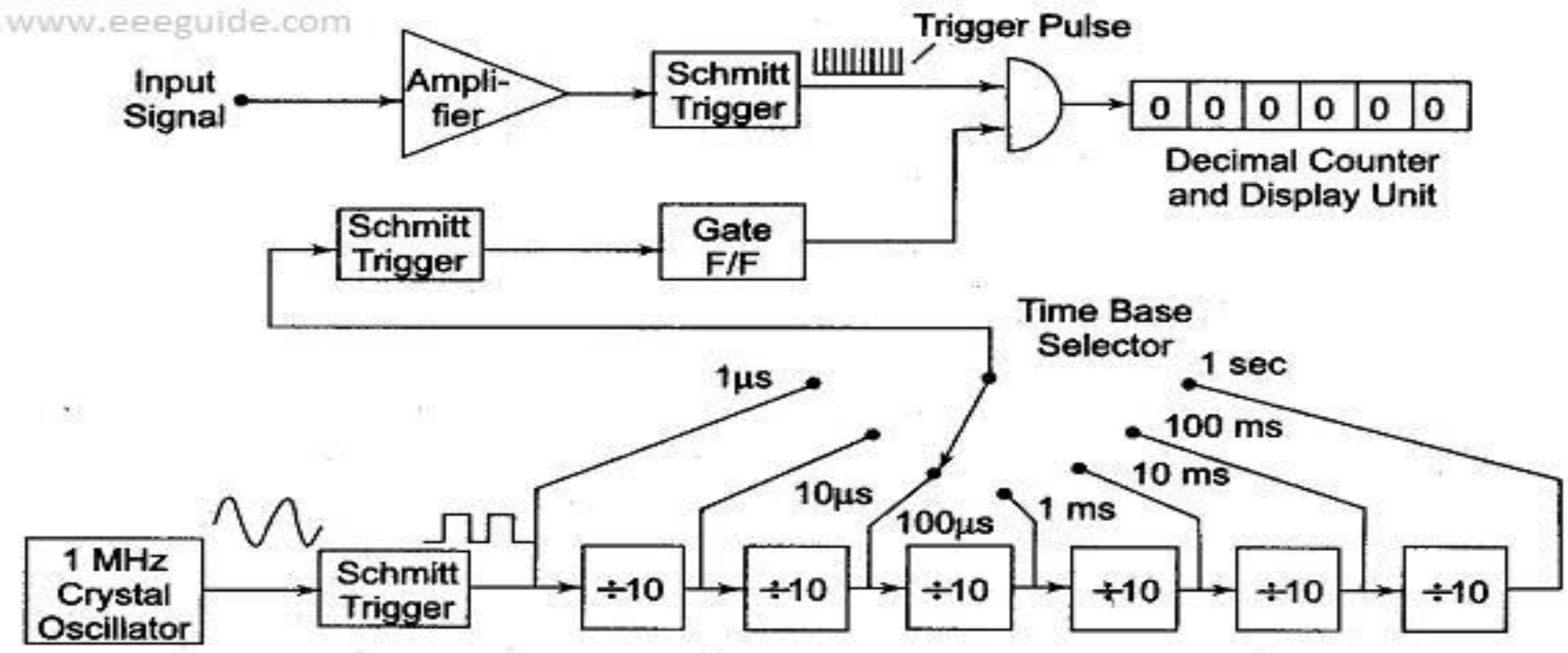


Fig. 6.7 Block Diagram of a Digital Frequency Meter

DIGITAL FREQUENCY METER

- The input signal is amplified and converted to a square wave by a Schmitt trigger circuit. In this diagram, the square wave is differentiated and clipped to produce a train of pulses, each pulse separated by the period of the input signal. The time base selector output is obtained from an oscillator and is similarly converted into positive pulses.
- The first pulse activates the gate control F/F. This gate control F/F provides an enable signal to the AND gate. The trigger pulses of the input signal are allowed to pass through the gate for a selected time period and counted.
- The second pulse from the decade frequency divider changes the state of the control F/F and removes the enable signal from the AND gate, thereby closing it.
- The decimal counter and display unit output corresponds to the number of input pulses received during a precise time interval; hence the counter display corresponds to the frequency.

Use of Digital Frequency Meter

1) For testing radio equipment
2) Measuring the temperature, pressure, and other physical values.
3) Measuring vibration, strain
4) Measuring transducers

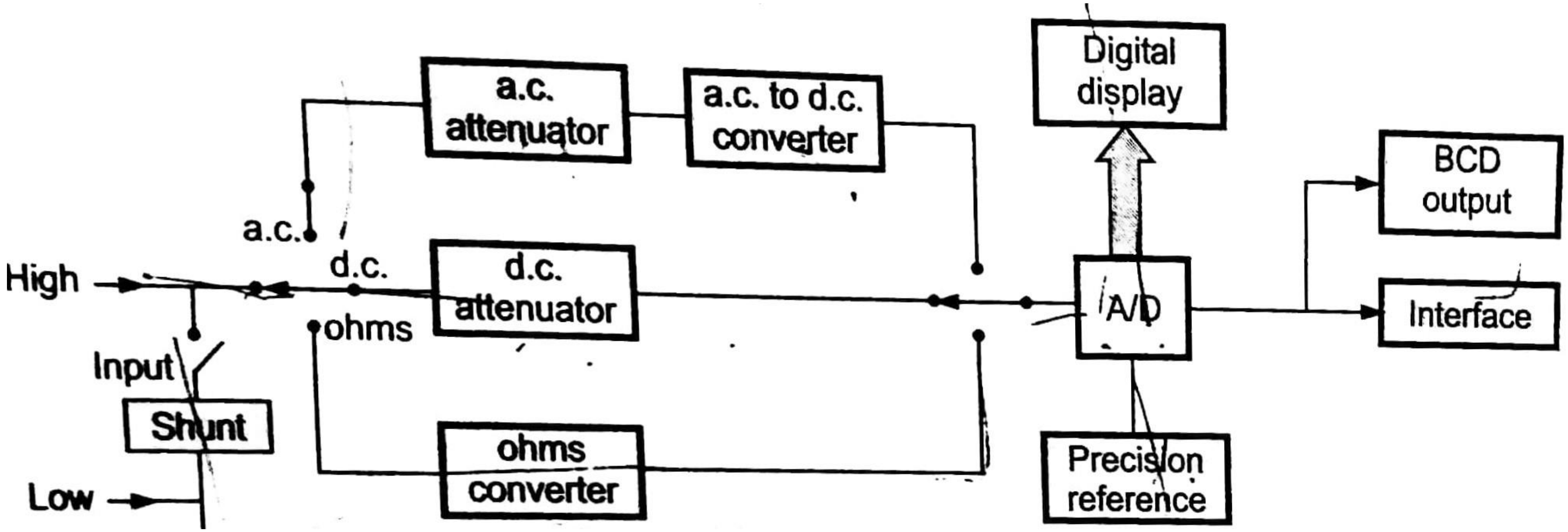
DIGITAL MULTIMETERS

DIGITAL MULTIMETERS

- Digital multimeters or DMMs can measure a variety of different parameters within an electrical circuit. The basic DMMs can measure amps, volts and ohms, as the older analogue meters did, but with the ease of incorporating further functionality into an integrated circuit, many digital multimeters are able to make a number of other measurements as well.
- Many of them include functions enabling measurement of capacitance, frequency, continuity (with a buzzer to facilitate easy measurements when looking at the circuit board), temperature, transistor functionality, and often a number of other measurements as well.

Digital Multimeters

- The digital multimeter is an instrument which is capable of measuring a.c. voltages, d.c. voltages, a.c. and d.c. currents and resistances over several ranges. The basic circuit of a 'digital multimeter is always a d.c. voltmeter as shown in the Fig(a).



Fig(a). Basic scheme of digital multimeter

- The current is converted to voltage by passing it through low shunt resistance. The a.c. quantities are converted to d.c. by employing various rectifier and filtering circuits, While for the resistance measurements the meter consists of a precision low current source that is applied across the unknown resistance while gives d.c. voltage.
- All the quantities are digitalised using analog to digital converter and displayed in the digital form on the display.

Digital Multimeters

- The analog multimeters require no power supply and they suffer less from electric noise and isolation problems but still the digital multimeters have following advantages over analog multimeters :

1) The accuracy is very high.
2) The input impedance is very high hence there is no loading effect.
3) An unambiguous reading at greater viewing distances is obtained.
4) The output available is electrical which can be used for interfacing with external equipment.
5) Due to improvement in the integrated technology, the prices are going down.
6) These are available in very small size.

The requirement of power supply, electric noise and isolation problems are the two limitations.

Digital Multimeters

- The basic building blocks of digital multimeter are several A/D converters, counting circuitry and an attenuation circuit.
- Generally dual slope integration type ADC is preferred in the multimeters.
- The single attenuator circuit is used for both a.c. and d.c. measurements in many commercial multimeters.
- The block diagram of a digital multimeter is shown in the Fig.(b)

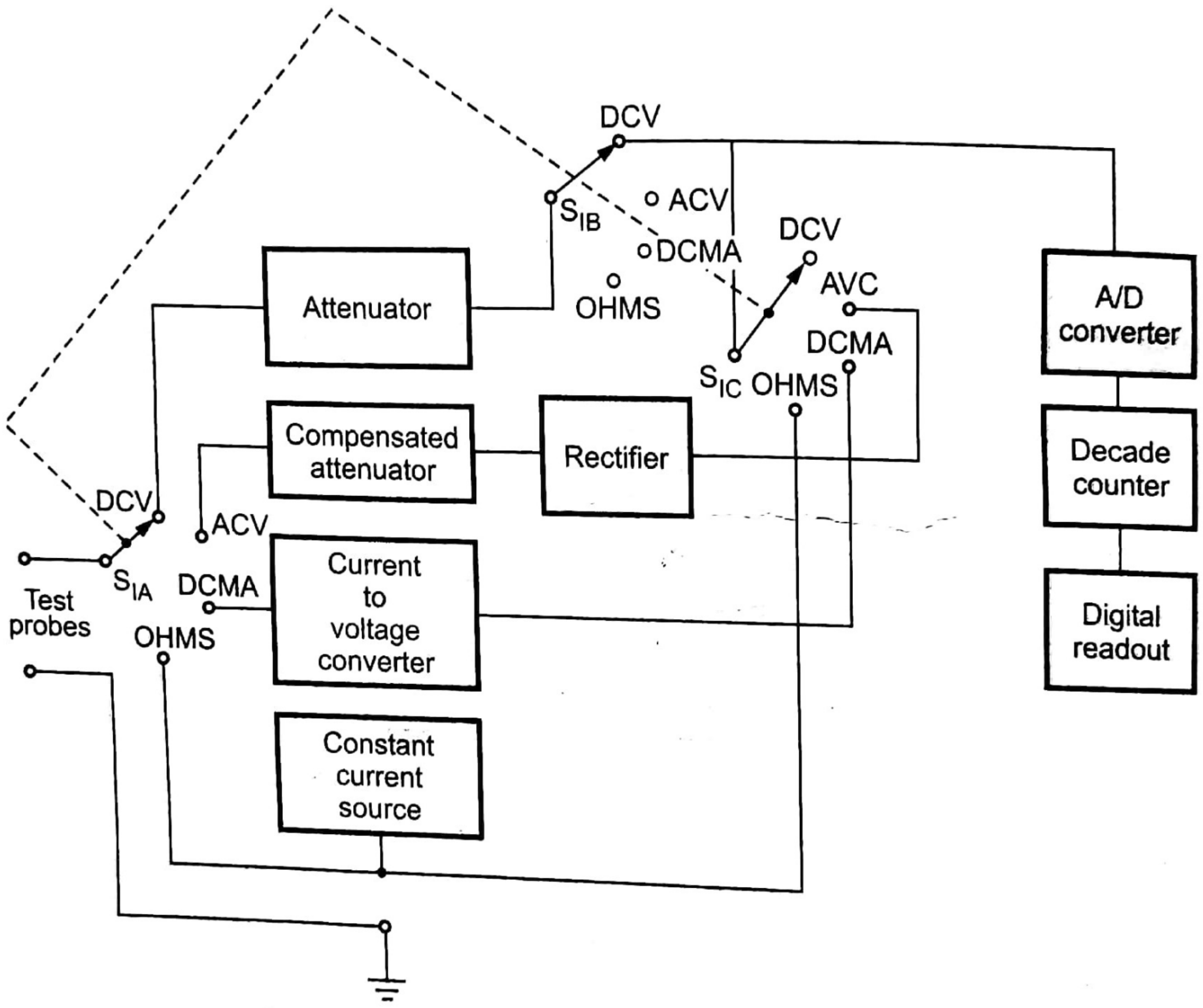
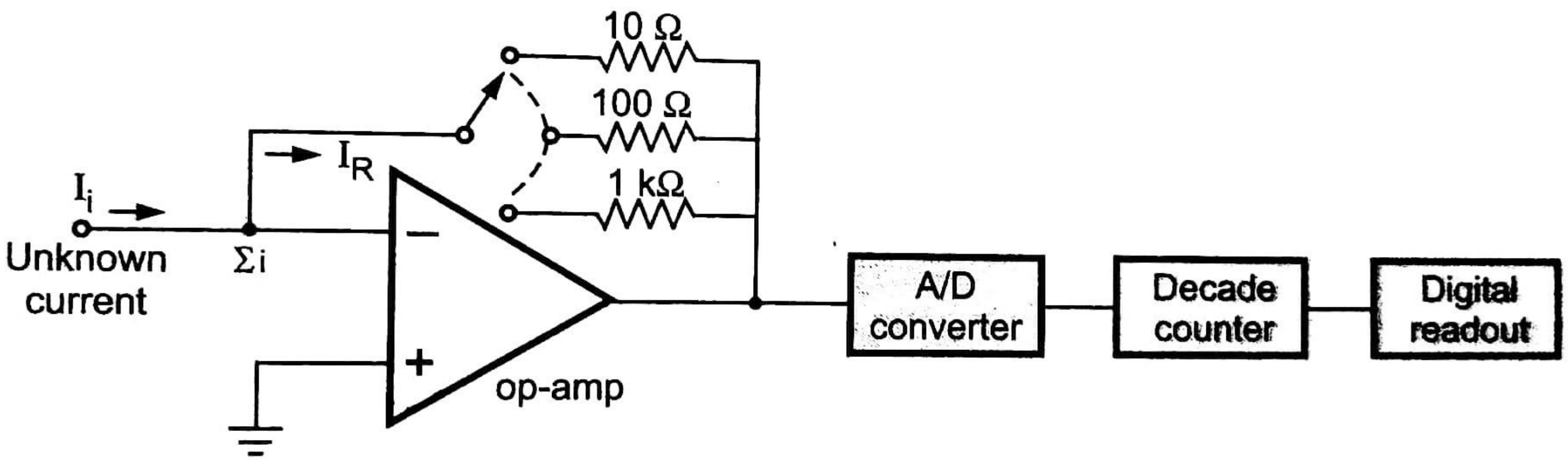


Fig.(b)The block diagram of a digital multimeter.

- As mentioned above basically it is a d.c. voltmeter. In order to measure unknown currents, current to voltage converter circuit is implemented. This is shown in the Fig(c).



Fig(c) Current to voltage converter

- The unknown current is applied to the summing junction Σi at the input of op-amp- As input current of op-amp is almost zero, the current I_R is almost same as I_i. This current I_R causes a voltage drop, which is proportional to the current to be measured.
- This voltage drop is the analog input to the analog to digital converter, thus providing a reading that is proportional to the unknown current

Digital Multimeters

- In order to measure the resistances, a constant current source is used. The known current is passed through the unknown resistance. The voltage drop across the resistance is applied to analog to digital converter hence providing the display of the value of the unknown resistance.
- To measure the a.c. voltages, the rectifiers and filters are used.
- The a.c. is converted to d.c and then applied to the analog to digital converter.

  In addition to the visual display, the output from the digital multimeters can also be used to interface with some other equipments.

DIGITAL TACHOMETER

Digital Tachometer

- A digital tachometer is a digital device that measures and indicates the speed of a rotating object. A digital tachometer is an optical encoder that determines the angular velocity of a rotating shaft or motor
- Used in laboratories to measure speed of motor/generator/engine

Different Types of Tachometers

- Based on data acquisition – contact or non contact types
  - Contact type – The wheel of the tachometer needs to be brought into contact with the rotating object
  - Non Contact type – The measurement can be made without having to attach the tachometer to the rotating object
- Based on the measurement technique – time based or frequency based technique of measurement
  - In Time Measurement – The tachometer calculates speed by measuring the time interval between the incoming pulses
  - In Frequency Measurement – The tachometer calculates speed by measuring the frequency of the incoming pulses
- They can also be classified as analog or digital type

DIGITAL TACHOMETER

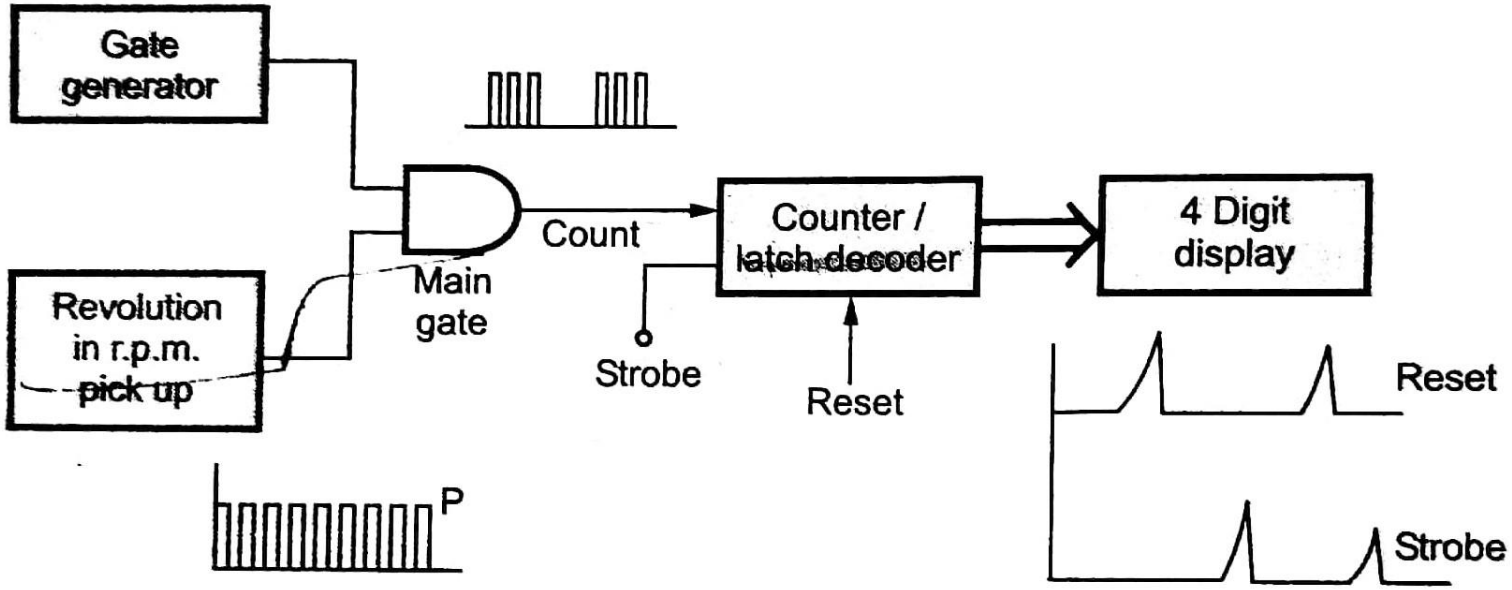
Non Contact (laser)type

Contact type

Digital Tachometer

- The technique used for the measurement of a speed of a rotating shaft of a running motor is same as that used in the conventional frequency counter.
- But in conventional frequency counter, the gate pulses are obtained in accordance with the schmitt trigger output while in the digital tachometer the gate period is selected in accordance with the r.p.m. calibration.

- A typical schematic diagram of digital tachometer is as shown in the Fig(a).



Fig(a) Digital tachometer circuit

Digital Tachometer

- Tachometer consists of an optical sensor which generates pulses proportional to the speed of rotation. It counts the number of pulses sensed by the sensor and number of pulses per second gives the rps and when multiplied by 60 gives rpm.
- Consider that the revolution per minute of a shaft be R.
- Assume that the number of pulses produced in one revolution of the rotating shaft be P.
- Then in one minute the number of pulses from the revolution pick up can be calculated as R x P.
- Thus the frequency of the signal is given by (R × P/60).
- Suppose the gate period is G measured in second, the the pulses counted are (Rx P x h/60).
- Now the direct reading is possible in rpm if only the gate period is (60 / p) and the pulses counted by the counter are R i.e.,

$$R \times P \times \left(\frac{60}{P}\right)/60 = R$$

➢ Comparison between Analog and Digital Tachometers

Sl. No.	Analog Tachometer	Digital Tachometer
1.	Has a needle and dial type of interface	Has a LCD or LED readout
2.	No provision for storage of readings	Memory is provided for storage
3.	Cannot compute average, deviation, etc	Can perform statistical functions like averaging, determination of shaft velocity, etc.

➢ Comparison between Contact and Non Contact Tachometers

Sl. No.	Contactless type tachometer	Non-contact type tachometer
1.	The tachometer has to be in physical contact with the rotating shaft	The tachometer does not need to be in physical contact with the rotating shaft
2.	Preferred where the tachometer is generally fixed to the machine	Preferred where the tachometer needs to be mobile
3.	Generally, optical encoder / magnetic sensor is attached to shaft of tachometer	Generally, laser is used or an optical disk id attached to rotating shaft and read by a IR beam or laser

Disadvantage of Analog Tachometer

1. Less accuracy
2. Difficult to take reading as pointer takes long time to get stable.
3. No provision of storage of values
4. Cannot compute average deviations

Advantage of Digital Tachometer

1. Accuracy is high as display is used
2. Easy to take reading as little time delay is provided.
3. Memory is provided for storage of values
4. Can compute average deviations

DIGITAL ENERGY METER

Digital Energy Meter

- An electric meter or energy meter is an essential device that goes with consumption of commercially distributed energy.
- It enables systematic pricing of energy consumed by individual consumer as it measures the amount of electrical energy consumed by a residence, business, or an electrically powered device.
- They are typically calibrated in billing units, the most common one being the Kilowatts hour, which is equal to the amount of energy used by a load of one kilowatt over a period of one hour, or 3,600,000 joules.

- Generally, electricity meters operate by continuously measuring the instantaneous voltage (volts) and current (amperes) and finding the product of these to give instantaneous electrical power (watts) which is then integrated against time to give energy used (Joules, Kilowatt-hours etc.).
- Meters for smaller services (such as small residential customers) can be connected directly in-line between source and customer.
- For larger loads, more than about 200 amps of load, current transformers are used, so that the meter can be located other than in line with the service conductors

DIGITAL ENERGY METER

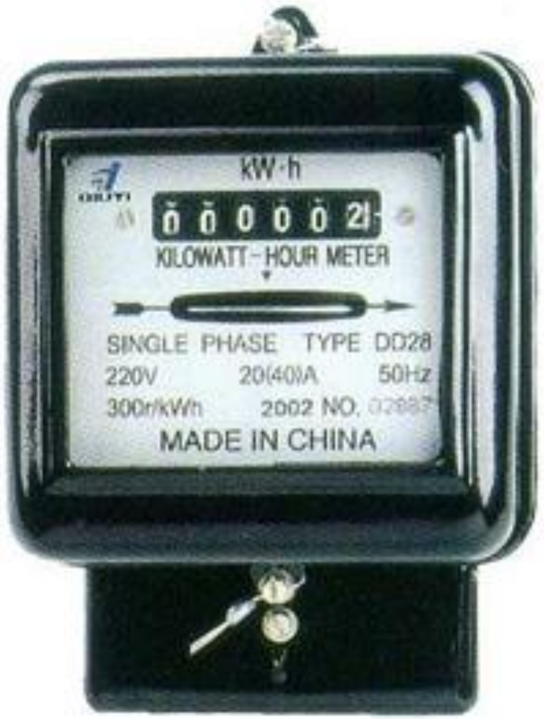
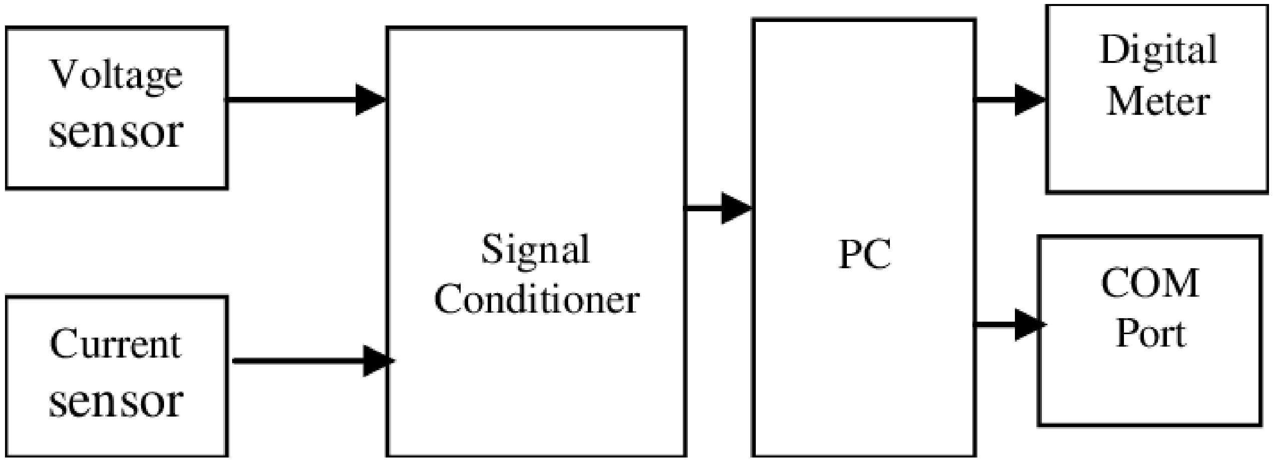


Image of Electronic Meter

Digital Energy Meter

- The block diagram for a digital meter shown in fig(a). here, two basic sensors are employed. These are voltage and current sensors. The voltage sensor built around a step down element and potential divider network senses both the phase voltage and load voltage.

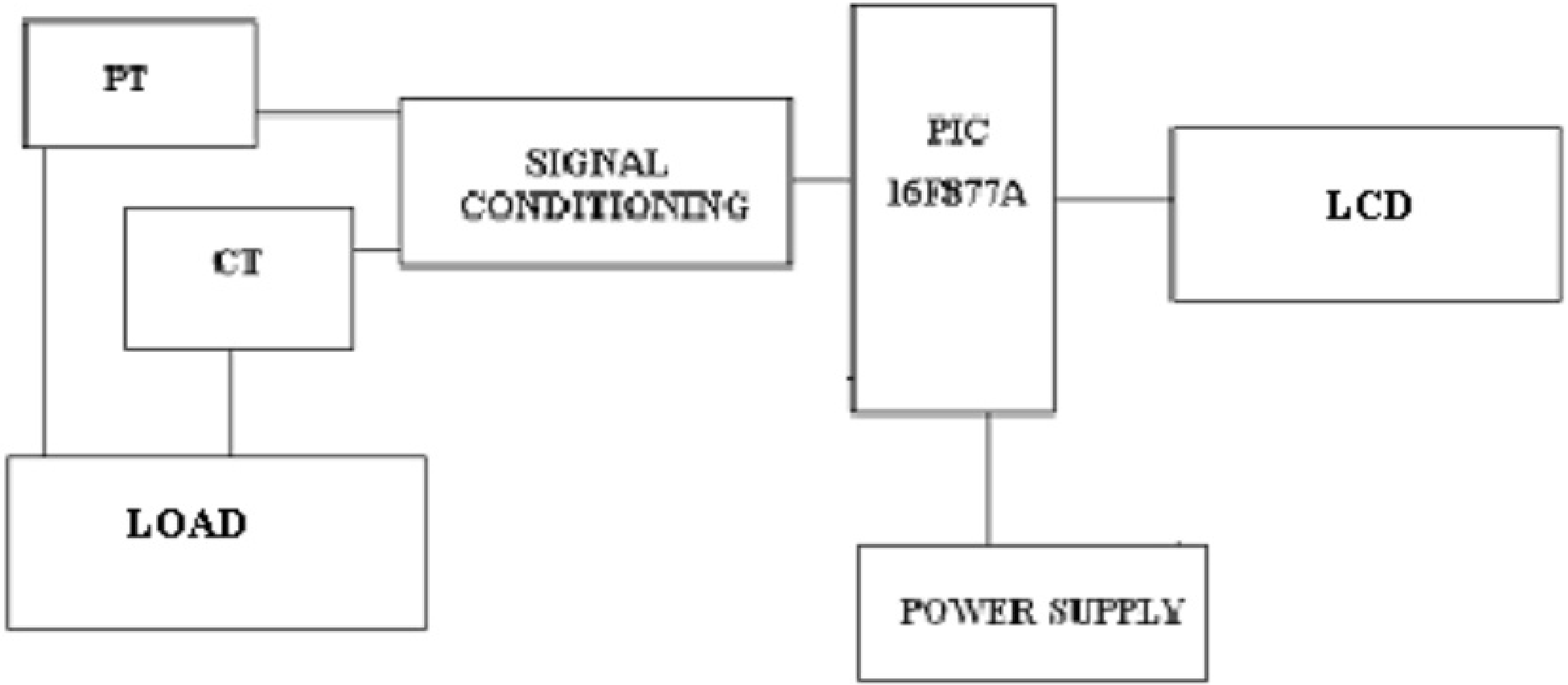
- The second sensor is a current sensor; this senses the current drawn by the load at any point in time. It is built around a current transformer and other active devices (such as voltage comparator) which convert the sensed current to voltage for processing.



Fig(a) block diagram

- The output from both sensors is then fed into a signal (or voltage) conditioner which ensures matched voltage or signal level to the control circuit, it also contain a signal multiplexer which enable sequential switching of both signal to the analogue input of the peripheral interface controller (PIC).

Digital Energy Meter



Fig(a) Simplified block diagram of Digital energy meter

Digital Energy Meter

- The control circuit centered on a PIC integrated circuit. The PIC is selected because it contain ten bit analogue to digital converter (ADC), very flexible to program and good for peripheral interfacing.
- The ADC converts the analogue signals to its digital equivalent; both signals from the voltage and current sensors are then multiplied by the means of embedded software in the PIC.
- Here the error correction is taken as the offset correction by determining the value of the input quality with short-circuited input and storing this value in the memory for use as the correction value device calibration. The PIC is programmed in C language.
- Such that apart from the multiplier circuit it simulates, it is able to use the received data to calculate power consumption per hour, as well as the expected charges.
- These are displayed on the liquid crystal display attached to the circuit.

Digital Energy Meter

- The digital energy meter has a power supply, a metering engine, a processing and communication engine.
- Digital Energy meters display the energy used on an LCD or LED display, and some can also transmit readings to remote places.
- In addition to measuring energy used, these can also record other parameters of the load and supply such as instantaneous and maximum rate of usage demands, voltages, power factor and reactive power used etc.

A **Smart Energy Meter**

- A **smart meter** is an electronic device that records information such as consumption of electric energy, voltage levels, current, and power factor.
- Smart meters communicate the information to the consumer for greater clarity of consumption behavior, and electricity suppliers for system monitoring and customer billing. Smart meters typically record energy near real-time, and report regularly, short intervals throughout the day.
- Smart meters enable two-way communication between the meter and the central system. Such an advanced metering infrastructure (AMI) differs from automatic meter reading (AMR) in that it enables two-way communication between the meter and the supplier.
- Communications from the meter to the network may be wireless, or via fixed wired connections such as power line carrier (PLC).
- Wireless communication options in common use include cellular communications, Wi-Fi (readily available).

Digital Energy Meter

Advantages Digital energy meter

- There are many methods of error correction in digital electricity meters which are usually based on the known methods of A/D converters error correction.
- Most of these methods use software correction based on calibration process. While in digital electricity meter, percentage error could be as low as 0.01%, in analogue meters it is usually above 0.05%.
- Secondly, the orientation problem associated with electromechanical energy meter is completely a nonissue in a digital energy meter. Hence installation is made easier.
- Thirdly, the user friendly display in the digital meters makes energy reading from time to time very easy.
- The fourth and the most serious setback of the electromechanical energy meter is its no-interface capability to external devices. This very set back is very serious in smart grid technology application.

DIGITAL LCR METER

DIGITAL LCR METER

- This type of meter is used to measure the resistance, inductance, capacitance and dissipation factor.
- The desired function can be selected by using a rotary switch. The various ranges available are (i) 200 μH/pF/Ω; (ii) 2000 μH/pF/Ω (ii) 20 mH/nF/kΩ; (iii) 200 mH/nF/kΩ (iv) 2H/μF/MΩ.

- With the help of this meter, the following ranges of various measurements can be made:
  - Resistance: From 200 Ω to 20 MΩ;
  - Inductance : From 2000 μH to 2H;
  - Capacitance : From 2000 pF to 2μF.

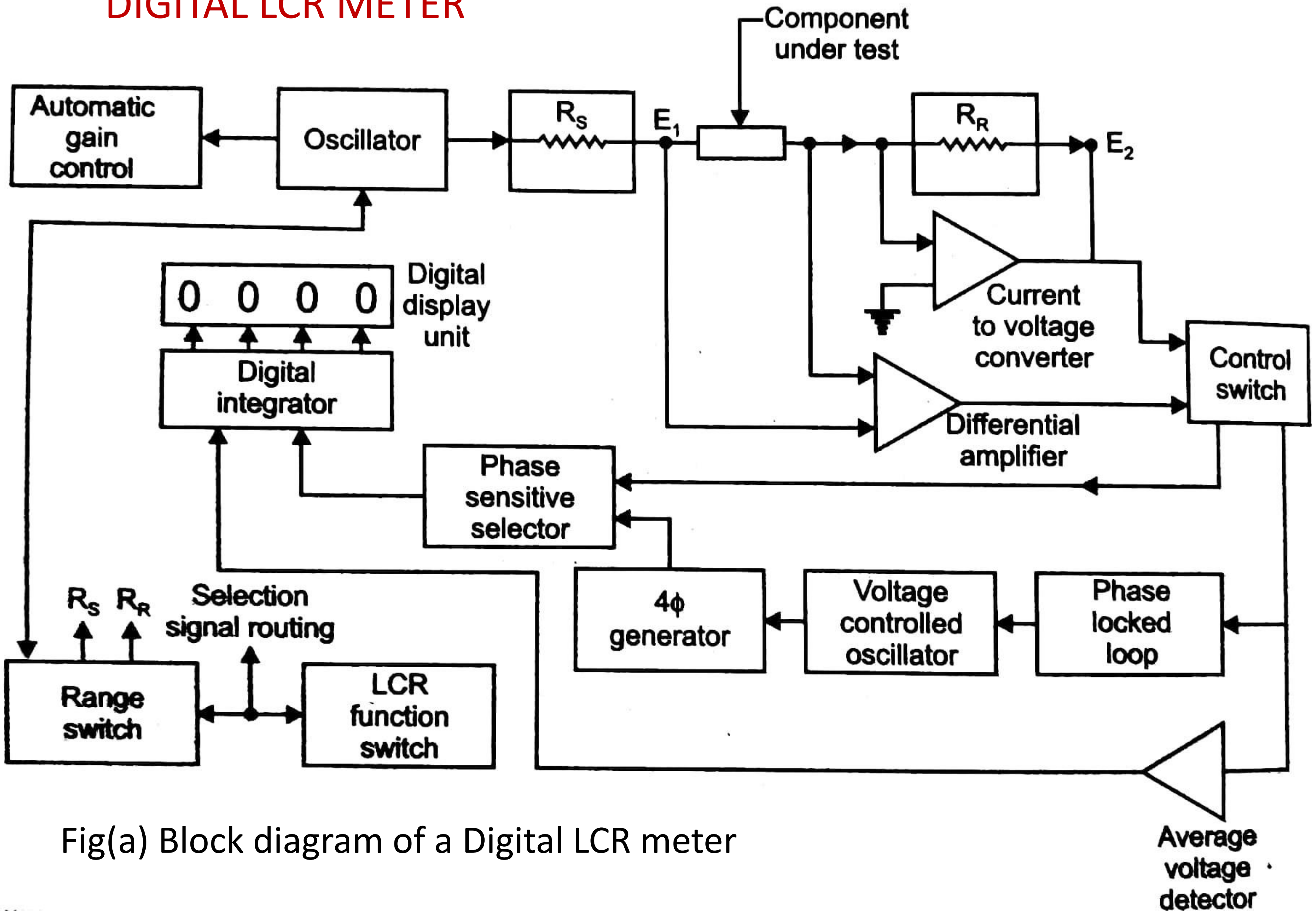
Fig(a) shows the block diagram of the digital LCR meter :

- In this meter, the basic principle used is to measure the voltage across the component and the current passing through the component under test, when the test signal is fed to the component.
- These processed voltage and current signals (from the component under test) are then fed to the digital integrator unit which finally enables a display unit to provide directly the value of the test component
- The oscillator (in built) applies the measuring test-signal to the test component, through a source resistor R_s (selectable). The typical test frequency is 1 kHz (for the above mentioned five ranges).

DIGITAL LCR METER

Digital LCR Meter, 20Hz~200kHz

DIGITAL LCR METER



Fig(a) Block diagram of a Digital LCR meter

DIGITAL LCR METER

- The signal current then flows to current-to-voltage converter, which is essentially an op-amp with range resistor R_R, connected in the feedback path.
- The op-amp drives the junction of component and R_R to a virtual ground and hence R_R does not change the current through the component. Thus signal current develops a voltage E_2 proportional to the current through the component.
- The voltage $E_1$and E_2 are the vector quantities and as such they define the characteristics of the component at a particular test frequency and signal level.

Mathematically : $E_1 \;\alpha\; V$ and $E_2 \;\alpha\; I$

Thus, *Capacitance,* $C \;\alpha\; \frac{I}{V} \;\alpha\; \frac{E_2}{E_1}$; *Inductance,* $L \;\alpha\; \frac{V}{I} \;\alpha\; \frac{E_1}{E_2}$

- The above ratios are adopted in measurement modes and are obtained by dual-slope integration method.
- For inductance measurements, a series equivalent circuit of an inductor is assumed while for capacitance measurements, a parallel equivalent circuit of capacitor is assumed for five ranges.
- The values of R_S and R_R are selected, based on the impedance of the unknown component. For the measurement of inductance, the component impedance is usually low and hence R_S is chosen much higher. This achieves a constant current drive to the component. The R_S decides the value of current.
- For capacitance measurement, the component impedance is high and R_S is chosen to be much low. Thus constant voltage drive is provided to the test component.

DIGITAL LCR METER

- voltage E_1 is applied as one input to 'differential amplifier': E_1 is then fed to a control switch, along with E_2. The greater of the two is fed to "average voltage detector (AVD) and lesser to the 'phase sensitive detector (PSD).
- The signal given to AVD is also given to 'phase locked loop (PLL) and voltage controlled oscillator (VCO). These two produce the clock signals, which are locked in phase with the reference signal. The clock signal is then divided into phase shifted 90° and 270° signals. These are given to PSD, which detector the phase angle.
- The D.C. voltage outputs, from AVD and PSD are then given to 'digital integrator unit' and final to 'digital display unit' where the component value is displayed.
- For wide range of values, the basic accuracy of this meter is $\pm$0.15% of the reading
- The range selection is fully automatic.The display used is normally 4 digit LEDs with automatic decimal point.
- The inductors and capacitors are automatically differentiated.

- **Commercial Digital LCR meter :**

Important features available in commercial digital LCR meter are :

1) Auto-ranging and auto-computing facility.
2) High basic accuracy, upto 0.2%
3) High frequency range, from 0.1 to 1 kHz.
4) Hold function.
5) Multipoint measurement techniques.
6) Series or parallel modes of operation.

Q - METER

Q - METER

➢ Introduction

- Q meter was developed by William D. Loughlin at Boonton Radio Corporation in the year 1934 in Boonton, New Jersey.
- The Q-meter instrument has become more popular in RF impedance measurement.
- There are different kinds of instruments available based on system usage. These are separated into two types like low-impedance injection & high-impedance injection.
- This device plays a key role in testing the RF circuits and also replaced in laboratories with other impedance measuring devices, although it is still in use among radio amateurs.

❑ **What is Q Meter?**

❑ **Definition:** A device that is used to measure the QF (quality factor) or storage factor of the circuit at radio frequencies is called the Q-meter. In the oscillatory system, the QF is one of the essential parameters, used to illustrate the relationships among the dissipated & stored energies.

- By using Q value, the overall efficiency can be evaluated for the capacitors as well as coils used in RF applications.

Q - METER

- The principle of this meter mainly depends on series resonance because the voltage drop is Q times than the applied voltage across the capacitor otherwise coil.
- When the fixed voltage is applied to an electric circuit, a voltmeter is used to adjust the capacitor's Q value to read directly.

- The total efficiency of capacitors & coils used for RF applications can be calculated with the help of Q value.

At resonance $X_L = X_C$ and $E_L = IX_L$, $E_C = IX_C$, $E = IR$

Where 'E' is an applied voltage
'E_C' is the capacitor voltage
'E_L' is an inductive voltage
'X_L' is the inductive reactance
'X_C' is the capacitive reactance
'R' is the coil resistance
'I' is circuit current

Thus, $Q = X_L/R = X_c/R = E_C/E$

- From the above 'Q'equation, if an applied voltage is kept stable so that the voltage across the capacitor can be calculated using a voltmeter to read 'Q' values directly.

Q - METER

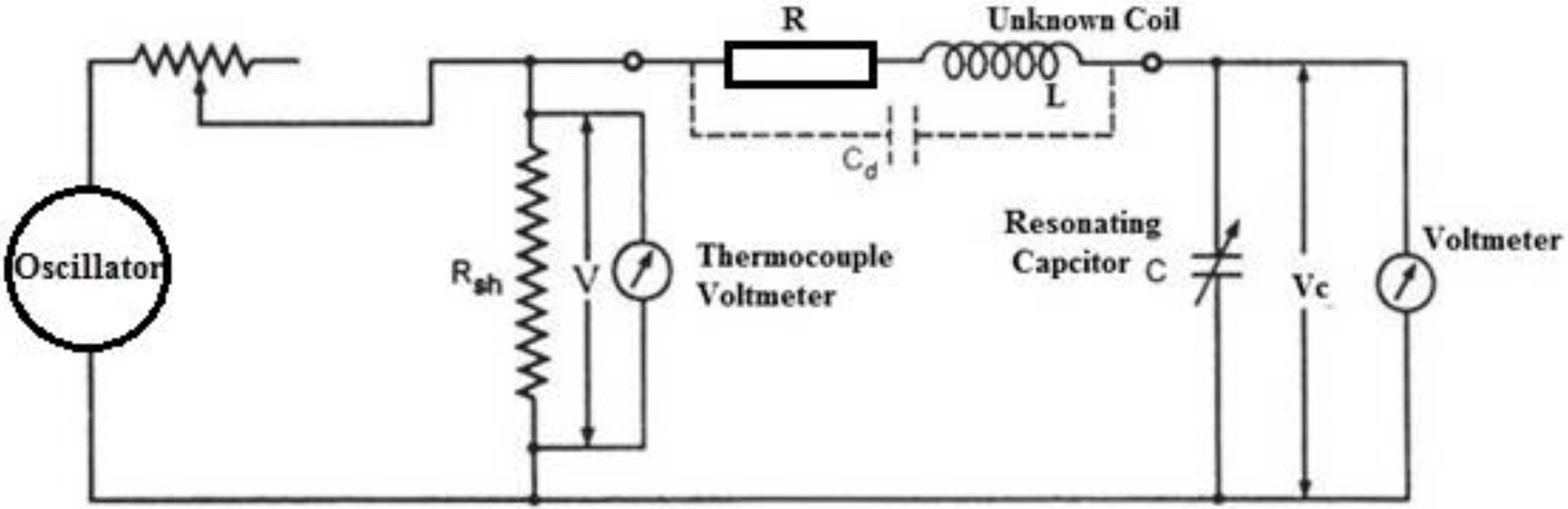
➢ Working Principle

- The **working principle of Q meter** is series resonant because the resonant exists within the circuit once the reactance of capacitance & reactance of inductance in the same magnitude.
- They induce energy to oscillate in between the fields of electric & magnetic of the inductor & capacitor respectively.
- This meter mainly depends on the feature of the capacitance, inductance & resistance of the resonant series circuit.

Q Meter Circuit

- The circuit diagram of the 'Q' meter is shown in fig(a). It is designed with an oscillator that uses the frequency that ranges from 50 kHz – 50 MHz. and provides current to a shunt resistance 'R_{sh}'with 0.02 ohms value.
- Here thermocouple meter is used to calculate the voltage across the shunt resistance whereas an electronic voltmeter is used to calculate the voltage across the capacitor. These meters can be calibrated to read 'Q' directly.

Q - METER



Fig(a) circuit diagram of the ‘Q’ meter

Q - METER

- In the circuit, the energy of the oscillator can be supplied to the tank circuit. This circuit can be adjusted for the resonance through unstable ‘C’ until the voltmeter reads the utmost(highest) value.
- The o/p voltage of resonance is ‘E’, equivalent to ‘E_C’ is E = Q × e and Q = E/e. Because ‘e’ is known so the voltmeter is adjusted to read ‘Q’ value directly.
- The coil is connected to the two test terminals of the instrument to determine the coil’s inductance
- This circuit is adjusted to resonance through changing either the oscillator frequency otherwise the capacitance.
- Once the capacitance is changed, then the frequency of the oscillator can be adjusted to a specified frequency & resonance is attained.
- If the value of capacitance is already fixed to a preferred value, then the frequency of the oscillator will be changed until resonance takes place.
- The reading of ‘Q’ on the o/p meter is multiplied through the setting of an index to get the actual ‘Q’ value.The coil’s inductance is calculated from known values of the coil frequency as well as the resonating capacitor.

- The specified Q is not the definite Q, as the losses of the voltmeter, inserted resistance & resonating capacitor are all incorporated in the circuit. Here, the definite ‘Q’ of the calculated coil is a bit larger than the specified Q.
- This dissimilarity is insignificant except wherever the coil’s resistance is relatively minute compared to the ‘R_{sh}’ resistance.

Applications of the Q-meter

The applications of Q-meter include the following.

1) It is used to measure the quality factor of the inductor.
2) By using this meter, unknown impedance can be measured using a series or shunt substitution method. If the impedance is small, the former technique is used and if it is large, then the latter technique is used.
3) It is used to measure small capacitor values.
4) By using this, inductance, effective resistance, self-capacitance, and bandwidth can be measured.

Q-Meter

➢ Determination of inductance (L) of the coil:

At resonance : $X_L = X_C$; $V_L = IX_L$; $V_C = IX_C$, $V = IR$

where,

X_L = Inductive reactance,
X_C = Capacitive reactance,
R = Coil resistance,
I = Circuit current,
V_L = Inductive voltage,
V_C = Capacitor voltage, and
V = Applied voltage.

Therefore, $$Q = \frac{X_L}{R} = \frac{X_C}{R}\left(= \frac{1}{\omega CR}\right) = \frac{V_C}{V}$$

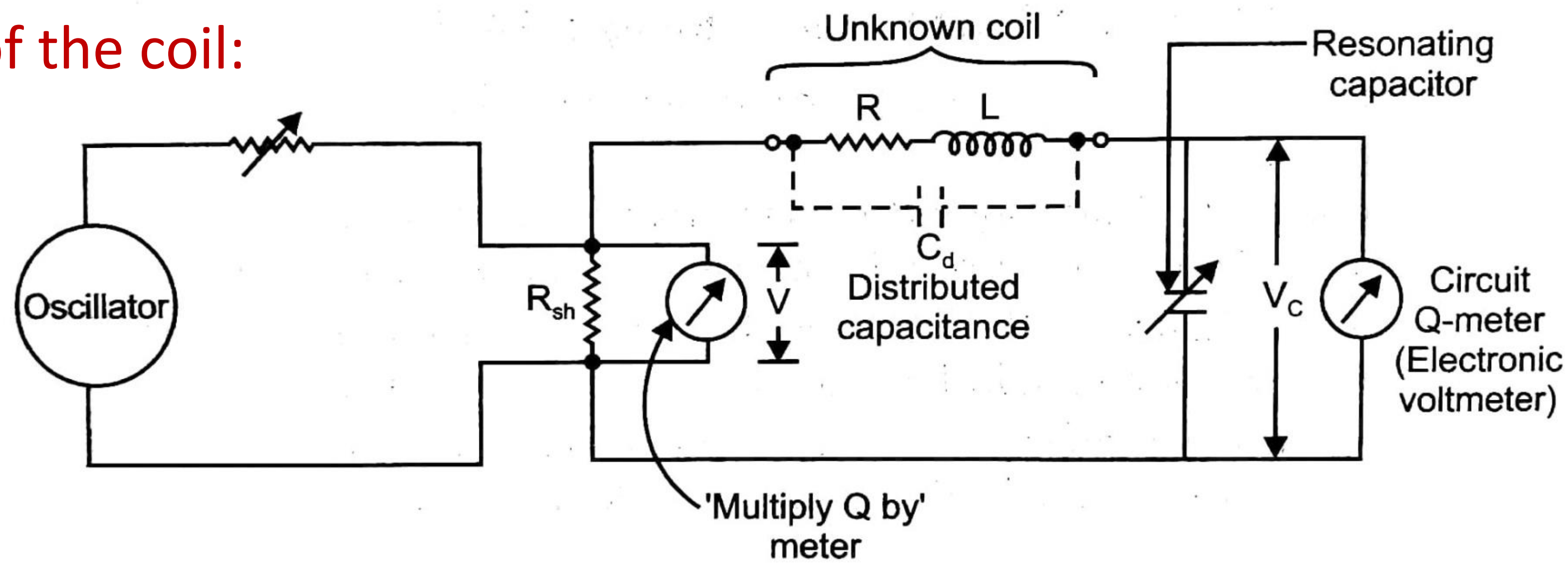


Fig. Circuit diagram of a Q-meter

- The inductance (L) of the coil can be determined by connecting it to the test terminals of the instrument.
- The circuit is tuned to resonance by varying either the capacitance or the oscillatory frequency. If the capacitance is varied, the oscillator frequency is adjusted to a given frequency and resonance is obtained. If the capacitance is pre-set to a desired value, the oscillator frequency is varied until resonance occurs.
- The Q reading on the output meter must be multiplied by index setting or the "Multiply Q by" switch to obtain the actual value.

Q-Meter

- The inductance of the coil can be calculated from known values of the coil frequency and resonating capacitor (C).

$$X_L = X_C$$

$$2\pi f L = \frac{1}{2\pi f C} \quad \text{or} \quad 4\pi^2 f^2 LC = 1$$

$$L = \frac{1}{(2\pi f)^2 C}$$

Phasor Diagram Of Q - METER

At resonant frequency f_0,

$$X_C = X_L$$

The value of capacitance reactance is

$$X_C = \frac{1}{2}\pi f_0 C = \frac{1}{\omega_0 C}$$

At inductive reactance,

$$X_L = \frac{1}{2}\pi f_0 L = \frac{1}{\omega_0 L}$$

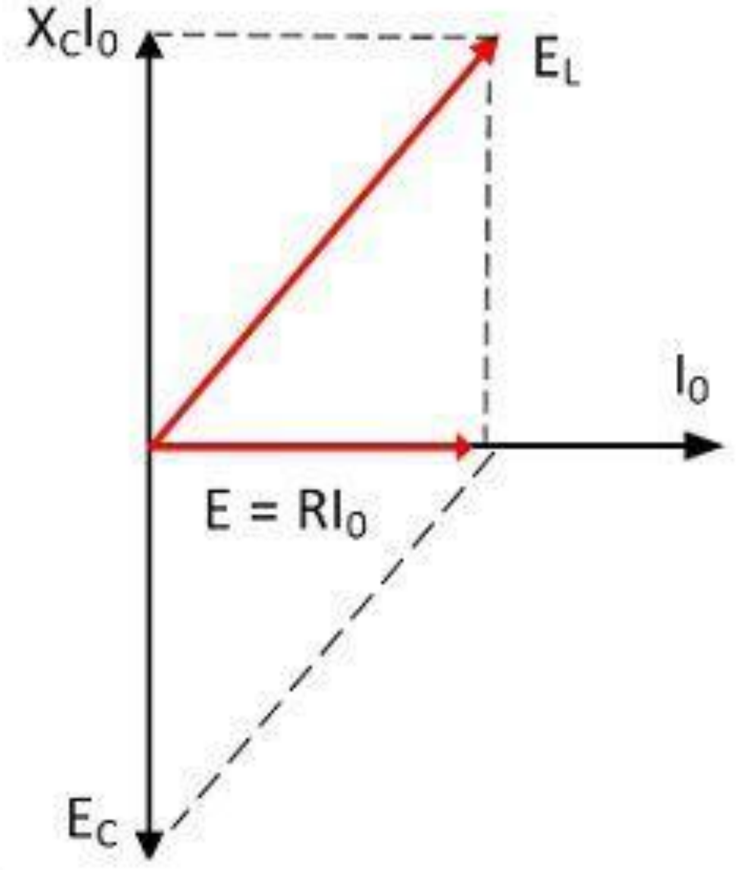
At the resonant frequency,

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

and current at resonance becomes

$$I_0 = \frac{E}{R}$$

The phasor diagram of the resonance is shown in the figure



Phasor Diagram

Circuit Globe

Q - METER

The voltage across the capacitor is expressed as

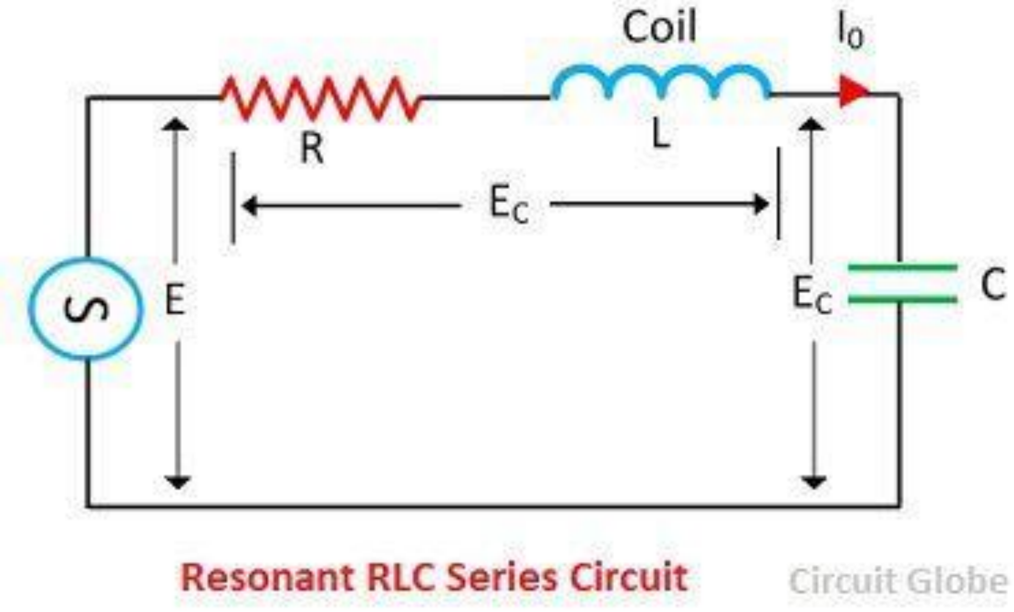
$$E_C = I_0 X_C = I_0 X_L = I_0 \omega_0 L$$

Input voltage

$$E = I_0 r$$

$$\frac{E_C}{E} = \frac{I_0 \omega_0 L}{I_0 R} = \frac{\omega_0 L}{R} = Q$$

$$E_0 = QE$$



The above equation shows that the input voltage E is Q times the voltage appears across the capacitor. The voltmeter is calibrated for finding the value of Q factor.

Applications of the Q-meter

- **Measurement of Inductance –** The inductance is measured by the equation shown below.

$$L = {}^{1}\!/_{4\pi^2 f_0^2 C}$$

The value of f_0 & C is required for calculating the value of inductance.

- **Measurement of Effective resistance –** The equation computes the value of effective resistance

$$R = \frac{\omega_0 L}{Q_{true}}$$

- **Measurement of Self-Capacitance –** The self-capacitance is determined by measuring the two capacitance at different frequencies. The capacitor is adjusted to the high value, and the circuit is resonated by adjusting the oscillator frequency. The resonance of the circuit is determined by the Q meter.

$$f_1 = \frac{1}{2\pi\sqrt{L(C_1 + C_d)}}$$

Applications of the Q-meter

Thus,

$$f_2 = \frac{1}{2\pi\sqrt{L(C_2 + C_d)}}$$

$$f_2 = 2f_1$$

$$\frac{1}{2\pi\sqrt{L(C_2 + C_d)}} = 2 \times \frac{1}{2\pi\sqrt{L(C_1 + C_d)}}$$

or distributed capacitance

$$C_d = \frac{C_1 - 4C_2}{3}$$

- **Measurement of Bandwidth –** The equation below calculates the bandwidth

For LC band pass circuits and filters:

$$Q = \frac{F}{BW}$$

Where F is the resonant frequency (center frequency) and BW is the filter bandwidth.

- **Measurement of Capacitance –**
  - The capacitance is determined by connecting the dummy coil across the terminal T_1 and T_2.
  - Let the capacitor under test is connected across the terminal T_3 and T_4.
  - The circuit is again resonated by varying the value of tuning capacitor C_2.
  - The value of testing capacitance is determined by subtracting the C_1 and C_2.

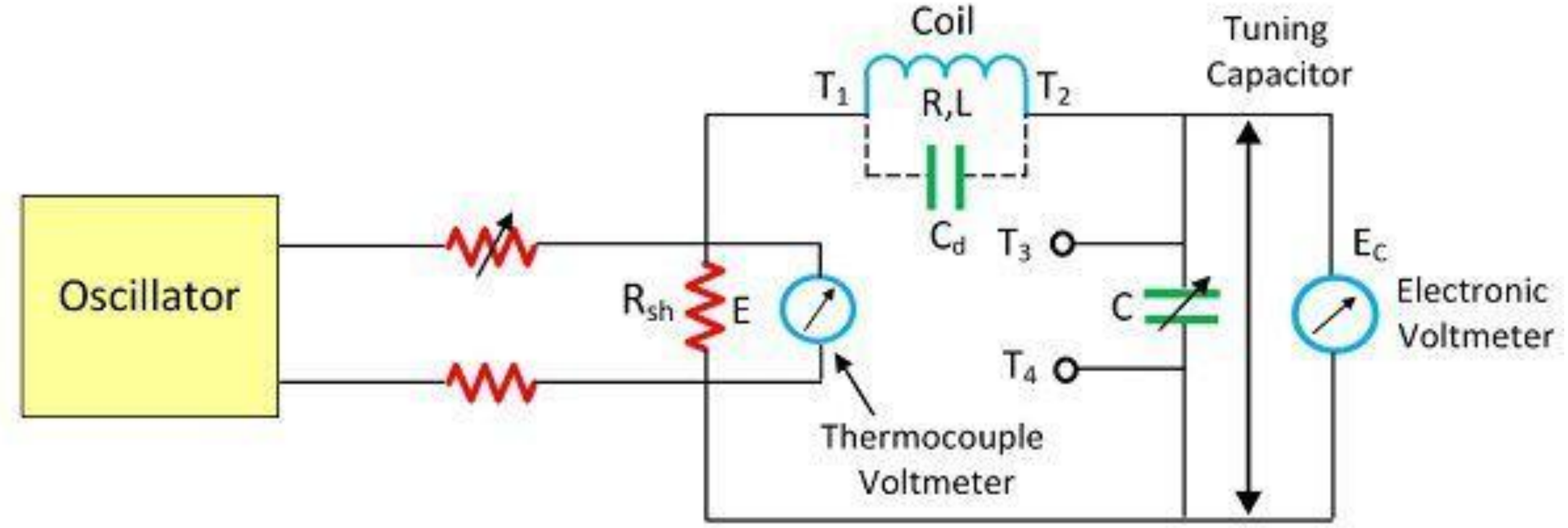
Applications of the Q-meter

The following are the applications of the Q-meter.

- **Measurement of Q (**Quality factor **)–** The circuit used for measurement of Q is shown in the figure.

  - The oscillator and tuning capacitor adjust to the desired frequency for obtaining the maximum value of E_0.
  - Under this condition, the value of the quality factor is expressed as

$$Q_{max} = \frac{\omega_0 L}{R}$$



Circuit of Q meter

Circuit Globe

Applications of the Q-meter

True value is given as

$$Q_{max} = \frac{\omega_0 L}{R}$$

$$Q_{true} = Q_{meas}\left(1 + \frac{R_{sh}}{R}\right)$$

- The value of the quality factor is obtained by the voltmeter which is connected across the capacitor.
- The measured value is the Q factor of the whole circuit and not only of the coil.
- Thus, errors occur in the reading because of the shunt resistance and distributed capacitance.

$$Q_{true} = Q_{meas}\left(1 + \frac{C_d}{C}\right)$$

- The above equations show that the measured value of the Q is smaller than the true value.
- The Q indicated is not the actual Q, because the losses of the resonating capacitor, voltmeter and inserted resistance are all included in the measuring circuit.
- The actual Q of the measured coil is somewhat greater than the indicated Q. This difference is negligible except where the resistance of the coil is relatively small compared to the inserted resistance R_{sh}

Q-Meter

1Q).A circuit consisting of an unknown coil, a resistance and a variable capacitor connected in series is tuned to resonance using a Q-meter. If the frequency is 400 kHz, the resonating capacitor is set at 220 pF, the resistance is 0.8Ω and the Q-meter indicates 110,
determine the effective inductance and resistance of the unknown coil.

Solution. *Given* : $f = 400\text{kHz}$; $C = 200\text{pF}$; $R_{sh} = 0.8\Omega$; $Q = 110$.

Inductance (L) ; resistance (R) :

Under *resonant conditions*, we have :

$$L_{coil} = \frac{1}{\omega^2 C} = \frac{1}{(2\pi \times 400 \times 1000)^2 \times 220 \times 10^{-12}}$$

$$= \mathbf{719.6\ \mu H\ (Ans.)}$$

$$R_{coil} = \frac{\omega L}{Q} - R_{sh} = \frac{2\pi \times 400 \times 1000 \times 719.6 \times 10^{-6}}{110} - 0.8$$

$$= \mathbf{16.44\ \Omega\ (Ans.)}$$

➢ Applications of the Q-meter

1) Applications : Some of the specialized uses of this instrument are to measure :

(i) Q of a coil;

(ii) Inductance and capacitance;

(iii) Distributed capacitance of a coil;

(iv) Q and power factor of a dielectric material;

(v) Mutual inductance of coupled circuits;

(vi) Coefficient of coupling;

(vii) Critical coupling

3Q).Calculate the value of distributed capacitance of a coil when the following measurements are made :

At frequency f_1 = 2 MHz, the tuning capacitor is set at 410 pF.
At frequency, f_2 = 5 MHz, the tuning capacitor is tuned at 50 pF.

Solution: Given : f_1 = 2MHz, C_1 = 410 pF; f_2 = 5MHz, C_2 = 50pF.

Distributed capacitance C_d :

We know that : $$f_1 = \frac{1}{2\pi\sqrt{L(C_1 + C_d)}}$$

and, $$f_2 = \frac{1}{2\pi\sqrt{L(C_2 + C_d)}}$$

Now, $$\frac{f_2}{f_1} = \frac{5}{2} = 2.5 \quad \text{or} \quad f_2 = 2.5\, f_1$$

$$\therefore \quad \frac{1}{2\pi\sqrt{L(C_2 + C_d)}} = 2.5 \times \frac{1}{2\pi\sqrt{L(C_1 + C_d)}}$$

$$\frac{1}{2\pi\sqrt{L(C_2 + C_d)}} = 2.5 \times \frac{1}{2\pi\sqrt{L(C_1 + C_d)}}$$

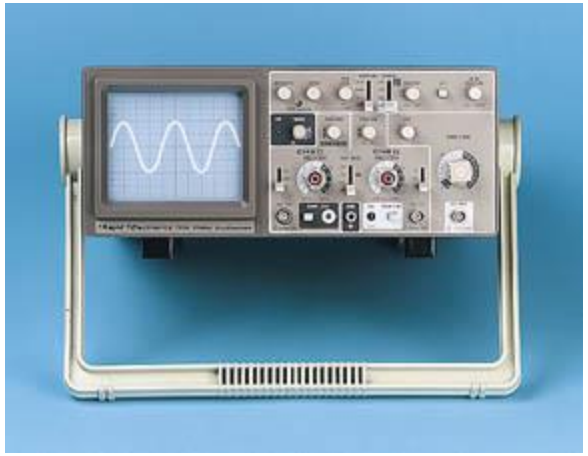
$$\frac{1}{C_2 + C_d} = \frac{6.25}{C_1 + C_d}$$ ...squaring both sides and simplifying

$$(C_1 + C_d) = 6.25(C_2 + C_d)$$

$$C_d = \frac{C_1 - 6.25\, C_2}{5.25} = \frac{410 - 6.25 \times 50}{5.25}$$

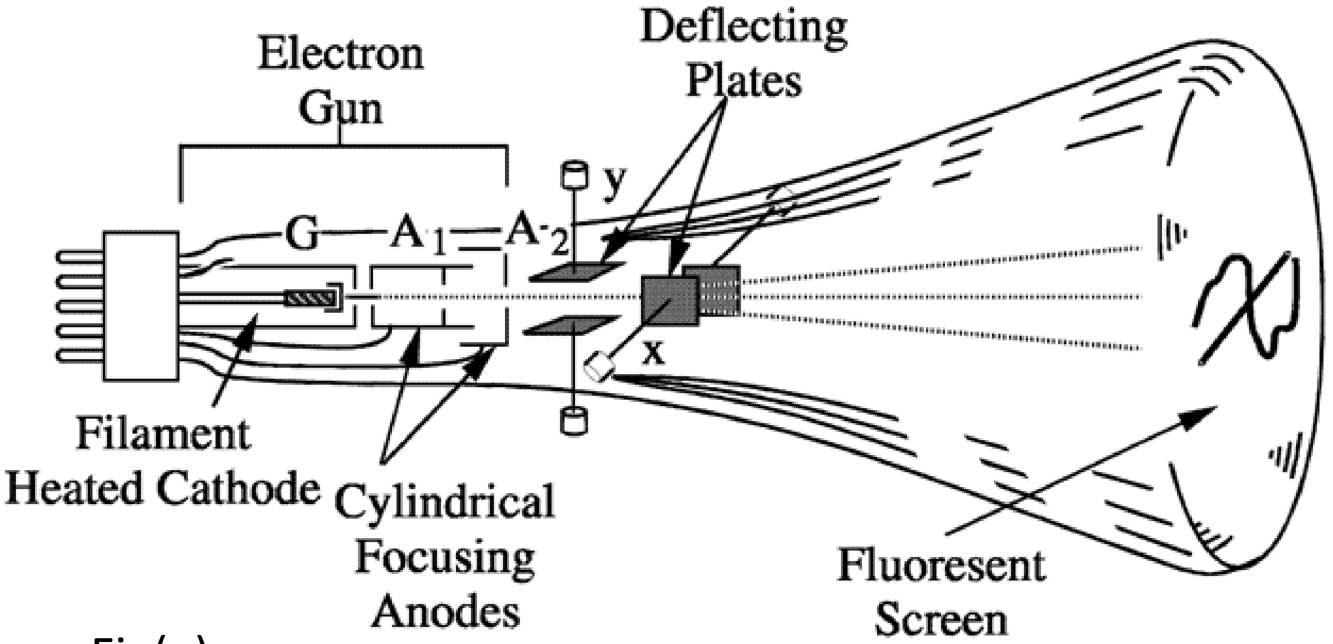
$$= \mathbf{18.57pF\ (Ans.)}$$

Lissajous Patterns of CRO or Cathode Ray Oscilloscope



CRO

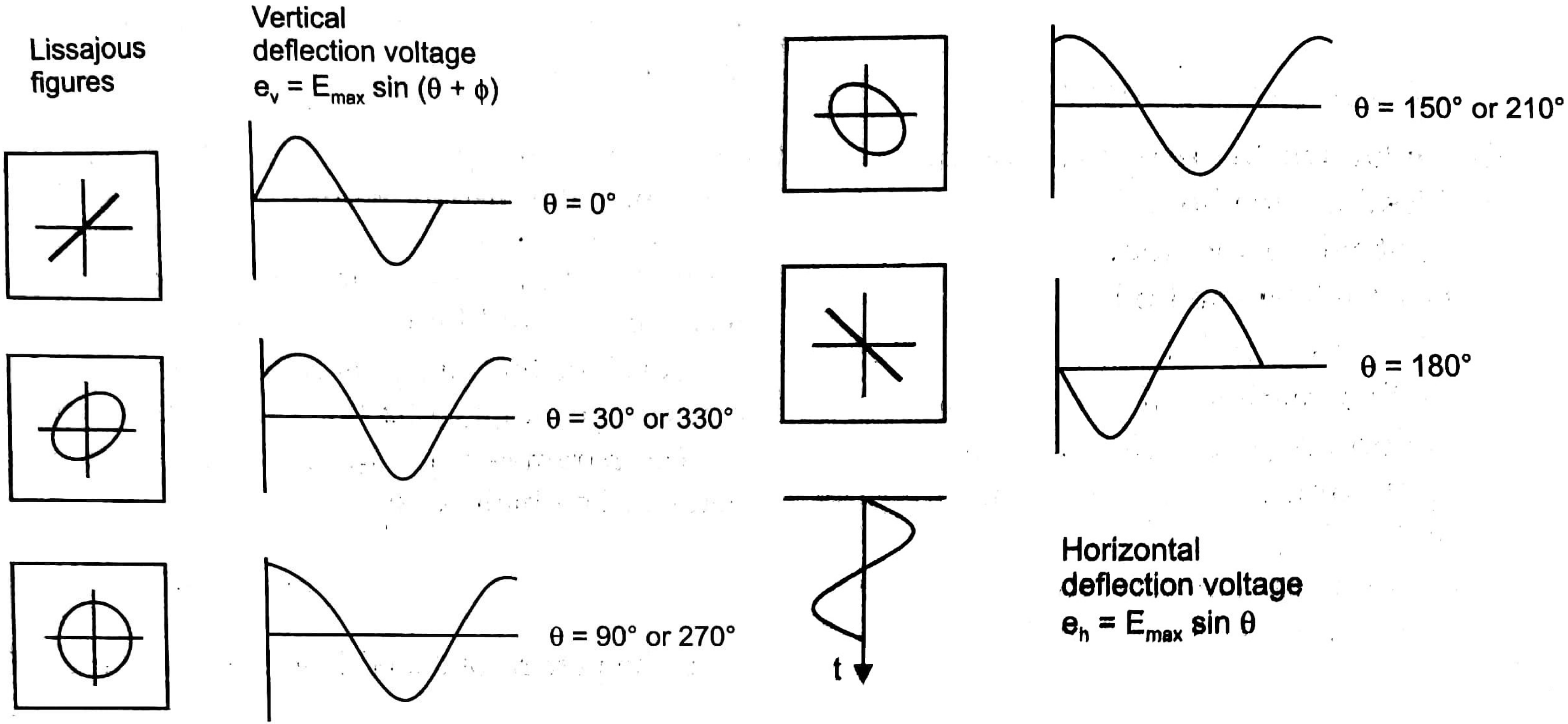
- CRO is very important electronic device. CRO is very useful to analyze the voltage wave form of different signals. The main part of CRO is CRT (**Cathode Ray Tube**). A simple CRT is shown in fig(a).-
- When both pairs of the deflection plates (horizontal deflection plates and vertical deflection plates) of CRO (Cathode Ray Oscilloscope) are connected to two sinusoidal voltages, the patterns appear at CRO screen are called the Lissajous pattern.
- Shape of these Lissajous pattern changes with changes of phase difference between signal and ratio of frequencies applied to the deflection plates (traces) of CRO.



Fig(a)

- Which makes these **Lissajous patterns** very useful to analysis the signals applied to deflection plated of CRO. These lissajous patterns have two Applications to analysis the signals.
- To calculate the phase difference between two sinusoidal signals having same frequency. To determine the ratio frequencies of sinusoidal signals applied to the vertical and horizontal deflecting plates.

CRO



Fig(b) Lissajous patterns for same frequency, different phase shift

➢ Phase-angle measurement

- An oscilloscope can be used to find the phase angle between the two sinusoidal quantities of the same frequency.
- One of the signals is fed to the Y-plates. The time-base generator is switched out and the second signal is fed to the X-plates.
- It is necessary that X and Y are of equal magnitudes.
- This fact can be checked by measuring the amplitudes separately.

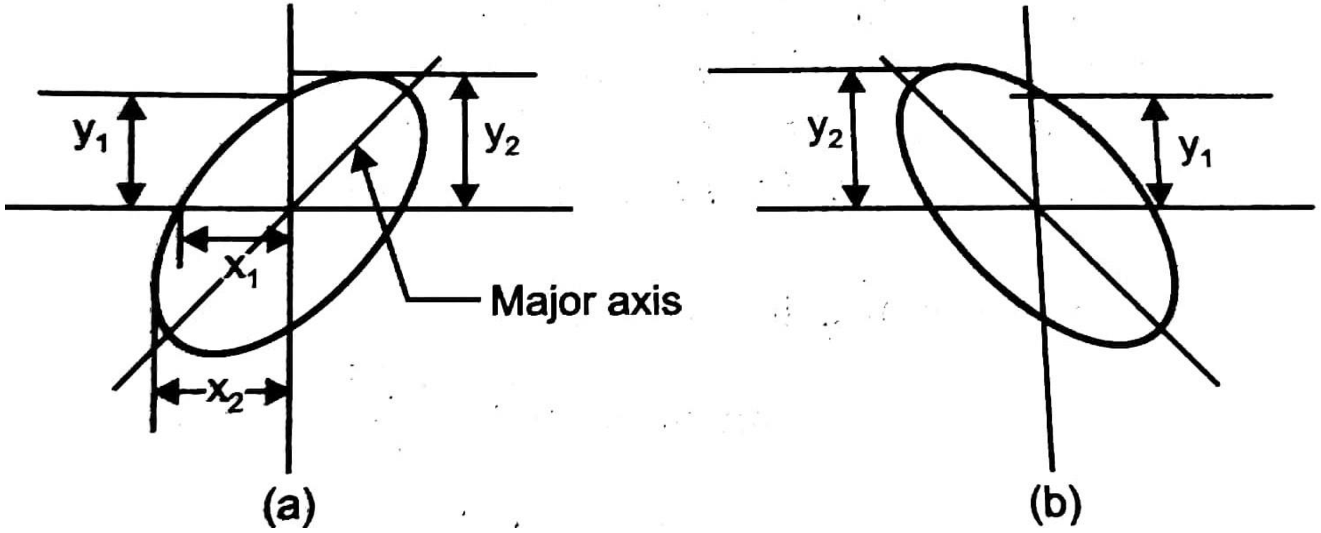


Fig.(1) Measurement of phase angle by using CRO

- If the two signals are in phase, the display would be a straight line at 45° to the horizontal. If the phase angle is 90°, the display would be a circle. For any other phase difference, the display would be an ellipse.
- If the phase difference is between 0° and 90° or between 270° and 360°, the ellipse would have its major axis in the first and third quadrants.
- For difference between 90° and 180° or 180° and 270°, the major axis would be in the second and fourth quadrants.

Phase-angle measurement

- Consider the Lissajous figure obtained on CRO with an unknown phase difference Ø as shown in Fig(1a). The frequency and amplitude of the two waves are same.
- The parameters y_1, y_2 or x_1, x_2 can be measured (See Fig.(1a);
- The phase angle then can be obtained as,

$$\Phi = \sin^{-1}\left(\frac{y_1}{y_2}\right) = \sin^{-1}\left(\frac{x_1}{x_2}\right)$$

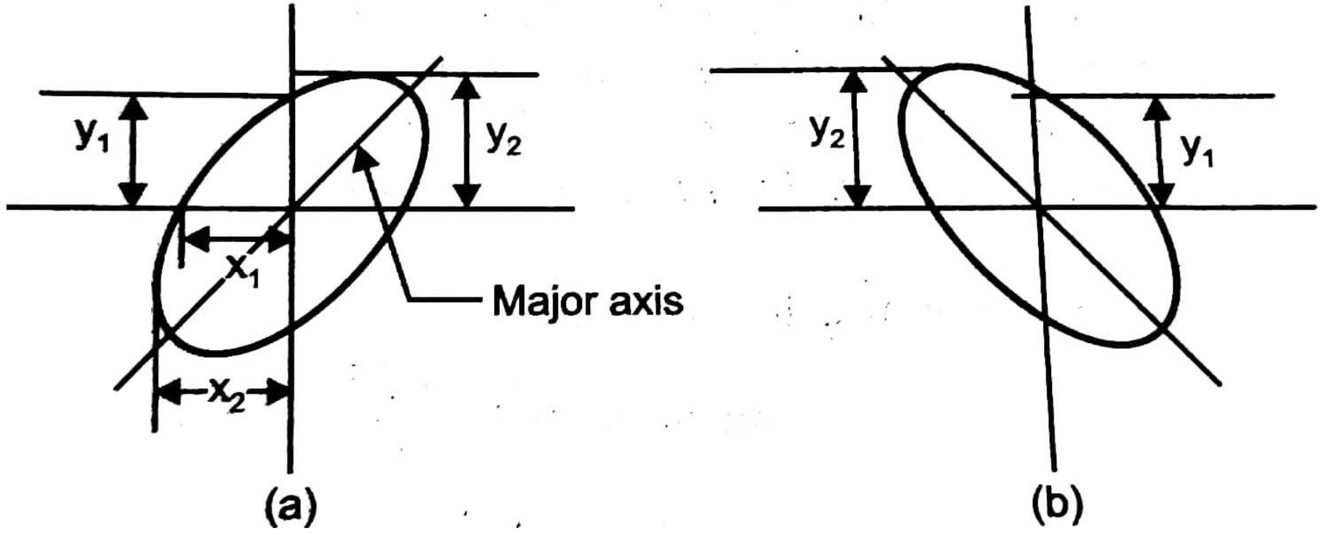


Fig.(1) Measurement of phase angle by using CRO

If the pattern obtained is as shown in Fig.(1b), then the phase angle is given by:

$$\Phi = 180° - \sin^{-1}\left(\frac{y_1}{y_2}\right)$$

Phase-angle measurement

1Q). Voltage E_1 is applied to the horizontal input and voltage E_2 is applied to the vertical input of a CRO. E_1 and E_2 have the same frequency. The trace is an ellipse. The slope of the major axis is positive. The maximum vertical value is 2.5 divisions and the point where the ellipse crosses the vertical axis is 1.25 divisions. The ellipse is symmetrical about the horizontal and vertical axis. Determine the possible phase angles of E_2 with respect to E_1

Solution:

Referring to Fig(1a),

$$\sin \Phi = \frac{y_1}{y_2} = \frac{1.25}{2.5} = 0.5$$

$$\therefore \quad \Phi = 30^\circ$$

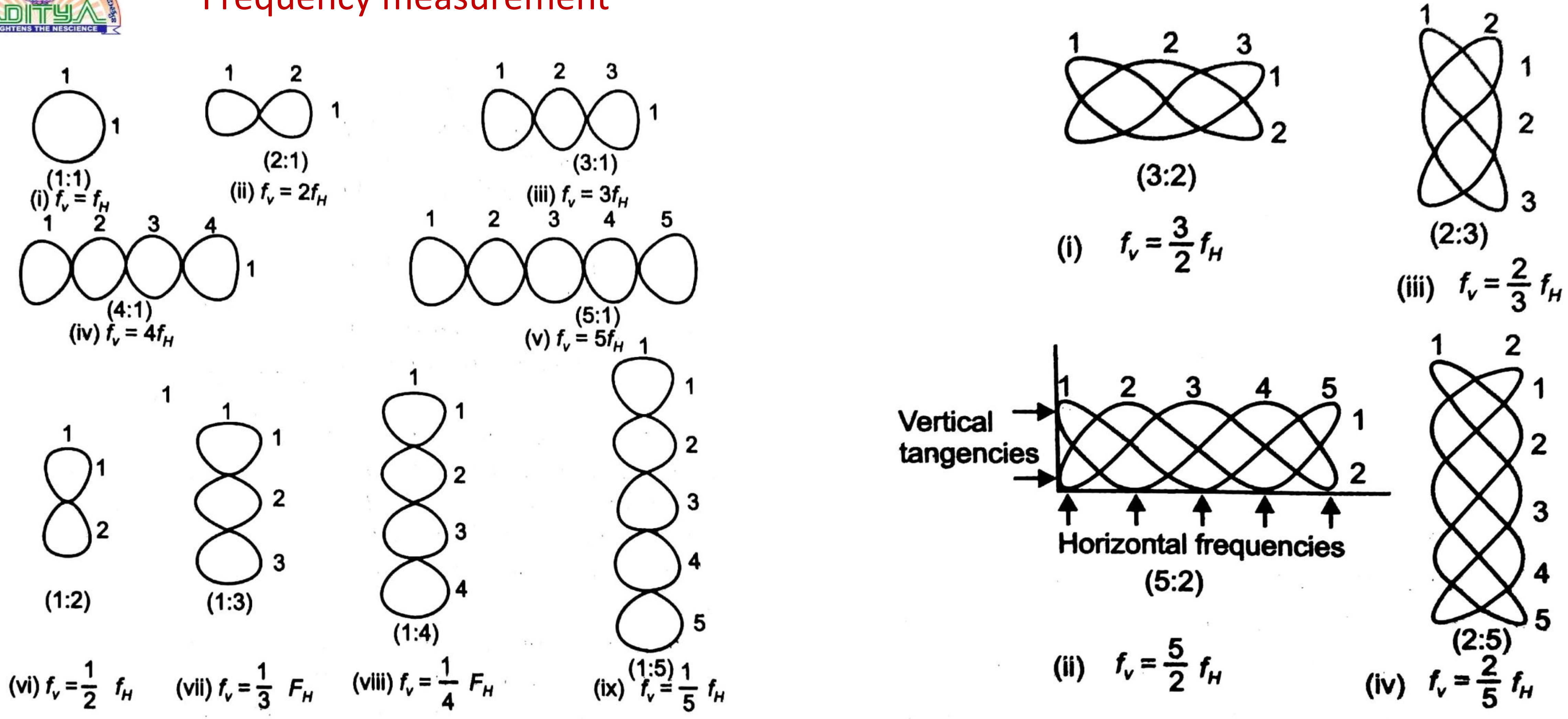
Thus possible phases are **30° or 330° (Ans.)**

➢ Frequency measurement

- The unknown frequency can be measured by the following methods:
  1. Lissajous method.
  2. Spot wheel method.
  3. Gear wheel method.
- **1. Lissajous method :**

- In this method of measurement a standard frequency is applied to one set of deflection plates (X-plates) of the CRT tube while the unknown frequency (of approximately the same amplifide) is simultaneously applied to the other set of plates (Y-plates).
- The resulting patterns depend on the integral and phase relationship between the two frequencies.
- The horizontal signal is designated as f_H and the vertical signal as f_v.

Frequency measurement



(*a*) For integral frequencies

Fig.2

(*b*) For non-integral frequencies

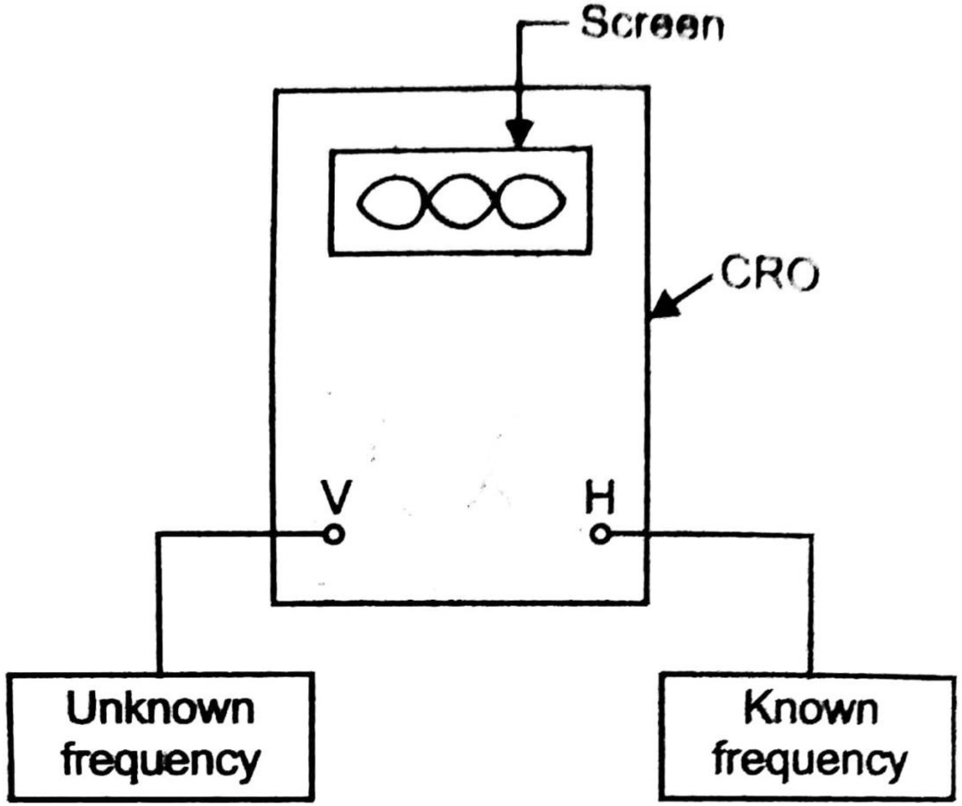
Typical Lissajous patterns are shown in Fig.2 (a & b) for sinusoidal frequencies which are equal, integral and in ratio.

Frequency measurement

➢ Measurement procedure :

1) Set up the oscilloscope and switch off the internal sweep (change to Ext.)
2) Switch off sync. control.
3) Connect the signal source as shown in Fig.(c)
4) Set the horizontal and vertical gain control for the desired width and height of the pattern.

- Keep frequency f_v constant and vary Frequency f_H ,noting that the pattern spins in alternate directions and changes shape.
- The pattern stands still whenever f_v and f_H are in an integral ratio (either even or odd)
  - When $f_v = f_H$ pattern stands still and is a single circle or ellipse.
  - When f_v= 2 f_H, a two loop horizontal pattern is obtained as shown in Fig(2)



Fig(c).Basic circuit for frequency measurements With lissajious patterns

- For determining the frequency from any Lissajous pattern, count the number of horizontal loops in the pattern, divide it by the number of vertical loops and multiply this quantity by f_H (known or standard frequency) .

- If the two frequencies being compared are not equal, but are fractionally related, a more complex stationary pattern results, whose form is dependent on the frequency ratio and the relative phase between the two signals.
- The fractional relationship between the two frequencies is determined by counting the number of cycles in the vertical and horizontal,

$$f_v = (\text{Fraction}) \times f_H$$

or $$\frac{f_v}{f_H} = \frac{\text{No. of horizontal tangencies}}{\text{No. of vertical tangencies}}$$

Example; It can be seen in Fig(2b) (ii) that number of horizontal tangencies is 5 and those of vertical tangencies is 2;

Hence, $$\frac{f_v}{f_H} = \frac{\text{No. of horizontal tangencies}}{\text{No. of vertical tangencies}} = \frac{5}{2} \quad \text{or} \quad f_v = 2.5\, f_H$$

The basic circuit for comparing two frequencies by Lissajous method is Illustrated in Fig(c).

➢ Limitations of Lissajous method

- The Lissajous method of frequency determination has limitations and is being discarded gradually because low-cost digital frequency counters are becoming increasingly available in market.
- Following are the two limitations of this method :

  1. The numerator and denominator of the frequency ratio must be whole number.

  2. The maximum ratio of frequencies that can be used is 10 : 1. Beyond that, the Lissajous patterns become too complex to analyze.

Thank you