UNIT-III

Formal Grammar:

* Introduction
* classification of formal Grammar.
    1. chomsky Hirearchy.
    2. Types.

* Introduction :—

Mathematically A formal Grammar is a tuple like

$G = (V, T, P, S)$ where,

V = finite and non Empty set of non-Terminal symbols (or) Variables.

variables are represented by upper case letters

T = finite and non Empty set of Terminal symbols Represented by lower case letters and some special symbols are there.

P = It is a set of production rules are of the form

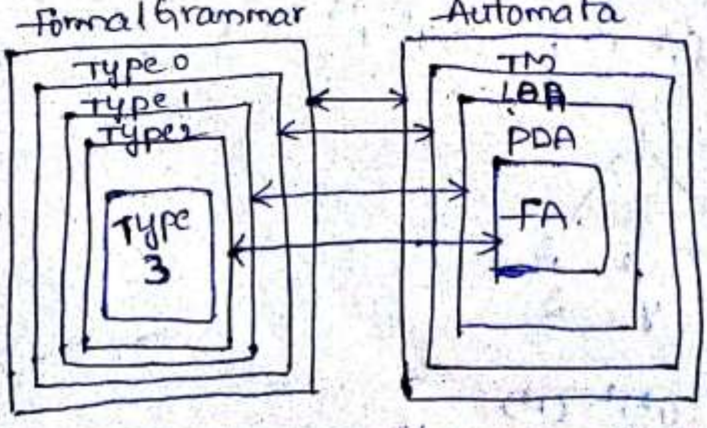
$$P \rightarrow \alpha \rightarrow \beta$$
$$\alpha \in V$$
$$\beta \in (V \cup T)^*$$

S → It is the starting symbol of the Grammar is always, a variable which is $S \in V$.

NOTE :— Grammar's are used to describe a language

* classification of Grammar :—

- using chomsky hierarchy.



Type 3 Grammar:—

* It is also called as Regular grammar.
* Type 3 Grammar is defined as $G=(V,T,P,S)$ where,

V -> set of variables.
T -> set of Terminals
P -> set of production rules are of the form

$\boxed{\begin{array}{l} A \to Ba \\ A \to a \end{array}}$ — According to left linear grammar

(or)

$\boxed{\begin{array}{l} A \to aB \\ A \to a \end{array}}$ — According to Right linear grammar.

Ex:- $A \to aB$
$A \to Ba$
$A \to a$
$A \to \epsilon$

where.
$(A,B) \in V$
$a \in T^*$

* Type 3 Grammar is used to generating Regular language
* Regular languages are recognised (or) accepted by finite automata. i.e; NFA (or) DFA.

Type 2 Grammar:—

* It is also called as Context-free grammar.
* The context-free grammar is defined as $G=(V,T,P,S)$

where V -> finite set of variables
T -> finite set of Terminals
P -> finite set of production rules are of the form

$$\alpha \to \beta$$

where $\alpha \in V$
$\beta \in (V \cup T)^*$

Ex:- $S \to aSa$
$S \to bSb$
$S \to ab$
$S \to \epsilon$

* Context-free grammars are used to generate context-free language.

* Context-free language recognised (or) Accepted by Pushdown Automata.

Type 1 Grammar :—

* It is also called as context - sensitive Grammar.
* A CSG is defined as $G = (V, T, P, S)$ where
  V = finite set of variables
  T = finite set of Terminals.
  P = set of production rules are of the form $\alpha \to \beta$
  where, $\alpha \in (V \cup T)^+$
  $\beta \in (V \cup T)^*$
  length of $|\alpha| \leq$ length of $|\beta|$

Ex:- $S \to aBb$
$bB \to aa$
$B \to b$

* CSG is used to generating Context - Sensitive language
* CSL recognised (or) Accepted by Linear Bounded-Automata

Type 0 Grammar :—

* It is also called also Recursive - Grammar (or) Recursive Enumerable grammar. (or) phrase structured Grammar.
* mathematically Recursive grammar is defined as
  $G = (V, T, P, S)$ where $V \to$ finite set of variables
  $T \to$ finite set of Terminals
  $P \to$ set of production rules. are of the form.
  $\alpha \to \beta$
  $\alpha \in (V \cup T)^{*+}$
  $\beta \in (V \cup T)^*$
  $|\alpha| \geq |\beta|$

Ex:- $S \to aAbB$
$aAbB \to aB$
$abB \to aA$
$A \to \epsilon$

* Recursive Grammars are used to generating recursive language (or) Recursive - Enumerable language (or) phrase structured language.

* Recursive languages are recognised are accepted by Turing machine.

Relationship b/w formal grammar and automata :—

1. Type 3 $\subseteq$ Type 2 $\subseteq$ Type 1 $\subseteq$ Type 0

2. FA $\subseteq$ PDA $\subseteq$ LBA $\subseteq$ TM

Context - Free Grammar :

* Introduction
* Design of CFL
* closure properties of CFL.

* Introduction :—

Content - free Grammar is a Grammar which is defined by four tuples like $G = (V, T, P, S)$ where,

V - It is finite and non-empty set of non-terminal symbols (or) variables.

T - finite and non-Empty set of Terminal symbols.

P - finite and non-Empty set of production rules are of the form $\alpha \to \beta$

$$\alpha \in V$$

$$\beta \in (V \cup T)^*$$

Ex :- $S \to aSa$

$S \to bSb$

$S \to a a | b b$

$S \to \epsilon$

$S \to$ It is starting symbol.

Context free - language :—

Let $G = (V, T, P, S)$ be a Context - free grammar. The CFG generating a language 'L' is called Context - free language

* It is denoted by L(G)..
* Context - free languages are organized by PDA.

Design of CFL :—

1) construct a CFL for the following set. {ϵ, a, aa, aaa, ... a^n}

sol:- Given set {ϵ, a, aa, aaa, aaaa, ---- a^n}

minimum string = ϵ
Next minimum string = a
Maximum string = a^n

$S \to a^n$
$\Downarrow$
$a \cdot a^{n-1} \Rightarrow S \to aS$
$\Downarrow$
$a \cdot a \cdot a^{n-2}$ $\quad S \to \epsilon$
$\quad S \to a$
$\Downarrow$
$a \cdot a \cdot a \cdot a^{n-3}$

CFG:- S
$S \to aS$
$S \to \epsilon$
$S \to a$

$L = \{a^n \mid n \geq 0\}$ //.

2) construct a CFL for the following set {ϵ, ab, aabb, ---}

sol:- Minimum String = ϵ
Next minimum string = ab
Maximum string = $a^n b^n$

$S \to a^n b^n$
$S \to a \underline{a^{n-1} \cdot b^{n-1}} b$
$S \to a a \underline{a^{n-2} b^{n-2}} bb$

$\therefore S \to aSb$
$S \to \epsilon$
$S \to ab$

$\therefore$ CFG :- $S \to aSb$
$S \to \epsilon$
$S \to ab$

$\therefore L = \{a^n b^n \mid n \geq 0\}$

3) construct a CFL for the following set $\{a, b, ab, aabb, aaabbb, \ldots\}$

sol:- Minimum string $= a/b$

Maximum string $= a^n b^n$

$S \rightarrow a^n b^n$

$\rightarrow a a^{n-1} b^{n-1} b \Rightarrow S \rightarrow aSb$

$S \rightarrow a/b$

$\rightarrow aaa^{n-2} b^{n-2} bb \quad S \rightarrow b.$

$\therefore$ CFG $S \rightarrow aSb$

$S \rightarrow a$

$S \rightarrow b$

$\therefore L = \{a^n b^n \mid n \geq 1\}$

4) construct a CFG to generate the language $L = \{a^n b^{2n} / n \geq 1\}$

sol:- Minimum string $= abb$

maximum string $= a^n b^{2n}$

$S \rightarrow a^n b^{2n}$

$S \rightarrow a a^{n-1} b^{2n-2} bb \Rightarrow S \rightarrow aSbb$

$S \rightarrow abb.$

$\therefore$ CFG $= S \rightarrow aSbb$

$S \rightarrow abb.$

5) construct CFG for the following CFL

$L = \{0^i 1^{i+1} \mid i \geq 0\}$

sol: $L = \{0^i 1^{i+1} \mid i \geq 0\}$

$= 0^i 1^i . 1$

$S \rightarrow A1$

$A \rightarrow 0^i 1^i$

$\rightarrow 0 0^{i-1} 1^{i-1} 1$

$\rightarrow 0A1$

$A \rightarrow 0A1$

$A \rightarrow \epsilon$

$A \rightarrow 01$

CFG: $S \rightarrow A1$

$A \rightarrow 0A1$

$A \rightarrow \epsilon$

$A \rightarrow 01$

6) construct a CFL from the following Language.

$L = \{a^m b^m c^n \mid m, n \geq 0\}$

sol:-

$\underbrace{a^m b^m}_{A} \underbrace{c^n}_{B}$

$A \to a^m b^m$

$\to a\, a^{m-1} b^{m-1} b$

$A \to aAb$

$A \to \epsilon$

$A \to ab$

$B \to c^n$

$B \to c c^{n-1}$

$B \to cB$

$B \to \epsilon$

$B \to c$

CFG:- $S \to AB$

$A \to aAb$

$A \to \epsilon$

$A \to ab$

$B \to cB$

$B \to \epsilon$

$B \to c$

Closure properties of CFL:-

- content free languages are closed under union
- " " " " " " concatenation.
- " " " " " " Kleene closure
- " " " " " " Reversal

- content free languages are not closed under Complement
- " " " " " " " Intersection
- " " " " " " " difference.

Derivation:-

* Introduction * Types of Derivation * Derivation tree

Derivation is a process of generating a string from a given grammar.

Derivation process can be represented graphically is called Derivation tree (or)

* left most derivation * Rightmost derivation.

Left most derivation:- with example

In this, we can replace a left most Variable to obtain the given input string.

Right most derivation:-

In this, at each step we can replace a Right most variable to obtain the given input string.

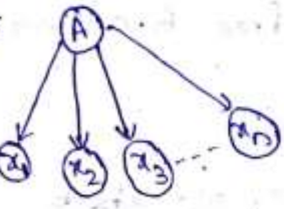
Derivation tree :-

Let $G = (V, T, P, S)$ be a CFG. Then there is a derivation tree for G If and only if.

* The root node of the Tree is labelled with start symbol of G
* All leaf nodes of Tree are labelled by Terminals (or) special symbols of G.
* The interior nodes are labelled by variables of G.
* If any production rule in G. is are the form.

$A \rightarrow x_1 x_2 x_3 --- x_n$ then the

derivation tree is



find the
i) left most derivation
ii) Right most derivation
iii) parse tree for the i/p string id+id*id

from the following grammar. $E \rightarrow E+E$

$E \rightarrow E*E$

$E \rightarrow id$.

Sol:- The given grammar is

$E \rightarrow E+E$

$E \rightarrow E*E$

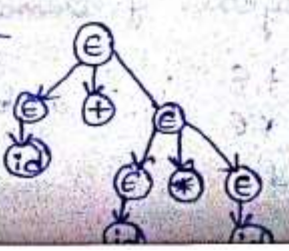
$E \rightarrow id$.

Input string. id+id*id.

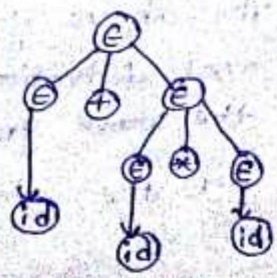
LMD:-
$E \rightarrow \underline{E}+E$
~~$\rightarrow E+E*E$~~
$\rightarrow id+\underline{E}$
$\rightarrow id+\underline{E}*E$
$\rightarrow id+id*\underline{E}$
$\rightarrow id+id*id$.

RMD:-
$E \rightarrow E+\underline{E}$
$\rightarrow E+E*\underline{E}$
$\rightarrow E+\underline{E}*id$
$\rightarrow \underline{E}+id*id$
$\rightarrow id+id*id$

Parse tree:-



Parse tree:-



Ambiguious Grammar:-

* A CFG $G=(V,T,P,S)$ which generates two (or) more parse trees for given i/p string is called Ambiguious grammar.

* That means an Ambegeous grammar has two or more left most derivations (or) right most derivation (or) parse tree.

Ex:- Prove that $S \to aSbS$ is ambegeous for the i/p
$S \to bSaS$
$S \to \epsilon$

string abab

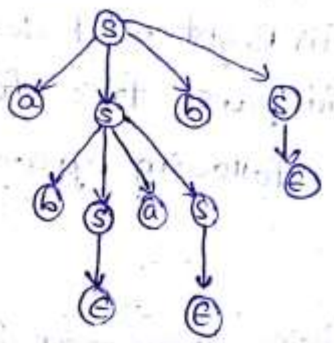
Sol:- The given Context free Grammar is $S \to aSbS$
$S \to bSaS$
$S \to \epsilon$.

The input string is $w = abab$

① LMD:

$S \to aSbS$
$\to abSaSbS$
$\to ab\epsilon aSbS$
$\to abaSbS$
$\to aba\epsilon bS$
$\to ababS$
$\to abab\epsilon$
$\to abab$

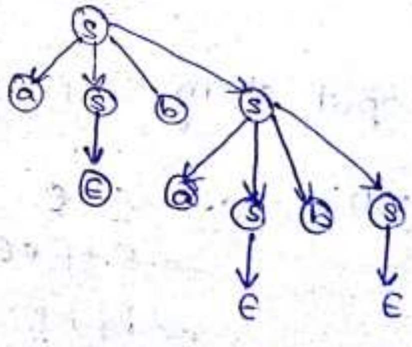
parsetree



② LMD:

$S \to aSbS$
$\to a\epsilon bS$
$\to abS$
$\to abaSbS$
$\to aba\epsilon bS$
$\to ababS$
$\to abab\epsilon$
$\to abab$

Parsetree



∴ The above grammar generates two parse trees (or) two left most derivation for the same i/p string $w=abab$. Hence, the above grammar is ambegeous grammar.

2) P.T the grammar $E \to E+E$
$E \to E*E$ is ambegeous for i/p
$E \to id$

string $id+id*id$.

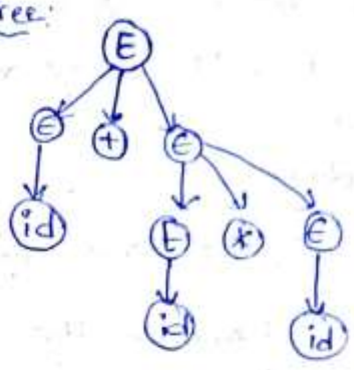
Sol:- The given context free grammar is $E \to E+E$
$E \to E*E$
$E \to id$

The input string is $w = id+id*id$.

① LMD:

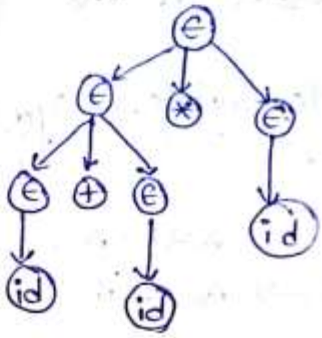
Parse tree:

$E \to E+E$
$\to id+E$
$\to id+E*E$
$\to id+id*E$
$\to id+id*id.$



② LMD:-

$E \to E*E$
$\to E+E*E$
$\to id+E*E$
$\to id+id*E$
$\to id+id*id.$



** (7m) Simplification of CFG:

* Introduction
* Methods
  1. Elimination of useless symbols.
  2. Elimination of E-productions
  3. Elimination of unit productions.

Introduction :-

It's means minimizing the no. of productions in the given CFG. that is reducing size of CFG. size of CFG is equal to no. of productions.

Methods :- $S \to AB$
$A \to a$
$A \to aA$
$B \to SB$

Elimination of useless symbols :-

useful symbol:- A variable is said to be useful if and only if

* It generates a terminal string
* It is used in derivation of a string at least one time

useless symbol :-

* A variable is said to be useless if and only if.
  * It doesn't generates a terminal string.
  * It doesn't used in derivation of a string at least one time.

Procedure :-

step 1 :- Determine useless symbols in the grammar.

step 2 :- Remove the productions which contains useless symbols in the grammar.

Ex :- Eliminate useless symbols from the following grammar.

$S \rightarrow AB \mid cA$
$B \rightarrow Bc \mid AB$
$A \rightarrow a$
$C \rightarrow aB \mid b$

sol :- The given CFG is

$S \rightarrow AB$
$S \rightarrow CA$
$B \rightarrow Bc$
$B \rightarrow AB$
$A \rightarrow a$
$C \rightarrow aB$
$C \rightarrow b$

In the given grammar 'B' doesn't generating a terminal string.

$S \rightarrow \underline{A}B$
$S \rightarrow a\underline{B}$
$\rightarrow a\underline{B}C$
$\rightarrow a\underline{A}BC$
$\rightarrow aaB\underline{C}$
$\rightarrow aaBb$
$\rightarrow aa\underline{A}Bb$
$\rightarrow aaaBb$
$\vdots$

so, ∴ 'B' is useless symbol.

so, we can eliminate the productions which contains 'B'.

∴ The reduced CFG is

$S \rightarrow cA \mid c \rightarrow b.$
$A \rightarrow a$

2) elimination of ϵ-production :-

ϵ-production :- A production is of the form $A \to \epsilon$ is called ϵ-production (or) NULL production.

procedure :-

step 1 :- If the grammar contains $A \to \epsilon$ then replace 'A' with ϵ in the remaining productions.

step 2 :- Remove $A \to \epsilon$ from the grammar.

Ex :- Remove ϵ-productions from the following grammar

$A \to 0B1 \mid 1B1$

$B \to 0B \mid 1B \mid \epsilon$

sol :- the given CFG is $A \to 0B1$

$A \to 1B1$

$B \to 0B$

$B \to 1B$

$B \to \epsilon$.

$A \to 0B1 \to 0\epsilon 1 \to 01$ $\quad \therefore A \to 0B1$, $A \to 01$

$B \to 0B \to 0\epsilon \to 0$ $\quad \therefore B \to 0B$, $B \to 0$

$A \to 1B1 \to 1\epsilon 1 \to 11$ $\quad \therefore A \to 1B1$, $A \to 11$

$B \to 1B \to 1\epsilon \to 1$ $\quad \therefore B \to 1B$, $B \to 1$

After eliminating $B \to \epsilon$ the resultant CFG is

$A \to 0B1$

$A \to 01$

$A \to 1B1$

$A \to 11$

$B \to 0B$

$B \to 0$

$B \to 1B$

$B \to 1$

14M

* **Normal forms :-**

* Introduction
* Types of Normal forms
    1. chomsky Normal Form (CNF)
    2. Greiback Normal Form (GNF).

Introduction :-

In CFG each production of the form $\alpha \rightarrow \beta$ where $\alpha \in V$ that means β contains any no. of non-terminal symbols $\beta \in (V \cup T)$ and any no. of terminal symbols. But, we need to have a grammar in specific form i.e; we can decide the no. of non-terminals and terminals on R.H.S of the grammar. This can be implemented by using "normalization of CFG".

Normalization :-

The process of Arranging the grammar with fixed no. of non-terminals and terminals on R.H.S of CFG is called normalization.

normal-forms are classified into two types

i) chomsky normal form.

ii) Greiback normal form

chomsky normal form :-

It is defined as $\alpha \to \beta$

non-terminal $\to$ Non-terminal. Non-terminal. (or) Non-terminal $\to$ Terminal.

conversion of CFG to CNF :-

procedure :-

step 1 :- simplify the CFG

step 2 :- convert the simplified CFG to CNF.

Ex:- convert the following CFG into chomsky normal form.

$S \to aaaaS$
$S \to aaaa$

Sol:- The given grammar is $S \to aaaaS$
$S \to aaaa$

consider a non-terminal A that derives Terminal a.

$\therefore$ The production rule is $A \to a$. is in CNF

$S \to aaaaS$

$S \to A\boxed{AAAS}$ can be replaced by P_1

$S \to AP_1$ is in CNF.

$P_1 \to A\boxed{AAS}$ can be replaced by P_2.

$P_1 \to AP_2$ is in CNF

$P_2 \to A\boxed{AS}$ can be replaced by P_3

$P_2 \to AP_3$ is in CNF

$P_3 \to AS$ is in CNF

$S \to aaaa$

$S \to A\boxed{AAA}$ can be replaced by P_4

$S \to AP_4$ is in CNF

$P_4 \to A\boxed{AA}$ can be replaced by P_5

$P_4 \to AP_5$ is in CNF

$P_5 \to AA$ is in CNF.

The resultant Grammar CNF is

$S \to AP_1$

$S \to AP_4$

$P_1 \to AP_2$

$P_2 \to AP_3$

$P_3 \to AS$

$P_4 \to AP_5$

$P_5 \to AA$

$A \to a$

2) Convert the given CFG to CNF. $S \to aSa$
$S \to bSb$
$S \to a$
$S \to b$.

Sol:- The given grammar is $S \to aSa$
$S \to bSb$.
$S \to a$
$S \to b$

It is already in simplified form.

Consider a non-terminal A that derives a terminal a and the non-Terminal B that derives the terminal b.

$\therefore$ The production rules is $A \to a$, $B \to b$. are in CNF.

(i) $S \to aSa$

$S \to A\boxed{SA}$ can be replaced by P_1

$S \to AP_1$ is in CNF.

$P_1 \to SA$ is in CNF.

(ii) $S \to bSb$

$S \to B\boxed{SB}$ can be Replaced by P_2

$S \to BP_2$ is in CNF

$P_2 \to SB$ is in CNF

(iii) $S \to a$ is in CNF

$S \to b$ is in CNF.

$\therefore$ The resultant grammar in CNF is

$S \to AP_1$

$S \to BP_2$

$S \to a$

$S \to b$

$P_1 \to SA$

$P_2 \to SB$

$A \to a$

$B \to b$.

Greibach Normal Form (GNF) :-

GNF is defined as

Non-terminal $\to$ Terminal. any no. of nonterminals

Non-terminal $\to$ Terminal.

Lemma 1:

Let CFG be $G=(V,T,P,S)$ and there is a production rule $A \to \alpha B$ and $B \to \beta_1 | \beta_2 | \beta_3 | \cdots | \beta_n$ then add the new production rule to GNF $A \to \alpha\beta_1 | \alpha\beta_2 | \alpha\beta_3 | \cdots | \alpha\beta_n$ to GNF.

$\therefore$ B is replaced by $B \to \beta_1 | \beta_2 | \cdots | \beta_n$

Lemma 2:

Let CFG be $G=(V,T,P,S)$ and there is production rule

$A \to A\alpha_1 | A\alpha_2 | \cdots | A\alpha_n | \beta_1 | \beta_2 | \cdots | \beta_n$ then the production rules are added to GNF.

$A \to \beta_1 | \beta_2 | \beta_3 | \cdots | \beta_n$

$A \to \beta_1 Z | \beta_2 Z | \beta_3 Z | \cdots | \beta_n Z$

$Z \to \alpha_1 | \alpha_2 | \alpha_3 | \cdots | \alpha_n$

$Z \to \alpha_1 Z | \alpha_2 Z | \alpha_3 Z | \cdots | \alpha_n Z$

Converting CFG into GNF :-

Procedure:-

step 1:- Simplify the CFG.

step 2:- Converting simplified CFG into GNF.

Ex:- Convert the given CFG to GNF. $S \to ABA$

$A \to aA | \epsilon$

$B \to bB | \epsilon$.

Sol:- The Given CFG is $S \to ABA$

$A \to aA$

$A \rightarrow \epsilon$

$B \rightarrow bB$

$B \rightarrow \epsilon$

Simplified of given CFG :-

(a) elimination of ϵ-productions :-

$A \rightarrow \epsilon \quad B \rightarrow \epsilon$

(1) $S \rightarrow ABA$
$S \rightarrow \epsilon BA$
$S \rightarrow BA$

(2) $S \rightarrow ABA$
$S \rightarrow AB\epsilon$
$S \rightarrow AB$

(3) $S \rightarrow ABA$
$S \rightarrow A\epsilon A$
$S \rightarrow AA$

(4) $S \rightarrow ABA$
$S \rightarrow \epsilon\epsilon A$
$S \rightarrow A$

(5) $S \rightarrow ABA$
$S \rightarrow \epsilon B \epsilon$
$S \rightarrow B$

$A \rightarrow aA \qquad B \rightarrow bB$

$A \rightarrow a\epsilon \qquad B \rightarrow b\epsilon$

$A \rightarrow a \qquad B \rightarrow b$

$\therefore$ After eliminating $A \rightarrow \epsilon$, $B \rightarrow \epsilon$ from the grammar the resultant grammar is

$S \rightarrow ABA \qquad A \rightarrow aA$

$S \rightarrow BA \qquad A \rightarrow a$

$S \rightarrow AB \qquad B \rightarrow bB$

$S \rightarrow AA \qquad B \rightarrow b$

$S \rightarrow A$ ✓

$S \rightarrow B$ ✓

Elimination of unit productions :-

The above grammar has two unit productions like

$S \rightarrow A$ × $\qquad S \rightarrow B$ ×

$S \rightarrow aA \qquad S \rightarrow bB \qquad \left[\because \text{since } A \rightarrow aA \quad B \rightarrow bB \right.$

$S \rightarrow a \qquad S \rightarrow b \qquad \left. \qquad A \rightarrow a \quad B \rightarrow b \right]$

$\therefore$ After elimination unit productions $S \rightarrow A$, $S \rightarrow B$ from the grammar. The resultant grammar is

$S \rightarrow ABA \qquad A \rightarrow aA$

$S \rightarrow BA \qquad A \rightarrow a$

$S \rightarrow AB \qquad B \rightarrow bB$

$S \rightarrow AA \qquad B \rightarrow b$

$S \rightarrow aA$

$S \rightarrow a$

$S \rightarrow bB$

$S \rightarrow b$

there is no useless production.

∴ The simplified CFG is

$S \to ABA$ $A \to aA$
$S \to BA$ $A \to a$
$S \to AB$ $B \to bB$
$S \to AA$ $B \to b$
$S \to aA$
$S \to a$
$S \to bB$
$S \to b$

Converting simplified CFG to GNF:

i) $S \to ABA$ $A \to aA$ ✓
$S \to aABA$ ✓ $A \to a$ ✓
$S \to aBA$ ✓ $B \to bB$ ✓

ii) $S \to BA$ $B \to b$ ✓
$S \to bBA$ ✓
$S \to bA$ ✓

iii) $S \to AB$
$S \to aAB$
$S \to aB$ ✓

iv) $S \to AA$
$S \to aAA$
$S \to aA$

v) $S \to aA$ ✓
$S \to a$ ✓

vi) $S \to bB$ ✓
$S \to b$ ✓

∴ The resultant grammar is in GNF is

$S \to aABA \mid aBA \mid bBA \mid bA \mid aAB \mid aB \mid aAA \mid aA \mid bB \mid a \mid b$
$A \to aA \mid a$
$B \to bB \mid b$.

② Convert the following CFG into GNF $S \to AA \mid 0$
$A \to SS \mid 1$

Sol: Given Grammar $S \to AA$
$S \to 0$
$A \to SS$
$A \to 1$

The simplified CFG is $S \to AA$
$S \to 0$
$A \to SS$
$A \to 1$

① S → AA | 0
S → SSA | 0
S → 0
S → 0Z
Z → SA
Z → SAZ
Z → SA
Z → 0A
Z → 0ZA
Z → 1AA
Z → 0A

② S → AA | 0
S → 1A | 0
S → 1A
S → 0

Z → SAZ
Z → 0AZ
Z → 0ZAZ
Z → 1AAZ
Z → 0AZ

③ A → SS
A → 0S
A → 0ZS
A → 1AS
A → 0S

∴ The resultant grammar is

S → 0 | 0Z | 1A
Z → 0A | 0ZA | 1AA | 0A | 0AZ | 0ZAZ | 1AAZ |
A → 0S | 0ZS | 1AS | 1

③ convert the given CFG to GNF
S → CA
A → a
C → aB | b

⇒ Given CFG is not a simplified Grammar
After eliminating the useless symbols the resultant CFG is.
S → CA
A → a
C → b

By Applying Lemma 1
S → CA
S → bA

∴ The resultant GNF is
S → bA
A → a
C → b

④ Convert the given CFG to GNF
S → SS
S → 0S1 | 01

The given CFG is a simplified CFG
The resultant grammar is
S → SS
S → 0S1
S → 01

Replaced 0 by A, 1 by B
then productions are
A → 0
B → 1

$S \rightarrow SS$

$S \rightarrow ASB$

$S \rightarrow AB$

Applying Lemma ①

① $S \rightarrow SS$

$S \rightarrow ASBS$

$S \rightarrow 0SBS$

② $S \rightarrow SS$

$S \rightarrow ABS$

$S \rightarrow 0BS$

③ $S \rightarrow ASB$ $\quad S \rightarrow AB$

$S \rightarrow 0SB$ $\quad S \rightarrow 0B$

The resultant grammar GNF is

$S \rightarrow 0SBS \mid 0BS \mid 0SB \mid 0B$

$A \rightarrow 0$

$B \rightarrow 1$

Pumping Lemma for CFL:-

pumping lemma is used for proving the given language is CFL (or) not.

Lemma:- let 'L' be any CFL, then there is a constant 'n' which depends only a part 'L' such that there exist a string. $w = uvxyz$ such that
1. $|vy| \geq 1$
2. $|vxy| \leq n$
3. for $\forall\, i \geq 0$ uv^ixy^iz is in L

Then 'L' is said to be CFL. otherwise it is not a CFL.

① prove that $L = \{a^n b^n c^n \mid n \geq 0\}$ is not a CFL.

The given language $L = \{a^n b^n c^n \mid n \geq 0\}$.

$L = \{\epsilon, abc, aabbcc, ----\}$

consider a constant n and the string $w = a^n b^n c^n$

consider a string $w \in L$

$w = abc$ for $n = 1$

$|w| = 3n$

for $i = 1$ $w = abc$

for $i = 2$

$a^n \quad b^n \quad c^n$

$a^{n-1} \quad a \quad b^{n-1} \quad b \quad c^n$

$u \quad v \quad x \quad y \quad z$

$w = uv^ixy^iz$

$w = uv^2xy^2z$

$w = a^{n-1} a^2 b^{n-1} b^2 c^n$

$w = a^{n+1} b^{n+1} c^n \notin L$

$\therefore$ The given language is not a CFL.